Two-Level Factorials (cont.)

Blocking and Confounding
Fractional Factorials
The concept of design Resolution

Blocking

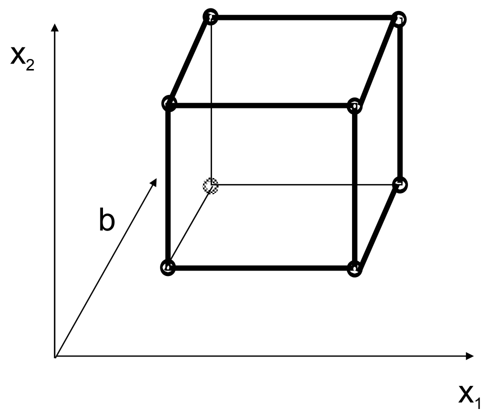
What if we need to clean the reactor every four runs?

Run	P	T	F	PT	PF	TF	PTF	block
1	-	-	-	+	+	+	-	I
2	+	-	-	-	-	+	+	II
3	-	+	-	-	+	-	+	II
4	+	+	-	+	-	-	-	I
5	-	-	+	+	-	-	+	II
6	+	-	+	-	+	-	-	I
7	-	+	+	-	-	+	-	I
8	+	+	+	+	+	+	+	II

This block will bias the PTF interaction.
The generator is **4=123**.

A simple Example in Blocking

- Do 4 experiments, at 2 levels of b, and estimate the effects of x_1 and x_2



Blocking

What if we need to clean the reactor every *two* runs?

Run	P	T	F	PT	PF	TF	PTF	b_1	b_2	block
1	-	-	-	+	+	+	-	-	+	II
2	+	-	-	-	-	+	+	+	+	IV
3	-	+	-	-	+	-	+	+	-	III
4	+	+	-	+	-	-	-	-	-	I
5	-	-	+	+	-	-	+	+	-	III
6	+	-	+	-	+	-	-	-	-	I
7	-	+	+	-	-	+	-	-	+	II
8	+	+	+	+	+	+	+	+	+	IV

This will bias the PTF, TF interactions <u>but also the P effect!!</u>

We used two generators: **4=123 (b_1) 5=23 (b_2)**
45 = 123x23 = 12233 = 1II =1

Blocking

For two runs/block (4 blocks) better to use this blocking:

Run	P	T	F	PT	PF	TF	PTF	b_1	b_2	blck
1	-	-	-	+	+	+	-	+	+	IV
2	+	-	-	-	-	+	+	-	-	I
3	-	+	-	-	+	-	+	-	+	II
4	+	+	-	+	-	-	-	+	-	III
5	-	-	+	+	-	-	+	+	-	III
6	+	-	+	-	+	-	-	-	+	II
7	-	+	+	-	-	+	-	-	-	I
8	+	+	+	+	+	+	+	+	+	IV

This blocking will bias the PT, PF and TF
We used two generators:
4=12 (b_1) 5=13 (b_2) 45 = 12x13 = I23 = 23
The main effects are still recognizable!

Examples on Blocking

- 2^4 design (16 runs) in 2 blocks of 8 runs:
- Need one generator: I = 12345 (i.e. block=5)

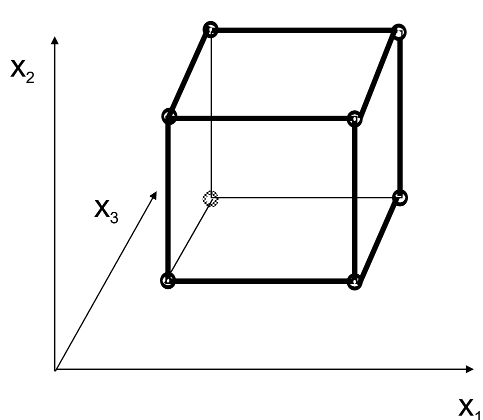
Run	A	B	C	D	block
1	-	-	-	-	
2	+	-	-	-	
3	-	+	-	-	
4	+	+	-	-	
5	-	-	+	-	
6	+	-	+	-	
7	-	+	+	-	
8	+	+	+	-	
9	-	-	-	+	
10	+	-	-	+	
11	-	+	-	+	
12	+	+	-	+	
13	-	-	+	+	
14	+	-	+	+	
15	-	+	+	+	
16	+	+	+	+	

Examples on Blocking

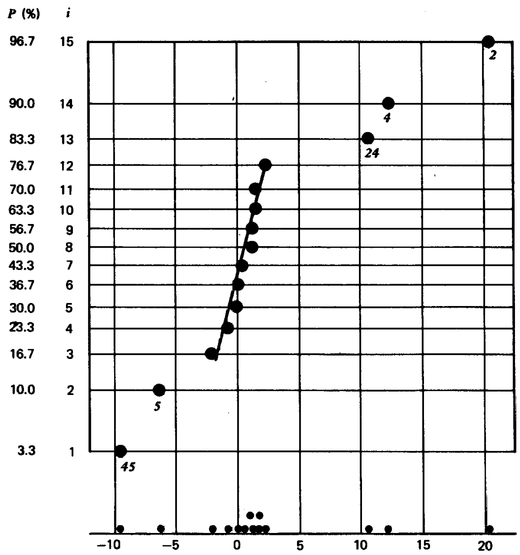
- 2^4 design (16 runs) in 4 blocks of 4 runs:
- Need two generators: B_1=124, B_2=134

Run	A	B	C	D	B1	B2
1	-	-	-	-		
2	+	-	-	-		
3	-	+	-	-		
4	+	+	-	-		
5	-	-	+	-		
6	+	-	+	-		
7	-	+	+	-		
8	+	+	+	-		
9	-	-	-	+		
10	+	-	-	+		
11	-	+	-	+		
12	+	+	-	+		
13	-	-	+	+		
14	+	-	+	+		
15	-	+	+	+		
16	+	+	+	+		

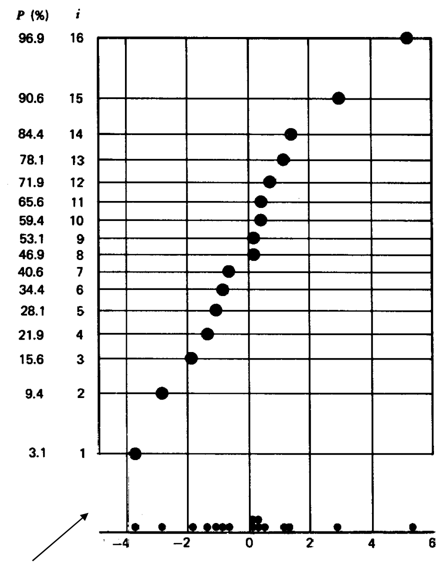
Can we measure the effect of the added variable?



Large Experiments have many insignificant Effects...



Effects of 2^5 experiment



Residuals after eliminating insignificant effects

Large experiments can be cut to half without significant conflicts...

relationship between column pairs	confounding pattern	estimate
1 = 2345	$l_1 \to 1 + 2345$	$l_1 = -2.0$
2 = 1345	$l_2 \to 2 + 1345$	$l_2 = 20.5$
3 = 1245	$l_3 \to 3 + 1245$	$l_3 = 0.0$
4 = 1235	$l_4 \to 4 + 1235$	$l_4 = 12.25$
5 = 1234	$l_5 \to 5 + 1234$	$l_5 = -6.25$
12 = 345	$l_{12} \to 12 + 345$	$l_{12} = 1.5$
13 = 245	$l_{13} \to 13 + 245$	$l_{13} = 0.5$
14 = 235	$l_{14} \to 14 + 235$	$l_{14} = -0.75$
15 = 234	$l_{15} \to 15 + 234$	$l_{15} = 1.25$
23 = 145	$l_{23} \to 23 + 145$	$l_{23} = 1.5$
24 = 135	$l_{24} \to 24 + 135$	$l_{24} = 10.75$
25 = 134	$l_{25} \to 25 + 134$	$l_{25} = 1.25$
34 = 125	$l_{34} \to 34 + 125$	$l_{34} = 0.25$
35 = 124	$l_{35} \to 35 + 124$	$l_{35} = 2.25$
45 = 123	$l_{45} \to 45 + 123$	$l_{45} = -9.50$
(I = 12345)	$[l_I \to \text{average} + \frac{1}{2}(12345)]$	$(l_I = 65.25)$

Fractional Factorials

- Do I need 8 runs to estimate 4 significant parameters?
- 2^{3-1} fractional factorial (half fraction)

The generating relation is **3=12**

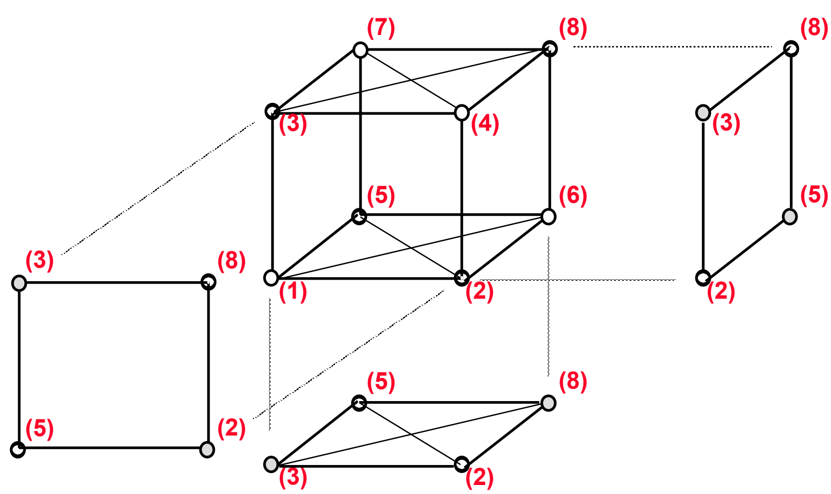
1=23 $\ell_1 = 1 + 23$
2=13 $\ell_2 = 2 + 13$
3=12 $\ell_3 = 3 + 12$
(I=123) $\ell_I = avg + 123x0.5$

Run	P	T	F	PT
1	-	-	+	+
2	+	-	-	-
3	-	+	-	-
4	+	+	+	+

Since 23, 13, 12 and 123 are insignificant, this design works. (This is a resolution III design.)

2^{3-1} Fractional Factorial

The 2^{3-1} fractional factorial is a complete factorial for any 2 of the 3 variables!



Combining Half Fractions

Run	P	T	F	PT	PF	TF	PTF
1	-	-	+	+	-	-	+
2	+	-	-	-	-	+	+
3	-	+	-	-	+	-	+
4	+	+	+	+	+	+	+

$\ell_1 = 1 + 23$
$\ell_2 = 2 + 13$
$\ell_3 = 3 + 12$
$\ell_I = \text{avg} + 123 \times 0.5$

Run	P	T	F	PT	PF	TF	PTF
5	+	+	-	-	+	-	-
6	-	+	+	+	+	+	-
7	+	-	+	+	-	+	-
8	-	-	-	-	-	-	-

$\ell'_1 = 1 - 23$
$\ell'_2 = 2 - 13$
$\ell'_3 = 3 - 12$
$\ell'_I = \text{avg} - 123 \times 0.5$

Designs of "Resolution R"

No p-factor confounded with anything less than R-p factors.

Example: I = 1 2 3 is a resolution III design. 2^{3-1}_{III}

Example: one-half fractional of the LPCVD experiment
I = 1 2 3 => 4 runs!
$\ell_1 = (1 + 2\,3)$ Peff = 0.32
$\ell_2 = (2 + 1\,3)$ Teff = 0.97
$\ell_3 = (3 + 1\,2)$ Feff = 0.15
$\ell_{avg} = (\text{Avg} + 0.5\ 123)$ Avg = 5.11

2^{5-1} Fractional Factorial

Run	A	B	C	D	E
1	-	-	-	-	+
2	+	-	-	-	-
3	-	+	-	-	-
4	+	+	-	-	+
5	-	-	+	-	-
6	+	-	+	-	+
7	-	+	+	-	+
8	+	+	+	-	-
9	-	-	-	+	-
10	+	-	-	+	+
11	-	+	-	+	+
12	+	+	-	+	-
13	-	-	+	+	+
14	+	-	+	+	-
15	-	+	+	+	-
16	+	+	+	+	+

Obtaining the Highest Possible Resolution

- Write a full factorial for the first k-1 variables
- Associate the k_{th} variable with +/- the interaction of the k-1 variables.

A fractional factorial of resolution R contains complete factorials in every set of R-1 variables!

Example: Modeling Plasma Etch Anisotropy

Parameter	Range	Units
RF Power	300-400	Watts
Pressure	200-300	mTorr
Electrode Spacing	1.2-1.8	cm
CCl_4 Flow	100-150	sccm
He Flow	50-200	sccm
O_2 Flow	10-20	sccm

- The original experiment included $2^{6-1} = 32$ runs (plus 3 center point replications).
- A $2^{6-2} = 16$ quarter fraction was used for anisotropy, because the measurements were costly and noisy.

Example: Designing the 2^{6-2} quarter fraction

5 = 123 6 = 234 2^{6-2}_{IV}

Run	1 Rf	2 Pr	3 El	4 F_1	5 F_2	6 F_3
1	-	-	-	-		
2	+	-	-	-		
3	-	+	-	-		
4	+	+	-	-		
5	-	-	+	-		
6	+	-	+	-		
7	-	+	+	-		
8	+	+	+	-		
9	-	-	-	+		
10	+	-	-	+		
11	-	+	-	+		
12	+	+	-	+		
13	-	-	+	+		
14	+	-	+	+		
15	-	+	+	+		
16	+	+	+	+		

Blocked Fractional Factorial Experiments

- Fractional factorials can be blocked. Just choose interactions that are unimportant.

5 = 123 6 = 234 2_{IV}^{6-2}

Run	1 Rf	2 Pr	3 El	4 F1	5 F2	6 F3	B1	B2	B3
1	-	-	-	-	-	-			
2	+	-	-	-	+	-			
3	-	+	-	-	+	+			
4	+	+	-	-	-	+			
5	-	-	+	-	+	+			
6	+	-	+	-	-	+			
7	-	+	+	-	-	-			
8	+	+	+	-	+	-			
9	-	-	-	+	-	+			
10	+	-	-	+	+	+			
11	-	+	-	+	+	-			
12	+	+	-	+	-	-			
13	-	-	+	+	+	-			
14	+	-	+	+	-	-			
15	-	+	+	+	-	+			
16	+	+	+	+	+	+			

Typical Designs are available...

number of variables	number of runs	degree of fractionation	type of design	method of introducing "new" factors	blocking (with no main effect or interaction confounded)	method of introducing blocks
5	16	$\frac{1}{2}$	2_V^{5-1}	$\pm 5 = 1234$	not available	
6	32	$\frac{1}{2}$	2_{VI}^{6-1}	$\pm 6 = 12345$	two blocks of 16 runs	$B_1 = 123$
7	64	$\frac{1}{2}$	2_{VII}^{7-1}	$\pm 7 = 123456$	eight blocks of 8 runs	$B_1 = 1357$ $B_2 = 1256$ $B_3 = 1234$
8	64	$\frac{1}{4}$	2_V^{8-2}	$\pm 7 = 1234$ $\pm 8 = 1256$	four blocks of 16 runs	$B_1 = 135$ $B_2 = 348$
9	128	$\frac{1}{4}$	2_{VI}^{9-2}	$\pm 8 = 13467$ $\pm 9 = 23567$	eight blocks of 16 runs	$B_1 = 138$ $B_2 = 129$ $B_3 = 789$
10	128	$\frac{1}{8}$	2_V^{10-3}	$\pm 8 = 1237$ $\pm 9 = 2345$ $\pm \overline{10} = 1346$	eight blocks of 16 runs	$B_1 = 149$ $B_2 = 12\overline{10}$ $B_3 = 89\overline{10}$
11	128	$\frac{1}{16}$	2_V^{11-4}	$\pm 8 = 1237$ $\pm 9 = 2345$ $\pm \overline{10} = 1346$ $\pm \overline{11} = 1234567$	eight blocks of 16 runs	$B_1 = 149$ $B_2 = 12\overline{10}$ $B_3 = 89\overline{10}$

Resolving Ambiguities in Fractional Factorials

Upon the completion of a fractional factorial, selected confoundings can be clarified with selected additional runs.

5 = 123 6 = 234

Suppose that 23 is significant. Since l_{23} = (23 + 15 + 64) we need a minimum of 2 additional runs. Because of potential blocking between the main experiment and the supplement, we actually need 3 additional runs. One choice is:

23	15	64	Rf	Pr	El	F1	F2	F3	Results
-	+	-							R_1
+	-	-							R_2
-	-	-							R_3

Finally, solve the system:

$$M - 0.5l_{23} + 0.5l_{15} - 0.5l_{64} = R_1$$
$$M + 0.5l_{23} - 0.5l_{15} - 0.5l_{64} = R_2$$
$$M - 0.5l_{23} - 0.5l_{15} - 0.5l_{64} = R_3$$
$$l_{23} + l_{15} + l_{64} = l_{23conf}$$

Conclusion

- Factorial experiments can accommodate blocking, if one controls the “conflicts” in estimating effects.
- Fractional factorial experiments take advantage of the insignificance of higher order terms, to accommodate many variables with few runs.
- Experiments can be done in stages, initially screening, and later analyzing important effects in detail.

(see chapter 13 in BHH or chapter 12 in Montgomery)