Lecture 3: Benefits of Scaling II

- **Announcements:**
- HW#1 passed out and online
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- **Today:**
- Reading: Senturia, Chapter 1
- Lecture Topics:
  - Benefits of Miniaturization
  - Examples
    - GHz micromechanical resonators
    - Chip-scale atomic clock
    - Thermal Circuits
    - Micro gas chromatograph
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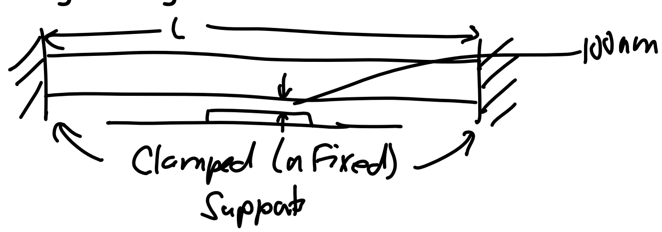
- **Last Time:**
- Going through module 2



→ Eq. for Resonance Freq.:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.03\sqrt{\frac{E}{\rho}}\frac{h}{L^2} \quad (1)$$

"giga"

where $E \triangleq$ Young's modulus [GPa]

$\rho \triangleq$ density [kg/m^3]

$h \triangleq$ thickness [m]

$L \triangleq$ length [m]

Example. $L = 40\mu m$, $h = 2\mu m$

polySi → $E = 150$ GPa, $\rho = 2300$ kg/m^3

$$\therefore f_0 = (1.03)\sqrt{\frac{150G}{2300}}\frac{2\mu}{(40\mu)^2} \rightarrow \boxed{f_0 = 10.4\,MHz}$$

acoustic velocity = 8,076 m/s

As $L\downarrow \rightarrow f_0\uparrow$

Scaling!

⇒ If we scale all dimensions equally by a scaling factor S:

$$f_0 \sim \frac{S}{S^2} = \frac{1}{S} \checkmark \rightarrow f_0\uparrow \text{ as we scale to smaller sizes!}$$

If we scale only L:

$$f_0 \sim \frac{1}{S^2} \rightarrow \text{even faster rise in } f_0!$$

Example. $L = 4\mu \rightarrow \boxed{f_0 = 1.04\,GHz!}$

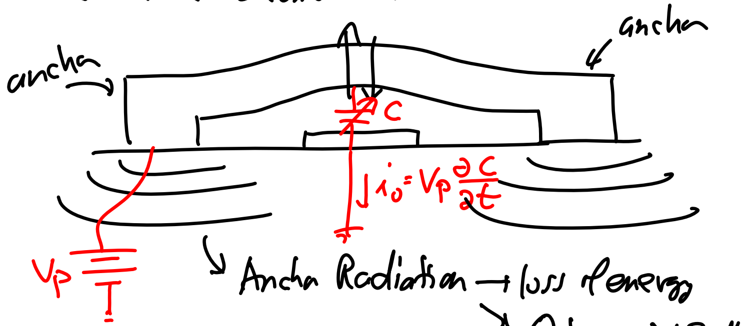
GHz freqs. possible!

Smaller → Faster!

Remarks.

① Eq. (1) not accurate when $L \approx h \approx w$.

② When L ≈ h (or when it isn't more than 10×h), anchor losses become an issue:

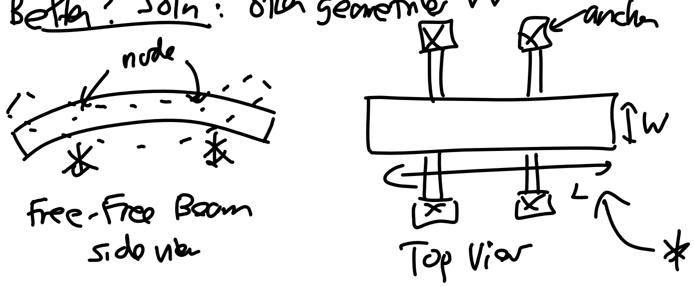


↳ Anchor Radiation → loss of energy
→ Q↓ — ✗ Bad! Problem!

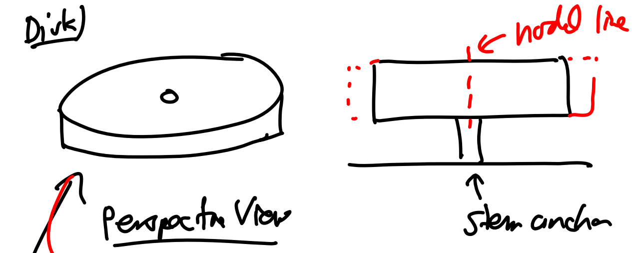
③ Soln: nanodimensions! ✗
↳ problem: power handling

④ Soln: use an array!

⑤ Better? Soln: other geometries ✓✓

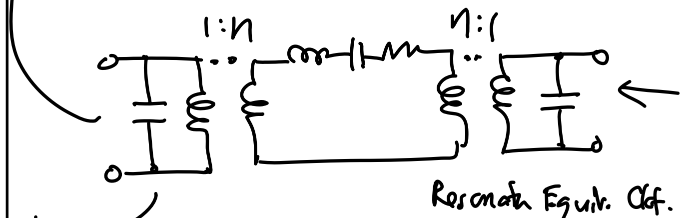


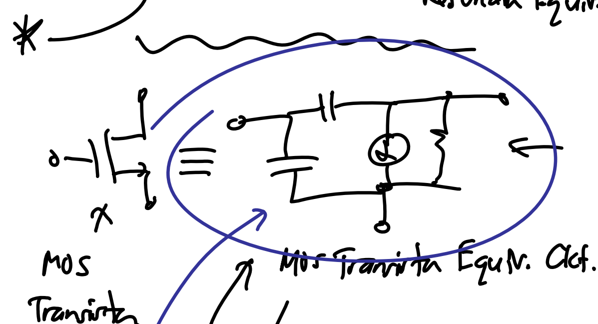
Disk)



6 GHz w/ Q's ~ 50,000
500 MHz

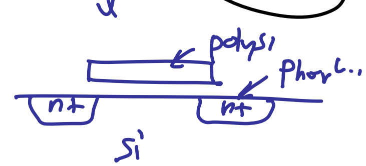
⑥ This device is a glorified LCR:





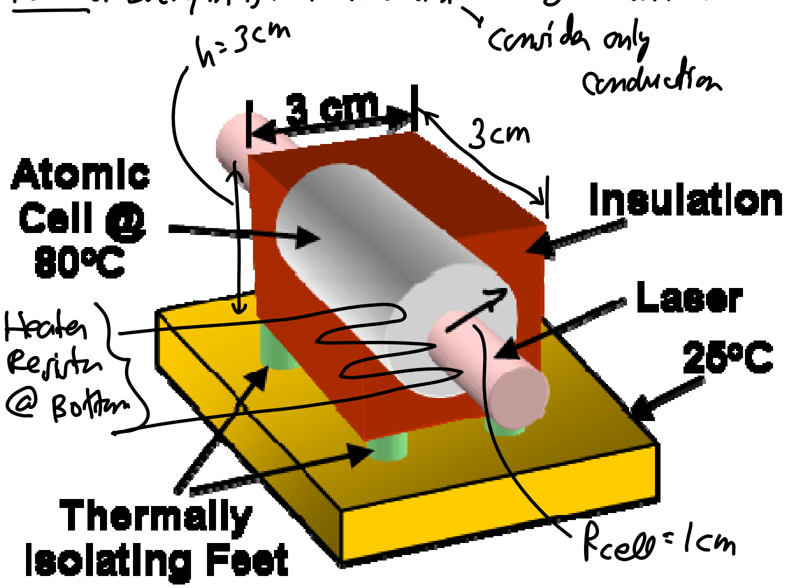
↳ If you can design large ckts. using this, then why not w/ this?

You absolutely can design ckts w/ mechanics! (and large ones)



Thermal Ckt. Modeling

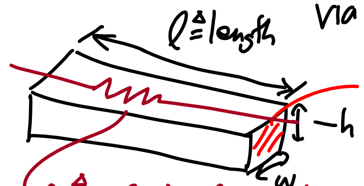
Assume: Everything is in vacuum → neglect convection
→ consider only conduction



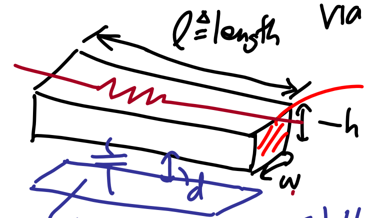
Example. Determine the power req'd to maintain the cell @ 80°C & the time req'd to get it there from a starting T = 25°C.

Review Electrical Resistance First

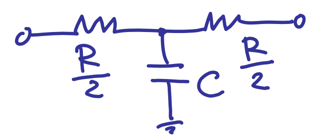
→ then attack thermal R via analogy

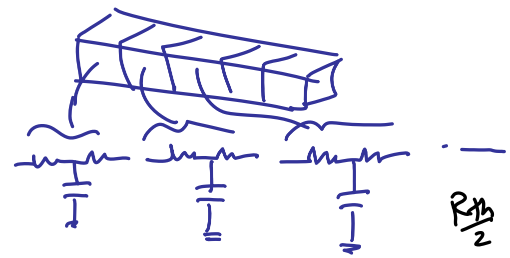


$R \triangleq$ electrical resistance $= \frac{\ell}{\sigma A}$ ← electrical conductivity

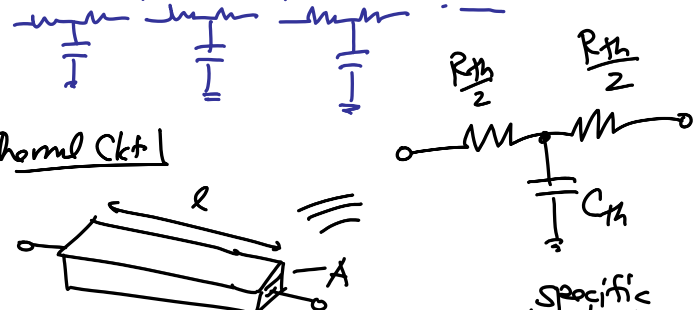


$C \triangleq$ capacitance $= \frac{\epsilon_0 W L}{d}$





Thermal Ckt



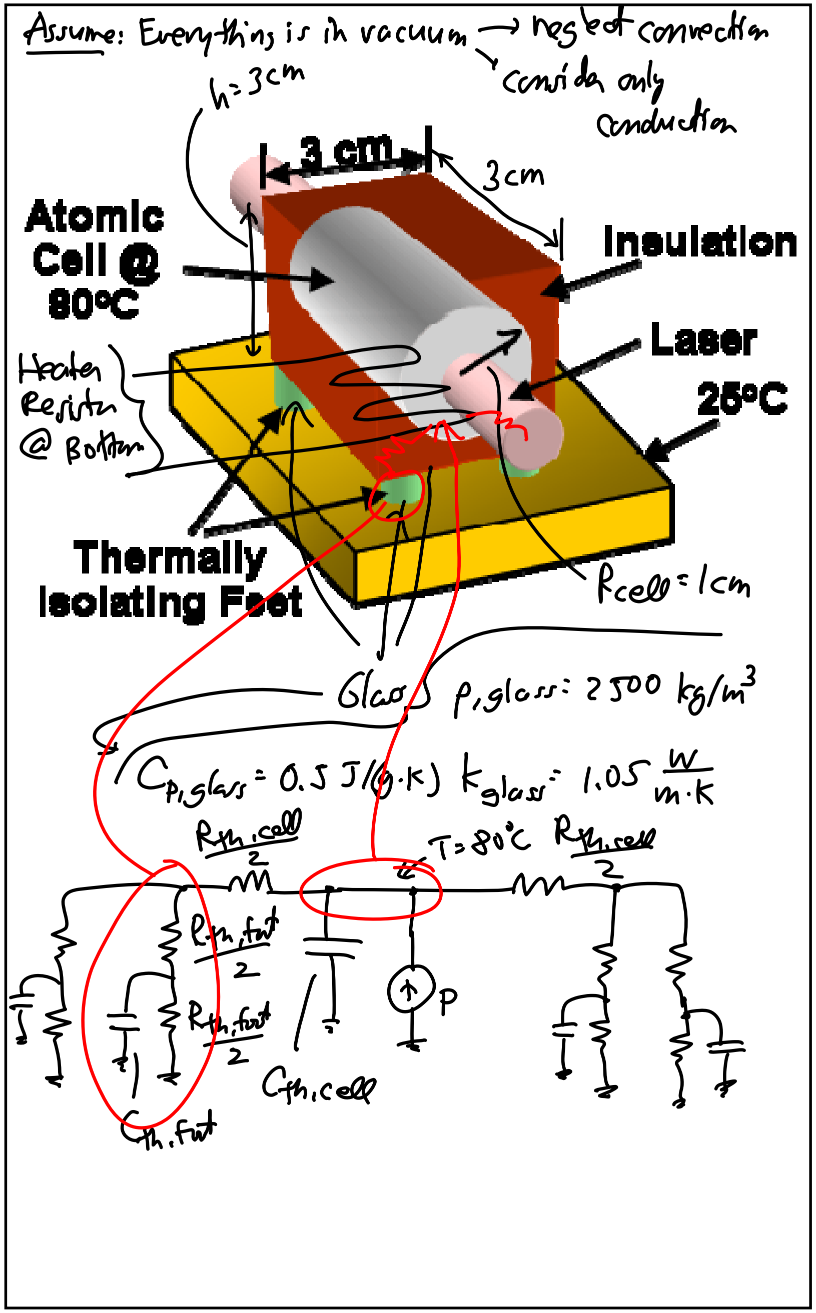
→ thermal capacitance: $C_{th} = \rho V c_p$

(ρ: density, V: volume, c_p: specific heat)

→ thermal resistance:

$R_{th} = \frac{\ell}{kA}$

(ℓ ← length, A ← cross-sectional area, k ← thermal conductivity)


Assume: Everything is in vacuum → neglect convection
→ consider only conduction
h=3cm
3 cm
3cm
Atomic Cell @ 80°C
Insulation
Laser
25°C
Heater Resistor @ Bottom
Thermally Isolating Feet
Rcell = 1cm
Glass
ρ, glass: 2500 kg/m³
Cp,glass = 0.5 J/(g·K)
kglass = 1.05 W/(m·K)
Rth,cell/2
T=80°C
Rth,cell/2
Rth,foot/2
Rth,foot/2
P
Cth,cell
Cth,foot