


EE C245 - ME C218
Introduction to MEMS Design
Fall 2010

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Lecture Module 12: Capacitive Transducers

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Lecture Outline

- Reading: Senturia, Chpt. 5, Chpt. 6
- Lecture Topics:
 - ↗ Energy Conserving Transducers
 - Charge Control
 - Voltage Control
 - ↗ Parallel-Plate Capacitive Transducers
 - Linearizing Capacitive Actuators
 - Electrical Stiffness
 - ↗ Electrostatic Comb-Drive
 - 1st Order Analysis
 - 2nd Order Analysis

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Basic Physics of Electrostatic Actuation

- **Goal:** Determine gap spacing g as a function of input variables
- First, need to determine the energy of the system
- Two ways to change the energy:
 - ↪ Change the charge q
 - ↪ Change the separation g

$$\Delta W(q, g) = V\Delta q + F_e\Delta g$$

$$dW = Vdq + F_e dg$$

- **Note:** We assume that the plates are supported elastically, so they don't collapse

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Stored Energy

• Here, the stored energy is the work done in increasing the gap after charging capacitor at zero gap

$$W = 0 + \int_0^g F_e dg'$$

No change in charge: $dq=0$

$$F_e = \left(\frac{q}{2}\right)\epsilon = \frac{1}{2} \frac{q^2}{\epsilon A}$$

(independent of g)

$$\therefore W = \int_0^g F_e dg' = F_e g \Big|_0^g = F_e g$$

$$\therefore W(g) = \frac{1}{2} \frac{q^2}{\epsilon A} g$$

zero gap → zero stored energy

For a capacitor C :
 $q = CV \rightarrow V = \frac{q}{C}$

$$W(q) = \int_0^q V dq = \int_0^q \left(\frac{q}{C}\right) dq$$

$$= \frac{1}{2} \frac{q^2}{C} = \frac{1}{2} \frac{q^2 g}{\epsilon A} = W(q)$$

Work done to charge C to q at fixed gap

Same answer when

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Charge-Control Case

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- Having found stored energy, we can now find the force acting on the plates and the voltage across them:

From $dW = Vdq + F_e dg$:

⇒ Force is given by:

$$F_e = \left. \frac{\partial W(q,g)}{\partial g} \right|_{q=\text{const.}} = \frac{\partial}{\partial g} \left(\frac{1}{2} \frac{q^2}{\epsilon A} g \right)$$

$$\therefore \boxed{F_e = \frac{1}{2} \frac{q^2}{\epsilon A}} \Rightarrow \text{independent of gap spacing!}$$

⇒ Voltage is given by:

$$V = \left. \frac{\partial W(q,g)}{\partial q} \right|_{g=\text{const.}} = \frac{\partial}{\partial q} \left(\frac{1}{2} \frac{q^2}{\epsilon A} g \right) = \frac{qg}{\epsilon A} \therefore \boxed{V = \frac{q}{C}} \Rightarrow \text{consistent w/ what we already know} \checkmark$$

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Voltage-Control Case

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- Practical situation: We control V
 - Charge control on the typical sub-pF MEMS actuation capacitor is difficult
 - Need to find F_e as a partial derivative of the stored energy $W = W(V,g)$ with respect to g with V held constant? But can't do this with present $W(q,g)$ formula
 - Solution:** Apply Legendre transformation and define the co-energy $W'(V,g)$

Effort (e.g., force, voltage, ...)

Energy $\rightarrow W(q) = \int_0^{q_1} e dq = \int_0^{q_1} \Phi(q) dq$

Co-Energy $\rightarrow W'(e) = \int_0^{e_1} q de = \int_0^{e_1} \Phi^{-1}(e) de$

⇒ Can define co-energy as: $W'(e) = eq - W(q)$ (from this plot)

For a linear system, these will be equal.

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Co-Energy Formulation

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- For our present problem (i.e., movable capacitive plates), the co-energy formulation becomes

$$W'(V, g) = qV - W(q, g)$$

Differentially, this becomes:

$$dW'(V, g) = (q dV + V dq) - dW(q, g)$$

But $[dW(q, g) = F_e dg + V dq]$

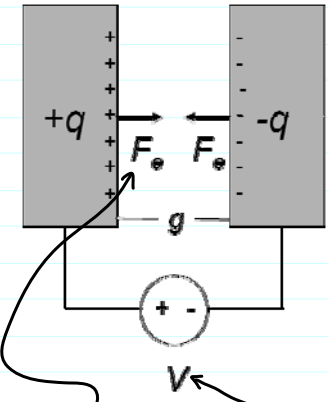
$dW'(V, g) = q dV - F_e dg$

Working Co-Energy Expression

From which:

Charge, $Q = \left. \frac{\partial W'(V, g)}{\partial V} \right|_{g = \text{const.}}$

Force, $F_e = - \left. \frac{\partial W'(V, g)}{\partial g} \right|_{V = \text{const.}}$ ⇒ this gives force as a function of applied voltage



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Electrostatic Force (Voltage Control)

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- Find co-energy in terms of voltage (w/ gap held constant)

$$W' = \int_0^V q(g, V') dV' = \int_0^V \left(\epsilon \frac{A}{g} \right) V' dV' = \frac{1}{2} \left(\frac{\epsilon A}{g} \right) V^2 = \frac{1}{2} C V^2$$

(as expected)

- Variation of co-energy with respect to gap yields electrostatic force:

$$F_e = - \left. \frac{\partial W'(V, g)}{\partial g} \right|_V = - \frac{1}{2} \left(- \frac{\epsilon A}{g^2} \right) V^2 = \frac{1}{2} \frac{C}{g} V^2$$

strong function of gap!

- Variation of co-energy with respect to voltage yields charge:

$$q = \left. \frac{\partial W'(V, g)}{\partial V} \right|_g = \left(\frac{\epsilon A}{g} \right) V = C V$$

as expected

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Spring-Suspended Capacitive Plate

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Charge Control of a Spring-Suspended C

Force generated by charge q supplied by current I :

$$F_e = \left. \frac{\partial W(q, g)}{\partial g} \right|_q = \frac{q^2}{2\epsilon A}$$

Restoring force of spring: $F_{\text{spring}} = kz = F_e$ (@ equilibrium)

And the gap:

$$g = g_0 - z = g_0 - \frac{F_e}{k} = \boxed{g_0 - \frac{1}{2} \frac{q^2}{\epsilon A} \frac{1}{k}} = g \Rightarrow \text{can increase } q \text{ and drive } g \rightarrow 0$$

initial gap

$$V = \frac{q}{C} = \frac{q}{\epsilon A} g = \boxed{\frac{q}{\epsilon A} \left(g_0 - \frac{1}{2} \frac{q^2}{\epsilon A} \frac{1}{k} \right)} = V \Rightarrow V \downarrow \text{ as } g \downarrow$$

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Voltage Control of a Spring-Suspended C

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fixed k fixed

Again, $F_{\text{spring}} = kz = F_e$
 But now:

$$F_e = \left. \frac{\partial W'(V, g)}{\partial g} \right|_q = \frac{1}{2} \frac{\epsilon A}{g^2} V^2$$

And the gap:

$$g = g_0 - z = g_0 - \frac{F_e}{k} = \boxed{g_0 - \frac{1}{2} \frac{\epsilon A}{g^2} V^2 = g} \Rightarrow g \text{ shows up on both sides!}$$

Charge: (for a stable gap)

$$q = \left. \frac{\partial W'(V, g)}{\partial V} \right|_g = CV = \boxed{\frac{\epsilon A}{g} V = q}$$

cubic nonlinearity in g !

If $V \uparrow \rightarrow g \downarrow \rightarrow F_e \uparrow$

Feedback!

\Rightarrow If loop gain > 1 , then this will go unstable!

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Stability Analysis

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- Net attractive force on the plate:

$$F_{\text{net}} = \underbrace{\frac{\epsilon AV^2}{2g^2}}_{F_e} - \underbrace{k(g_0 - g)}_{F_{\text{spring}}}$$

- An increment in gap dg leads to an increment in net attractive force dF_{net}

$$dF_{\text{net}} = \frac{\partial F_{\text{net}}}{\partial g} dg = \left[-\frac{\epsilon AV^2}{g^3} + k \right] dg$$

$F_{\text{net}} \downarrow \rightarrow dF_{\text{net}} = (-)$

If $g \downarrow \rightarrow dg = (-)$, then for stability need \rightarrow otherwise, plate collapses

Thus, need this = (+) \Rightarrow $\boxed{k > \frac{\epsilon AV^2}{g^3}}$ (for a stable uncollapsed state)

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Pull-In Voltage V_{PI}

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- V_{PI} = voltage at which the plates collapse
- The plate goes unstable when

$$k = \frac{\epsilon A V_{PI}^2}{g_{PI}^3} \quad (1) \quad \text{and} \quad F_{net} = 0 = \frac{\epsilon A V_{PI}^2}{2g_{PI}^2} - k(g_0 - g_{PI}) \quad (2)$$

$2g_{PI} \leftarrow \text{pull-in gap}$

- Substituting (1) into (2):

$$0 = \frac{\epsilon A V_{PI}^2}{2g_{PI}^2} - \frac{\epsilon A V_{PI}^2}{g_{PI}^3} (g_0 - g_{PI})$$

$$\frac{g_0 - g_{PI}}{g_{PI}} = \frac{1}{2} \rightarrow g_0 = \frac{3}{2} g_{PI}$$

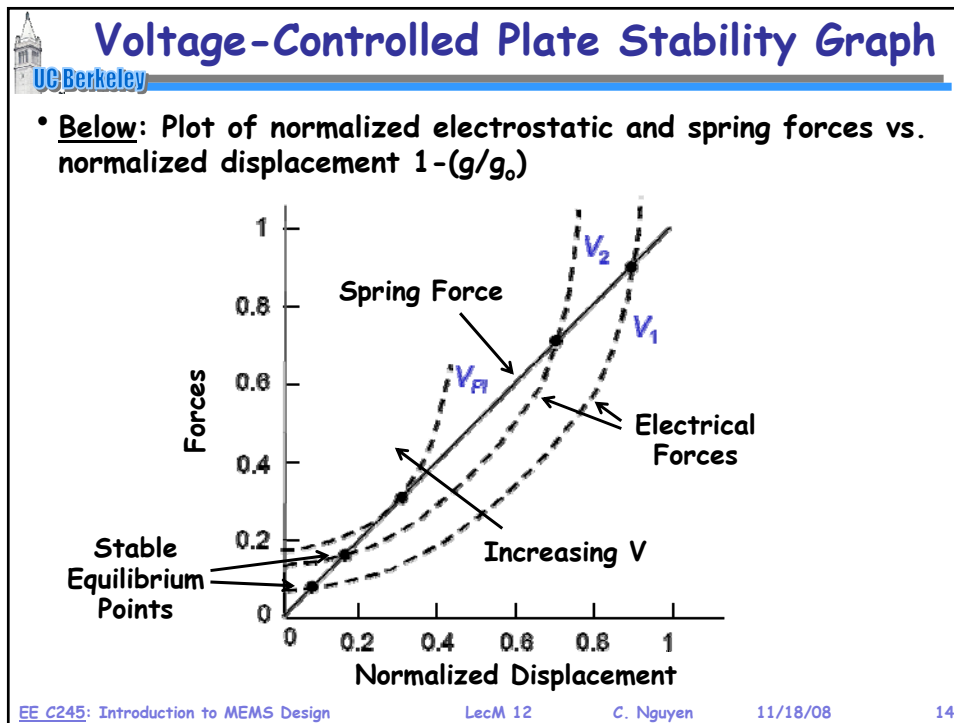
$$\therefore g_{PI} = \frac{2}{3} g_0$$


$$V_{PI} = \sqrt{\frac{k g_{PI}^3}{\epsilon A}}$$

$$\therefore V_{PI} = \sqrt{\frac{8}{27} \frac{k g_0^3}{\epsilon A}}$$

When a gap is driven by a voltage to (2/3) its original spacing, collapse will occur!

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Advantages of Electrostatic Actuators

- Easy to manufacture in micromachining processes, since conductors and air gaps are all that's needed → low cost!
- Energy conserving → only parasitic energy loss through I^2R losses in conductors and interconnects
- Variety of geometries available that allow tailoring of the relationships between voltage, force, and displacement
- Electrostatic forces can become very large when dimensions shrink → electrostatics scales well!
- Same capacitive structures can be used for both drive and sense of velocity or displacement
- Simplicity of transducer greatly reduces mechanical energy losses, allowing the highest Q's for resonant structures

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Problems With Electrostatic Actuators

- Nonlinear voltage-to-force transfer function
- Relatively weak compared with other transducers (e.g., piezoelectric), but things get better as dimensions scale

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Linearizing the Voltage-to-Force Transfer Function

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Linearizing the Voltage-to-Force T.F.

- Apply a DC bias (or polarization) voltage V_p together with the intended input (or drive) voltage $v_i(t)$, where $V_p \gg v_i(t)$

$v(t) = V_p + v_i(t)$

$$F_e(t) = \frac{\partial W}{\partial x} = \frac{\partial}{\partial x} \left(\frac{1}{2} C [v(t)]^2 \right)$$

$$= \frac{1}{2} \frac{\partial C}{\partial x} [v(t)]^2 = \frac{1}{2} (V_p + v_i(t))^2 \frac{\partial C}{\partial x}$$

$$= \frac{1}{2} [V_p^2 + 2V_p v_i(t) + [v_i(t)]^2] \frac{\partial C}{\partial x}$$


$$[V_p \gg v_i(t)] \Rightarrow F_e(t) = \underbrace{\frac{1}{2} V_p^2 \frac{\partial C}{\partial x}}_{\text{DC Offset}} + \underbrace{V_p \frac{\partial C}{\partial x} v_i(t)}_{\text{AC drive signal}}$$

$$C(x) = \frac{\epsilon A}{g_0 \cdot x} = C_0 \left(1 - \frac{x}{g_0}\right)^{-1} \approx C_0 \left(1 + \frac{x}{g_0}\right)$$

$[x \ll g_0] \quad \sim \text{const.} \therefore \text{linear}$

$$\therefore \frac{\partial C}{\partial x} \approx \frac{C_0}{g_0} \Rightarrow F_e(t) = \frac{1}{2} \frac{C_0}{g_0} V_p^2 + V_p \frac{C_0}{g_0} v_i(t)$$

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Differential Capacitive Transducer

• The net force on the suspended center electrode is

$$F_{net} = F_{er}(t) - F_{el}(t)$$

Do the math.


Assume matched gaps.

$$F_{net}(t) = \frac{1}{2} \frac{\partial C}{\partial x} \{ [V_R(t)]^2 - [V_L(t)]^2 \}$$

$$= \frac{1}{2} \frac{\partial C}{\partial x} \{ (V_p^2 + 2V_p V(t) + [V(t)]^2) - (V_p^2 - 2V_p V(t) + [V(t)]^2) \}$$

$$\therefore F_{net}(t) = 2V_p \frac{\partial C}{\partial x} V(t) = 2V_p \frac{C_0}{g_0} N(t) \Rightarrow \text{Linear w/ } V(t)! \text{ (gap match limited)}$$

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Remaining Nonlinearity (Electrical Stiffness)

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Parallel-Plate Capacitive Nonlinearity

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- Example:** clamped-clamped laterally driven beam with balanced electrodes
- Nomenclature:**

V_a or v_A

$v_a = |v_a| \cos \omega t$

V_A

t

V_a or $v_A = V_A + v_a$

Total Value

DC Component (upper case variable; upper case subscript)

AC or Signal Component (lower case variable; lower case subscript)

Conductive Structure

Electrode

k_m

d_1

m

x

F_{d1}

v_1

V_1

V_P

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Parallel-Plate Capacitive Nonlinearity

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- Example:** clamped-clamped laterally driven beam with balanced electrodes
- Expression for $\partial C/\partial x$:**

$$C_1(x) = \frac{\epsilon A}{d_1 + x} = C_0 \left(1 + \frac{x}{d_1}\right)^{-1} \rightarrow \frac{\partial C_1}{\partial x} = -\frac{C_0}{d_1} \left(1 + \frac{x}{d_1}\right)^{-2}$$

[Expand the Taylor series further]

$$\frac{\partial C_1}{\partial x} = -\frac{C_0}{d_1} \left(1 + A_1 x + A_2 x^2 + A_3 x^3 + \dots\right)$$

where

$$A_1 = -\frac{2}{d_1}$$

$$A_2 = \frac{3}{d_1^2}$$

$$A_3 = -\frac{4}{d_1^3}$$

⋮

Conductive Structure

Electrode

k_m

d_1

m

x

F_{d1}

v_1

V_1

V_P

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Parallel-Plate Capacitive Nonlinearity

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- Thus, the expression for force from the left side becomes:

$$F_{d1} = \frac{1}{2} \frac{\partial C}{\partial x} (V_P - V_1 - N_i)^2 = \frac{1}{2} \frac{\partial C}{\partial x} (V_{P1} - N_i)^2$$

{small displacements: $x \ll d_1$ }

$$F_{d1} = \frac{1}{2} \left(-\frac{C_{o1}}{d_1} \right) (1 + A_1 x) (V_{P1}^2 - 2V_{P1} N_i + N_i^2)$$

$$= \frac{1}{2} \left(-\frac{C_{o1}}{d_1} \right) \left\{ V_{P1}^2 - 2V_{P1} N_i + N_i^2 + A_1 V_{P1}^2 x - 2A_1 V_{P1} x N_i + A_1 x N_i^2 \right\}$$

@ resonance: $x = \frac{Q F_{d1}}{j k} \approx \frac{Q}{j k} \frac{\partial C}{\partial x} V_{P1} V_1$

Thus:

$$N_i = |N_i| \cos \omega_o t \rightarrow x = |x| \sin \omega_o t$$

\uparrow
 x 90° phase-shifted from N_i

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Parallel-Plate Capacitive Nonlinearity

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- Retaining only terms at the drive frequency:


$$F_{d1}|_{\omega_o} = \underbrace{V_{P1} \frac{C_{o1}}{d_1}}_{\text{Drive force arising from the input excitation voltage at the frequency of this voltage}} |v_1| \cos \omega_o t + \underbrace{V_{P1}^2 \frac{C_{o1}}{d_1^2}}_{\text{Proportional to displacement}} |x| \sin \omega_o t$$

90° phase-shifted from drive, so in phase with displacement

- These two together mean that this force acts against the spring restoring force!
- ↳ A negative spring constant
- ↳ Since it derives from V_p , we call it the electrical stiffness, given by:

$$k_e = V_{P1}^2 \frac{C_{o1}}{d_1^2} = V_{P1}^2 \frac{\epsilon A}{d_1^3}$$

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Electrical Stiffness, k_e

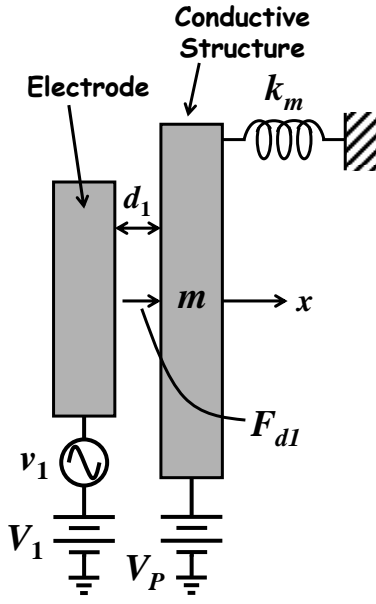
- The electrical stiffness k_e behaves like any other stiffness
- It affects resonance frequency:

$$\omega_o' = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_m - k_e}{m}}$$


$$= \sqrt{\frac{k_m}{m} \left(1 - \frac{k_e}{k_m}\right)^{1/2}}$$

$$\omega_o' = \omega_o \left(1 - \frac{V_{P1}^2 \epsilon A}{k_m d_1^3}\right)^{1/2}$$

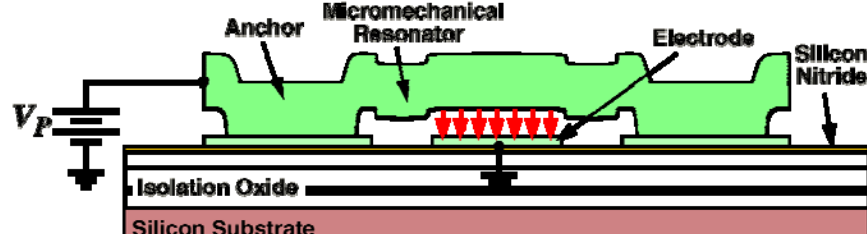
Frequency is now a function of dc-bias V_{P1}



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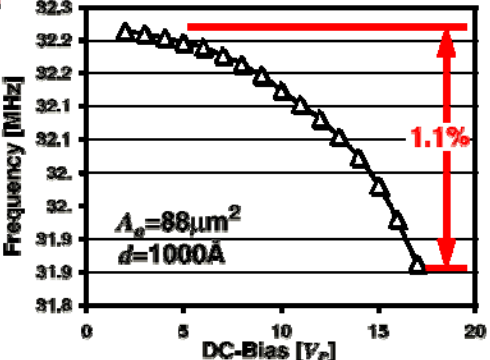
Voltage-Controllable Center Frequency



- Quadrature force \Rightarrow voltage-controllable electrical stiffness:

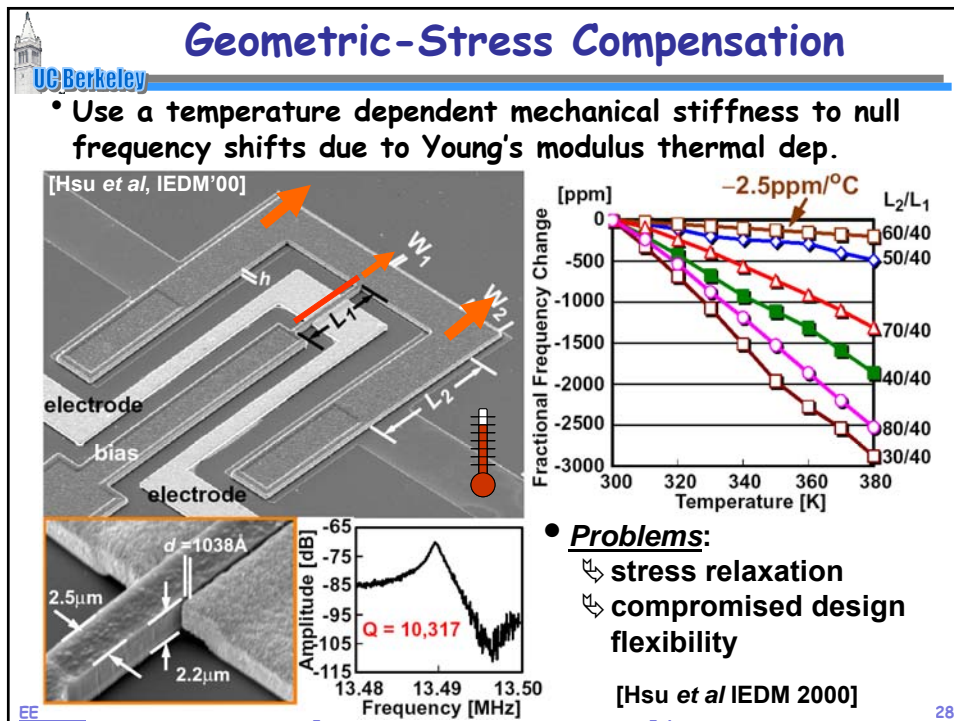
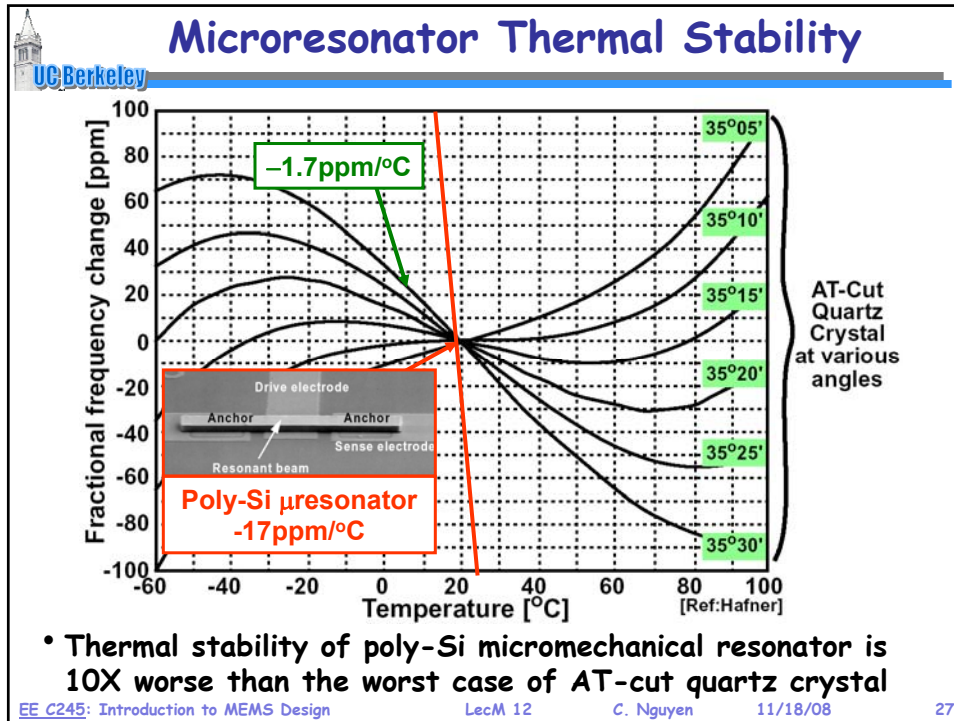
$$k_e = \frac{\epsilon_o A_o}{d^3} V_P^2$$

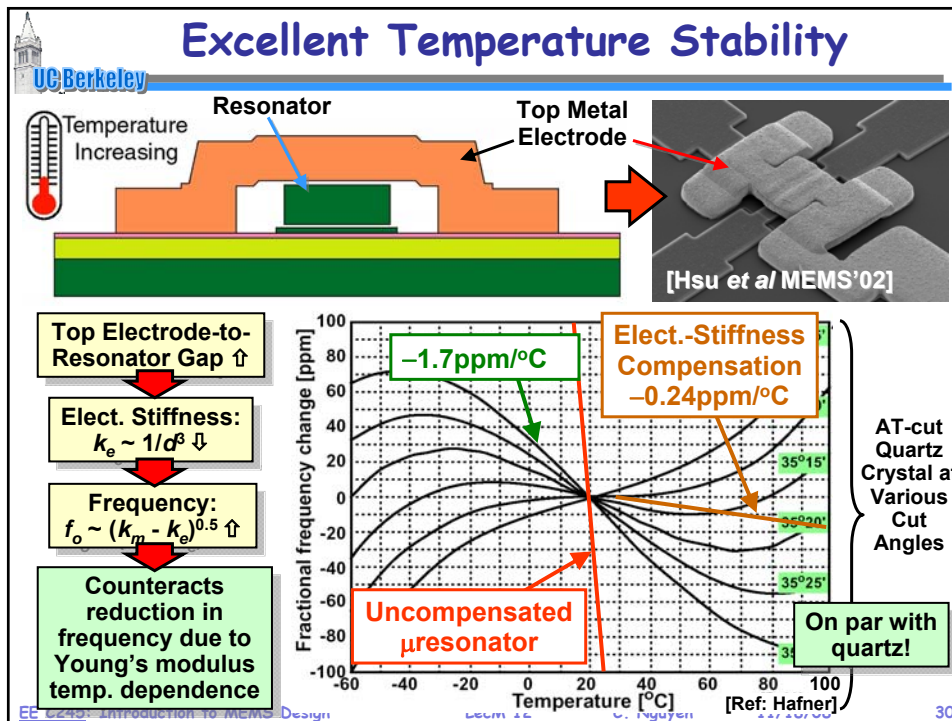
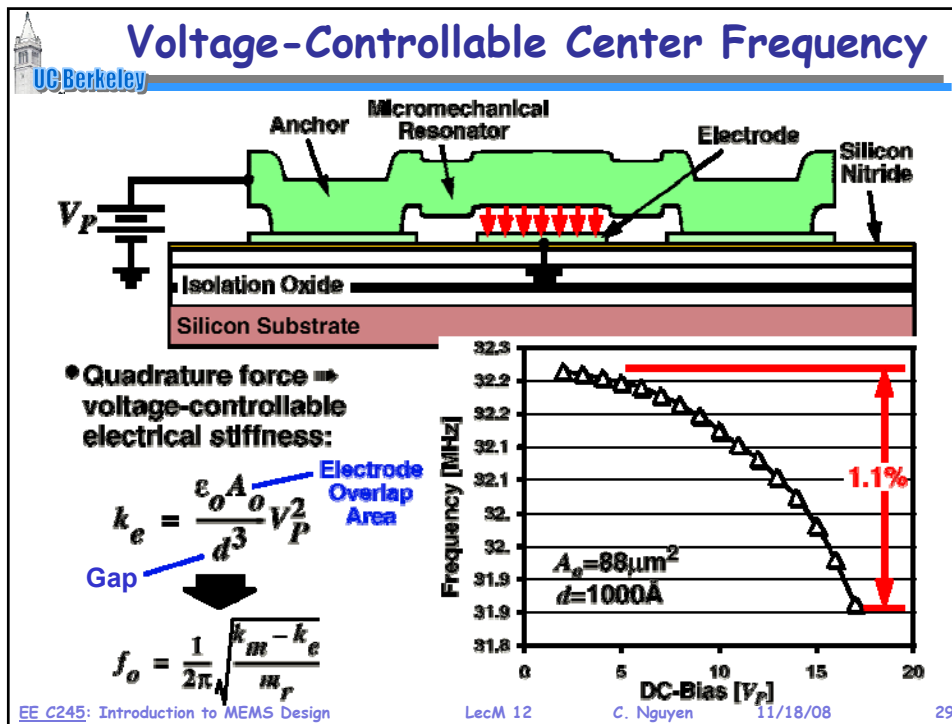
Electrode Overlap Area
Gap

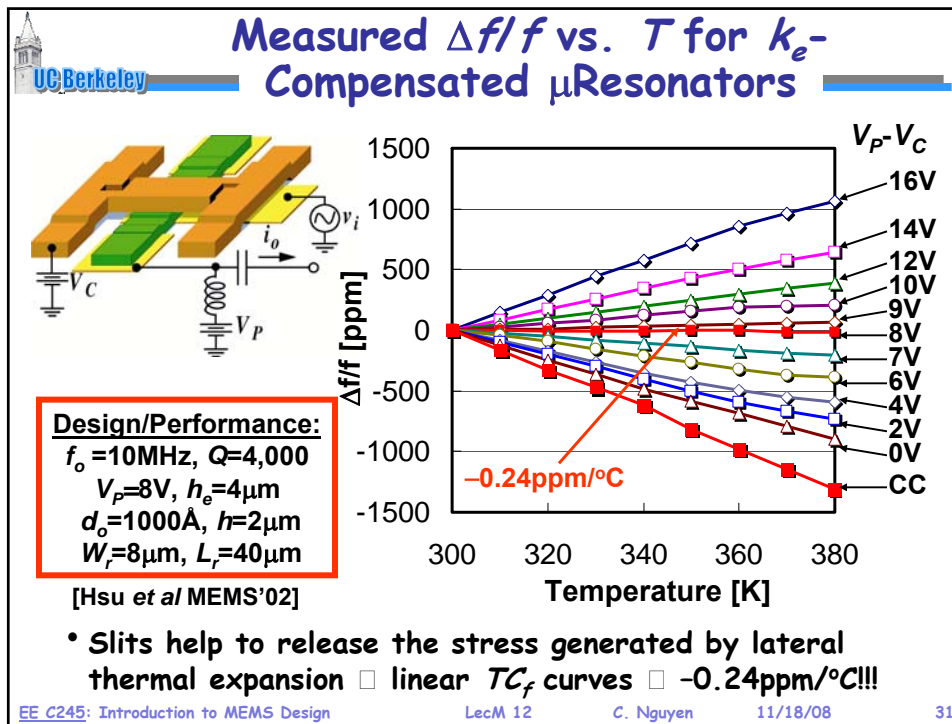
$$f_o = \frac{1}{2\pi} \sqrt{\frac{k_m - k_e}{m_r}}$$


$A_o = 88 \mu\text{m}^2$
 $d = 1000 \text{ \AA}$

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Can One Cancel k_e w/ Two Electrodes?


- What if we don't like the dependence of frequency on V_p ?
- Can we cancel k_e via a differential input electrode configuration?
- If we do a similar analysis for F_{d2} at Electrode 2:

Subtracts from the F_{d1} term, as expected

$$F_{d2}|_{\omega_o} = -V_{P2} \frac{C_{o2}}{d_2} |v_2| \cos \omega_o t + V_{P2}^2 \frac{C_{o2}}{d_2^2} |x| \sin \omega_o t$$

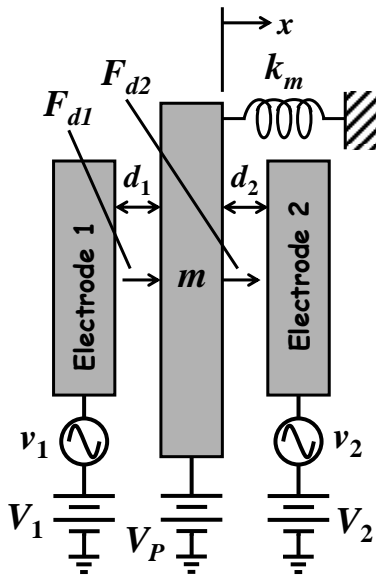
Adds to the quadrature term $\rightarrow k_e$'s add, no matter the electrode configuration!

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Problems With Parallel-Plate C Drive

- Nonlinear voltage-to-force transfer function
 - ↪ Resonance frequency becomes dependent on parameters (e.g., bias voltage V_P)
 - ↪ Output current will also take on nonlinear characteristics as amplitude grows (i.e., as x approaches d_0)
 - ↪ Noise can alias due to nonlinearity
- Range of motion is small
 - ↪ For larger motion, need larger gap ... but larger gap weakens the electrostatic force
 - ↪ Large motion is often needed (e.g., by gyroscopes, vibromotors, optical MEMS)



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Electrostatic Comb Drive

Electrostatic Comb Drive

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Electrostatic Comb Drive

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- Use of comb-capacitive transducers brings many benefits
 - ↳ Linearizes voltage-generated input forces
 - ↳ (Ideally) eliminates dependence of frequency on dc-bias
 - ↳ Allows a large range of motion

Comb-Driven Folded Beam Actuator

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Comb-Drive Force Equation (1st Pass)

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Top View

Side View

$$C(x) = \frac{2\epsilon_0 x h}{d} \rightarrow \left[\frac{\partial C}{\partial x} = \frac{2\epsilon_0 h}{d} \right]$$

$$F_d = \frac{\partial W'}{\partial x} = \frac{1}{2} \frac{\partial C}{\partial x} (V_p - V_i)^2 = \frac{2}{2} \frac{\epsilon_0 h}{d} (V_p^2 - 2V_p V_i + V_i^2) \approx -2V_p \frac{\epsilon_0 h}{d} V_i = F_d$$

When $V_i = (+) \rightarrow F_d = (-)$ ✓

↳ But wait! This ignores other practical effects! (No dependence on λ ! LINEAR!)

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Lateral Comb-Drive Electrical Stiffness

Top View

Side View

- Again: $C(x) = \frac{2Nshx}{d} \rightarrow \frac{\partial C}{\partial x} = \frac{2Nsh}{d}$
- No $(\partial C/\partial x)$ x-dependence \rightarrow no electrical stiffness: $k_e = 0!$
- Frequency immune to changes in V_p or gap spacing!

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Typical Drive & Sense Configuration

2-port Lateral Microresonator

N_f : # shuttle fingers

Simple Analysis:

$$F_{d1} = \frac{1}{2} \frac{\partial C_1}{\partial x} (V_1 - V_{P1})^2 = \frac{1}{2} \left(-\frac{\epsilon_0 h}{d} \right) (N_1^2 - 2V_{P1}V_1 + V_{P1}^2) (2N_f)$$

$$F_{d2} = \frac{1}{2} \frac{\partial C_2}{\partial x} (V_2 - V_{P2})^2 = \frac{1}{2} \left(\frac{\epsilon_0 h}{d} \right) (N_2^2 - 2V_{P2}V_2 + V_{P2}^2) (2N_f)$$

$$\therefore F_{net} = F_{d1} + F_{d2} = \frac{1}{2} \left(\frac{\epsilon_0 h}{d} \right) (N_2^2 - N_1^2 - 2(V_{P2}V_2 - V_{P1}V_1) + V_{P2}^2 - V_{P1}^2) (2N_f)$$

For $V_1 = V_2, V_1 = -V_2$

$$F_{net} = 2(2N_f) \left(\frac{\epsilon_0 h}{d} \right) V_{P1} V_1$$

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Comb-Drive Force Equation (2nd Pass)

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- In our 1st pass, we accounted for
 - ↪ Parallel-plate capacitance between stator and rotor
- ... but neglected:
 - ↪ Fringing fields
 - ↪ Capacitance to the substrate
- All of these capacitors must be included when evaluating the energy expression!

The diagram illustrates a comb-drive actuator with stator and rotor fingers. The stator fingers are labeled 'Stator' and the rotor fingers are labeled 'Rotor'. The gap between the stator and rotor fingers is labeled 'g', and the thickness of the fingers is labeled 't'. The distance from the stator fingers to the ground plane is labeled 'L', and the distance from the rotor fingers to the ground plane is labeled 'x'. The ground plane is labeled 'Ground Plane'. The electrical circuit shows two voltage sources, V_s and V_r , connected to the stator and rotor fingers respectively. The ground plane is connected to ground. The diagram also shows the distance from the stator fingers to the ground plane, labeled z_0 .

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Comb-Drive Force With Ground Plane Correction

UC Berkeley

- Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane → modifies the capacitive energy

$$F_{e,x} = \frac{\partial W'}{\partial x} = \frac{1}{2} \frac{dC_{sp}}{dx} V_s^2 + \frac{1}{2} \frac{dC_{rp}}{dx} V_r^2 + \frac{1}{2} \frac{dC_{rs}}{dx} (V_s - V_r)^2$$

The diagram shows an equivalent circuit model for the comb-drive actuator. It consists of three nodes: stator (s), rotor (r), and ground plane (p). The stator node is connected to the rotor node by a capacitor with value $\frac{1}{2} C_{rs}$. The rotor node is connected to the ground plane node by a capacitor with value C_{rp} . The stator node is connected to the ground plane node by a capacitor with value $\frac{1}{2} C_{sp}$. The rotor node is connected to a voltage source V_r and the ground plane node is connected to a voltage source V_s . The ground plane node is also connected to ground.

[Gary Fedder, Ph.D., UC Berkeley, 1994]

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Capacitance Expressions

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- Case: $V_r = V_p = 0V$
- C_{sp} depends on whether or not fingers are engaged

$$C_{sp} = N[C'_{sp,e}x + C'_{sp,u}(L-x)]$$

$$C_{rs} = NC'_{rs}x$$

Capacitance per unit length

Region 2

Region 3

[Gary Fedder, Ph.D., UC Berkeley, 1994]

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Comb-Drive Force With Ground Plane Correction

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- Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane → modifies the capacitive energy

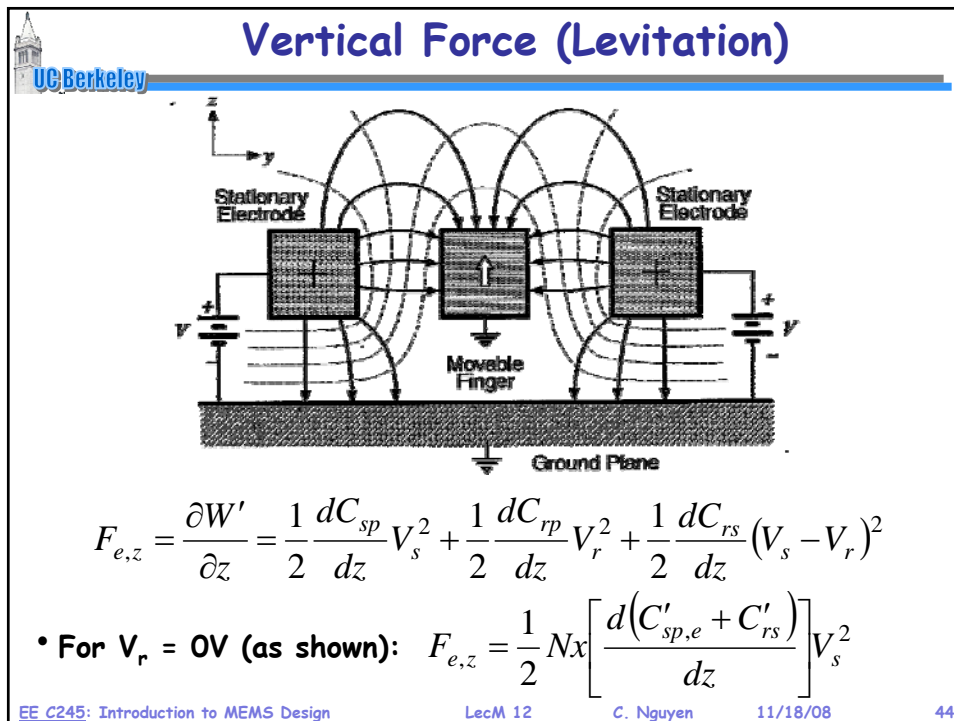
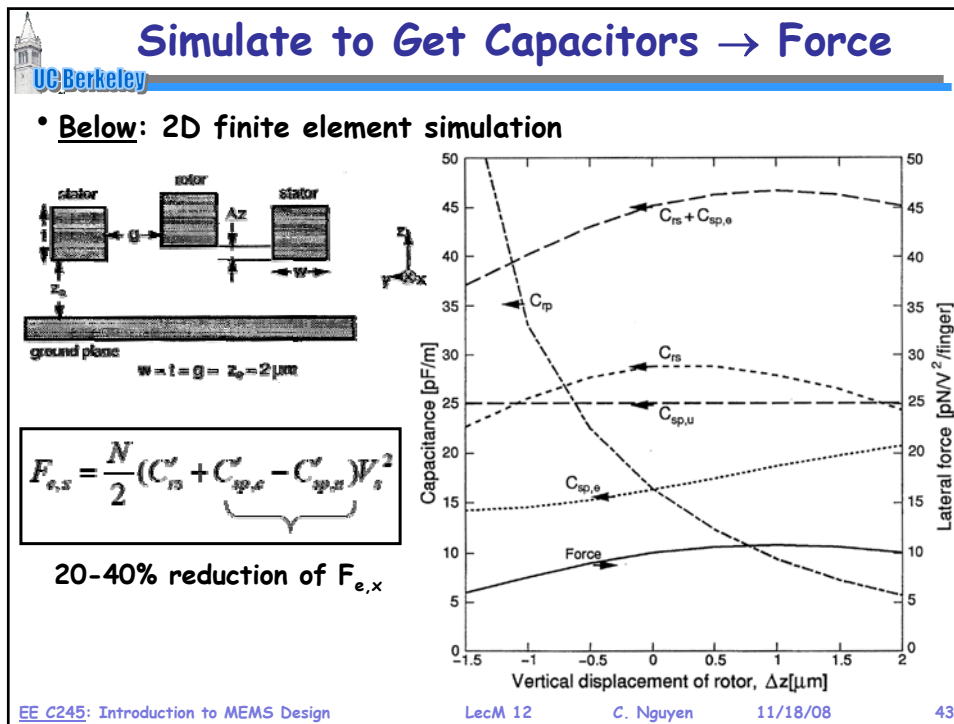
$$F_{e,x} = \frac{\partial W'}{\partial x} = \frac{1}{2} \frac{dC_{sp}}{dx} V_s^2 + \frac{1}{2} \frac{dC_{rp}}{dx} V_r^2 + \frac{1}{2} \frac{dC_{rs}}{dx} (V_s - V_r)^2$$

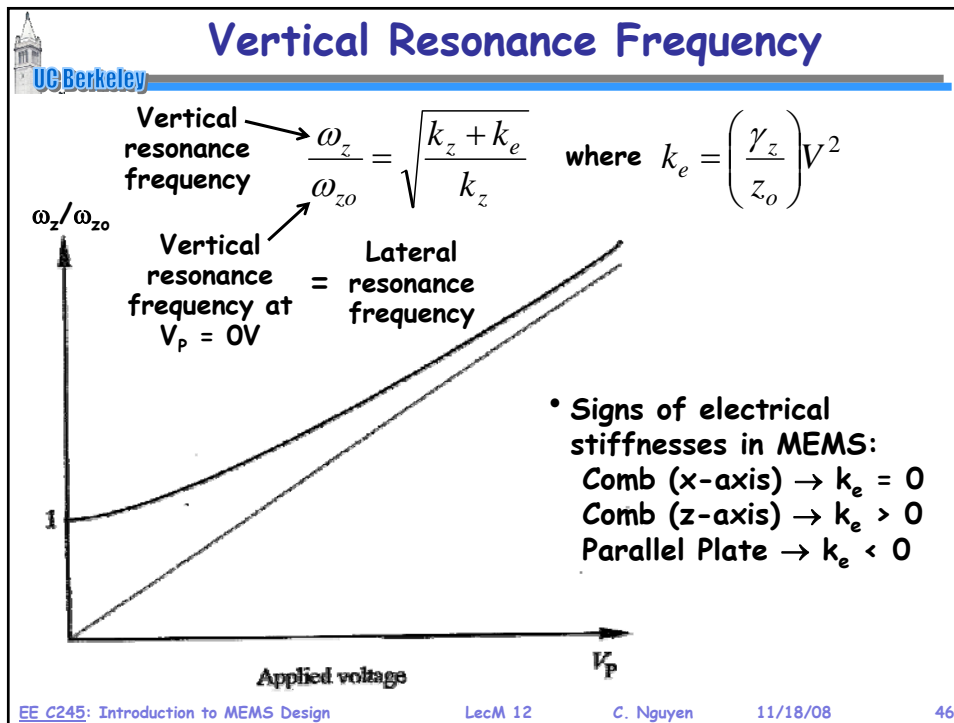
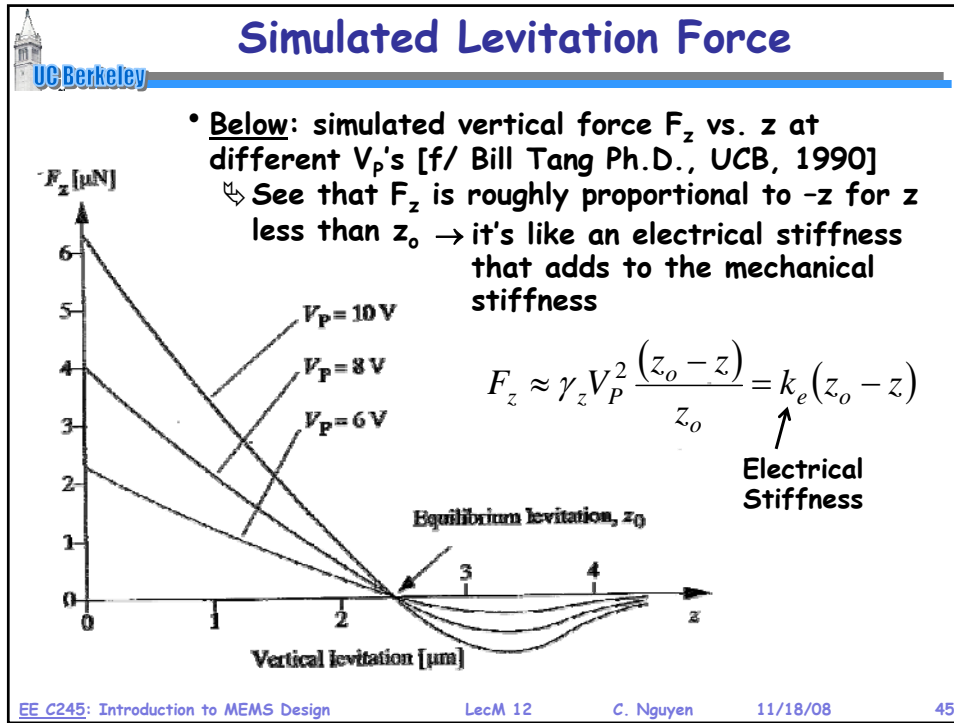
$$F_{e,x} = \frac{N}{2} (C'_{rs} + C'_{sp,e} - C'_{sp,u}) V_s^2$$

(for $V_r = V_p = 0$)

[Gary Fedder, Ph.D., UC Berkeley, 1994]

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Suppressing Levitation

- Pattern ground plane polysilicon into differentially excited electrodes to minimize field lines terminating on top of comb
- Penalty: x-axis force is reduced

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Force of Comb-Drive vs. Parallel-Plate

gap = $g = 1 \mu\text{m}$,
 thickness = $t = 2 \mu\text{m}$
 finger length = $L = 100 \mu\text{m}$
 overlap length $x = 75 \mu\text{m}$

- Comb drive (x-direction)
 $\hookrightarrow V_1 = V_2 = V_s = 1V$

$$F_{e,x} = \frac{\epsilon_0 t}{g} V_s^2$$
- Differential Parallel-Plate (y-direction)
 $\hookrightarrow V_1 = 0V, V_2 = 1V$

$$F_{e,y} = \frac{1}{2} \frac{\epsilon_0 t x}{g^2} V_2^2$$

$$\frac{F_{e,y}}{F_{e,x}} = \frac{\frac{\epsilon_0 t x}{g^2} V_2^2}{\frac{2 \epsilon_0 t}{g} V_s^2} = \frac{1}{2} \frac{x}{g}$$

Parallel-plate generates a much larger force; but at the cost of linearity

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