

EE C245 - ME C218

Introduction to MEMS Design


Fall 2010

Prof. Clark T.-C. Nguyen

Dept. of Electrical Engineering & Computer Sciences
University of California at Berkeley
Berkeley, CA 94720

Lecture Module 15: Gyros, Noise, & MDS

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 1



Lecture Outline

- Reading: Senturia, Chpt. 14, Chpt. 16, Chpt. 21
- Lecture Topics:
 - ↳ Gyroscopes
 - ↳ Gyro Circuit Modeling
 - ↳ Minimum Detectable Signal (MDS)
 - Noise
 - Angle Random Walk (ARW)

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 2

UC Berkeley

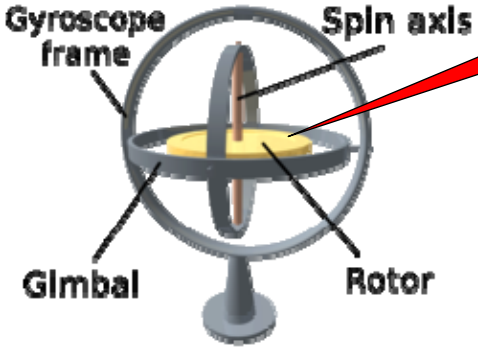
Gyroscopes

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 3


UC Berkeley

Classic Spinning Gyroscope

- A gyroscope measures rotation rate, which then gives orientation → very important, of course, for navigation
- Principle of operation based on conservation of momentum
- Example: classic spinning gyroscope



Rotor will preserve its angular momentum (i.e., will maintain its axis of spin) despite rotation of its gimbal chassis



EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 4

Vibratory Gyroscopes

UC Berkeley

- Generate momentum by vibrating structures
- Again, conservation of momentum leads to mechanisms for measuring rotation rate and orientation
- **Example:** vibrating mass in a rotating frame

Mass at rest y x y' x'

Driven into vibration along the y -axis

$C(t)$

y -displaced mass

Capacitance between mass and frame = constant

Rotate 30°

Get an x' component of motion $C(t_2) > C(t_1)$

$C(t_1)$ $C(t_2)$

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 5

Basic Vibratory Gyroscope Operation

UC Berkeley

Principle of Operation

- Tuning Fork Gyroscope:

Input Rotation $\vec{\Omega}$ z \vec{v} \vec{a}_c

Driven Vibration @ f_0

Coriolis (Sense) Response

Coriolis Torque

x y z

Side View: $\vec{v} = \vec{v}$

not fore on support support = 0

very little anchor dissipation

Top View: $\vec{v} = \vec{v}$

Detect motion out-of-the plane of the tuning fork as rotation!

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 6

Basic Vibratory Gyroscope Operation

Principle of Operation

- Tuning Fork Gyroscope:

The diagram shows a tuning fork structure with a central stem and two prongs. A coordinate system (x, y, z) is centered at the base. The z-axis is vertical, the x-axis is horizontal, and the y-axis is out of the page. The central stem rotates about the z-axis with angular velocity $\vec{\Omega}$. The prongs vibrate in the x-direction with velocity \vec{v} . This vibration is driven at frequency f_o . The Coriolis acceleration \vec{a}_c is shown as a vector pointing in the y-direction. The Coriolis force \vec{F}_c is shown as a vector pointing in the x-direction. The Coriolis displacement \vec{x} is shown as a vector pointing in the x-direction. The Coriolis torque is shown as a vector pointing in the z-direction.

Drive/Sense Response Spectra:

The graph shows Amplitude on the y-axis and frequency ω on the x-axis. The Drive Response curve is a broad peak centered at f_o . The Sense Response curve is a sharp peak centered at f_o . A red dot marks the peak of the Sense Response curve at f_o . A green arrow points from the red dot to the text f_o (@ T_1).

Equations:

$$\vec{a}_c = 2\vec{v} \times \vec{\Omega}$$

$$\vec{F}_c = m\vec{a}_c = \frac{m\vec{a}_c}{k} = \frac{\vec{a}_c}{\omega_r^2}$$

$$\vec{x} = \frac{\vec{F}_c}{k} = \frac{m\vec{a}_c}{k} = \frac{\vec{a}_c}{\omega_r^2}$$

Labels: Drive Velocity, Rotation Rate, Beam Mass, Beam Stiffness (in sense direction), Sense Frequency, Coriolis Acceleration, Coriolis Force, Coriolis Displacement, Coriolis Torque.

Handwritten notes: "same frequency" (red), "sense direction" (green), "drive direction" (green).

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 7

Vibratory Gyroscope Performance

Principle of Operation

- Tuning Fork Gyroscope:

The diagram is identical to the one in the previous slide, showing the tuning fork structure and the coordinate system (x, y, z).

Equations:

$$\vec{x} = \frac{\vec{F}_c}{k} = \frac{m\vec{a}_c}{k} = \frac{\vec{a}_c}{\omega_r^2} \quad \vec{a}_c = 2\vec{v} \times \vec{\Omega}$$

Labels: Beam Mass, Beam Stiffness, Sense Frequency, Driven Velocity.

- To maximize the output signal x , need:
 - ↺ Large sense-axis mass
 - ↺ Small sense-axis stiffness
 - (Above together mean low resonance frequency)
 - ↺ Large drive amplitude for large driven velocity (so use comb-drive)
 - ↺ If can match drive freq. to sense freq., then can amplify output by Q times

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 8

MEMS-Based Gyroscopes

Tuning Fork Gyroscope [Ayazi, GA Tech.]

Tuning Fork Gyroscope [Draper Labs.]

Vibrating Ring Gyroscope [Michigan]

Nuclear Magnetic Resonance Gyro [NIST]

Labels in diagrams: Laser, Polarizer, Rb/Xe Cell, Photodiode, $\dot{\theta}$, 3.2 mm, 1 mm, 1 mm.

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 9

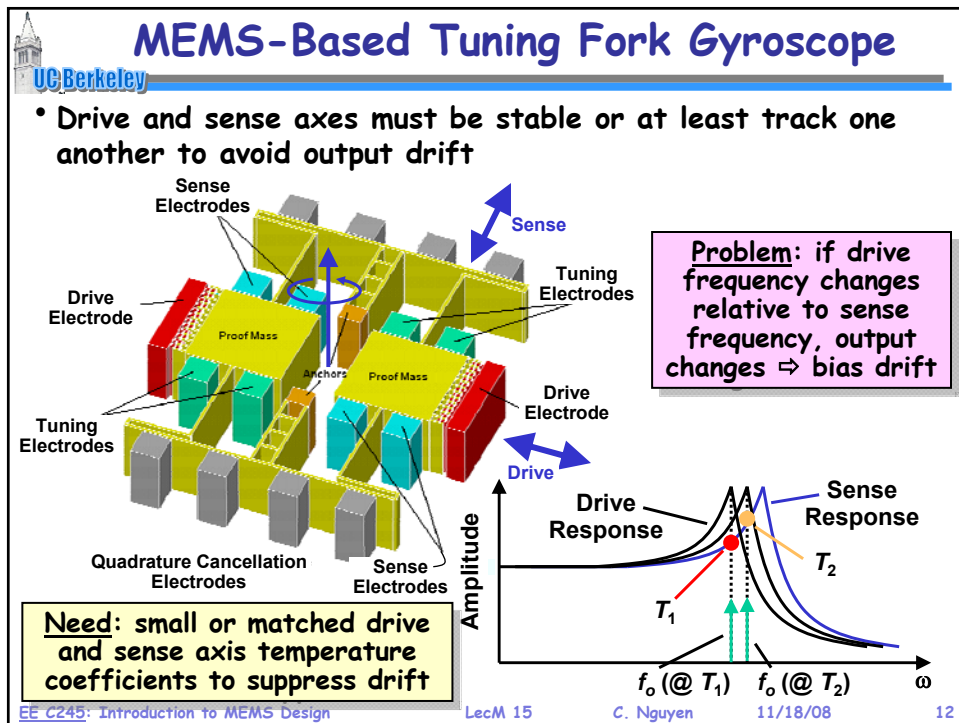
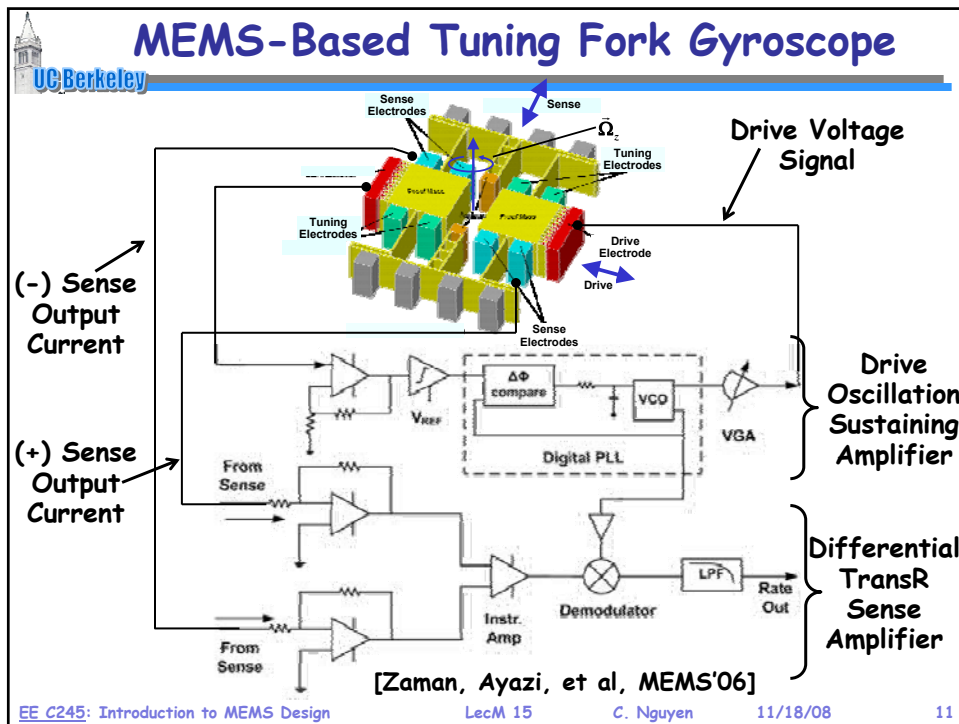
MEMS-Based Tuning Fork Gyroscope

Drive Mode

Sense Mode

- In-plane drive and sense modes pick up z-axis rotations
- Mode-matching for maximum output sensitivity
- From [Zaman, Ayazi, et al, MEMS'06]

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 10



Mode Matching for Higher Resolution

UC Berkeley

- For higher resolution, can try to match drive and sense axis resonance frequencies and benefit from Q amplification

Problem: mismatch between drive and sense frequencies \Rightarrow even larger drift!

Need: small or matched drive and sense axis temperature coefficients to make this work

EE C245: Introduction to MEMS Design
LecM 15
C. Nguyen
11/18/08
13

Issue: Zero Rate Bias Error

UC Berkeley

- Imbalances in the system can lead to zero rate bias error

Mass imbalance \Rightarrow off-axis motion of the proof mass

Drive imbalance \Rightarrow off-axis motion of the proof mass

Output signal in phase with the Coriolis acceleration

Quadrature output signal that can be confused with the Coriolis acceleration

EE C245: Introduction to MEMS Design
LecM 15
C. Nguyen
11/18/08
14

Nuclear Magnetic Res. Gyroscope

UC Berkeley

- The ultimate in miniaturized spinning gyroscopes?
 - ↳ from CSAC, we may now have the technology to do this

-20°
0°
20°

Better if this is a noble gas nucleus (rather than e-), since nuclei are heavier ⇒ less susceptible to B field

-20°
0°
20°

Atoms Aligned Nuclear Spins

Soln: Spin polarize Xe¹²⁹ nuclei by first polarizing e- of Rb⁸⁷ (a la CSAC), then allowing spin exchange

Challenge: suppressing the effects of B field

Laser
Polarizer
Rb/Xe Cell
Photodiode
3.2 mm
1 mm
1 mm
 $\dot{\theta}$

EE C245: Introduction to MEMS Design
LecM 15
C. Nguyen
11/18/08
15

MEMS-Based Tuning Fork Gyroscope

UC Berkeley

Sense Electrodes
Tuning Electrodes
Drive Electrode
Drive
Sense
 $\dot{\Omega}_z$

Drive Voltage Signal

(-) Sense Output Current

(+) Sense Output Current

From Sense

Instr. Amp

Demodulator

Digital PLL

$\Delta\Phi$ compare

VCO

VGA

V_{REF}

Drive Oscillation Sustaining Amplifier

Differential TransR Sense Amplifier

Rate Out

[Zaman, Ayazi, et al, MEMS'06]

EE C245: Introduction to MEMS Design
LecM 15
C. Nguyen
11/18/08
16

UC Berkeley

Determining Sensor Resolution

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 17

UC Berkeley

MEMS-Based Tuning Fork Gyroscope

(-) Sense Output Current

(+) Sense Output Current

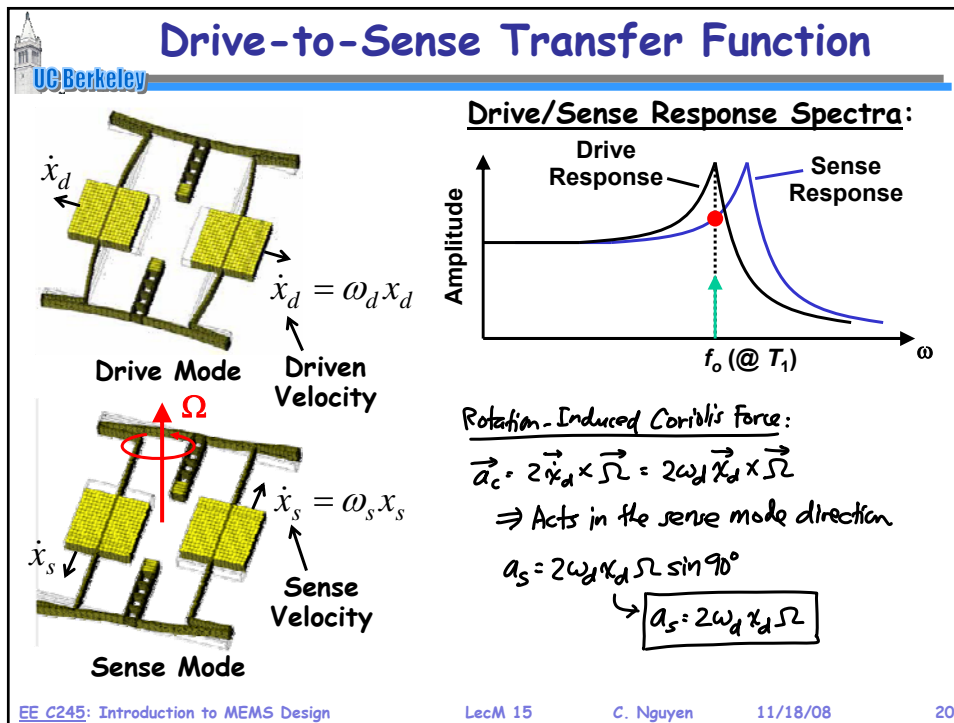
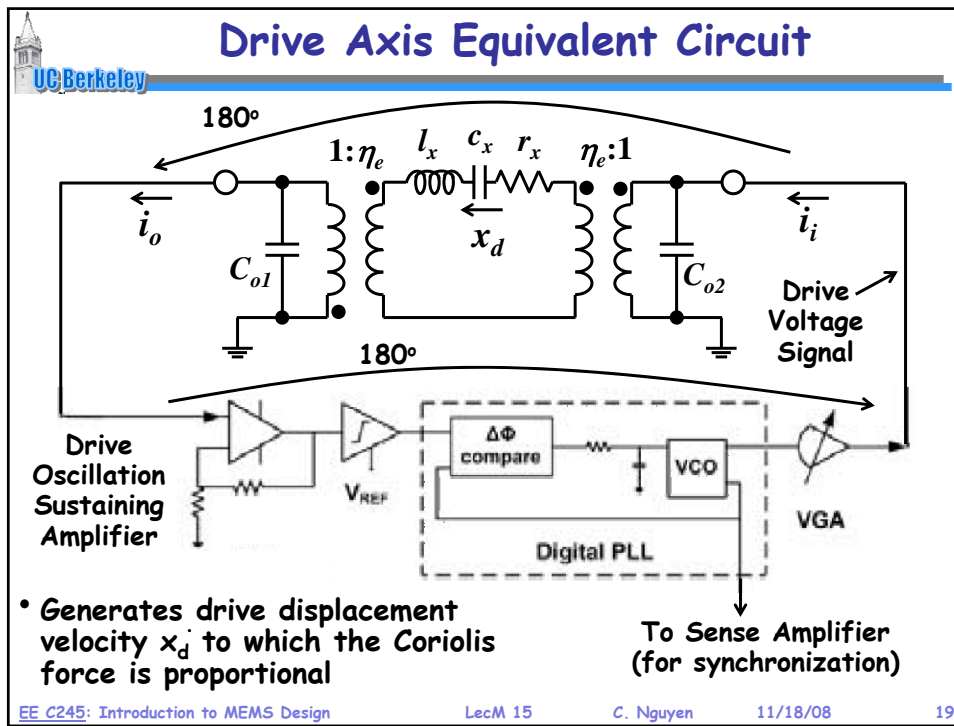
Drive Voltage Signal

Drive Oscillation Sustaining Amplifier

Differential TransR Sense Amplifier

[Zaman, Ayazi, et al, MEMS'06]

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 18



Gyro Readout Equivalent Circuit (for a single tone)

$\vec{F}_c = m\vec{a}_c = m \cdot (2\vec{\dot{x}}_d \times \vec{\Omega})$

Gyro Sense Element Output Circuit **Signal Conditioning Circuit (Transresistance Amplifier)**

- Easiest to analyze if all noise sources are summed at a common node

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 21

Minimum Detectable Signal (MDS)

- **Minimum Detectable Signal (MDS):** Input signal level when the signal-to-noise ratio (SNR) is equal to unity

Sensor **Signal Conditioning Circuit** **Output**

Includes desired output plus noise

- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 22

Move Noise Sources to a Common Point

UC Berkeley

- Move noise sources so that all sum at the input to the amplifier circuit (i.e., at the output of the sense element)
- Then, can compare the output of the sensed signal directly to the noise at this node to get the MDS

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 23

Gyro Readout Equivalent Circuit (for a single time)

UC Berkeley

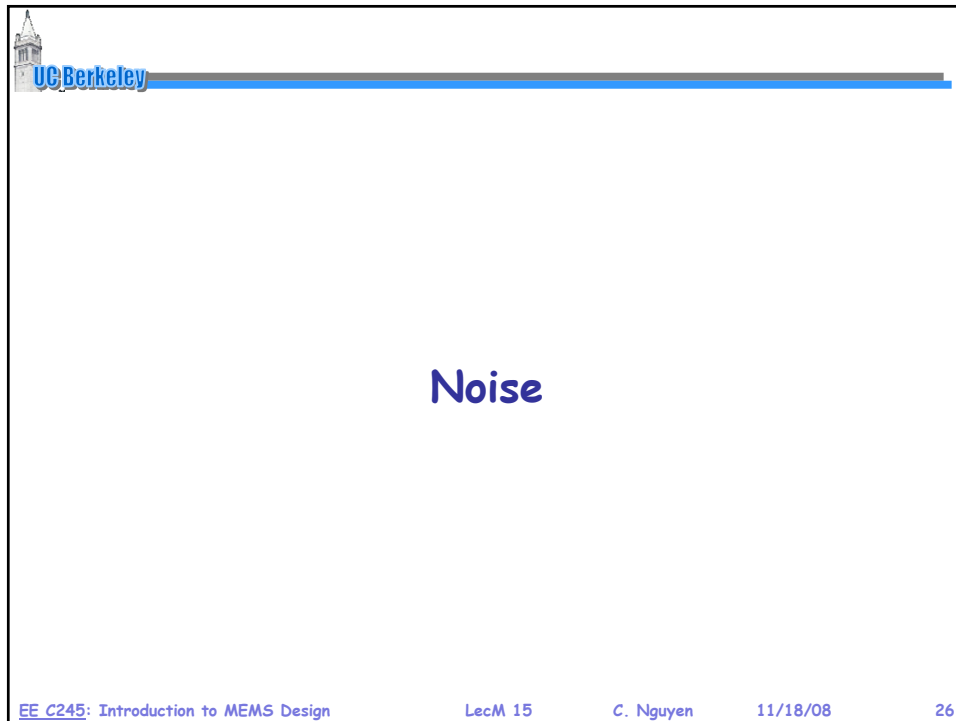
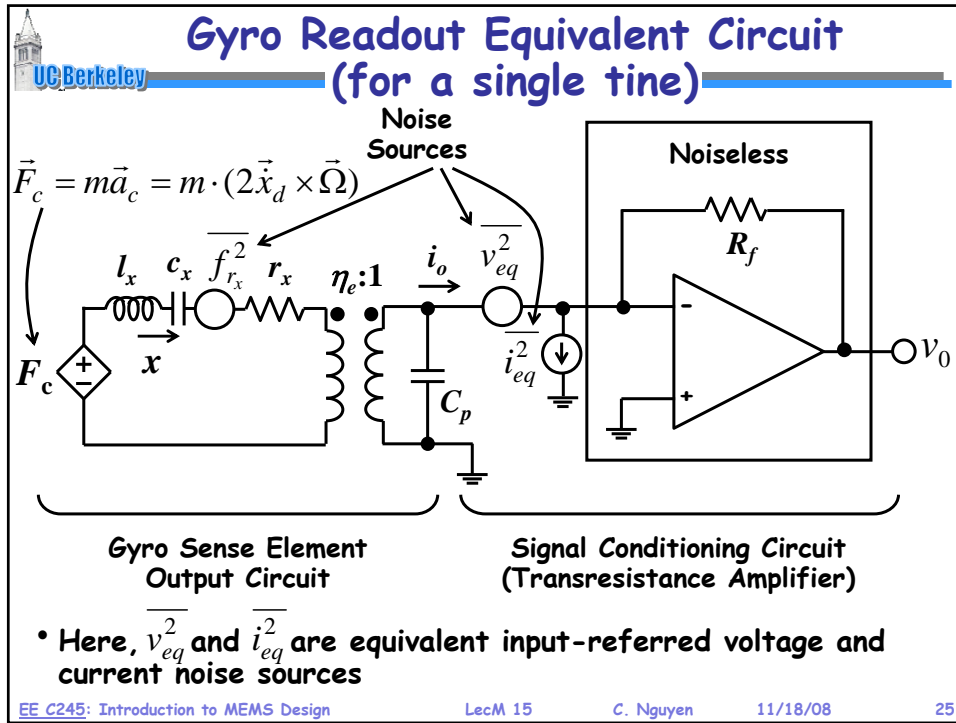
Noise Sources


$$\vec{F}_c = m\vec{a}_c = m \cdot (2\vec{\dot{x}}_d \times \vec{\Omega})$$

Gyro Sense Element Output Circuit Signal Conditioning Circuit (Transresistance Amplifier)

- Easiest to analyze if all noise sources are summed at a common node

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 24

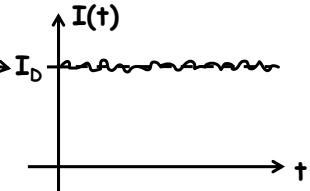




Noise

- **Noise:** Random fluctuation of a given parameter $I(t)$
- In addition, a noise waveform has a zero average value

Avg. value
(e.g. could be DC current)




- We can't handle noise at instantaneous times
- But we can handle some of the averaged effects of random fluctuations by giving noise a power spectral density representation
- Thus, represent noise by its mean-square value:

Let $i(t) = I(t) - I_D$

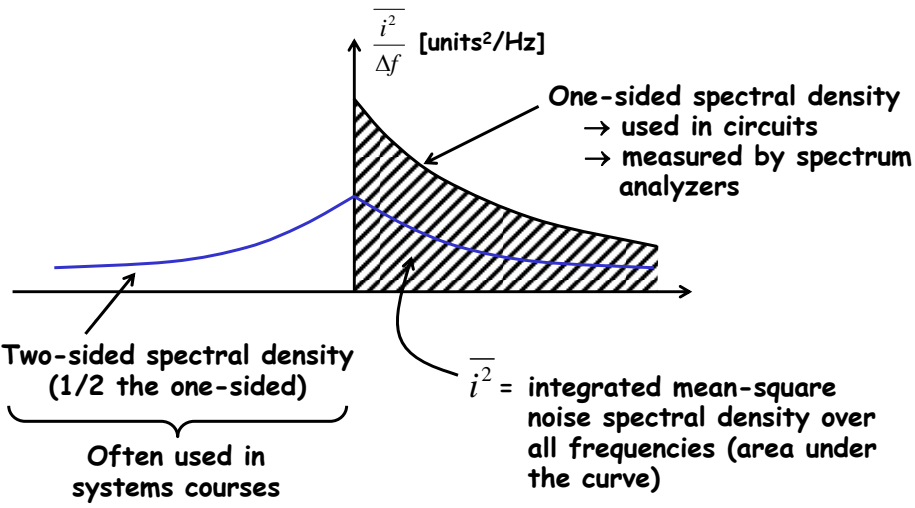
Then $\overline{i^2} = \overline{(I - I_D)^2} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T |I - I_D|^2 dt$

EE C245: Introduction to MEMS Design
LecM 15
C. Nguyen
11/18/08
27



Noise Spectral Density

- We can plot the spectral density of this mean-square value:




One-sided spectral density
 → used in circuits
 → measured by spectrum analyzers

Two-sided spectral density
 (1/2 the one-sided)
 Often used in systems courses

$\overline{i^2} =$ integrated mean-square noise spectral density over all frequencies (area under the curve)

EE C245: Introduction to MEMS Design
LecM 15
C. Nguyen
11/18/08
28

Circuit Noise Calculations



Deterministic

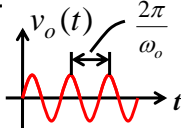
Inputs $v_i(j\omega)$

Outputs $v_o(j\omega)$

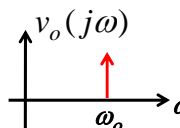
Linear Time-Invariant System $H(j\omega)$

Random $S_i(\omega)$ $S_o(\omega)$

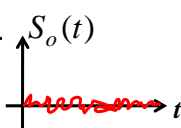
No j → noise has random phase, so j is pointless!



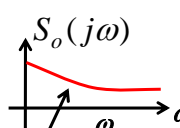
$v_o(t)$



$v_o(j\omega)$



$S_o(t)$



$S_o(j\omega)$

Mean square spectral density

- **Deterministic:** $v_o(j\omega) = H(j\omega)v_i(j\omega)$
- **Random:** $S_o(\omega) = [H(j\omega)H^*(j\omega)]S_i(\omega) = |H(j\omega)|^2 S_i(\omega)$


$$\sqrt{S_o(\omega)} = |H(j\omega)|\sqrt{S_i(\omega)}$$

Root mean square amplitudes

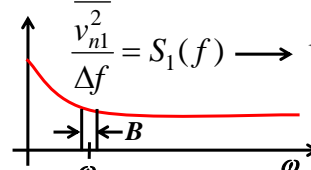
→ How is it we can do this?

EE C245: Introduction to MEMS Design
LecM 15
C. Nguyen
11/18/08
29

Handling Noise Deterministically

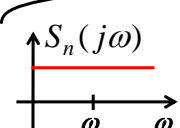


Can do this for noise in a tiny bandwidth (e.g., 1 Hz)

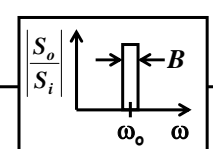


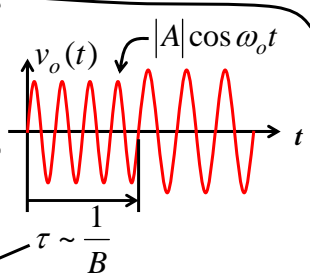
$\frac{v_{n1}^2}{\Delta f} = S_1(f) \rightarrow v_{n1} = \sqrt{S_1(f) \cdot B}$

Can approximate this by a sinusoidal voltage generator (especially for small B, say 1 Hz)



$S_n(j\omega)$





$v_o(t) = |A| \cos \omega_o t$

$\tau \sim \frac{1}{B}$

[This is actually the principle by which oscillators work → oscillators are just noise going through a tiny bandwidth filter]

Why? Neither the amplitude nor the phase of a signal can change appreciably within a time period 1/B.

EE C245: Introduction to MEMS Design
LecM 15
C. Nguyen
11/18/08
30

Systematic Noise Calculation Procedure

General Circuit With Several Noise Sources

- Assume noise sources are uncorrelated
- 1. For $\overline{i_{n1}^2}$ replace w/ a deterministic source of value

$$i_{n1} = \sqrt{\frac{\overline{i_{n1}^2}}{\Delta f}} \cdot (1 \text{ Hz})$$

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 31

Systematic Noise Calculation Procedure

- Calculate $v_{on1}(\omega) = i_{n1}(\omega)H(j\omega)$ (treating it like a deterministic signal)
- Determine $\overline{v_{on1}^2} = \overline{i_{n1}^2} \cdot |H(j\omega)|^2$
- Repeat for each noise source: $\overline{i_{n1}^2}, \overline{v_{n2}^2}, \overline{v_{n3}^2}$
- Add noise power (mean square values)

$$\overline{v_{onTOT}^2} = \overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \dots$$

$$v_{onTOT} = \sqrt{\overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \dots}$$

↑
Total rms value

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 32

UC Berkeley

Determining Sensor Resolution

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 33

UC Berkeley

Example: Gyro MDS Calculation

$$\vec{F}_c = m\vec{a}_c = m \cdot (2\vec{\dot{x}}_d \times \vec{\Omega})$$

- The gyro sense presents a large effective source impedance
 - ↳ Currents are the important variable; voltages are "opened" out
 - ↳ Must compare i_o with the total current noise i_{eqTOT} going into the amplifier circuit

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 34

Example: Gyro MDS Calculation (cont)

UC Berkeley

$$\vec{F}_c = m\vec{a}_c = m \cdot (2\vec{\dot{x}}_d \times \vec{\Omega})$$

• First, find the rotation to i_o transfer function:

$$\dot{x}_s = \frac{\omega_s Q}{k_s} \Theta_s(j\omega_d) F_s = \frac{\omega_s Q}{k_s} \cdot 2\omega_d \kappa_d \Omega m \cdot \Theta(j\omega_d)$$

$[F_s = F_c = 2\omega_d \kappa_d \Omega m]$

$$\dot{x}_s = 2 \frac{\omega_d}{\omega_s} Q \kappa_d \Theta(j\omega_d) \cdot \Omega$$

EE C245: Introduction to MEMS Design
LecM 15
C. Nguyen
11/18/08
35

Example: Gyro MDS Calculation (cont)

UC Berkeley

$$i_o = \eta_e \dot{x}_s = 2 \frac{\omega_d}{\omega_s} Q \kappa_d \eta_e \Theta(j\omega_d) \cdot \Omega \rightarrow i_o = A\Omega$$

$A \triangleq \text{scale factor}$

$$\text{Where } A = 2 \frac{\omega_d}{\omega_s} Q \kappa_d \eta_e \Theta(j\omega_d)$$

When $\Omega = \Omega_{\min} \triangleq \text{MDS}$, $i_o = i_{eqTOT}$ ← input-referred noise current entering the sense amplifier → in pA/√Hz

$$\therefore i_{eqTOT} = A\Omega_{\min} \rightarrow \Omega_{\min} = \frac{i_{eqTOT}}{A} \left(\frac{3600s}{hr} \right) \left(\frac{180^\circ}{\pi} \right) \left[\left(\frac{\%hr}{\sqrt{Hz}} \right) \right]$$

$$\text{Angle Random Walk: ARW} = \frac{1}{60} \Omega_{\min} \left[\frac{\circ}{\sqrt{hr}} \right]$$

↪ Easier to determine directional error as a function of elapsed time.

EE C245: Introduction to MEMS Design
LecM 15
C. Nguyen
11/18/08
36

Example: Gyro MDS Calculation (cont)

UC Berkeley

$\vec{F}_c = m\vec{a}_c = m \cdot (2\vec{\dot{x}}_d \times \vec{\Omega})$

• Now, find the i_{eqTOT} entering the amplifier input:

$$i_{eqTOT}^2 = i_s^2 + i_{eq}^2 \rightarrow i_{eqTOT}^2 = i_s^2 + i_f^2 + i_{ia}^2 + \frac{N_{ia}^2}{R_f^2}$$

$\frac{f_{rx}^2}{\Delta f} = 4kTR_x$

Brownian motion noise of the sense element \rightarrow determined entirely by the noise in $r_x \rightarrow f_{rx}^2$
 \hookrightarrow easiest to convert to an all electrical equiv. ckt.

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 37

Example: Gyro MDS Calculation (cont)

UC Berkeley

Where $L_x = \frac{R_x}{\eta_e}$, $C_x = \eta_e^2 C_x$, $R_x = \frac{r_x}{\eta_e}$

$$\therefore i_s = N_{R_x} \left(\frac{1}{R_x} \right) |H(j\omega_d)|^2 \rightarrow \frac{i_s^2}{\Delta f} = 4kTR_x \left(\frac{1}{R_x^2} \right) |H(j\omega_d)|^2$$

$$\Rightarrow \frac{i_s^2}{\Delta f} = \frac{4kT}{R_x} |H(j\omega_d)|^2$$

Thus:

$$\frac{i_{eqTOT}^2}{\Delta f} = \frac{4kT}{R_x} |H(j\omega_d)|^2 + \frac{4kT}{R_f} + \frac{i_{ia}^2}{\Delta f} + \frac{N_{ia}^2}{\Delta f} \left(\frac{1}{R_f^2} \right)$$

Learn to get there from EE240.
 \hookrightarrow or just get them from a data sheet ...

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 38

LF356 Op Amp Data Sheet

UC Berkeley

LF155/LF156/LF256/LF257/LF355/LF356/LF357 JFET Input Operational Amplifiers

General Description

These are the first monolithic JFET input operational amplifiers to incorporate well matched, high voltage JFETs on the same chip with standard bipolar transistors (BI-FET™ Technology). These amplifiers feature low input bias and offset currents/low offset voltage and offset voltage drift, coupled with offset adjust which does not degrade drift or common-mode rejection. The devices are also designed for high slew rate, wide bandwidth, extremely fast settling time, low voltage and current noise and a low 1/f noise corner.

Common Features

- Logarithmic amplifiers
- Photocell amplifiers
- Sample and Hold circuits
- Low input bias current: 30pA
- Low Input Offset Current: 3pA
- High input impedance: $10^{12}\Omega$
- Low input noise current: $0.01 \text{ pA}/\sqrt{\text{Hz}}$
- High common-mode rejection ratio: 100 dB
- Large dc voltage gain: 106 dB

Features

Advantages

- Replace expensive hybrid and module FET op amps
- Rugged JFETs allow blow-out free handling compared with MOSFET input devices
- Excellent for low noise applications using either high or low source impedance—very low 1/f corner
- Offset adjust does not degrade drift or common-mode rejection as in most monolithic amplifiers
- New output stage allows use of large capacitive loads (5,000 pF) without stability problems
- Internal compensation and large differential input voltage capability

Applications

- Precision high speed integrators
- Fast D/A and A/D converters
- High impedance buffers
- Wideband, low noise, low drift amplifiers

Uncommon Features

	LF155/ LF355	LF156/ LF256/ LF356	LF257/ LF357 ($A_v=5$)	Units
Extremely fast settling time to 0.01%	4	1.5	1.5	μs
Fast slew rate	5	12	50	V/ μs
Wide gain bandwidth	2.5	5	20	MHz
Low input noise voltage	20	12	12	nV/ $\sqrt{\text{Hz}}$

Handwritten notes:

- $\sqrt{\frac{0.2 \text{ pA}}{\Delta f}} = 0.01 \text{ pA}/\sqrt{\text{Hz}}$
- $\sqrt{\frac{12 \text{ nV}}{\Delta f}} = 12 \text{ nV}/\sqrt{\text{Hz}}$

8/08 39

Example ARW Calculation

UC Berkeley

Example Design:

Sensor Element:

- $m = (100\mu\text{m})(100\mu\text{m})(20\mu\text{m})(2300\text{kg}/\text{m}^3) = 4.6 \times 10^{-10} \text{ kg}$
- $\omega_s = 2\pi(15\text{kHz})$
- $\omega_d = 2\pi(10\text{kHz})$
- $k_s = \omega_s^2 m = 4.09 \text{ N/m}$
- $x_d = 20 \mu\text{m}$
- $Q_s = 50,000$
- $V_p = 5\text{V}$
- $h = 20 \mu\text{m}$
- $d = 1 \mu\text{m}$

Sensing Circuitry:

- $R_f = 100\text{k}\Omega$
- $i_{ia} = 0.01 \text{ pA}/\sqrt{\text{Hz}}$
- $v_{ia} = 12 \text{ nV}/\sqrt{\text{Hz}}$

The diagram shows a 3D perspective of a MEMS gyroscope. It features a central proof mass (yellow) with sense electrodes (red) and tuning electrodes (green) around it. Drive electrodes (blue) are also visible. The diagram is annotated with labels for 'Sense Electrodes', 'Proof Mass', 'Tuning Electrodes', and 'Drive Electrodes'. A coordinate system with Ω_z is shown.

8/08 40

Example ARW Calculation (cont)

UC Berkeley

Get rotation rate to output current scale factor:

$$A = 2 \frac{\omega_d}{\omega_s} Q_s \kappa_d \eta_e |\Phi(j\omega_d)| = 2 \left(\frac{10\text{K}}{15\text{K}} \right) (50\text{K}) (20\mu) (5) (2000\epsilon_0) (0.000024) = \underline{2.83 \times 10^{-12} \text{ C}}$$

$$\left[\begin{aligned} \Phi(j\omega_d) &= \frac{(j\omega_d)(\omega_s/Q_s)}{-\omega_d^2 + \frac{j\omega_d\omega_s}{Q_s} + \omega_s^2} = \frac{j(10\text{K})(15\text{K})/(50\text{K})}{(15\text{K})^2 - (10\text{K})^2 + \frac{j(10\text{K})(15\text{K})}{50\text{K}}} = \frac{j(3\text{K})}{1.25 \times 10^8 + j(3\text{K})} \\ \Rightarrow |\Phi(j\omega_d)| &= \frac{3\text{K}}{\sqrt{(1.25 \times 10^8)^2 + (3\text{K})^2}} = 0.000024 \quad \underline{8.854 \times 10^{-8} \text{ F/m}} \end{aligned} \right]$$

$$\left[\begin{aligned} \frac{\partial C}{\partial x} = \frac{C_0}{d} = \frac{\epsilon_0 h \omega_p}{d} = \frac{\epsilon_0 (20\mu)(100\mu)}{(1\mu)^2} = 2000\epsilon_0 \rightarrow \eta_e = V_p \frac{\partial C}{\partial x} = 5(2000\epsilon_0) \\ \underline{8.854 \times 10^{-12} \text{ F/m}} \end{aligned} \right]$$

Assume electrode covers the whole sidewall.

Then, get noise:

$$\frac{\overline{i_{eq}^2}}{\Delta f} = \frac{4kT}{R_x} |\Phi(j\omega_d)|^2 + \frac{4kT}{R_f} + \frac{\overline{i_{ia}^2}}{\Delta f} + \frac{\overline{N_{ie}^2}}{\Delta f} \left(\frac{1}{R_f} \right)$$

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 41

Example ARW Calculation (cont)

UC Berkeley

$$\left[R_x = \frac{\omega_s m}{Q_s \eta_e^2} = \frac{2\pi(15\text{K})(4.6 \times 10^{-10})}{(50\text{K})(8.854 \times 10^{-8})^2} = 110.6 \text{ k}\Omega \right]$$

$$\frac{\overline{i_{eq}^2}}{\Delta f} = \frac{(1.66 \times 10^{-29})}{(110.6\text{K})} (0.000024)^2 + \frac{(1.66 \times 10^{-29})}{1\text{M}} + (0.01\text{p})^2 + \frac{(12\text{n})^2}{(1\text{M})^2}$$

$\rightarrow 8.64 \times 10^{-35} \text{ A}^2/\text{Hz}$ $1.66 \times 10^{-26} \text{ A}^2/\text{Hz}$ $1 \times 10^{-28} \text{ A}^2/\text{Hz}$ $1.44 \times 10^{-28} \text{ A}^2/\text{Hz}$
 sensor element noise insignificant Noise from R_f dominates!

$$\therefore \frac{\overline{i_{eq}^2}}{\Delta f} = 1.68 \times 10^{-26} \text{ A}^2/\text{Hz} \rightarrow i_{eq}^2 = \sqrt{\frac{\overline{i_{eq}^2}}{\Delta f}} = 1.30 \times 10^{-13} \text{ A}/\sqrt{\text{Hz}}$$

$$\therefore \Omega_{\min} = \frac{i_{eq}^2}{A} \left(\frac{3600\text{s}}{\text{hr}} \right) \left(\frac{180^\circ}{\pi} \right) = \frac{1.30 \times 10^{-13}}{2.83 \times 10^{-12}} (3600) \left(\frac{180}{\pi} \right) = \underline{9448 (\%/\text{hr})/\sqrt{\text{Hz}}}$$

And finally:

$$\text{ARW} = \frac{1}{60} \Omega_{\min} = \frac{1}{60} (9448) = \underline{157 \%/\text{hr}} = \text{ARW} \Rightarrow \text{Almost turned around in 1 hour!}$$

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 42

What if $\omega_d = \omega_s$?

UC Berkeley

If $\omega_d = \omega_s = 15\text{kHz}$, then $|\Theta(j\omega_d)| = 1$ and

$$A = 2 \frac{\omega_d}{\omega_s} Q_s X_d \eta_e |\Theta(j\omega_d)| = 2 Q_s X_d \eta_e = 2(50k)(20\mu)(5)(2000\epsilon_0) = 1.77 \times 10^{-7} \text{C}$$

$$\frac{\dot{i}_{eqTOT}^2}{\Delta f} = \frac{(1.66 \times 10^{-29})^2}{(110.6k)^2} + \frac{(1.66 \times 10^{-29})^2}{1M} + (0.01p)^2 + \frac{(12n)^2}{(1M)^2}$$

$\swarrow 1.51 \times 10^{-25} \text{A}^2/\text{Hz}$
 $\swarrow 1.66 \times 10^{-26} \text{A}^2/\text{Hz}$
 $\swarrow 1 \times 10^{-28} \text{A}^2/\text{Hz}$
 $\swarrow 1.44 \times 10^{-28} \text{A}^2/\text{Hz}$

Now, the sensor element dominates!

$$\therefore \frac{\dot{i}_{eqTOT}^2}{\Delta f} = 1.67 \times 10^{-25} \text{A}^2/\text{Hz} \rightarrow \dot{i}_{eqTOT} = \sqrt{\frac{\dot{i}_{eqTOT}^2}{\Delta f}} = 4.08 \times 10^{-13} \text{A}/\sqrt{\text{Hz}}$$

$$\therefore \Sigma_{min} = \frac{\dot{i}_{eqTOT}}{A} \left(\frac{3600s}{hr} \right) \left(\frac{180^\circ}{\pi} \right) = \frac{4.08 \times 10^{-13}}{1.77 \times 10^{-7}} (3600) \left(\frac{180}{\pi} \right) = 0.476 (\%/hr)/\sqrt{\text{Hz}}$$

And finally:

$$ARW = \frac{1}{60} \Sigma_{min} = \frac{1}{60} (0.476) = 0.0079 \%/hr = ARW \Rightarrow \text{Navigation grade!}$$

EE C245: Introduction to MEMS Design LecM 15 C. Nguyen 11/18/08 43