UNIVERSITY OF CALIFORNIA
College of Engineering
Department of Electrical Engineering
and Computer Sciences

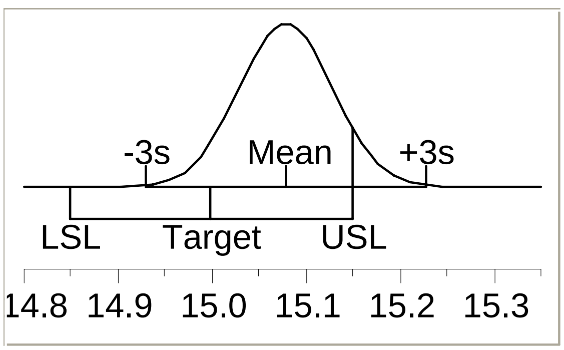
C. SPANOS **EECS 290H**
Fall 2005

PROBLEM SET No. 3
Due on Thursday, September 29th, 2005

1. Suppose the concentration of particles produced in an etching operation on any given day is normally distributed with a mean of 15.08 particles/ft^3 and a standard deviation of 0.05 particles/ft^3. The specifications on the process call for a concentration of 15.00 +/- 0.15 particles/ft^3. What fraction of etching systems conform to specifications?

   I essentially want to find the area under the standadrd normal distribution that started at mean-4.6sigma and goes to mean+1.4sigma. From the table, I find that this area is 91.92%

Specification	*Value*	*Portion*	*% Actual*
Lower Spec Limit	*14.85*	*Below LSL*	*0.0000*
Upper Spec Limit	*15.15*	*Above USL*	*8.0970*
Spec Target	*15*	*Total Outside*	*8.0970*



2. A process that produces high-density memory devices is subject to catastrophic defects that obey a Poison distribution with a mean of 4 defects/die.

   a) Calculate the functional yield of the process (assume that it takes just one defect to destroy one die).

   Yield is equivalent to probability of no defect:
   $P(0) = (e^{-4}\, 4^0)/0! = 0.0183$, or a mere 1.8%.

   b) If redundant columns are introduced, how many columns are needed to ensure a yield of 50%? (Assume that one redundant column is needed to repair exactly one defect).

   This is equivalent to finding the minimum number of defects n, so that:
   $P(0) + P(1) +...+ P(n) > 0.50$

This is done next:

n	*P*	*Cumulative*
0	*0.0183*	*0.0183*
1	*0.0733*	*0.0916*
2	*0.1465*	*0.2381*
3	*0.1954*	*0.4335*
4	*0.1954*	*0.6289*

So, we need at least 4 redundant columns to achieve 50% yield.

5. Suppose we are interested in calibrating a chemical vapor deposition furnace. The furnace will be shut down for repairs if significant difference is found between the thermocouples that are measuring the deposition temperature at the two ends of the furnace tube. The following temperatures have been measured during several test runs:

Thermocouple 1 (^oC)	Thermocouple 2 (^oC)
606.5	604.0
605.0	604.5
605.5	605.5
605.5	605.7
606.2	605.5
606.5	605.2
603.7	606.0
607.7	606.5
607.7	607.7
604.2	604.2

a) Do these temperatures have the same mean on both ends of the tube? ($\alpha = 0.05$)

In order to answer this we will use the unpaired t test. First, however, we need to establish if the two groups have identical variances or not. The two sigmas are estimated to be 1.34 and 1.11, respectively. These two estimates are close enough so that the actual moments can be assumed identical. (This assumption will be tested at part (b) with the application of the F-distribution test.) The result of the unpaired t-test is:

Data File: Untitled Data

Independent Samples...

Variable:	Column 1	Column 2
Mean:	6.0585e+2	6.0548e+2
Std. Deviation:	1.3385e+0	1.1134e+0
Observations:	10	10

t-statistic:	6.7204e-1	Hypothesis:
Degrees of Freedom:	18	Ho: μ1 = μ2
Significance:	5.10e-1	Ha: μ1 ¥ μ2

So, the conclusion is that the two means are the same at the 5% level of significance.

b) Find the 90% confidence interval of the ratio of the two variances (L/R).

The estimated ratio of variances is 1.457, and this is a random number from the F distribution with 9 and 9 degrees of freedom. From the statistical tables we can find the 5% and 95% points of the $F_{9,9}$ distribution, and they are 0.315 and 3.18, respectively. (Only the 95% point is given by the table, while the 5% point is found by the relationship: $F_\alpha = 1/F_{1-\alpha}$). Using these values, the 90% confidence interval is found to be [0.453, 4.63]. (By the way, since this interval includes 1.0, the hypothesis of variance equality cannot be rejected at the 10% level of significance)

c) Find the sample size needed to confirm a 1°C deviation with a β of 10% when the σ is known to be 1°C.

What we want to do here is find the sample size that with α=5% will give us a β of 10%, when the variances are assumed identical and the means are assumed to be different by 1°C. The two distributions that will be compared then will be:

$$N \cdot \left(\mu, \frac{\sigma^2}{n}\right)$$

$$N \cdot \left(\mu + 1, \frac{\sigma^2}{n}\right)$$

The statistic that will be used to test this hypothesis will be:

$$\overline{x_1} - \overline{x_2} \sim \begin{cases} N\left(0, \frac{2\sigma^2}{n}\right) & H_0 \\ N\left(1, \frac{2\sigma^2}{n}\right) & H_1 \end{cases}$$

This means that if I set 5% limits on H_0, the upper 2.5% limit of H_0 should correspond to the lower 10% limit of H_1. This leads to the following equation that can be solved to find n.

$$0 + 1.960\sqrt{\frac{2\sigma^2}{n}} = 1 - 1.28\sqrt{\frac{2\sigma^2}{n}} \Rightarrow n = 21$$