Two-Level Factorials

Treatments and Blocks
Two-Way Analysis of Variance - MANOVA
The Importance of Transformation
Two-Level Factorials

What is a factorial?

- So far, we analyzed data based on the type of treatment they received.
- This analysis is known as "one-way" analysis of variance.
- Now we discuss data that can be classified in more than one "ways". Each of these ways is known as a "factor".
- In the beginning we will call the first classification a "treatment" and the second classification a "block".

Randomized Block Designs

- In general we "block" the effect that we want to eliminate so that we can see the effects of the "treatment".

		treatment 1	2	⋯	t	⋯	k	block average
block	1	y_{11}	y_{21}	⋯	y_{t1}	⋯	y_{k1}	
	2	y_{12}	y_{22}	⋯	y_{t2}	⋯	y_{k2}	
	⋮	⋮	⋮		⋮		⋮	⋮
	i	y_{1i}	y_{2i}	⋯	y_{ti}	⋯	y_{ki}	$\bar{y}_i$
	⋮	⋮	⋮		⋮		⋮	⋮
	n	y_{1n}	y_{2n}	⋯	y_{tn}	⋯	y_{kn}	
treatment average				⋯	$\bar{y}_t$	⋯		$\bar{y}$ = grand average

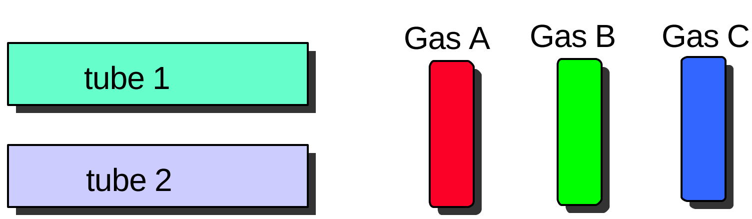
$$y_{ti} = \mu + \beta_i + \tau_t + \varepsilon_{ti}$$

$$y_{ti} = \bar{y} + (\bar{y}_i - \bar{y}) + (\bar{y}_t - \bar{y}) + (y_{ti} - \bar{y}_t - \bar{y}_i + \bar{y})$$

Example: Particle Contamination Study

- We have two LPCVD tubes and three gas suppliers. We are interested in finding out if the choice of the gas supplier makes any difference in terms of average particle counts on the wafers.

- Experiment: we run three full loads on each tube, one for each gas. We report the average particle adders for each load (excluding the first wafer in the boat…)



A Simple Two-Way ANOVA for particles...

		Treatment (Gas) A	B	C	
Block (Tube)	1	7	36	2	*15*
	2	13	44	18	*25*
		10	*40*	*10*	*20*

Decompose according to the equation:

$$y_{ti} = \mu + \beta_i + \tau_t + \varepsilon_{ti}$$

$$y_{ti} = \bar{y} + (\bar{y}_i - \bar{y}) + (\bar{y}_t - \bar{y}) + (y_{ti} - \bar{y}_t - \bar{y}_i + \bar{y})$$

7	36	2
13	44	18

20	20	20
20	20	20

-5	-5	-5
5	5	5

-10	20	-10
-10	20	-10

2	1	-3
-2	-1	3

$$S = S_A + S_B + S_T + S_R$$

Sum Sq 3,778 =

DFs 6 =

The Two-Way ANOVA

Decompose particle counts to global average, deviations from that average due to machine choice and gas source choice. What remains is the **residual**.

$$y_{ti} = \mu + \beta_i + \tau_t + \boxed{\varepsilon_{ti}}$$ **moments**

$$y_{ti} = \bar{y} + (\bar{y}_i - \bar{y}) + (\bar{y}_t - \bar{y}) + \boxed{(y_{ti} - \bar{y}_t - \bar{y}_i + \bar{y})}$$ **estimates**

Two-Way ANOVA Table

Source of Variance	Sum of sq	DFs	Mean sq
Average	S_A	1	
bet. blocks	S_B	n-1	s_B^2
bet. treatm	S_T	k-1	s_T^2
residuals	S_R	(n-1)(k-1)	s_R^2
total	S	nk	

Expected values: s^2_B : $\sigma^2 + k \sum \beta^2_i \;/ (n-1)$

s^2_T : $\sigma^2 + n \sum \tau^2_t \;/ (k-1)$

Assumptions:
- additivity
- IIND of residuals

MANOVA for our Example

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	3	1350.0000	450.000	32.1429
Error	2	28.0000	14.000	Prob > F
C Total	5	1378.0000		0.0303

Effect Test

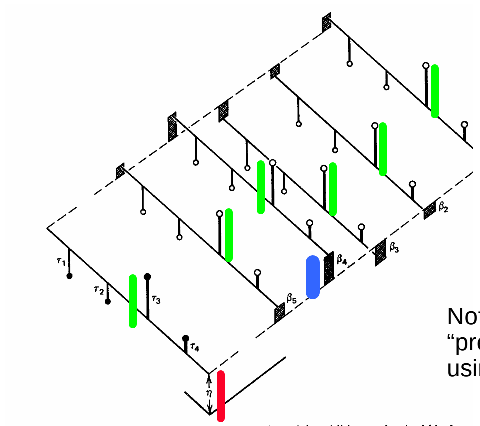
Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Block	1	1	150.0000	10.7143	0.0820
Treatment	2	2	1200.0000	42.8571	0.0228

The Model of the two-way ANOVA

$$y_{ti} = \mu + \beta_i + \tau_t + \varepsilon_{ti}$$

$$\hat{y}_{ti} = \mu + \beta_i + \tau_t$$

This model assumes additivity as well as IIND residuals:



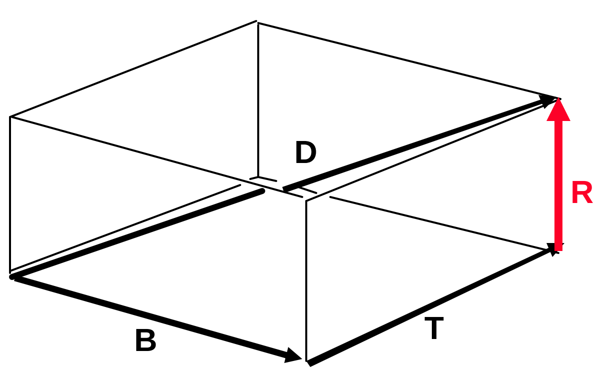
Note that this model "predicts" 20 values using 4+5+1=10 parameters.

Geometric Interpretation of the two-way ANOVA

D = **Y** - **A**

D = **B** + **T** + **R**

Easy to prove that **R** ⊥ **T** and **A** ⊥ **R**.



A Two-Way Factorial Design

- When both effects are of interest, then the same arrangement is called a factorial design.
- The only addition to our model is the interaction term.

$$y_{tij} = \mu_{ti} + \varepsilon_{tij}$$

$$\mu_{ti} = \mu + \tau_t + \beta_i + \boxed{\omega_{ti}}$$

moments

$$y_{tij} = \bar{y}_{ti} + (y_{tij} - \bar{y}_{ti})$$

$$\bar{y}_{ti} = \bar{y} + (\bar{y}_t - \bar{y}) + (\bar{y}_i - \bar{y}) + \boxed{(\bar{y}_{ti} - \bar{y}_t - \bar{y}_i + \bar{y})}$$

estimates

ANOVA Table for Two-Way Factorials

Source of Var	Sum of sq	DFs	Mean sq
bet. bake	S_B	n-1	s_B^2
bet. etch	S_T	k-1	s_T^2
interaction	S_I	(n-1)(k-1)	s_I^2
residuals	S_e	nk(m-1)	s_E^2
total	S	nkm-1	

Expected values:

s_B^2 : $\sigma^2 + mk\,\Sigma\,\beta^2_i \;/\; (n-1)$

s_T^2 : $\sigma^2 + mn\,\Sigma\,\tau^2_t \;/\; (k-1)$

s_I^2 : $\sigma^2 + m\Sigma\Sigma\,\omega^2_{ti} \;/\; (n-1)(k-1)$

Assumptions: residuals IIND

Factorial for our Example

Effect Test

Source	Nparm	DF	Sum of Squares	F-Ratio	Prob > F
Block	1	1	150.0000	•	•
Treatment	2	2	1200.0000	•	•
Block *Treatmen	2	2	28.0000	•	•

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	5	1378.0000	275.600	•
Error	0	0.0000	•	Prob > F
C Total	5	1378.0000		•

What are your conclusions?

Two way factorial experiment for DLeff.

Etch Rec	Bake	DLeff	Etch Rec	Bake	DLeff
1	1	0.31	1	2	0.40
2	1	0.82	2	2	0.49
3	1	0.43	3	2	0.31
4	1	0.45	4	2	0.71
1	1	0.45	1	2	0.23
2	1	1.10	2	2	1.24
3	1	0.45	3	2	0.40
4	1	0.71	4	2	0.38
1	1	0.46	1	3	0.22
2	1	0.88	2	3	0.30
3	1	0.63	3	3	0.23
4	1	0.66	4	3	0.30
1	1	0.43	1	3	0.21
2	1	0.72	2	3	0.37
3	1	0.76	3	3	0.25
4	1	0.62	4	3	0.36
1	2	0.36	1	3	0.18
2	2	0.92	2	3	0.38
3	2	0.44	3	3	0.24
4	2	0.56	4	3	0.31
1	2	0.29	1	3	0.23
2	2	0.61	2	3	0.29
3	2	0.35	3	3	0.22
4	2	1.02	4	3	0.33

Anova Table for 2-way DLeff Factorial

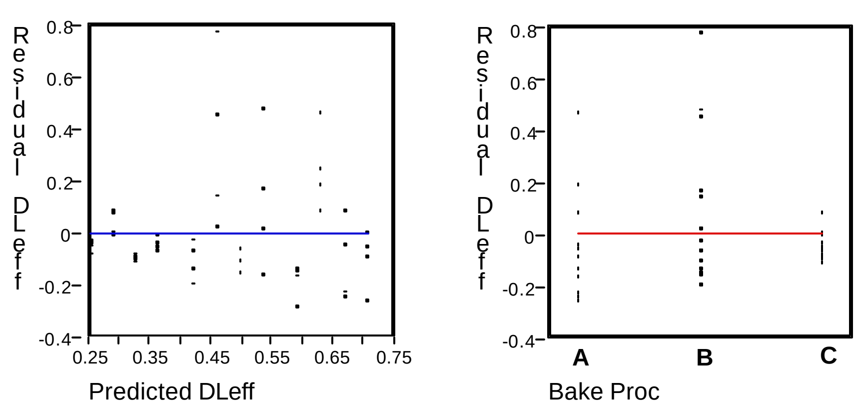
Effect Test

Source	Nparm	DF	Sum of Squares	F Ratio	Prob>F
Etch Recipe	3	3	0.9212063	13.8056	0.0000
Bake Proc	2	2	1.0330125	23.2217	0.0000
Etch Rec*Bake Pro	6	6	0.2501375	1.8743	0.1123

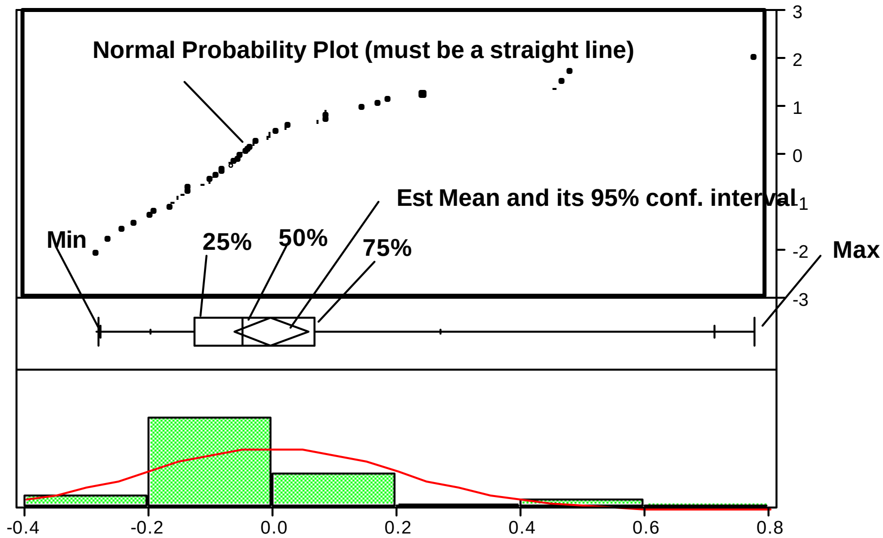
Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	11	2.2043563	0.200396	9.0097
Error	36	0.8007250	0.022242	Prob>F
C Total	47	3.0050813		0.0000

Diagnostics for DLeff



Residual Statistics for DLeff



Need for Transformation

<u>Variance Stabilization</u>

If the variance is a function of μ, then the appropriate transformation is needed to correct it. In general:

$$\sigma_y \text{ prop } \mu^\alpha$$
$$Y = y^\lambda$$
$$\lambda = 1 - \alpha$$

where σ can be determined empirically.

<u>Non-additivity</u>

An empirically selected transformation can be tested using a formal Anova-based test.

Anova Table for 2-way ln(DLeff) Factorial

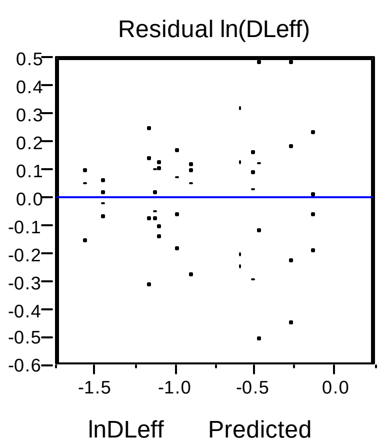
Effect Test

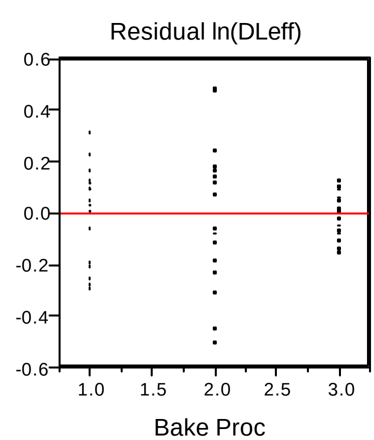
Source	Nparm	DF	Sum of Squares	F Ratio	Prob>F
Etch Recipe	3	3	3.5571735	21.9295	0.0000
Bake Proc	2	2	5.2374726	48.4324	0.0000
Etch Rec*Bake Pro	6	6	0.3957467	1.2199	0.3189

Analysis of Variance

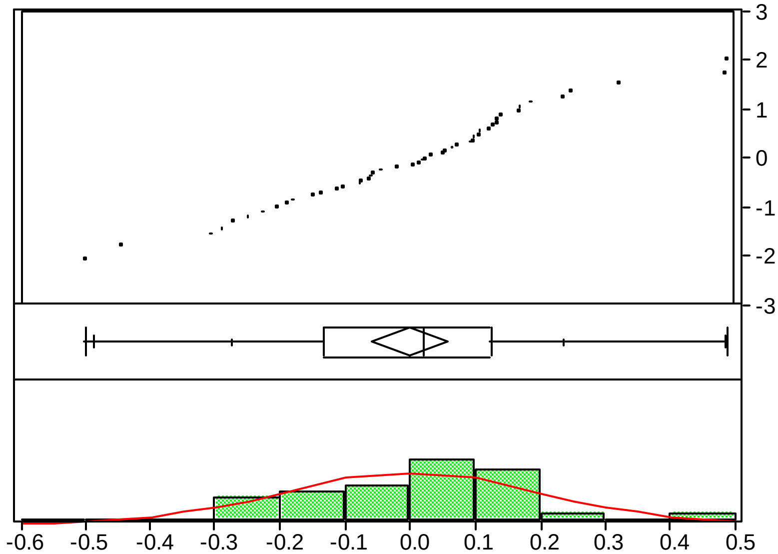
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	11	9.190393	0.835490	15.4520
Error	36	1.946516	0.054070	Prob>F
C Total	47	11.136909		0.0000

Diagnostics for ln(DLeff)

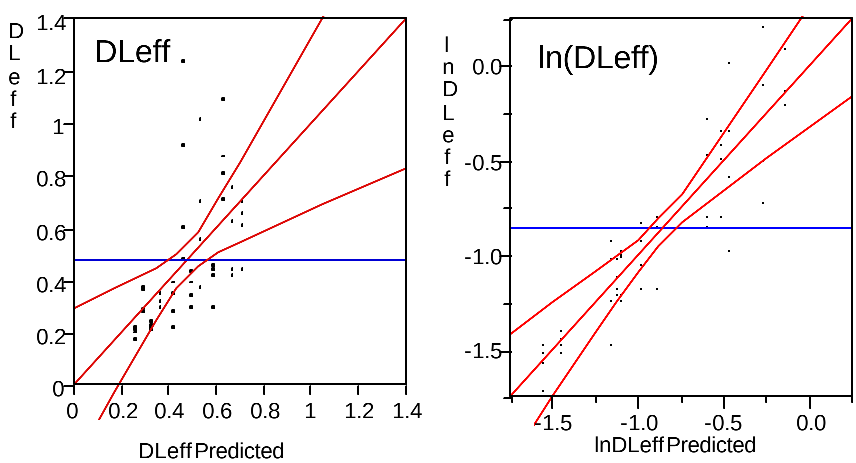




Residual Statistics for ln(DLeff)



Improvement due to Transformation



Other Blocked Arrangements

- Sometimes we might have to deal with more than one blocking variables.
- Example: testing the effect of a develop recipe on wafers and resists that come from different vendors.
- Attach two types of labels, A,B.. and I, II... and combine them to balance the blocking effects:

blocks

<- treatments

rows	columns 1	2	3	4	5
I	*A*	*B*	*C*	*D*	*E*
II	*C*	*D*	*E*	*A*	*B*
III	*E*	*A*	*B*	*C*	*D*
IV	*B*	*C*	*D*	*E*	*A*
V	*D*	*E*	*A*	*B*	*C*

5 x 5 *Latin Square* design.

Measuring the Effect of Variables

- The first step is always to find whether any variable has an effect on the outcome.
- Nothing more can be done for qualitative (categorical) variables such as recipe type, vendor name etc.

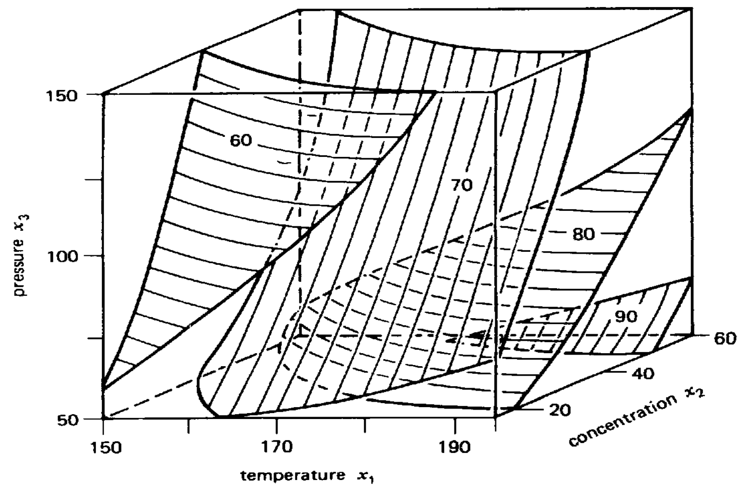
$$\hat{y}_{ti} = \mu + \tau_t + \beta_i + \omega_{ti}$$

- Some times we deal with quantitative variables (temperature, pressure).
- Once the effect has been confirmed, the next step is to build empirical quantitative models of the process:

$$y = f(x_1, x_2, \ldots, x_n)$$

Quantitative Models of the Process

they can be used to visualize, control, design, diagnose etc.



How can such a model be "extracted" from the process?

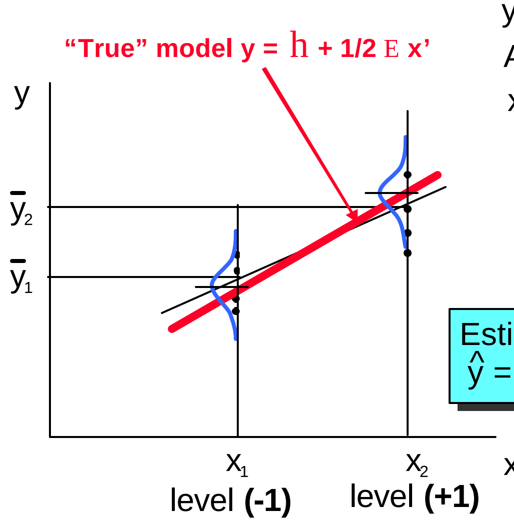
A Simple 1-Factor, 2-Level Factorial

$\text{Effect}_x = \bar{y}_2 - \bar{y}_1$

$\bar{y} = (\bar{y}_2 + \bar{y}_1) / 2$

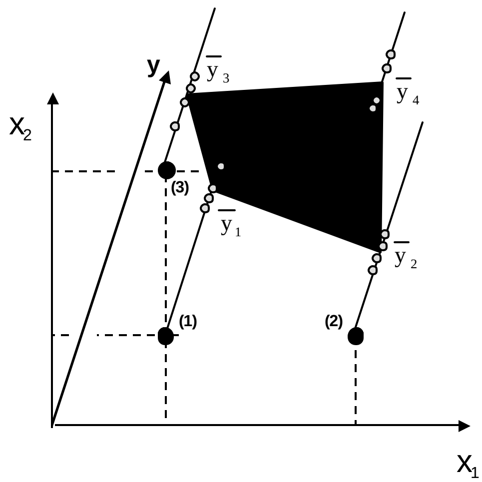
$\text{Avg}_x = (x_2 + x_1) / 2$

$x' = 2 (x - \text{Avg}_x) / (x_2 - x_1)$



Estimated model:
$\hat{y} = \text{Avg}_y + 1/2\ \text{Effect}_x\ x'$

A Simple 2-Factor 2-Level Factorial

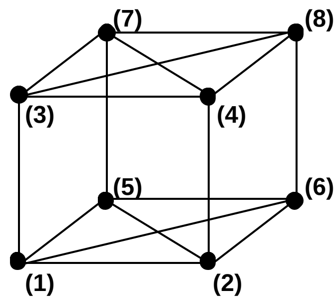


Calculations for the 2 Factor Factorial

- Average response
- Effect for x_1
- Effect for x_2
- Interaction of x_1 and x_2
- Model

General Factorial Designs at Two Levels

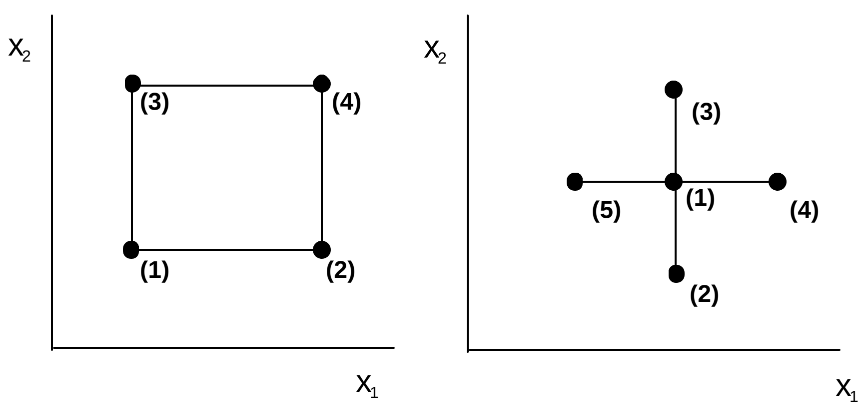
Note on Economy in experimentation:



Simple, few runs, can be augmented.

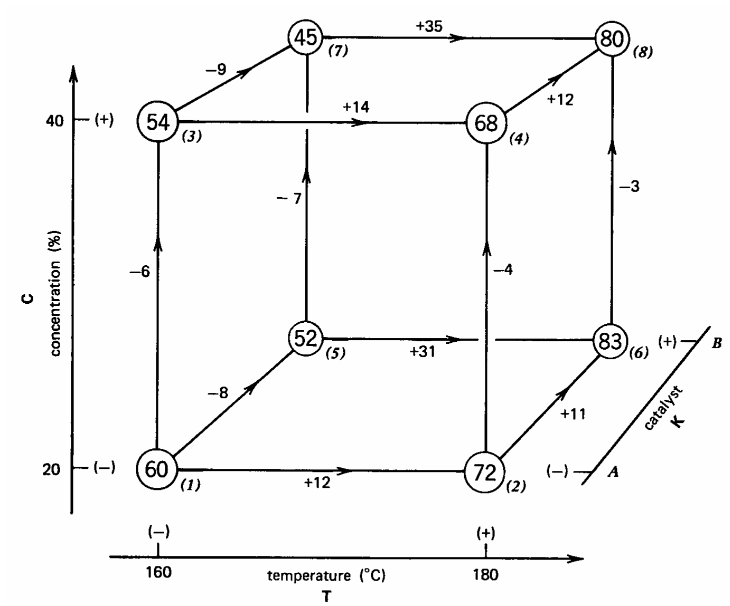
$$2^k$$
$$l_1 \times l_2 \times \ldots l_k$$

Factorial vs the "one-factor-at-a-time" experiment

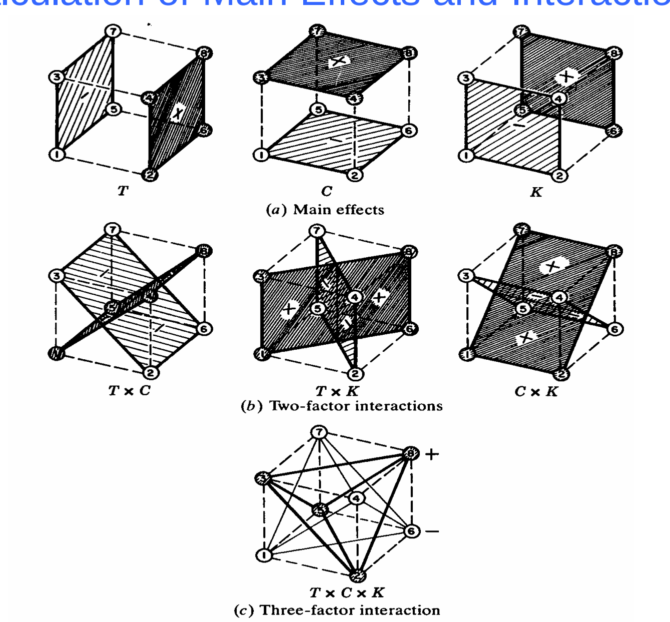


How many runs do we need for the same precision?

2^3 Pilot Plant Example



Calculation of Main Effects and Interactions



Two Level Factorial for Polysilicon Deposition

Pressure (P) 300-550 mtorr
Temperature (T) 605-650 C ^{o}
Silane flow (F) 100-250 sccm
Dep. Rate R in Å/min

P	T	F	R
-	-	-	94.80
+	-	-	110.96
-	+	-	214.12
+	+	-	255.82
-	-	+	94.14
+	-	+	145.92
-	+	+	286.71
+	+	+	340.52

Effects

AVG	**192.87**
P	**40.86**
T	**162.83**
F	**47.90**
PT	6.89
PF	11.93
TF	**30.75**
PTF	-5.88

$\sigma_{experiment}$ (estimate from replicates) = 9.05 (8)
σ_{mean} (estimate from replicates) = 3.20
σ_{effect} (estimate from replicates) = 6.40

Variance Estimation via replicated runs:

$$s^2 = \frac{\nu_1 s_1^2 + \nu_2 s_2^2 + \ldots + \nu_g s_g^2}{\nu_1 + \nu_2 + \ldots + \nu_g}$$

$$V(\text{effect}) = V(\bar{y}_+ - \bar{y}_-) = \left(\frac{1}{4} + \frac{1}{4}\right)\sigma^2 = \frac{1}{2}\sigma^2$$

in general:

$$V(\text{effect}) = \frac{4}{N}\sigma^2$$

Via insignificant high order effects:

High order effects can be seen as samples from one distribution:

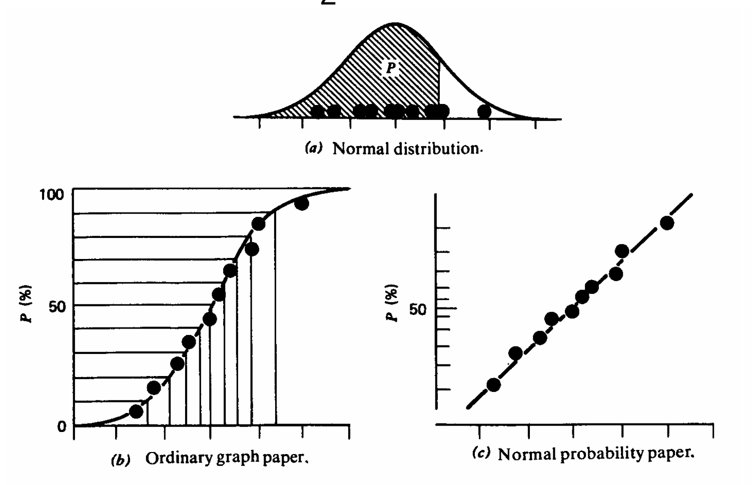
$$N\left(0, \frac{4}{N}\sigma^2\right)$$

Sum of squares can be used to estimate σ^2
Example from previous page:
σ_{effect} (estimate from insignificant effects) = 8.65 (3)

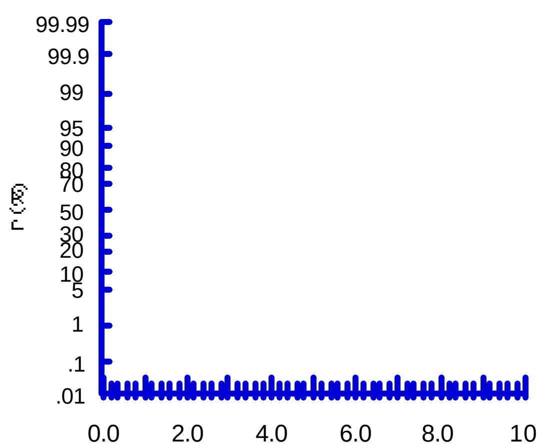
Normal Probability Plots

Used to identify insignificant effects.

$$P_i = 100(i - \frac{1}{2})/m \qquad i = 1,2, ..., m$$



Normal Probability Plot - Example



Normal Probability Plot - 2^5 Factorial Example

average = 65.5

1 =	− 1.375	123 =	1.50
2 =	19.5	124 =	1.375
3 =	− 0.625	125 =	−1.875
4 =	10.75	134 =	−0.75
5 =	− 6.25	135 =	−2.50
		145 =	0.625
12 =	1.375	235 =	0.125
13 =	0.75	234 =	1.125
14 =	0.875	245 =	−0.250
15 =	0.125	345 =	0.125
23 =	0.875		
24 =	13.25	1234 =	0.0
25 =	2.0	1245 =	0.625
34 =	2.125	2345 =	−0.625
35 =	0.875	1235 =	1.5
45 =	−11.0	1345 =	1.0
		12345 =	−0.25

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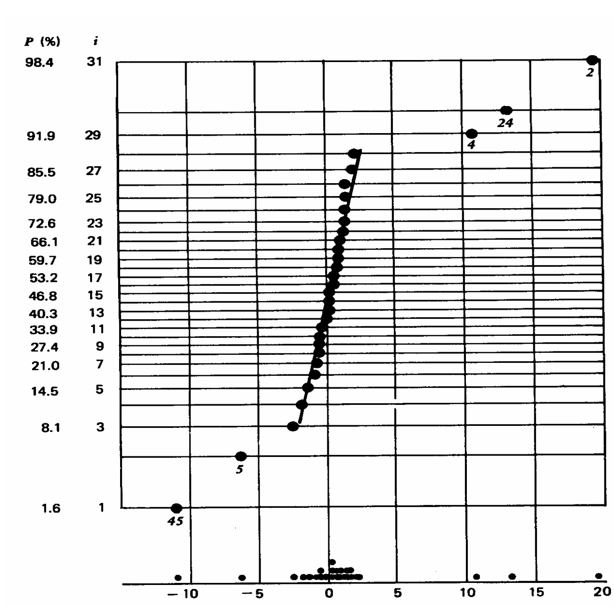
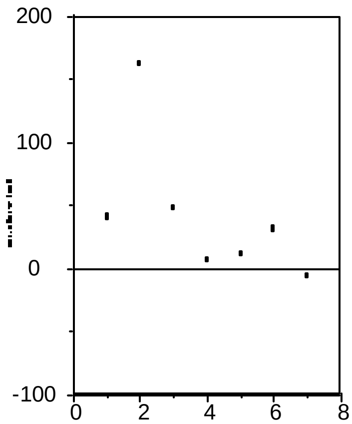
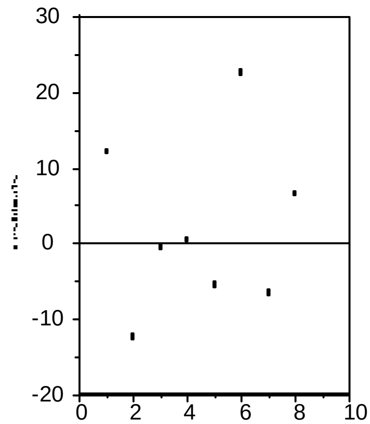
Normal Probability Plot - 2^5 Factorial Example



Polysilicon Deposition Example (cont.)

$$\hat{y} = 192.9+20.4P'+81.4T'+23.9F'+15.4T'F'$$





Using a Transformation to improve the model

ln of Deposition Rate R in ln(Å/min)

P	T	F	lnR
-	-	-	**4.55**
+	-	-	**4.71**
-	+	-	**5.37**
+	+	-	**5.54**
-	-	+	**4.54**
+	-	+	**4.98**
-	+	+	**5.66**
+	+	+	**5.83**

Effects	
AVG	**5.15**
P	**0.24**
T	**0.90**
F	**0.21**
PT	-0.06
PF	0.07
TF	0.08
PTF	-0.08

$\sigma_{experiment}$ (estimated from replications) = .028 (8)
σ_{mean} (estimated from replications) = .010
σ_{effect} (estimated from replications) = .020
σ_{effect} (estimated from insignificant effects) = .073 (4)

Quick Calculation of Effects - Contrasts

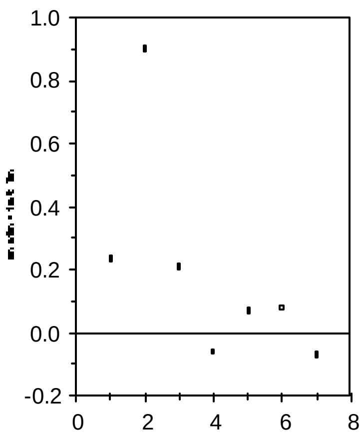
Mean	P	T	F	PT	PF	TF	PTF	Avg.
+	-	-	-	+	+	+	-	4.55
+	+	-	-	-	-	+	+	4.71
+	-	+	-	-	+	-	+	5.37
+	+	+	-	+	-	-	-	5.54
+	-	-	+	+	-	-	+	4.54
+	+	-	+	-	+	-	-	4.98
+	-	+	+	-	-	+	-	5.66
+	+	+	+	+	+	+	+	5.83
8	4	4	4	4	4	4	4	

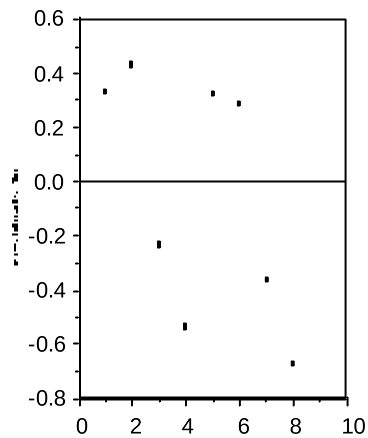
Quick Calculation of Effects - Yate's

	P	T	F	Avg.	(1)	(2)	(3)	d	est	eff
1	-	-	-	4.55	9.26	20.17	41.18	8	5.15	AVG
2	+	-	-	4.71	10.91	21.01	0.94	4	0.24	P
3	-	+	-	5.37	9.52	0.33	3.62	4	0.90	T
4	+	+	-	5.54	11.49	0.61	-0.26	4	-0.06	PT
5	-	-	+	4.54	0.16	1.65	0.84	4	0.26	F
6	+	-	+	4.98	0.17	1.97	0.28	4	0.07	PF
7	-	+	+	5.66	0.44	0.01	0.32	4	0.08	TF
8	+	+	+	5.83	0.17	-0.27	-0.28	4	-0.08	PTF

Effects, interactions and residuals must be checked...

$$\hat{y} = 51.15+.12P'+.45T'+.11F'$$





Conclusions

- From One way ANOVA to <u>M</u>ANOVA.
- From “table” models to continuous models.
- The most important concepts are:
  - There is a “true” model (with unknown “moments”)
  - There is an “estimated” model.
  - Each estimated model parameter is a “statistic”.
  - Critical underlying assumptions must be checked.
  - Residuals must be Independently, Identically, Normally Distributed (IIND).

(See chapter 7 and chapter 10 in BHH)