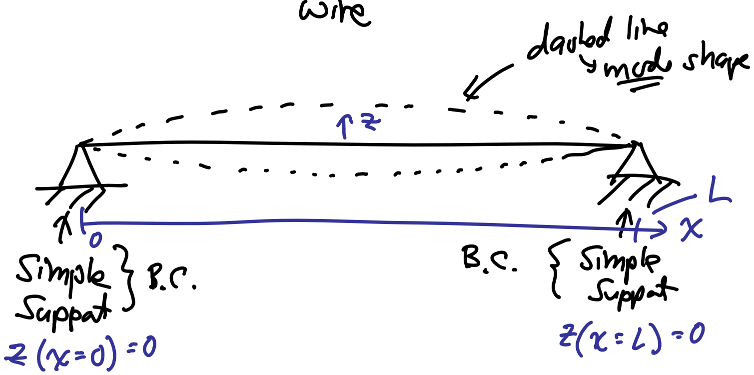
Lecture 2: Benefits of Scaling

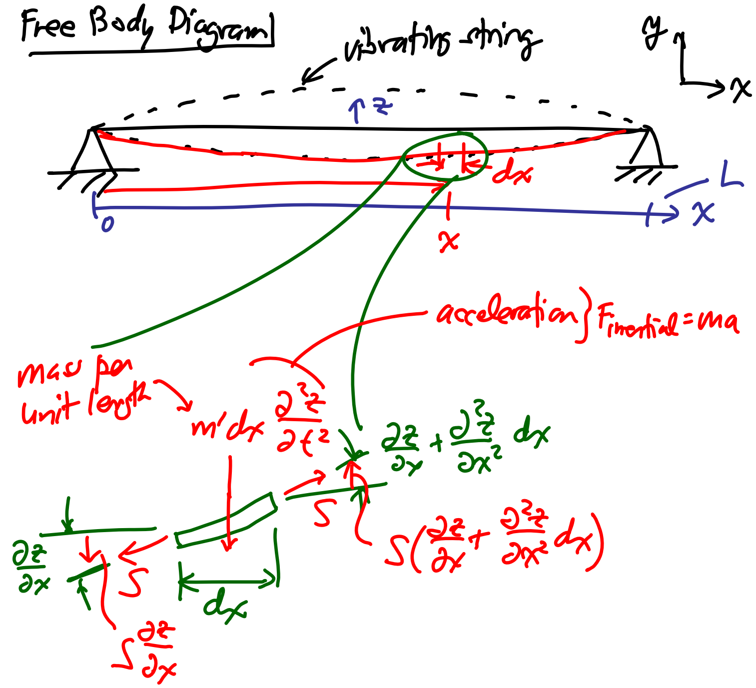
- Announcements:
- Discussion Section Change?
  - M ~~6-7~~? ~~No!~~ → we're not D'ing anything...
- TA Office Hours: in 481 Cory
- Modules
- The required textbook is the Senturia text.
- ------------------------------------
- Today:
- Reading: Senturia, Chapter 1
- Lecture Topics:
  - Benefits of Miniaturization
  - Examples
    - —GHz micromechanical resonators
    - —Chip-scale atomic clock
    - —Micro gas chromatograph
- ------------------------------------
- Start going through module 2

Scaling of Guitar Strings

guitar string ≡ transversely vibrating stretched wire



Free Body Diagram



⇒ condition for dynamic equilibrium:

$$S\left(\frac{\partial z}{\partial x}+\frac{\partial^2 z}{\partial x^2}dx\right)-S\frac{\partial z}{\partial x}-m'dx\frac{\partial^2 z}{\partial t^2}=0$$

Solve ↓

$$\boxed{f_i = \frac{i}{2L}\sqrt{\frac{S}{m'}}}$$

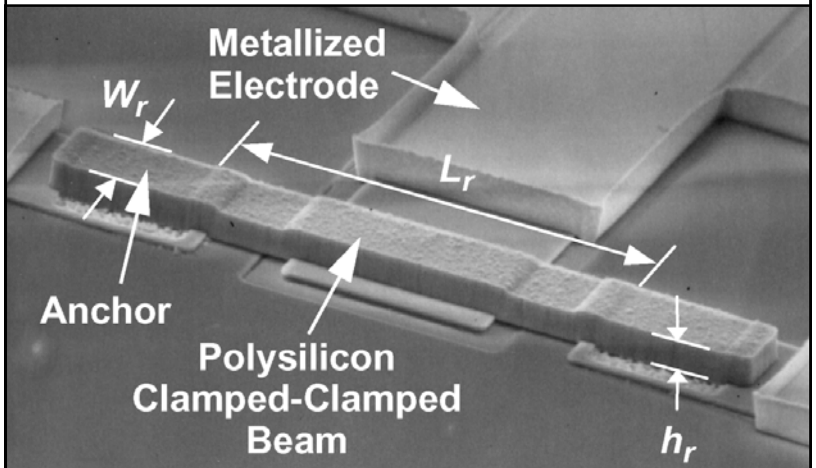
frequency

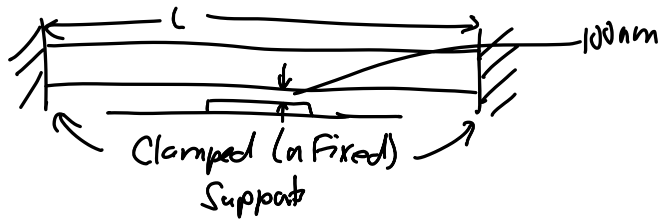
tension (force per unit cross-sectional area)

mass per unit length

Good approx for a guitar string...

Clamped-Clamped Beam





→ Eq. for Resonance Freq.:

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.03\sqrt{\frac{E}{\rho}}\,\frac{h}{L^2} \quad \text{(1)}$$

← "sigs"

where $E \triangleq$ Young's modulus [GPa]

$\rho \triangleq$ density [kg/m^3]

$h \triangleq$ thickness [m]

$L \triangleq$ length [m]

Example. $L = 40\mu m$, $h = 2\mu m$

polySi → $E = 150$ GPa, $\rho = 2300$ kg/m^3

$$\therefore\ f_o = (1.03)\underbrace{\sqrt{\frac{150G}{2300}}}_{}\ \frac{2\mu}{(40\mu)^2} \rightarrow \boxed{f_o = 10.4\,MHz}$$

acoustic velocity = 8,076 m/s

As $L\downarrow \rightarrow f_o\uparrow$

Scaling

⇒ If we scale all dimensions equally by a scaling factor S:

$f_o \sim \frac{S}{S^2} = \frac{1}{S}$ ✓ → $f_o\uparrow$ as we scale to smaller sizes!

If we scale only L:

$f_o \sim \frac{1}{S^2}$ → even faster rise in f_o!

Example. $L = 4\mu$ → $\boxed{f_o = 1.046\,GHz\ !}$

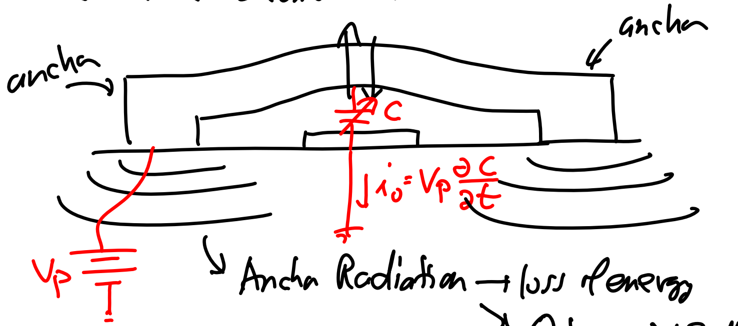
GHz freqs. possible!

Smaller → Faster!

Remarks.

① Eq. (1) not accurate when $L \approx h \approx W$.

② When L ≉ h (or when it isn't more than 10×h), anchor losses become an issue:

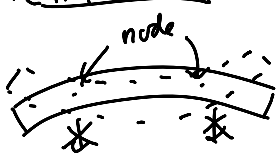


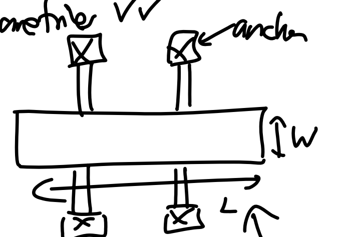
↳ Anchor Radiation → loss of energy
→ Q↓ — ✗ Bad!
Problem!

③ Soln: nanodimensions! ✗
↳ problem: power handling

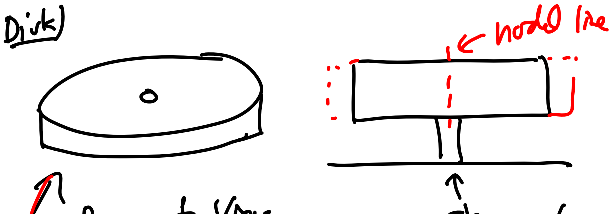
④ Soln: use an array!

⑤ Better? Soln: other geometries ✓✓



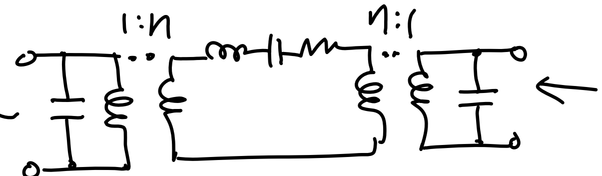


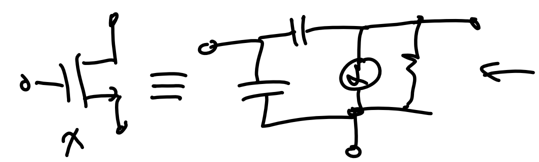
Disk)



6 GHz w/ Q's ~ 50,000
500MHz

⑥ This device is a glorified LCR:





↳ If you can design large ckt. using this, then why not w/ this?

You absolutely can design ckts w/ mechanics! (and large ones)