Lecture 4: Benefits of Scaling III

- Announcements:
- HW#1 passed out last time → due Tuesday, next week
- Neel Shah Office Hours: move to 9:30-11 a.m. (from the original 10:30-12 noon)
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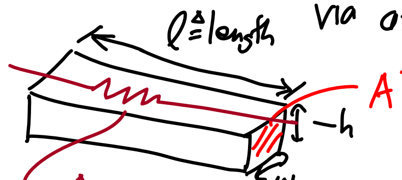
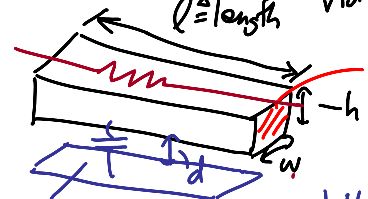
- Today:
- Reading: Senturia, Chapter 1
- Lecture Topics:
  - Benefits of Miniaturization
  - Examples
    - GHz micromechanical resonators
    - Chip-scale atomic clock
    - Thermal Circuits
    - Micro gas chromatograph
- -----------------------------------
- Last Time:
- Covering thermal circuit modeling …

Review Electrical Resistance First → then attack thermal R via analogy

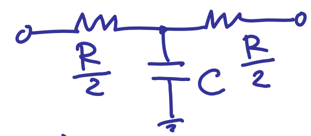


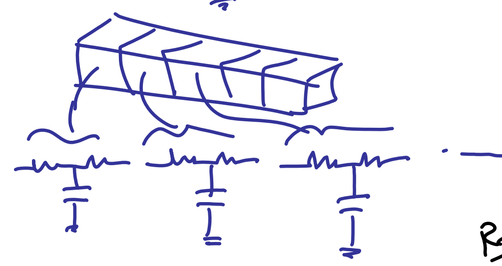
$R \triangleq$ electrical resistance $= \frac{l}{\sigma A}$ ← electrical conductivity



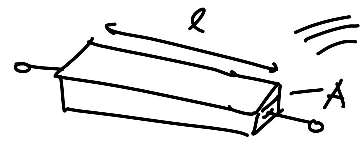
$C \triangleq$ capacitance $= \frac{\epsilon_0 WL}{d}$ → Stores energy (charge energy)

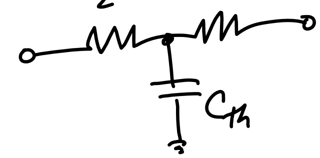
$E = \frac{1}{2}CV^2$ ↑ voltage across the capacitor





Thermal Ckt





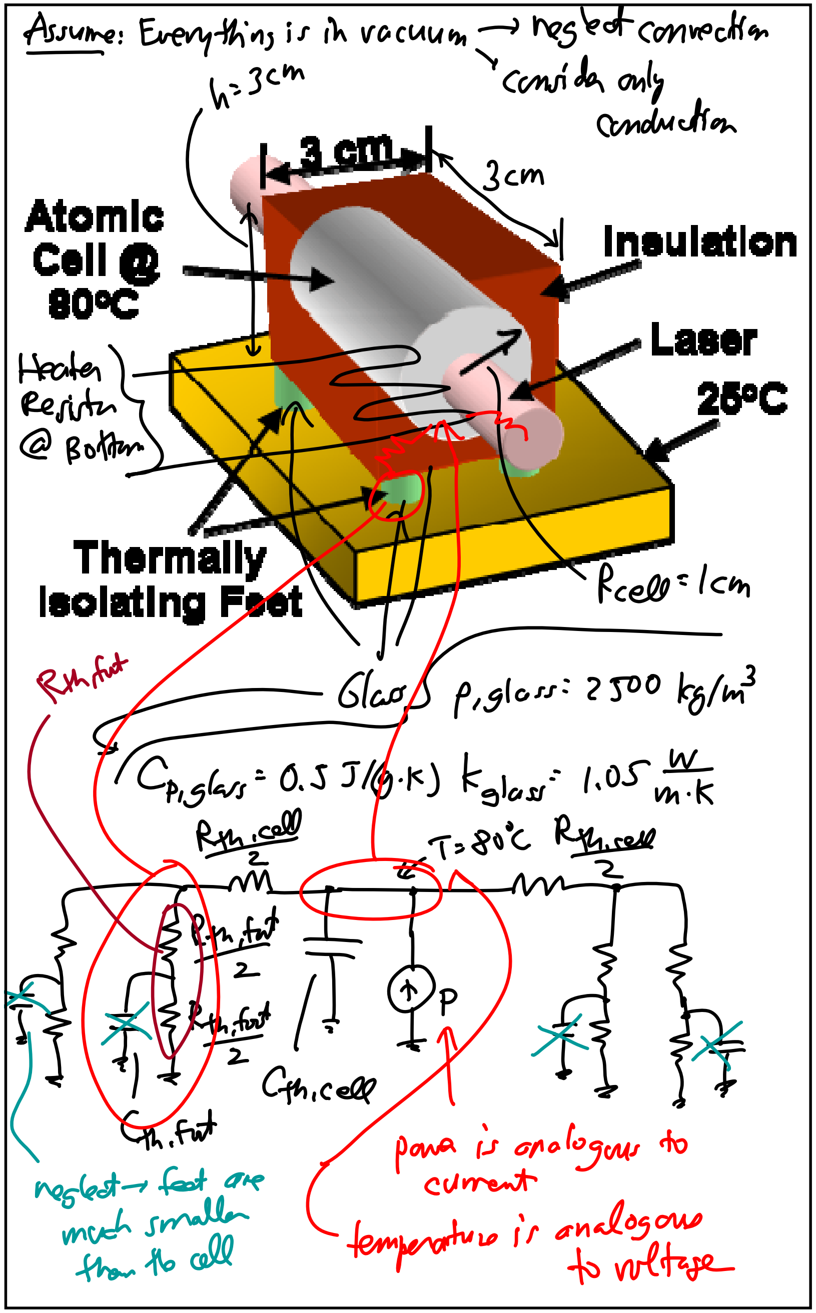
→ thermal capacitance: $C_{th} = \rho V c_p$ → Stores thermal energy

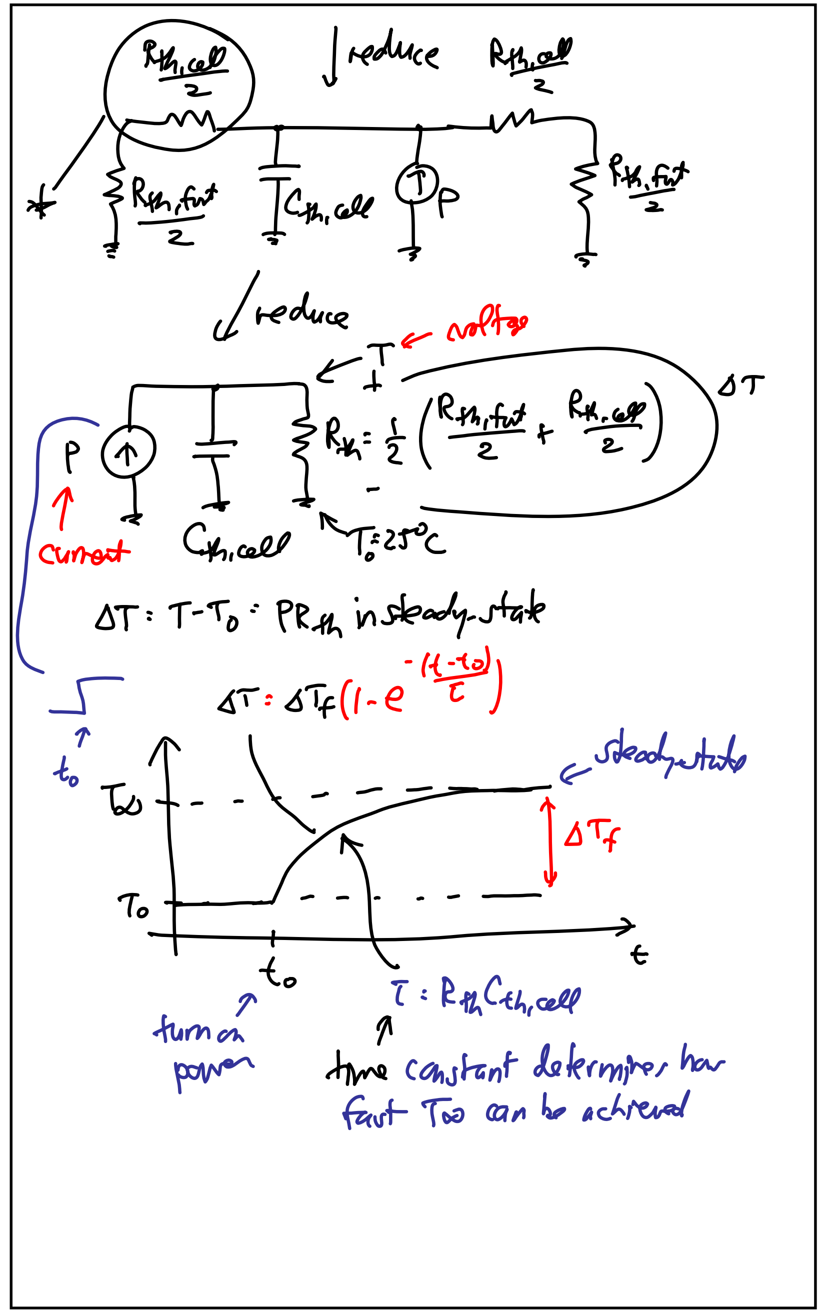
(ρ: density; V: volume; c_p: specific heat)

→ thermal resistance:

$$R_{th} = \frac{l}{kA}$$

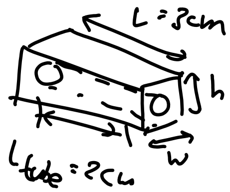
(l ← length; A ← cross-sectional area; k ← thermal conductivity)


Assume: Everything is in vacuum → neglect convection
consider only conduction
h=3cm
3 cm
3cm
Atomic Cell @ 80°C
Insulation
Laser
25°C
Heater Resistor @ Bottom
Thermally Isolating Feet
Rcell=1cm
Rth,feet
Glass
ρ,glass: 2500 kg/m³
Cp,glass = 0.5 J/(g·K)
kglass = 1.05 W/m·K
Rth,cell/2
T=80°C
Rth,cell/2
Rth,feet/2
Rth,feet/2
Cth,cell
P
Cth,feet
neglect → feet are much smaller than the cell
power is analogous to current
temperature is analogous to voltage


Rth,cell/2
reduce
Rth,cell/2
Rth,feet/2
Cth,cell
P
Rth,feet/2
reduce
T ← voltage
P
current
Rth = 1/2 (Rth,feet/2 + Rth,cell/2)
ΔT
Cth,cell
T0=25°C
ΔT = T−T0 = PRth in steady-state
ΔT = ΔTf(1 − e^(−(t−t0)/τ))
t0
steady-state
T∞
ΔTf
T0
t0
t
turn on power
τ = RthCth,cell
time constant determines how fast T∞ can be achieved

Find $C_{th,cell}$:

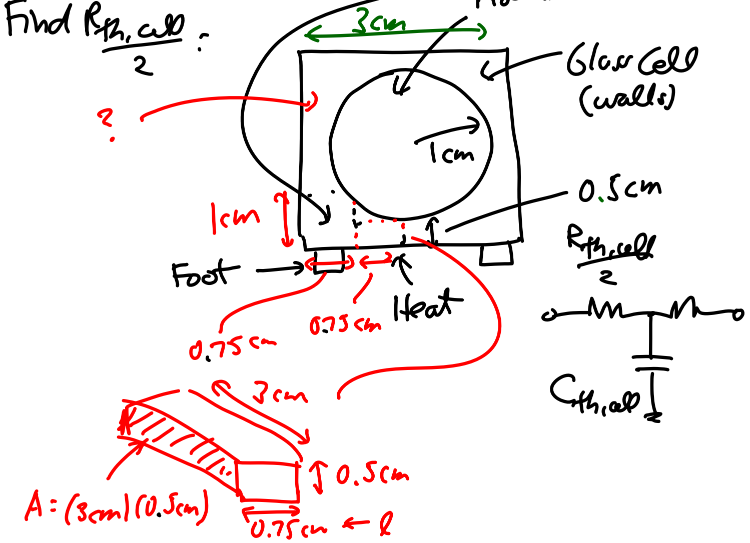
→ first find the cell volume:



$$V_{cell} = hWL - \pi R_{tube}^2 \cdot L_{tube}$$
$$= (3\,cm)(3\,cm)(3\,cm) - \pi(1\,cm)^2(2\,cm)$$
$$= 20.7\,cm^3$$

$$C_{th,cell} = \rho_{glass} V_{cell} C_{p,glass} \quad \leftarrow \text{specific heat}$$
$$= \left(2500\,\frac{kg}{m^3}\right)\left(1000\,\frac{g}{kg}\right)\left(\frac{1}{100^3}\,\frac{m^3}{cm^3}\right) \times (20.7\,cm^3)\left(0.5\,\frac{J}{g\cdot K}\right)$$
$$\Rightarrow C_{th,cell} = 25.9\,J/K$$

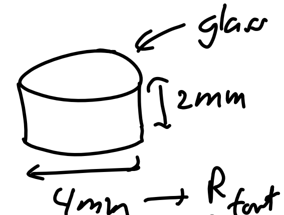
Find $\frac{R_{th,cell}}{2}$:



$$R_{th} = \frac{l}{kA}: \quad \frac{R_{th,cell}}{2} = \frac{\frac{3}{4}}{k(3)(\frac{1}{2})} + \frac{\frac{3}{4}}{k(3)(1)} = \frac{1}{k}\left(\frac{1}{2} + \frac{1}{4}\right) = \frac{3}{8}\frac{1}{k}$$

$$\therefore \frac{R_{th,cell}}{2} = \frac{3}{8}\frac{1}{1.05} \times \left(100\,\frac{cm}{m}\right) = 35.7\,K/W$$

Find $R_{th,foot}$:



4mm → $R_{foot} = 2mm$ (↑ radius)

$$A_{foot} = \pi R_{foot}^2$$
$$L_{foot} = 2mm$$

$$R_{th,foot} = \frac{L_{foot}}{k_{glass} \cdot A_{foot}} = \frac{2mm}{\left(1.05\,\frac{W}{m\cdot K}\right)\pi(2mm)^2} = 151.6\,K/W$$

Then:

$$R_{th} = \frac{1}{2}\left(\frac{R_{th,foot}}{2} + \frac{R_{th,cell}}{2}\right) = \frac{1}{2}\left(\frac{151.6}{2} + 35.7\right)$$
$$\rightarrow R_{th} = 55.8\,K/W$$

⇒ find power req'd to maintain T_∞ in steady state:

$$P = \frac{T_\infty - T_0}{R_{th}} = \frac{(80-25)}{55.8} = 0.99W \sim 1W$$

→ find the time constant:

$\tau = R_{th} \cdot C_{th,cell} = 24\ \text{min.}$

It takes ~3τ to reach steady state ∴ must wait 72 min. before using this clock!

More Examples