Robust Design

A New Definition of Quality.
The Signal-to-Noise Ratio.
Orthogonal Arrays.

The Taguchi Philosophy

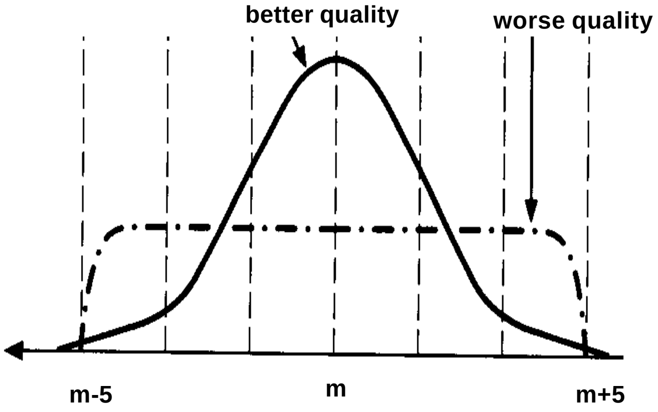
Taguchi starts with a new definition of Quality:

> Quality is related to the total *loss* to society due to functional and environmental variance

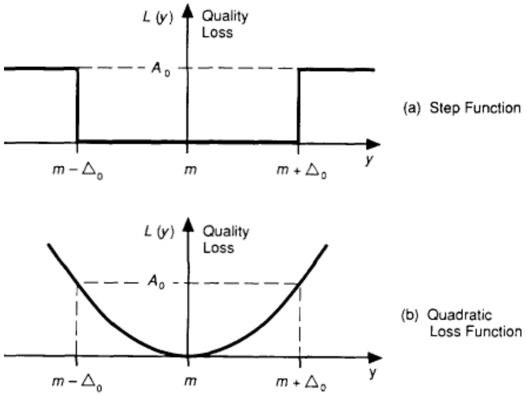
Taguchi's method focuses on Robust Design through use of:

- S/N Ratio to *quantify* quality
- Orthogonal Arrays to *investigate* quality

Meeting the specs vs. hitting the target



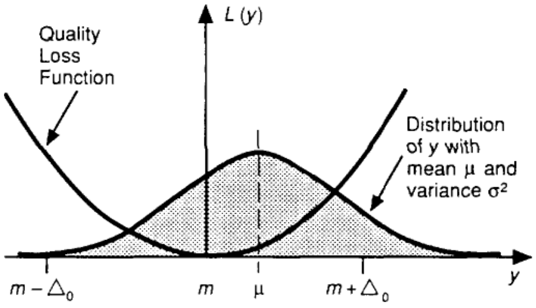
Quadratic Loss Function:



$$L(y) = k\,(y - m)^2$$

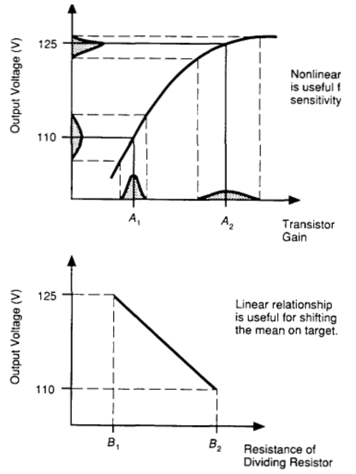
Quadratic Loss Function on Normal Distribution

Average quality loss due to μ and σ:

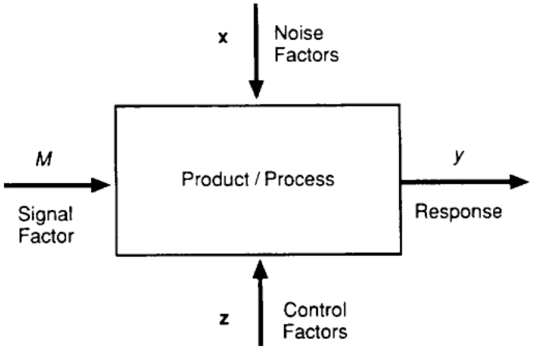


$$E(Q) = k\,[(\mu\text{-}m)^2 + \sigma^2]$$

Exploiting non-linearity:



Parameters are classified according to function:



Orthogonal Arrays

During Regression Analysis, an orthogonal arrangement of the experiment gave us independent model parameter estimates:

$$b = (X^TX)^{-1}X^Ty \qquad V(b) = (X^TX)^{-1}\sigma^2$$

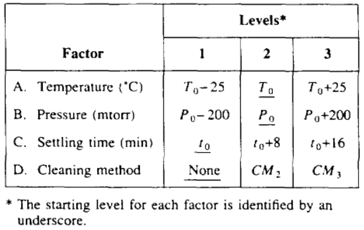
Orthogonal arrays have the same objective:

For every two columns all possible factor combinations occur equal times.

$L_4(2^3)$ $L_9(3^4)$ $L_{12}(2^{11})$ $L_{18}(2^1 \times 3^7)$

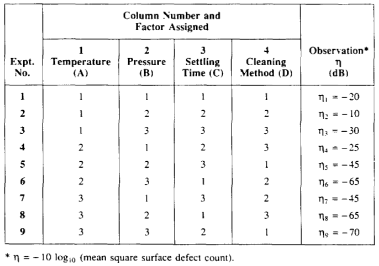
Simple CVD experiment for defect reduction

max $n = -10 \log_{10}(\text{MSQ def})$



Factor	Levels* 1	2	3
A. Temperature (°C)	T_0-25	$\underline{T_0}$	T_0+25
B. Pressure (mtorr)	P_0-200	$\underline{P_0}$	P_0+200
C. Settling time (min)	$\underline{t_0}$	t_0+8	t_0+16
D. Cleaning method	<u>None</u>	CM_2	CM_3

* The starting level for each factor is identified by an underscore.

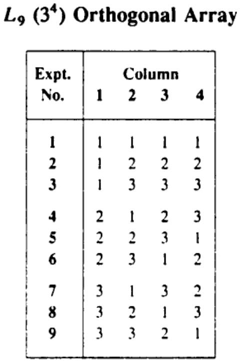


Expt. No.	Column Number and Factor Assigned: 1 Temperature (A)	2 Pressure (B)	3 Settling Time (C)	4 Cleaning Method (D)	Observation* η (dB)
1	1	1	1	1	$\eta_1 = -20$
2	1	2	2	2	$\eta_2 = -10$
3	1	3	3	3	$\eta_3 = -30$
4	2	1	2	3	$\eta_4 = -25$
5	2	2	3	1	$\eta_5 = -45$
6	2	3	1	2	$\eta_6 = -65$
7	3	1	3	2	$\eta_7 = -45$
8	3	2	1	3	$\eta_8 = -65$
9	3	3	2	1	$\eta_9 = -70$

* $\eta = -10 \log_{10}$ (mean square surface defect count).

Simple CVD experiment for defect reduction (cont)

Using the L_9 orthogonal array: $\boxed{L_9\ (3^4)}$



L_9 (3^4) Orthogonal Array

Expt. No.	Column 1	2	3	4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

Estimation of Factor Effects (ANOM)

$$m = \frac{1}{9}[\eta_1+\eta_2+\eta_3+\ldots+\eta_9]$$

$$m_{A_1} = \frac{1}{3}[\eta_1+\eta_2+\eta_3]$$

$$m_{A_2} = \frac{1}{3}[\eta_4+\eta_5+\eta_6]$$

$$m_{A_3} = \frac{1}{3}[\eta_7+\eta_8+\eta_9]$$

...

$$m_{B_2} = \frac{1}{3}[\eta_2+\eta_5+\eta_8]$$

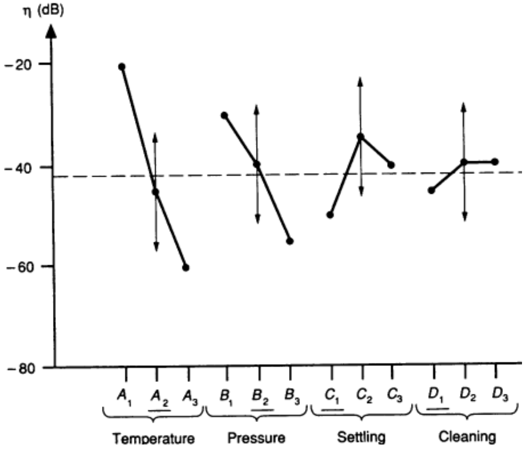
...

$$m_{D_3} = \frac{1}{3}[\eta_3+\eta_4+\eta_8]$$

$$\eta(A_i,B_j,C_k,D_l) = \mu+\alpha_i+\beta_j+\gamma_k+\delta_l+e$$

$$\sum\alpha_i = 0 \quad \sum\beta_i = 0 \quad \sum\gamma_i = 0 \quad \sum\delta_i = 0$$

Analysis of CVD defect reduction experiment



ANOVA for CVD defect reduction experiment

Grand total sum of squares: $\sum_{i=1}^{9} \eta_i^2 = 19{,}425\ (\text{dB})^2$

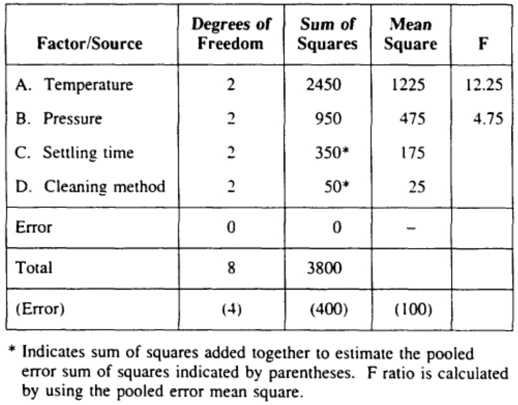
Sum of squares due to mean: $\sum_{i=1}^{9} m^2 = 15{,}625\ (\text{dB})^2$

Total sum of squares: $\sum_{i=1}^{9} (\eta_i - m)^2 = 3{,}800\ (\text{dB})^2$

Sum of squares due to A: $3 \sum_{i=1}^{3} (m_{A_i} - m)^2 = 2{,}450\ (\text{dB})^2$

Sum of squares due to error: $\sum_{i=1}^{9} e_i^2 = ???\ (\text{dB})^2$

ANOVA for CVD defect reduction experiment (cont)



Factor/Source	Degrees of Freedom	Sum of Squares	Mean Square	F
A. Temperature	2	2450	1225	12.25
B. Pressure	2	950	475	4.75
C. Settling time	2	350*	175	
D. Cleaning method	2	50*	25	
Error	0	0	–	
Total	8	3800		
(Error)	(4)	(400)	(100)	

* Indicates sum of squares added together to estimate the pooled error sum of squares indicated by parentheses. F ratio is calculated by using the pooled error mean square.

Estimation of Error Variance

The experimental error is estimated from the ANOVA residuals.

It is then used to estimate the error of the effects and to determine their significance at the 5% level.

Confirmation Experiment

Once the optimum choice has been made, it is tested by performing a confirmation run.

This run is used to "validate" the model as well as confirm the improvements in the process.

Variance of prediction (for the model)

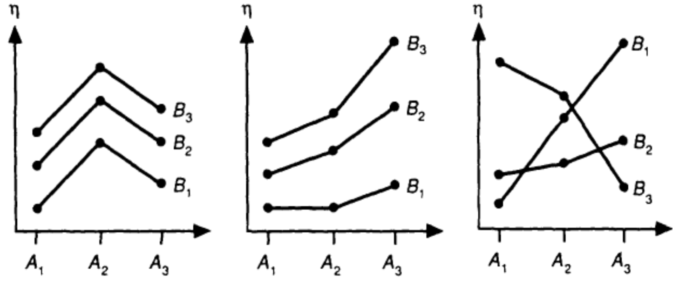
$$\frac{1}{n_0}\sigma_e^2 = \left[\frac{1}{n} + \left[\frac{1}{n_{A_1}} - \frac{1}{n}\right] + \left[\frac{1}{n_{B_1}} - \frac{1}{n}\right]\right]\sigma_e^2$$

$$\sigma_{pred}^2 = \frac{\sigma_e^2}{n_0} + \frac{\sigma_e^2}{n_r}$$

This gives us +/-2σ limits on the confirmation experiment.

The additive model

Since we assumed additive model, we must make sure that there are no interactions:



(a) No Interaction (b) Synergistic Interaction (c) Antisynergistic Interaction

---

Example: Large CVD experiment.

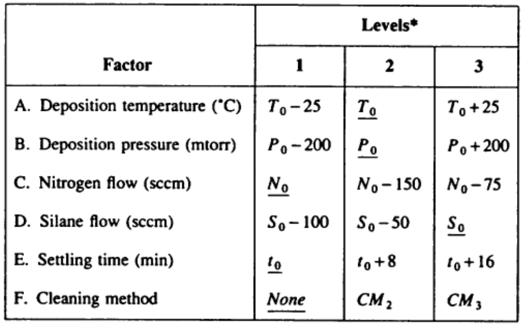
Objectives:

a) reduce defects $n = -10 \log_{10}(\text{MSQ Def})$

b) maximize S/N of rate $n' = 10 \log_{10}(\mu^2 / \sigma^2)$

c) adjust poly thickness to a 3600 Å target.

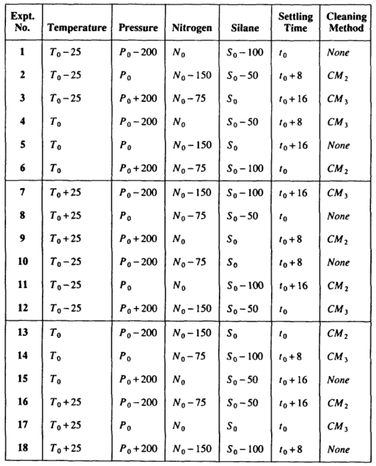
Choosing the Control Factors



Factor	Levels*		
	1	2	3
A. Deposition temperature (°C)	T_0-25	$\underline{T_0}$	T_0+25
B. Deposition pressure (mtorr)	P_0-200	$\underline{P_0}$	P_0+200
C. Nitrogen flow (sccm)	$\underline{N_0}$	N_0-150	N_0-75
D. Silane flow (sccm)	S_0-100	S_0-50	$\underline{S_0}$
E. Settling time (min)	$\underline{t_0}$	t_0+8	t_0+16
F. Cleaning method	$\underline{None}$	CM_2	CM_3

* Starting levels are identified by underscore.

Using the L18 orthogonal array...



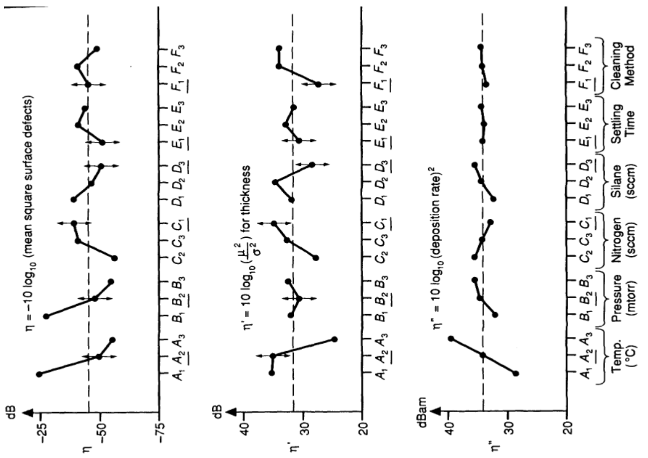
Expt. No.	Temperature	Pressure	Nitrogen	Silane	Settling Time	Cleaning Method
1	T_0-25	P_0-200	N_0	S_0-100	t_0	None
2	T_0-25	P_0	N_0-150	S_0-50	t_0+8	CM_2
3	T_0-25	P_0+200	N_0-75	S_0	t_0+16	CM_3
4	T_0	P_0-200	N_0	S_0-50	t_0+8	CM_3
5	T_0	P_0	N_0-150	S_0	t_0+16	None
6	T_0	P_0+200	N_0-75	S_0-100	t_0	CM_2
7	T_0+25	P_0-200	N_0-150	S_0-100	t_0+16	CM_3
8	T_0+25	P_0	N_0-75	S_0-50	t_0	None
9	T_0+25	P_0+200	N_0	S_0	t_0+8	CM_2
10	T_0-25	P_0-200	N_0-75	S_0	t_0+8	None
11	T_0-25	P_0	N_0	S_0-100	t_0+16	CM_2
12	T_0-25	P_0+200	N_0-150	S_0-50	t_0	CM_3
13	T_0	P_0-200	N_0-150	S_0	t_0	CM_2
14	T_0	P_0	N_0-75	S_0-100	t_0+8	CM_3
15	T_0	P_0+200	N_0	S_0-50	t_0+16	None
16	T_0+25	P_0-200	N_0-75	S_0-50	t_0+16	CM_2
17	T_0+25	P_0	N_0	S_0	t_0	CM_3
18	T_0+25	P_0+200	N_0-150	S_0-100	t_0+8	None

Data summary for large CVD experiment:

Expt. No.	Experiment Condition Matrix* e	A	B	C	D	E	e	F	Surface Defects η (dB)	Thickness μ (Å)	Thickness η′ (dB)	Deposition Rate η″ (dBam)
1	1	1	1	1	1	1	1	1	0.51	1958	35.22	23.23
2	1	1	2	2	2	2	2	2	−37.30	5255	35.76	31.27
3	1	1	3	3	3	3	3	3	−45.17	5965	36.02	32.34
4	1	2	1	1	2	2	3	3	−25.76	2121	42.25	31.15
5	1	2	2	2	3	3	1	1	−62.54	4572	21.43	37.27
6	1	2	3	3	1	1	2	2	−62.23	2891	32.91	33.89
7	1	3	1	2	1	3	2	3	−59.88	3375	21.39	37.68
8	1	3	2	3	2	1	3	1	−71.69	4527	22.84	40.46
9	1	3	3	1	3	2	1	2	−68.15	3946	30.60	41.21
10	2	1	1	3	3	2	2	1	−3.47	3415	26.85	27.89
11	2	1	2	1	1	3	3	2	−5.08	2535	38.80	26.02
12	2	1	3	2	2	1	1	3	−54.85	5781	38.06	31.82
13	2	2	1	2	3	1	3	2	−49.38	2723	32.07	34.50
14	2	2	2	3	1	2	1	3	−36.54	2852	43.34	33.20
15	2	2	3	1	2	3	2	1	−64.18	3201	37.44	34.76
16	2	3	1	3	2	3	1	2	−27.31	3105	31.86	37.71
17	2	3	2	1	3	1	2	3	−71.51	4074	22.01	40.45
18	2	3	3	2	1	2	3	1	−72.00	3596	18.42	39.22

* Empty column is denoted by e.

Data analysis for large CVD experiment (cont)



ANOVA tables for large CVD experiment:

Factor	Average η by Factor Level (dB) 1	2	3	Degree of Freedom	Sum of Squares	Mean Square	F
A. Temperature	− 24.23	− 50.10	− 61.76	2	4427	2214	27
B. Pressure	− 27.55	− 47.44	− 61.10	2	3416	1708	21
C. Nitrogen	− 39.03	− 55.99	− 41.07	2	1030	515	6.4
D. Silane	− 39.20	− 46.85	− 50.04	2	372	186	2.3
E. Settling time	− 51.52	− 40.54	− 44.03	2	378	189	2.3
F. Cleaning method	− 45.56	− 41.58	− 48.95	2	164†	82	
Error				5	405†	81	
Total				17	10192		
(Error)				(7)	(569)	(81)	

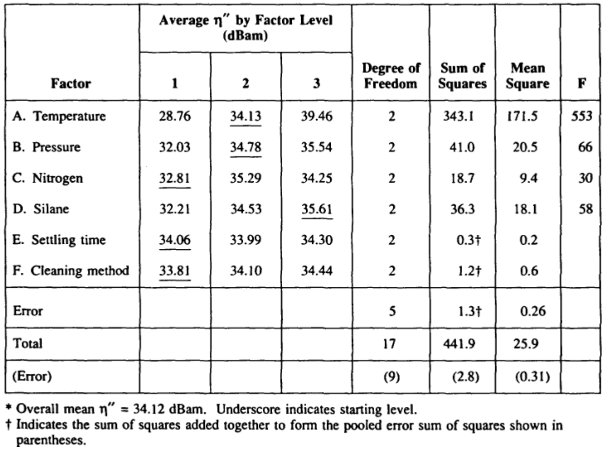
* Overall mean η = −45.36 dB. Underscore indicates starting level.
† Indicates the sum of squares added together to form the pooled error sum of squares shown in parentheses.

ANOVA tables for large CVD experiment (cont)

Factor	Average η′ by Level (dB) 1	2	3	Degree of Freedom	Sum of Squares	Mean Square	F
A. Temperature	35.12	34.91	24.52	2	440	220	16
B. Pressure	31.61	30.70	32.24	2	7†	3.5	
C. Nitrogen	34.39	27.86	32.30	2	134	67	5.0
D. Silane	31.68	34.70	28.17	2	128	64	4.8
E. Settling time	30.52	32.87	31.16	2	18†	9	
F. Cleaning method	27.04	33.67	33.85	2	181	90.5	6.8
Error				5	96†	19.2	
Total				17	1004	59.1	
(Error)				(9)	(121)	(13.4)	

* Overall mean η′ = 31.52 dB. Underscore indicates starting level.
† Indicates the sum of squares added together to form the pooled error sum of squares shown in parentheses.

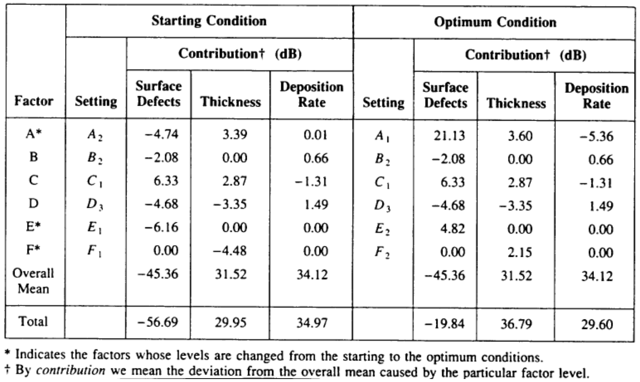
ANOVA tables for large CVD experiment (cont)



Factor	Average η″ by Factor Level (dBam) 1	2	3	Degree of Freedom	Sum of Squares	Mean Square	F
A. Temperature	28.76	34.13	39.46	2	343.1	171.5	553
B. Pressure	32.03	34.78	35.54	2	41.0	20.5	66
C. Nitrogen	32.81	35.29	34.25	2	18.7	9.4	30
D. Silane	32.21	34.53	35.61	2	36.3	18.1	58
E. Settling time	34.06	33.99	34.30	2	0.3†	0.2	
F. Cleaning method	33.81	34.10	34.44	2	1.2†	0.6	
Error				5	1.3†	0.26	
Total				17	441.9	25.9	
(Error)				(9)	(2.8)	(0.31)	

* Overall mean η″ = 34.12 dBam. Underscore indicates starting level.
† Indicates the sum of squares added together to form the pooled error sum of squares shown in parentheses.

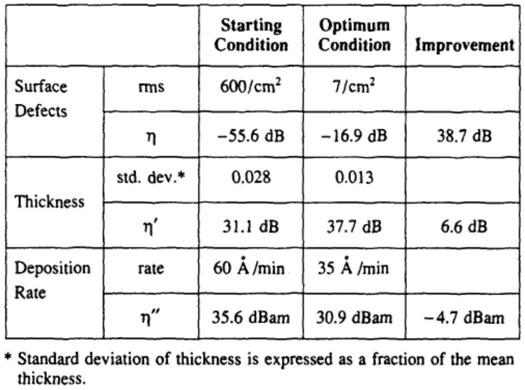
Prediction Using the Additive Model



	Starting Condition				Optimum Condition			
		Contribution† (dB)				Contribution† (dB)		
Factor	Setting	Surface Defects	Thickness	Deposition Rate	Setting	Surface Defects	Thickness	Deposition Rate
A*	A_2	−4.74	3.39	0.01	A_1	21.13	3.60	−5.36
B	B_2	−2.08	0.00	0.66	B_2	−2.08	0.00	0.66
C	C_1	6.33	2.87	−1.31	C_1	6.33	2.87	−1.31
D	D_3	−4.68	−3.35	1.49	D_3	−4.68	−3.35	1.49
E*	E_1	−6.16	0.00	0.00	E_2	4.82	0.00	0.00
F*	F_1	0.00	−4.48	0.00	F_2	0.00	2.15	0.00
Overall Mean		−45.36	31.52	34.12		−45.36	31.52	34.12
Total		−56.69	29.95	34.97		−19.84	36.79	29.60

* Indicates the factors whose levels are changed from the starting to the optimum conditions.
† By *contribution* we mean the deviation from the overall mean caused by the particular factor level.

Verification for large CVD experiment



		Starting Condition	Optimum Condition	Improvement
Surface Defects	rms	600/cm^2	7/cm^2	
	η	−55.6 dB	−16.9 dB	38.7 dB
Thickness	std. dev.*	0.028	0.013	
	η′	31.1 dB	37.7 dB	6.6 dB
Deposition Rate	rate	60 Å/min	35 Å/min	
	η″	35.6 dBam	30.9 dBam	−4.7 dBam

* Standard deviation of thickness is expressed as a fraction of the mean thickness.

for further reading: Quality Engineering Using Robust Design by Madhav S. Phadke
Prentice Hall 1989

Why use S/N Ratios?

- They lead to an optimum through a monotonic function.

- They help improve additivity of the effects.

Taguchi vs. RSM

Taguchi	RSM
Small number of runs	Explicit control of Interactions
Engineering Intuition	Statistical Intuition
“Complete” package	Training Issues
Additive Models	More General Models
Orthogonal Arrays	Fractional Factorials
A “Philosophy”	A Tool

Design of Experiments

- Comparison of Treatments
- Blocking and Randomization
- Reference Distributions
- ANOVA
- MANOVA
- Factorial Designs
- Two Level Factorials
- Blocking
- Fractional Factorials
- Regression Analysis
- Robust Design