UNIVERSITY OF CALIFORNIA
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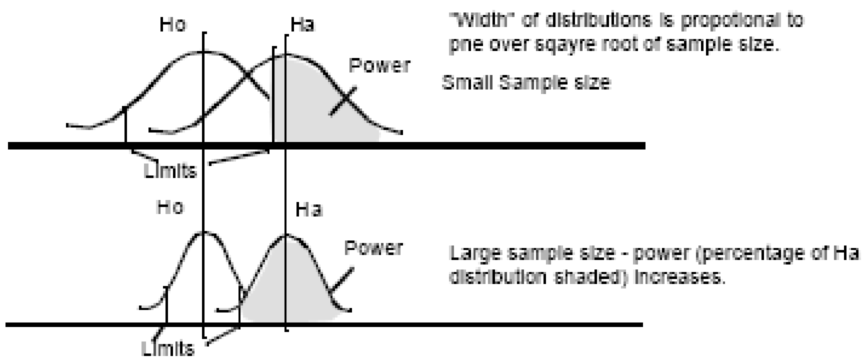
C. SPANOS **Special Issues in Semiconductor Manufacturing** **EECS 290H**
Fall 2005

PROBLEM SET No. 4
Official Solutions

1. Answer the following questions (briefly explain your answers)
 a) What is a "test statistic" in the context of Hypothesis testing?

A test statistic is the "score" one calculates from the data. Usefull test statistics follow known, well defined distributions under the null, and possibly known distribution(s) under the alternative hypothesis. The test consists of seeing how likely it is, that the test statistic actually came from the null hypothesis distribution.

 b) How is the *power* of a test related to the *sample size* of a test, assuming that the test statistic is an average of independent samples, and it follows the normal distribution?



The power of the test should be proportional to the square root of the sample size. This is because the two distributions (the null and the alternate) will reduce their variance at one over the square root of the sample size.

c) Is the power (P) of a test independent of the type I error?

No. One can trade power for type one error and vice versa. The power of the test depends on the criterion for Ho rejection, and this criterion (limit) is directly dependent on the type I error a.

2. Two types of photoresist (A and B) were used on a total of 8 randomly selected wafers, 4 for A and 4 for B. After patterned with a DUV stepper (using the same mask for all), the average line-width was measured in nm:

A	97.23	96.28	98.17	95.03
B	101.30	101.17	101.09	101.32

Obtain a 95% confidence interval for the ratio of sigmas. State your assumptions carefully.

The Assumptions are that my data are Independently, Identically and Normally Distributed (IIND). The calculations are straightforward, but this assumption is not supported by the analysis - i.e. the sigma of A is likely smaller than the sigma of B:

$$\frac{s_A^2/\sigma_A^2}{s_B^2/\sigma_B^2} \sim F_{3,3} \Rightarrow \frac{1}{F_{3,3(\alpha/2)}} \le \frac{s_A^2/\sigma_A^2}{s_B^2/\sigma_B^2} \le F_{3,3(\alpha/2)} \Rightarrow \frac{1}{F_{3,3(0.025)}}\frac{s_B^2}{s_A^2} \le \frac{\sigma_B^2}{\sigma_A^2} \le F_{3,3(0.025)}\frac{s_B^2}{s_A^2} \Rightarrow$$

$$\frac{1}{15.44}\cdot\frac{0.012}{1.802} \le \frac{\sigma_B^2}{\sigma_A^2} \le 15.44\cdot\frac{0.012}{1.802} \Rightarrow 0.0004 \le \frac{\sigma_B^2}{\sigma_A^2} \le 0.10282 \Rightarrow 0.0208 \le \frac{\sigma_A}{\sigma_B} \le 0.32065$$

3. You are considering introducing a new etch recipe, hoping that it will reliably shrink your after-etch CD. (Remember that each nm you shave off the CD gives you about $15/microprocessor you sell!). You have determined that the sigma of the process is 1nm, and it will not be affected by the new process. Using a mask with 90 nm lines, and the old etch recipe, the average measured polysilicon line-width is 75 nm. You will be willing to switch to the new etch recipe only if you see a 2 nm improvement (i.e. your measured pattern goes from 75nm to 73nm) with α=0.05 and β=0.10. How many samples done with the new process do you need for this experiment?

According to the null and to the alternate hypothesis, the limit "x" that will be used to implement the (one sided) test has as follows:

$$z_{1-0.05} = \frac{x-75}{\sigma/\sqrt{n}}$$

$$z_{0.90} = \frac{x-73}{\sigma/\sqrt{n}}$$

Substituting the proper z values from the normal probability tables, we form one equation with only n (the sample size) as the unknown.

$$-1.64\frac{1}{\sqrt{n}} + 75 = 1.29\frac{1}{\sqrt{n}} + 73 \Rightarrow n = 2.146$$

From the above I conclude that n must be at least 3.

4. We now have three types of resist to compare: A, B and C. We would like to use an ANOVA table in order to test the hypothesis of equivalence of the three treatments. A few wafers from the first group have already been measured. Look at the 4 measurements below and estimate the total number of wafers we need to measure if we would like to detect deviations in the order of 1nm between groups with a power of 80%, while the type I error is kept at 5%.

A:	96.96	96.59	97.26	97.47

Explain all implicit assumptions.

OK, so this is a tough problem. The idea here is that if we assume (to simplify the solution) an equal number of samples n in each group, then, assuming that the groups are not the same, we have the following: First, we estimate the sigma of the process to be 0.382 and the σ^2 = 0.146. *According to the known ANOVA formulae, if we had completed the experiment, then* s_t^2 *would be estimating the following number:*

$$s_T^2 = \frac{\sum_{t=1}^{3} n(\bar{y}_t - \bar{y})^2}{3-1} \sim \left(\sigma^2 + \frac{\sum_{t=1}^{3} n\tau_t^2}{3-1} \right)$$

The question is what would be the sampling distribution of s_t^2*? A good approximation is to use the* <u>*non-central*</u> x^2 *distribution with 2 degrees of freedom for this. So, under the null and the alternate hypothesis, the formula that will give me the solution is:*

$$F_{2,\,3(n-1),\,a} = \frac{\sigma^2 + n\,0.01^2}{\sigma^2} F_{2,\,3(n-1),\,1-\beta}$$

Where α*=0.05 and* β*=0.20. The total number of measurements will be 3xn (the number of groups multiplied by the number of samples per group.)*

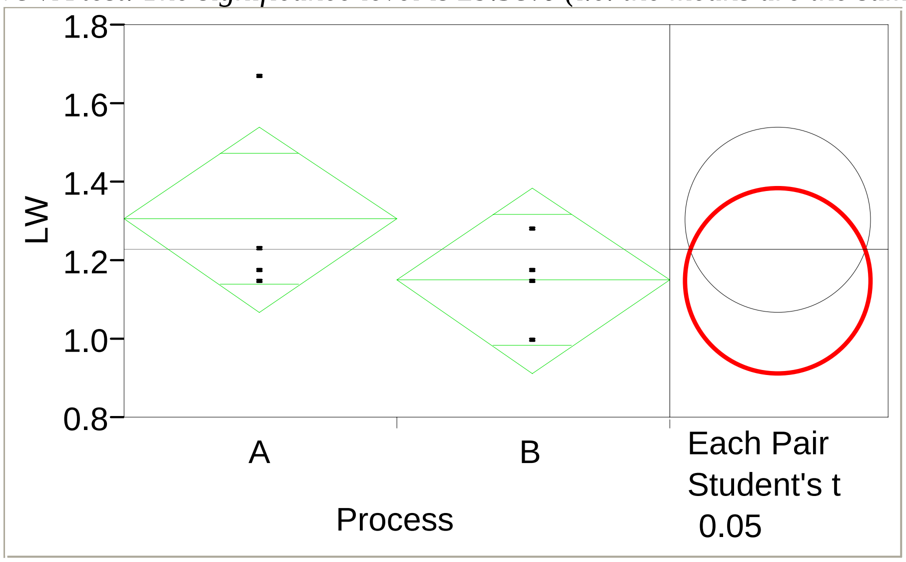
Another way to approach this problem is to consider its "equivalent": suppose that we just need to compare just <u>*two*</u> *groups in order to decide whether their two means are the same or not. We also want to set up the power of the test in order to detect a certain amount of deviation. Let us also assume that the sigma is known as estimated. Given all this we can approach this problem as we did in problem 3. We could use the normal distribution (we could also use the student-t, but it gets a bit more complicated.). The only difference is that while in problem 3 we were trying to determine a mean deviation of 2 sigma, here we are trying to determine a mean deviation of a about 3-sigma (1/0.382)…*

5. To compare two photolithography processes (A and B), 4 of 8 wafers were randomly assigned to each. The electrically measured line width of several NMOS transistors gave the following averages (in µm):

A:	1.176	1.230	1.146	1.672
B:	1.279	1.000	1.146	1.176

Assuming that the processes have the same standard deviation, calculate the significance for the comparison of means.

This is standard ANOVA – here are the JMP results done graphically, using the student-t test and using the one-way ANOVA test. The significance level is 29.58% (i.e. the means are the same at the 5% level).



t-Test

	Difference	t-Test	DF	Prob > |t|
Estimate	0.15575	1.145	6	0.2958
Std Error	0.13603			
Lower 95%	-0.17711			
Upper 95%	0.48861			

Assuming equal variances

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Process	1	0.04851612	0.048516	1.3109	0.2958
Error	6	0.22206475	0.037011		
C. Total	7	0.27058087			

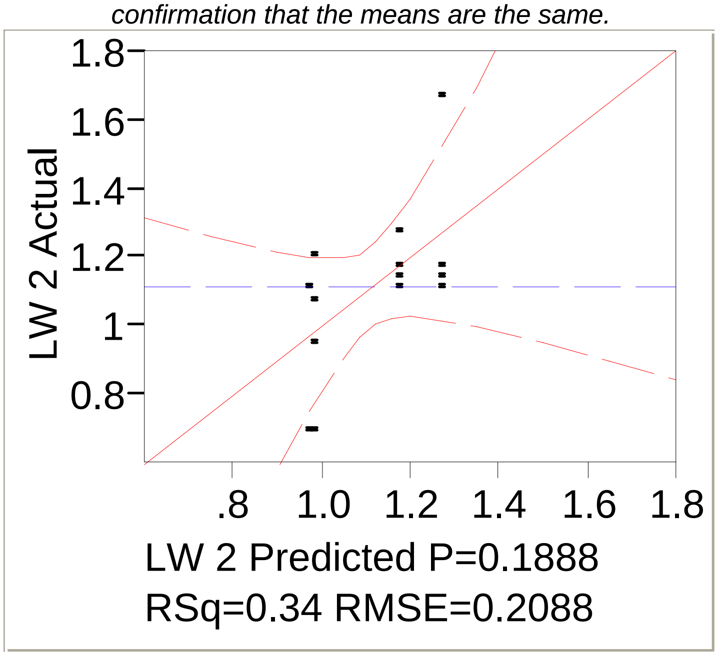
6. Suppose there are now four photolithography processes to compare (A, B, C, and D). Using 15 wafers, the measurements are (in µm):

	I	II	III	IV
A	1.176	1.146	1.672	1.114
B	1.279	1.146	1.176	1.114
C	0.954	1.079	1.204	0.699
D	0.699	1.114	1.114	

Calculate the full ANOVA table and find the level of significance for rejecting the hypothesis of equality. Explain any assumptions and perform the necessary diagnostics on the residuals.

This is also standard stuff. The level of significance for rejecting is 18.88%, so at the 5% level I will accept that the means are the same. My basic assumption is the IIND assumption: all residuals are indentically, independently and normally distributed. Here is the JMP analysis:

Actual by Predicted Plot – *The confidence interval of the model includes the horizontal line. This is a graphical confirmation that the means are the same.*



Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	3	0.24801618	0.082672	1.8960
Error	11	0.47964542	0.043604	Prob > F
C. Total	14	0.72766160		0.1888

Residual by Predicted Plot – *this plot is given as a graphical confirmation of the IIND Assumption. In this case I can see one point that might be an outlier. I tried to exclude t and redo the analysis without it, but this did not change the overall conclusion…*