UNIVERSITY OF CALIFORNIA
College of Engineering
Department of Electrical Engineering
and Computer Sciences

C. SPANOS **EECS 290H**
Fall 2005

PROBLEM SETs No. 1 and 2
Due on Tuesday, September 27th, 2005

1. A manufacturing facility has a yield that is controlled purely by random defects. The density of these random defects depends on the design rule used. More specifically, for a 90nm design rule, the density is 0. 1/cm^2, while for a 65nm design rule, the density is 0. 5/cm^2. (Use the simple exponential yield model).
    a) A given product takes 4.0cm^2. Further, 90% of this area is using 90nm design rules, while the rest 10% is using the 65nm design rules. Estimate the yield of this product.
    b) This product can be redesigned (shrunk) to take only 2.0cm^2, but now 50% of the chip has to use the agressive 65nm design rules. Estimate the yield of the redesigned product.
    c) What would be the ratio of good die per wafer of the redesigned product to that of the original product?

2. Consider the effect of defects on IC interconnect. The figure below illlustrates the impact of defect size on critical area for circular defects of diameter x. The area in which the center of such defects must fall to cause a failure increases linearly as a function of defect size. It can be expressed as

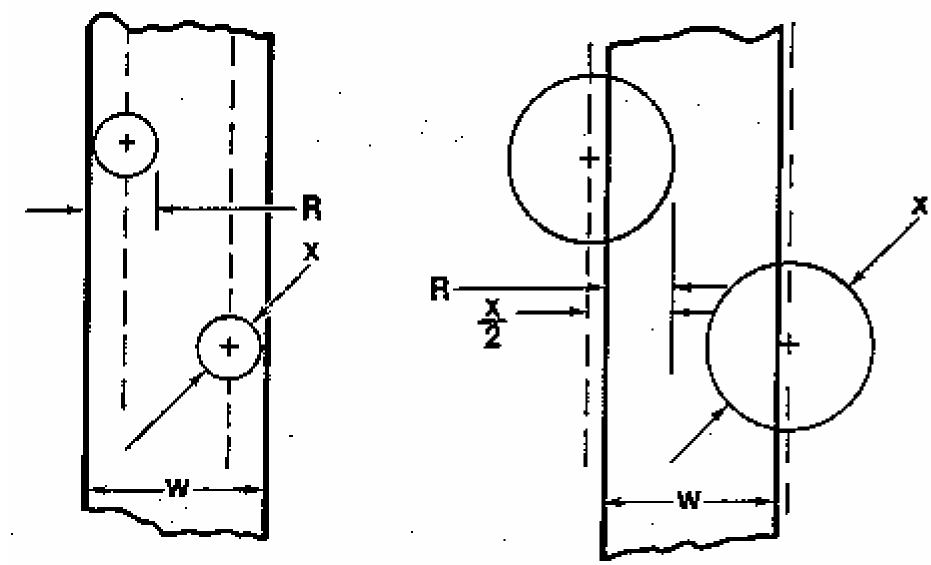
$$A_c(x) = L(x + w - 2R)$$

$$R \le x \le \infty$$

where L is the interconnect length and R is the allowable gap in the interconnect line. Suppose that the normalized probability density function of defect sizes is given by

$$g(x) = \frac{X_U^2 X_L^2}{x^3 (X_U^2 - X_L^2)}$$

where X_L and X_U are the lower and upper limits of the range of defect sizes, respectively.



(a) If $R \geq X_L$, find an expression for the average critical area (A_{AV}) by evaluating the integral

$$A_{AV} = \int_{X_L}^{X_U} A_c(x) g(x) dx$$

(b) Show that as the upper limit on defect size approaches infinity,

$$A_{AV} = \frac{L X_L^2 w}{2R^2}$$

3. Suppose we are given the joint distribution of several parameters which vary in an IC manufacturing process, and we would like to evaluate the impact of these variations on the overall performance of the IC by evaluating its parametric yield. For example, the drive current in mA of a MOSFET in saturation (I_{Dsat}) is given by

$$I_{Dsat} = \frac{k}{2}(V_{GS} - V_T)^2$$

where k is the device transconductance parameter, V_{GS} is the gate-source voltage, and V_T is the threshold voltage. Suppose that V_T is subject to variation, and that variation ultimately impacts I_{Dsat}. There is a way to find (analytically) the pdf of y, $f_y(y)$, if $y = g(x)$ and if the pdf of x is known, as shown here:

Fundamental Theorem

To find $f_y(y)$ for a given y we solve the equation

$$y = g(x)$$

for x in terms of y. If $x_1, x_2, \ldots, x_n, \ldots$ are all its *real* roots,

$$y = g(x_1) = \cdots = g(x_n) = \cdots$$

then

$$f_y(y) = \frac{f_x(x_1)}{|g'(x_1)|} + \cdots + \frac{f_x(x_n)}{|g'(x_n)|} + \cdots$$

where

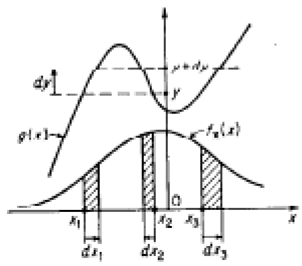
$$g'(x) = \frac{dg(x)}{dx}$$

Clearly, the numbers $x_1, \ldots, x_n, \ldots$ depend on y. If, for a certain y the equation

$$y = g(x)$$

has no real roots, then

$$f_y(y) = 0$$



(a) Find the *analytical* expression for the pdf for the drive current, if the V_T is *uniformly* distributed between 0.3 and 0.8V, and if all other parameters are deterministic.

(b) If k = 1 mA/V^2, determine the parametric yield for a large population of transistors that achieve drive currents between 1.5 and 2.0 mA, if V_{GS} is 2.5V.

4. Solve problem 3 empiricaly using a Monte Carlo method invoving 10,000 samples.
   a. Do this the “easy way” this time: i.e. generate the random samples of V_T, (uniformly distributed between 0.3 and 0.8V) and use the I_{Dsat} formula to generate the respective I_{Dsat} samples. Produce a histogram and compare to the actual pdf that you found in problem 4 (k = 1 mA/V^2 , V_{GS} is 2.5V). (Matlab will be a great tool for this problem!)
   b. Use your Monte Carlo solution to estimate the yield as defined in problem 3b for various sample sizes. Plot the *estimated* yield against the following Monte Carlo sample sizes:

   n = 10, 100, 500, 1000, 5000 and 10000.