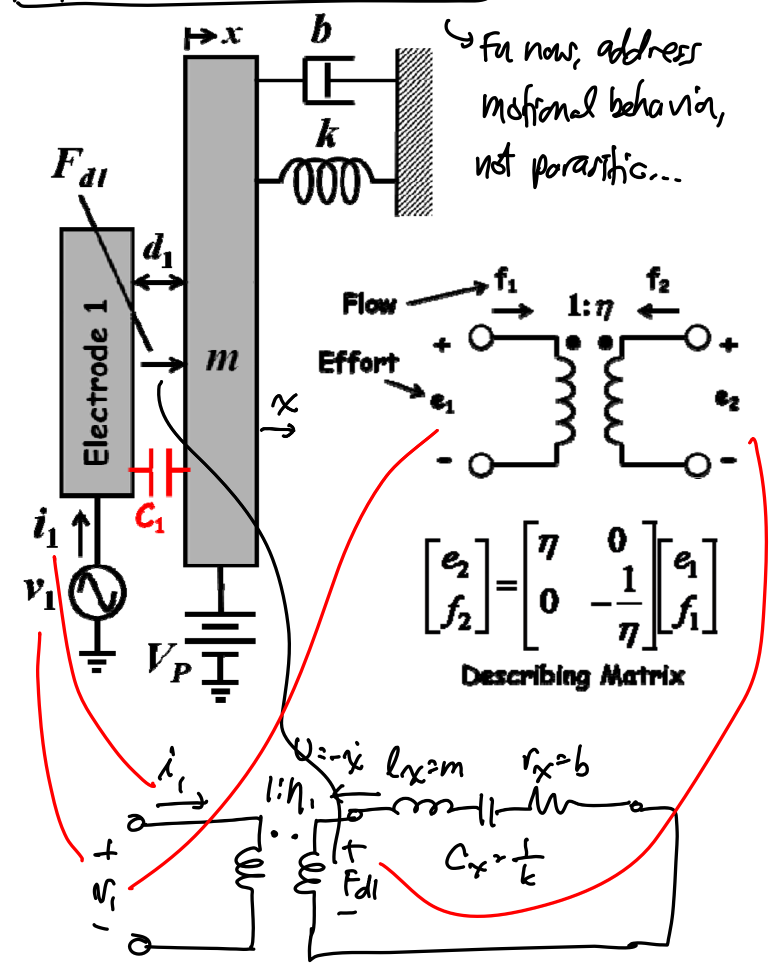
Lecture 24: Equivalent Circuits II

- Announcements:
- HW#7 will be online soon (tomorrow morning)
  - It's color, so will not be handed out
  - Get it from the website
- Reminder: 3rd project slide due Dec. 3
- No class Thursday (Thanksgiving)
- ------------------------------------
- Reading: Senturia, Chpt. 6, Chpt. 14
- Lecture Topics:
  - Input Modeling
    - Force-to-Velocity Equiv. Ckt.
    - Input Equivalent Ckt.
  - Current Modeling
    - Output Current Into Ground
    - Input Current
    - Complete Electrical-Port Equiv. Ckt.
  - Impedance & Transfer Functions
- ------------------------------------
- Last Time:
  - Started electrical equivalent circuits using Module 13

Input Electrical Equiv. Ckt.



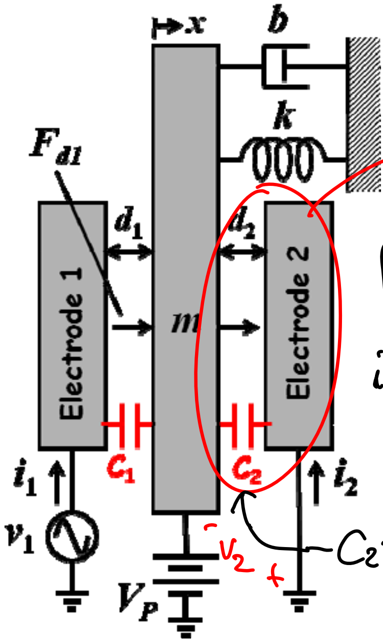
$$\underbrace{F_{d1}}_{e_2} = \underbrace{V_P \frac{\partial C_1}{\partial x}}_{\eta_1} \underbrace{v_1}_{e_1}$$

$$\boxed{\eta_1 = \left| V_P \frac{\partial C_1}{\partial x} \right|}$$

$\frac{\epsilon_0 A}{d^2}$ ↳ gap↓ → η_1↑

V_P↑ → η_1↑

Output Current Into Ground



Want this model.

$[q = CV]$

$i = \frac{dq}{dt} = C\frac{dV}{dt} + V\frac{dC}{dt}$ ← fine

$C_2 = f(t)$

$i_2 = C_2(x,t)\cancel{\frac{dV_2(t)}{dt}} + V_2(t)\frac{dC_2(x,t)}{dt}$

$[V_2(t) = -V_P] \Rightarrow i_2 = -V_P\frac{dC_2}{dt} = -V_P\frac{\partial C}{\partial x}\frac{\partial x}{\partial t}$

In phasor form: $I_2(j\omega) = -V_P\frac{\partial C_2}{\partial x}(j\omega X)$

$$\boxed{I_2(j\omega) = -j\omega V_P\frac{\partial C_2}{\partial x}X}$$

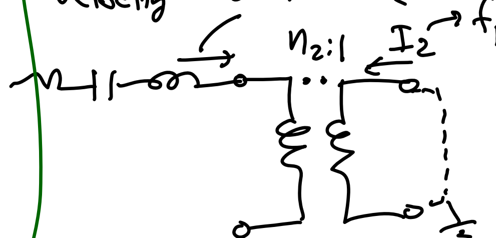
↑ motional current

$I_2(j\omega) = -j\omega V_P\frac{\partial C_2}{\partial x}X = -V_P\frac{\partial C_2}{\partial x}U$

↑ velocity

90° phase lag (f) (f) → $I_2 = (f)$ when $x = (f)$

Velocity → $U = \dot{x}$ ← f_2

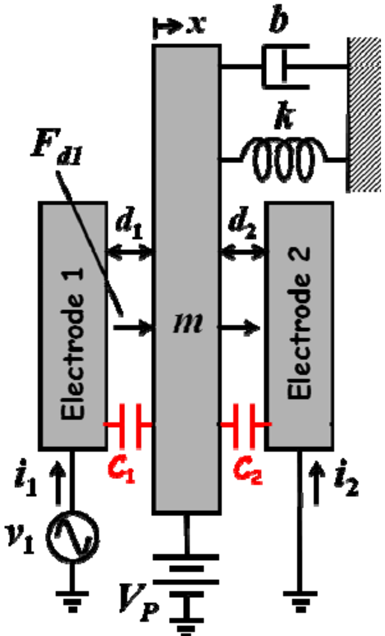


$f_2 = -\frac{1}{\eta_2}f_1 \rightarrow f_1 = -\eta_2 f_2$

$[f_1 = I_2,\ f_2 = U] \Rightarrow I_2 = -\eta_2 U$

$\therefore \boxed{\eta_2 = \left|V_P\frac{\partial C_2}{\partial x}\right|}$

Input Current Expression



Get $I_1(j\omega)$:

$$i_1(t) = C_1(x,t)\frac{dV_1(t)}{dt} + V_1(t)\frac{dC_1(x,t)}{dt}$$

$$[V_1(t) = v_1 - V_P] \Rightarrow i_1 = C_1\frac{dv_1}{dt} + (v_1 - V_P)\frac{\partial C_1}{\partial x}\frac{\partial x}{\partial t}$$

$f(t)$

due to mass motion!

$$\therefore I_1(j\omega) = \underbrace{j\omega C_1 V_1}_{\text{Feedthrough Current}} + \cancel{j\omega v_1 \frac{\partial C_1}{\partial x}X} \underbrace{- j\omega V_P \frac{\partial C_1}{\partial x}X}_{\text{Motional Current}}$$

@ DC: $x = \dfrac{F_{d1}}{k} = -\dfrac{1}{k}V_P\left(\dfrac{\partial C_1}{\partial x}\right)v_1$

@ resonance: $x = \dfrac{QF_{d1}}{jk} = -\dfrac{Q}{jk}V_P\dfrac{\partial C_1}{\partial x}v_1 = X$

90° phase lag

Thus: (@ resonance) ω_0

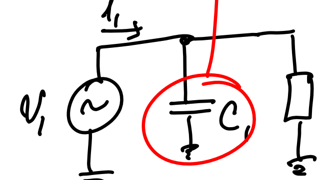
$$I_1(j\omega) = j\omega_0 C_1 V_1 + j\omega_0\left(V_P\frac{\partial C_1}{\partial x}\right)^2\frac{Q}{jk}V_1$$

$$i_P = j\omega_0 C_1 V_1 + \underbrace{\omega_0\frac{Q}{k}\eta_{e1}^2 V_1}_{i_x}$$

$|I_1|$, i_P, i_x, ω_0, ω

90° phase shifted from V_1 ↓ This is a capacitor in shunt at the input!

In phase w/ V_1. ↓ This is an effective resistance seen "looking into electrode 1" @ ω_0



Motional Resistance!

over

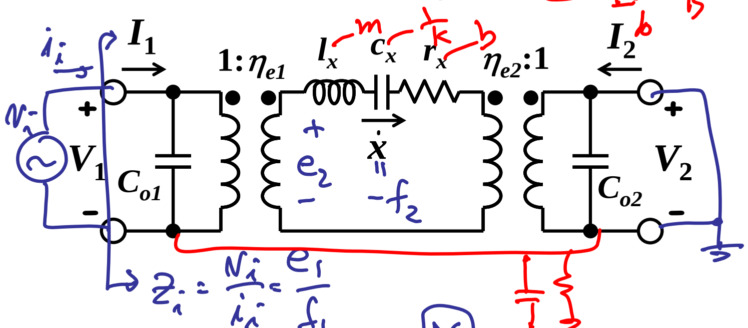
Motional Resistance:

$$R_{x1} = \frac{V_1}{I_1} = \frac{k}{\omega_o Q \eta_{e1}^2} = \frac{m\omega_o}{Q\eta_{e1}^2} = \boxed{\frac{b}{\eta_{e1}^2} = R_{x1}}$$

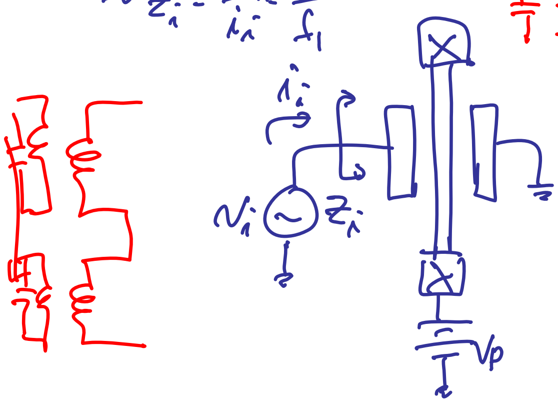
"motion"

The equivalent ckt. better get this right!

Input Impedance Info Part I



$Z_i = \frac{v_i}{i_i} = \frac{e_1}{f_1}$



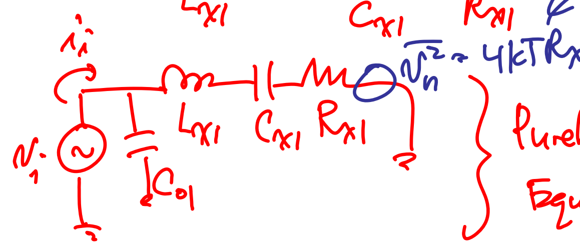
$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix} \Rightarrow \begin{cases} e_2 = \eta e_1 \to e_1 = \frac{e_2}{\eta} \\ f_2 = -\frac{1}{\eta} f_1 \to f_1 = -\eta f_2 \end{cases}$$

$$\frac{e_1}{f_1} = \frac{e_2}{\eta}\left(\frac{1}{-\eta f_2}\right) = -\frac{1}{\eta^2}\frac{e_2}{f_2} \to \frac{v_i}{i_i} = Z_i = -\frac{1}{\eta_{e1}^2}\frac{F_{d2}}{(-\dot{x}_2)} = \frac{1}{\eta_{e1}^2} Z_x$$

$$Z_i = \frac{1}{\eta_{e1}^2}\left(j\omega l_x + \frac{1}{j\omega c_x} + r_x\right)$$

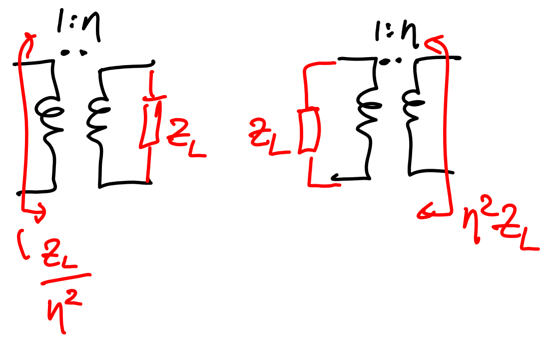
$$= j\omega\underbrace{\left(\frac{l_x}{\eta_{e1}^2}\right)}_{L_{x1}} + \frac{1}{j\omega\underbrace{(\eta_{e1}^2 c_x)}_{C_{x1}}} + \underbrace{\frac{r_x}{\eta_{e1}^2}}_{R_{x1}}$$

to model noise!

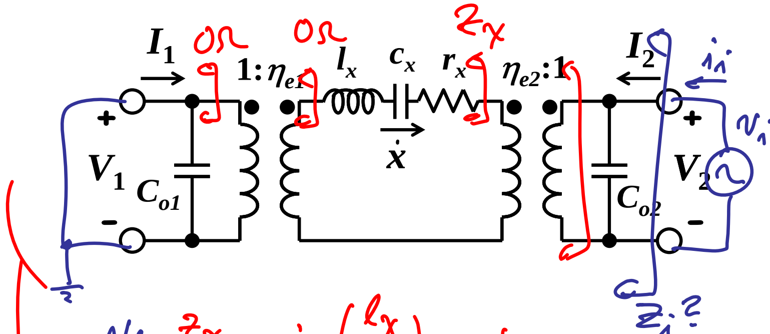


Purely Electrical Equiv. Ckt.

Xformer Reflection Analysis



Input Impedance Into Port 2



$$z_1 = \frac{v_1}{i_x} = \frac{z_x}{\eta_{e2}^2} = j\omega\underbrace{\left(\frac{l_x}{\eta_{e2}^2}\right)}_{L_{x2}} + \frac{1}{j\omega\underbrace{(\eta_{e2}^2 c_x)}_{C_{x2}}} + \underbrace{\frac{r_x}{\eta_{e2}^2}}_{R_{x2}}$$