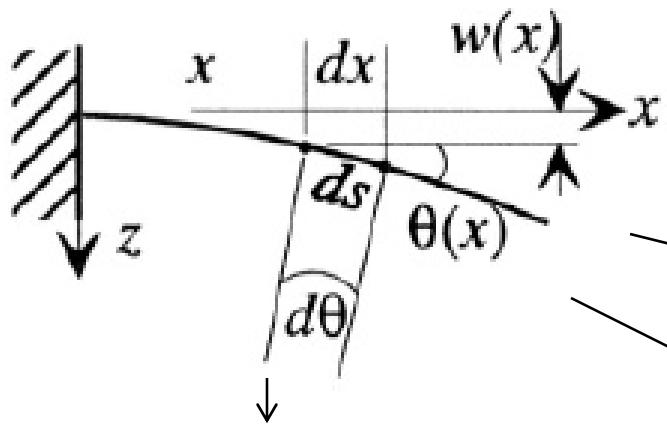


# derivation of beam bending equation



new segment length  $ds$

$$ds = \frac{dx}{\cos \theta}$$

small angle  
approximation

$$ds \approx dx$$

$w(x)$  – neutral axis as a function of position along the original beam  $x$

slope of  $w(x)$

$$\frac{dw}{dx} = \tan \theta$$

radius of curvature  $\rho$

$$ds = \rho d\theta$$

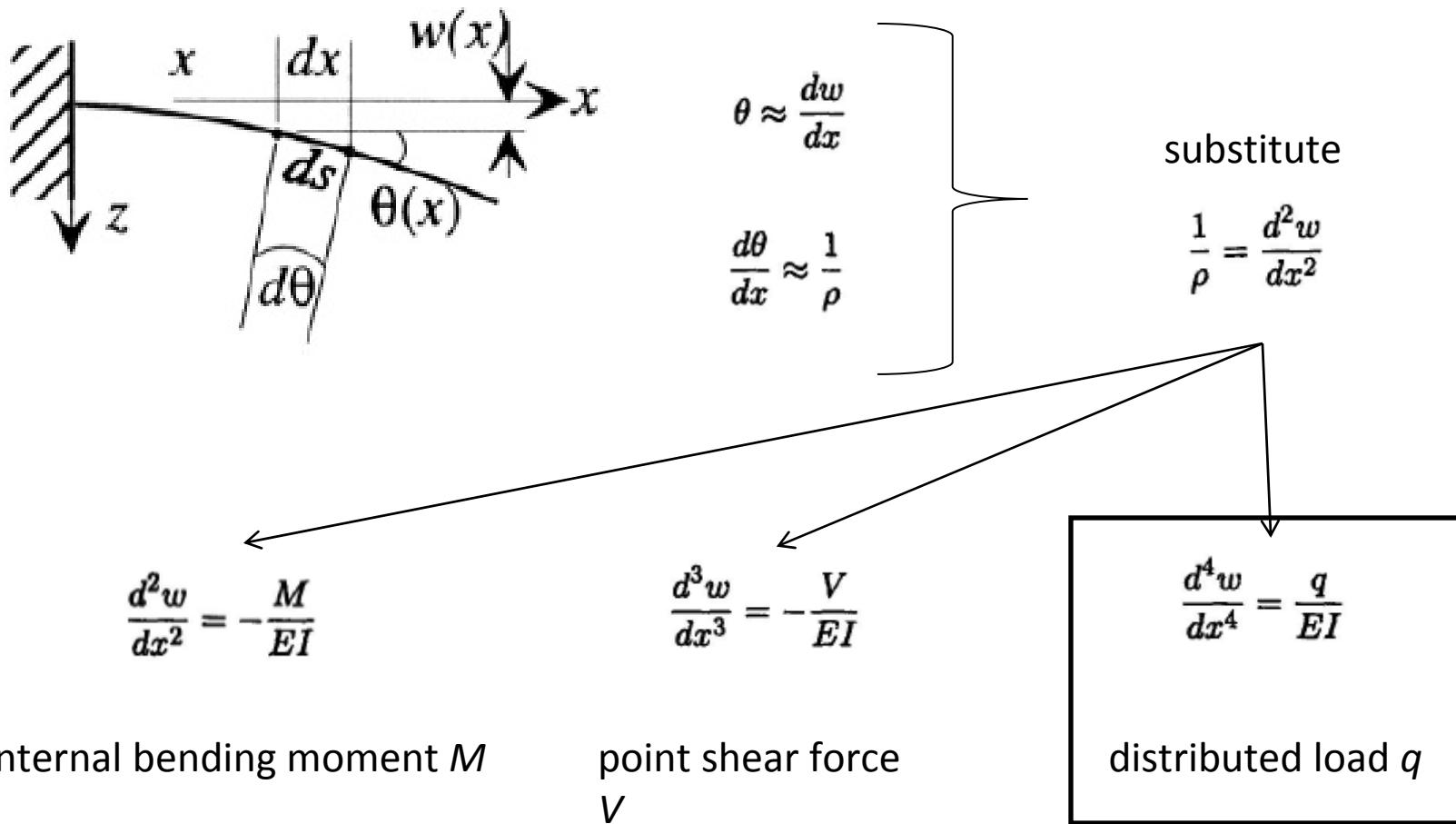


$$\theta \approx \frac{dw}{dx}$$



$$\frac{d\theta}{dx} \approx \frac{1}{\rho}$$

# derivation of beam bending equation



internal bending moment  $M$

point shear force  
 $V$

distributed load  $q$

$$\frac{d^4 w}{dx^4} = \frac{q}{EI}$$



how to solve?

- Laplace or Fourier transforms
- eigenfunction expansion
- guess and check

$$w = A + Bx + Cx^2 + Dx^3 + Ex^4$$



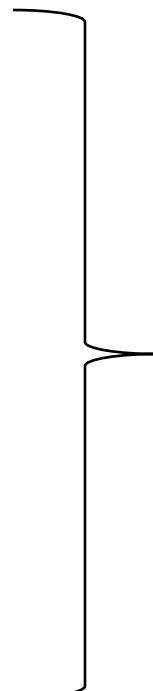
trial solution

$$\frac{dw}{dx} = B + 2Cx + 3Dx^2 + 4Ex^3$$

$$\frac{d^2 w}{dx^2} = 2C + 6Dx + 12Ex^2$$

$$\frac{d^3 w}{dx^3} = 6D + 24Ex$$

$$\frac{d^4 w}{dx^4} = 24E$$



boundary conditions

$$P = \frac{EI}{M}$$

$$\hookrightarrow \boxed{I = \frac{PM}{E}}$$