Multivariate Control and Model-Based SPC

T^2, evolutionary operation, regression chart.

Multivariate Control

Often, many variables must be controlled at the same time. Controlling p *independent* parameters with parallel charts:

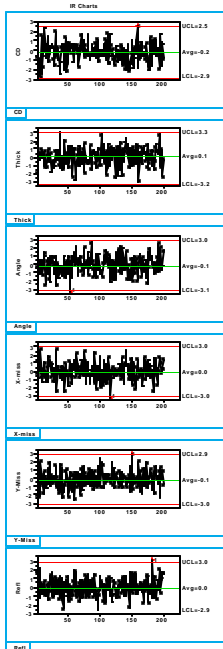$$\alpha' = 1 - (1 - \alpha)^p$$
$$P\{\text{all in control}\} = (1 - \alpha)^p$$

If the parameters are *correlated*, the type I (false alarms) and type II (missed alarms) rates change.
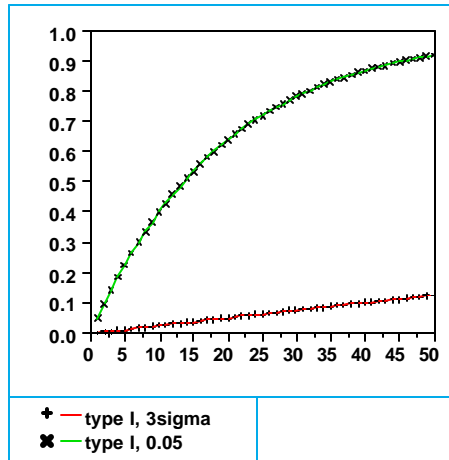
We need is a single comparison test for many variables. In one dimension, this test is based on the student t statistic:

$$t = \frac{(\overline{x} - \mu_0)}{s_{\overline{x}}} = \frac{(\overline{x} - \mu_0)}{\sqrt{s^2/n}} \sim t(n-1)$$

Monitoring Multiple Un-correlated Variables

Type I Error vs Number of Variables

+ — type I, 3sigma
✖ — type I, 0.05

Multivariate Control (cont.)

To compare p mean values to an equal number of targets we use the T^2 statistic:

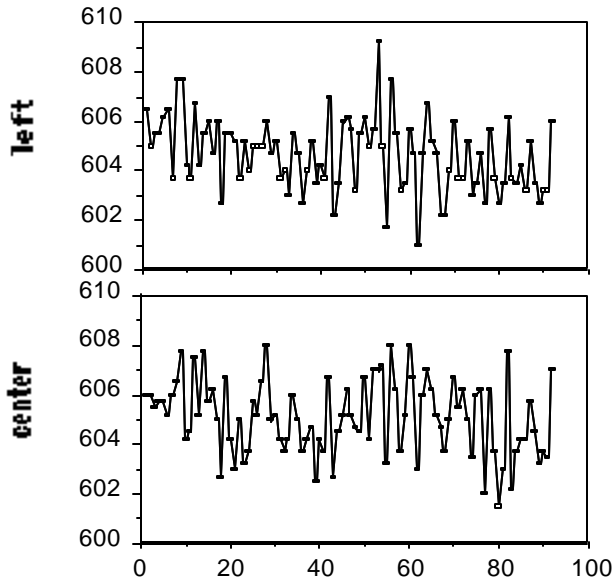$$T^2 = n \, (\overline{\mathbf{x}} - \overline{\overline{\mathbf{x}}})' \, \mathbf{S}^{-1} \, (\overline{\mathbf{x}} - \overline{\overline{\mathbf{x}}})$$

$$T^2_{\alpha, \, p, \, n-1} = \frac{p(n-1)}{n-p} F_{\alpha, \, p, \, n-p}$$

$$\text{with} \quad \overline{\mathbf{x}} = \begin{bmatrix} \overline{x}_1 \\ \overline{x}_2 \\ \dots \\ \overline{x}_p \end{bmatrix}, \quad \overline{\overline{\mathbf{x}}} = \begin{bmatrix} \overline{\overline{x}}_1 \\ \overline{\overline{x}}_2 \\ \dots \\ \overline{\overline{x}}_p \end{bmatrix}$$

S is the covariance matrix, $\overline{x}$ are the means for the last sample and $\overline{\overline{x}}$ the global means. We get an alarm when the T^2 exceeds a critical value (set by the F-statistic).
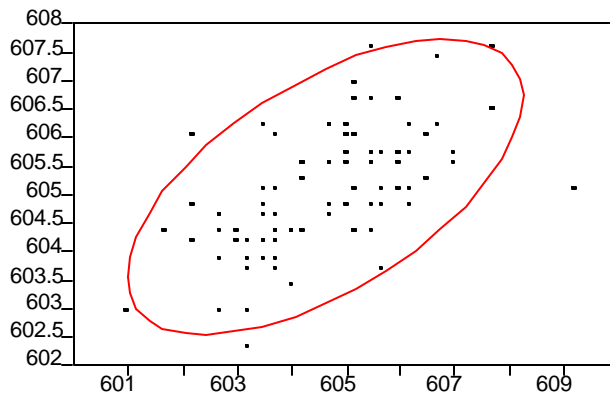
Example: Center and left temps are correlated

Examining the two Variables Together

Correlations

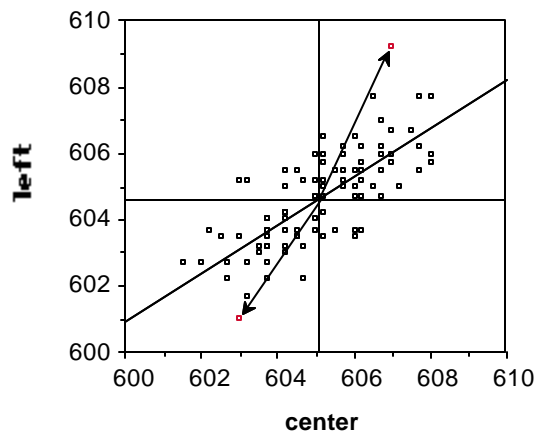| Variable	Cent Temp	Left Temp
Cent Temp	1.0000	0.6094
Left Temp	0.6094	1.0000

Example (cont.)

Since left and center are correlated, with estimated $\sigma = 1.15$, $\rho = 0.61$, their deviation from the target 605 can be determined by a single plot:
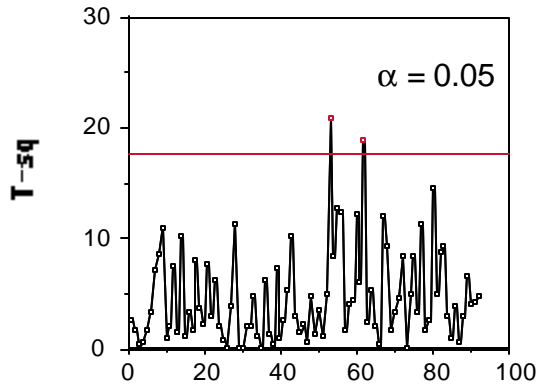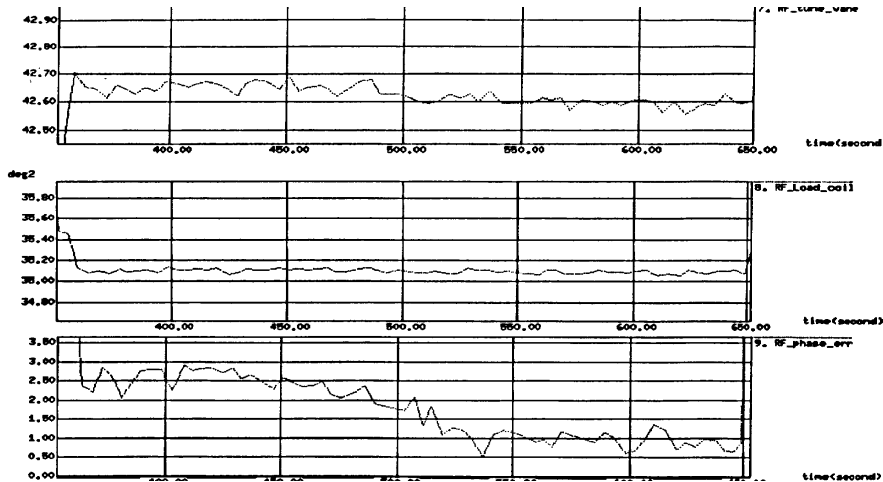
Example (cont.)

For two parameters, another graphical representation is possible:

Example - Multivariate Control of Plasma Etch
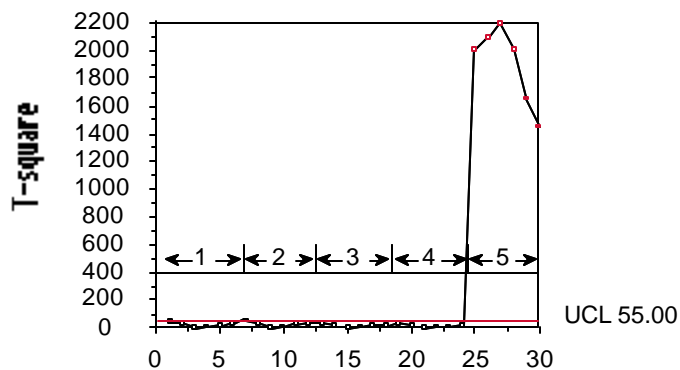
Five strongly correlated parameters* can be collected during the process:

* Tune vane, load coil, phase error, plasma imp. and peak-to-peak voltage.

Example - Multivariate SPC of Plasma Etch (cont.)

The first 24 samples were recorded during the etching of 4 "clean" wafers. The last 6 are out of control and they were recorded during the etching of a "dirty" wafer.

Evolutionary Operation - An SPC/DOE Application

A process can be optimized on-line, by inducing small changes and accepting the ones that improve its quality.

EVOP can be seen as an on-line application of designed experiments.

Example: Assume a two-parameter process:

$$y = f(x_1, x_2) + e$$

and assume the following approximate model:

$$y - a x_1 + b x_2 + c x_1 x_2$$

If we knew a, b, and c, we would know how to change the process in order to bring y closer to the specifications.

Of course this model will only be applicable for a <u>narrow range</u> of the input parameters.
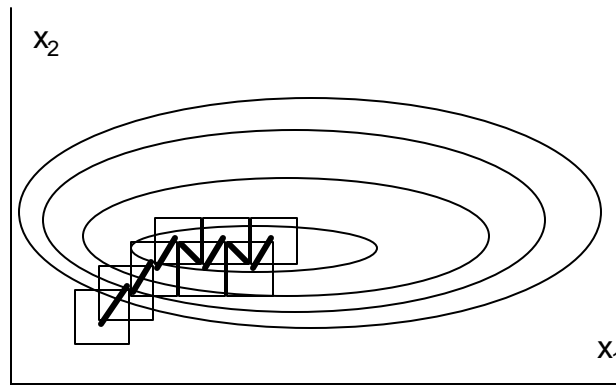
Evolutionary Operation (cont.)

design a 2^2 factorial experiment
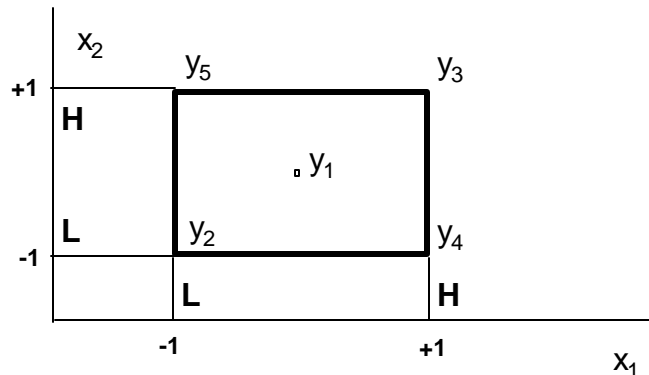
(Note that x_1 and x_2 are scaled so that they take the values -1, +1, at the edges of the experiment).

Evolutionary Operation (cont.)

Once the effects are known, choose the best corner of the
box and start a new experiment around it:

The process terminates when I find a box whose corners are
no better than its center.

Evolutionary Operation (cont.)

The values of a, b and c (or the respective "effects" and
"interactions") can be estimated:

$$a = \text{Eff}_{x1} = \frac{1}{2}[(y_3+y_4) - (y_2+y_5)]$$
$$b = \text{Eff}_{x2} = \frac{1}{2}[(y_3+y_5) - (y_2+y_4)]$$
$$c = \text{Int}_{x1x2} = \frac{1}{2}[y_2+y_3 - (y_4+y_5)]$$

This calculation is repeated for n-cycles until one of the
effects emerges as a significant factor.

Evolutionary Operation (cont.)

To decide whether an effect is significant, we need a good estimate of the <u>process sigma</u>.

The sigma of the process can be estimated from the difference of the last average and the new value at each of the experimental points. This value is distributed as:

$$N\left(0, \sigma^2 \frac{n}{n-1}\right)$$

The 95% confidence interval of each effect is:

$$\pm \left(\frac{2}{\sqrt{n}}s\right)$$

and of the change-in-mean effect is:

$$\pm \left(\frac{1.78}{\sqrt{n}}s\right)$$

Example - Use EVOP on a cleaning solution

The yield from the first 4 cycles of a chem. process is shown below. The variables are % conc. (x_1) at 30 (L), 31 (M), 32 (H) and temp. (x_2) at 140 (L), 142 (M), 144 F (H).

Cycle	Y_1 M-M	Y_2 L-L	Y_3 H-H	Y_4 H-L	Y_5 L-H
1	60.7	69.8	60.2	64.2	57.5
2	69.1	62.8	62.5	64.6	58.3
3	66.6	69.1	69.0	62.3	61.1
4	60.5	69.8	64.5	61.0	60.1
avg Y	64.2	67.9	64.1	63.0	59.3

$$\text{conc. effect} = \frac{1}{2}[(\bar{y}_3+\bar{y}_4) - (\bar{y}_2+\bar{y}_5)] = -0.025$$

$$\text{temp. effect} = \frac{1}{2}[(\bar{y}_3+\bar{y}_5) - (\bar{y}_2+\bar{y}_4)] = -3.800$$

$$\text{interaction} = \frac{1}{2}[\bar{y}_2+\bar{y}_3 - (\bar{y}_4+\bar{y}_5)] = 4.825$$

$$\text{chng in mean} = \frac{1}{5}[\bar{y}_2+\bar{y}_3+\bar{y}_4+\bar{y}_5 - 4\bar{y}_1] = -0.540$$

Example - EVOP on a cleaning solution (cont.)

We use the *range* of consecutive differences in order to estimate the sigma of the process:

$$\text{Take range of } (\ y_i^j - \overline{y}_i^{j-1}) \text{ for i = 1,2,3,4,5}$$
$$\text{ave rage for j = 2,3,4} \quad \overline{R}_D = 7.53$$
$$\text{and}$$
$$\frac{\overline{R}_D}{d_2} = \widehat{\sigma}\sqrt{n/(n\text{-}1)}, \text{ i.e. } \widehat{\sigma} = 2.787$$
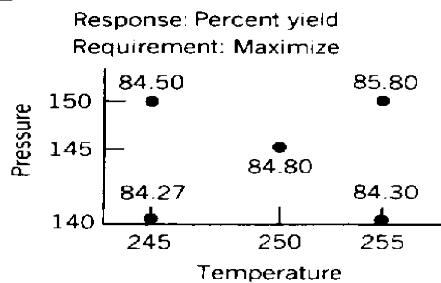
The 95% confidence limits for concentration, temperature and their interaction are:

$$+/-\ 2\,\sigma\,/\sqrt{n} = +/-\ 2.787$$

So, temperature and interaction are significant. Their signs dictate moving to point 2 (L-L).

Lecture 15: Multivariate and Model-based SPC

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EVOP Monitoring in the Fab

EVOP information board—cycle three

Response: Percent yield
Requirement: Maximize

Effects with 95% Error Limits	Error Limits for Averages: ± 0.67	
Temperature	0.67	± 0.67
Pressure	0.87	± 0.67
$T \times P$	0.64	± 0.67
Change in mean	0.07	± 0.60

Standard deviation: 0.58

Lecture 15: Multivariate and Model-based SPC

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Regression Chart - Model Based SPC

In typical SPC, we try to establish that certain process responses stay on target.

What happens if there is one assignable cause that we know and we can quantify?

If, for example, the deposition rate of poly is a function of the time since the last tube cleaning, it will never be "in control".

In cases like this, we build a <u>regression</u> model of the response vs the known effect, and we try to establish that the regression model remains valid throughout the operation.

Limits around the regression line are set according to the *prediction error* of the model.

A t-statistic is used to *update* the model whenever necessary.

Regression chart (cont.)

Polysilicon Deposition Rate
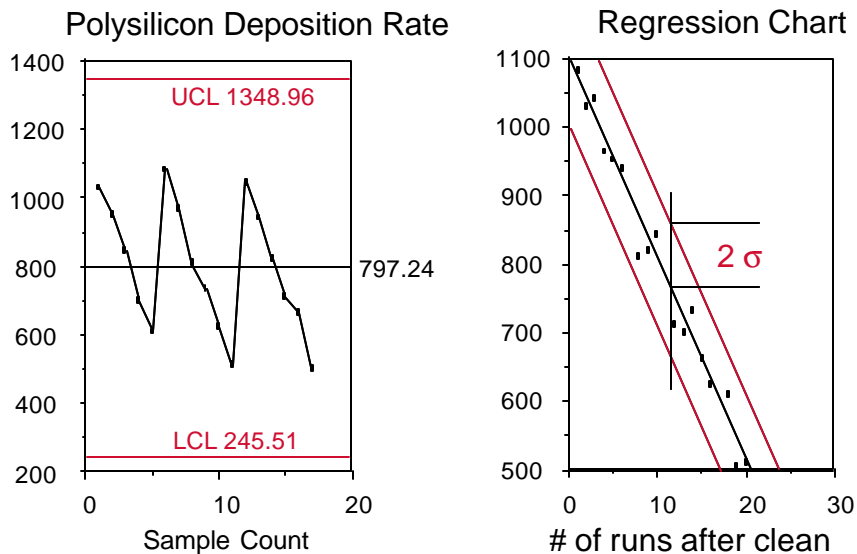
Regression Chart

Regression Chart (cont.)

- The regression chart can be generalized for complex equipment models.

- An empirical model is built to describe the changing aspects of the process.

- The difference between prediction and observation can be used as the control statistic

- If the control statistic becomes consistently different than zero, its value can be used to update the model.

Model Test and Adaptation

LPCVD Model

$\ln(Ro) = A + B \ln(P) + C(1/T) + D(1/Q)$
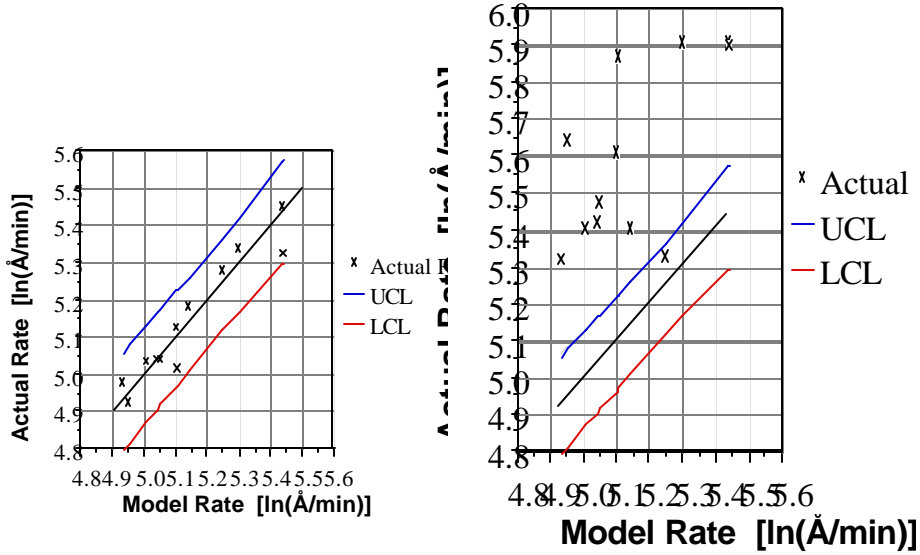
After substitution, the equation used is:

$Y = A + Bx_B + Cx_C + Dx_D$
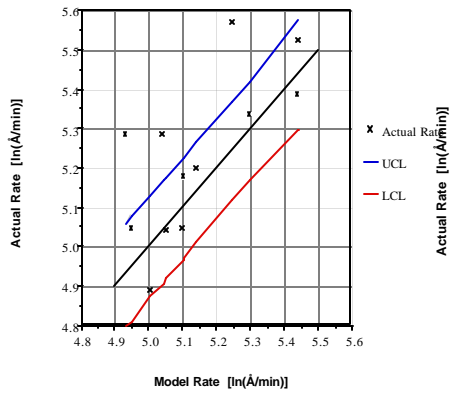
Control Limits: $Y +/- (t * s)$

Cumulative $t = \sum_{i=1}^{n} \frac{(Y_i - y_i)}{s_y}$

Regression Chart (Example)
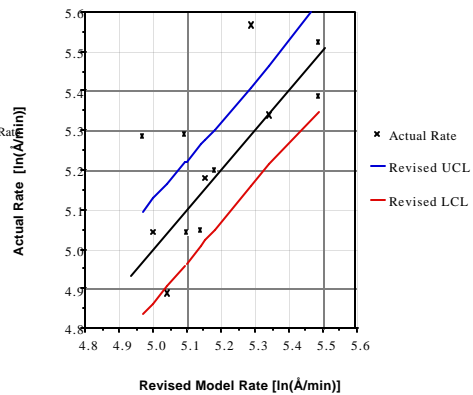
Regression Chart (Example)

Outdated Model   Updated Model

Summary so far...

As we move from classical, human operator oriented techniques, to more automated CIM based approaches:

- We need to increase sensitivity (reduce type II error), without increasing type I error. (CUSUM, EWMA).

- We need to distinguish between abrupt and gradual changes. (Choice of EWMA shape).

- We need to accommodate multiple sensor readings (T^2 chart).

- We need to accommodate multiple recipes and products in each process (EVOP, model-based SPC).