Robust Power Allocation in Wireless Communication Systems.

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1 Introduction

Spread Spectrum Systems where each user spreads his information onto a larger bandwidth have attained importance in wireless communication. The conventional matched filter receiver suffers from the near-far problem: a strong user (high power user or a user close to the base station) can easily make the communication of the weak user (low power user or a user at the edge of the cell) impossible. So, power control is necessary for fair allocation of resources. This report is aimed at one such scheme.

2 System Model and Power Allocation

We consider the communication from the mobile users to the central base station. Let us consider a system with K users. We restrict ourselves to a DS-CDMA system with a processing gain of N. Let $\mathbf{s}_i \in \mathbb{R}^N$ denote the signature sequence of user i and x_i the information symbol to be transmitted. The received signal $\mathbf{y} \in \mathbb{R}$ is given by

$$\mathbf{y} = \sum_{i=1}^{K} \sqrt{\alpha_i}\mathbf{s}_i x_1 + \mathbf{n} \tag{1}$$

where $\mathbf{n} \in \mathbb{R}$ is the additive white Gaussian noise with zero mean and a covariance matrix $\sigma^2 I_N$ and α_i is the path gain of the i^{th} user.

The conventional matched filter receiver is obtained by projecting the received vector onto the users' signature sequence. This estimate might be used by the channel decoder for further processing. Therefore, we consider the output to be a soft decision,

$$\hat{x}_i = \mathbf{s}_i^t \mathbf{y} \tag{2}$$

Since we are interested in soft decisions, the Signal-to-interference ratio (SIR) is an appropriate performance measure.

$$SIR_i = \frac{\alpha_i \left(\mathbf{s}_i^t \mathbf{s}_i\right)^2 p_i}{\sum_{j \neq i} \alpha_j \left(\mathbf{s}_i^t \mathbf{s}_j\right)^2 p_j + \sigma^2} \tag{3}$$

where $p_i = \mathbb{E}\left[x_i^2\right]$ is the power of user i.

Now, the aim is to allocate powers p_i such that we maximize the product of the attained SIRS of the users. Practical constraints limit the maximum power, P_i^{max}. Acquiring and time synchronization require a minimum power, P_i^{min}. So, the problem becomes,

$$\begin{aligned} \max \quad & \prod_{i=1}^{K} SIR_i \\ & SIR_i = \frac{\alpha_i \left(\mathbf{s}_i^t \mathbf{s}_i\right)^2 p_i}{\sum_{j \neq i} \alpha_j \left(\mathbf{s}_i^t \mathbf{s}_j\right)^2 p_j + \sigma^2} \\ & P_i^{min} \leq p_i \leq P_i^{max} \text{ for } i = 1, \ldots, K \end{aligned} \tag{4}$$

In the following, we will reformulate the above into a geometric program and also consider a robust version of it. The original problem is equivalent to,

$$\begin{aligned} \min \quad & t \\ & t = \prod_{i=1}^{K} t_i \\ & \sum_{j=1}^{K} \alpha_j \left(\mathbf{s}_i^t \mathbf{s}_j\right)^2 p_j + \sigma^2 - \alpha_i (1 + t_i) p_i = 0 \quad \text{for } i = 1, \ldots, K \\ & p_i \leq P_i^{max} \text{ for } i = 1, \ldots, K \\ & p_i \geq P_i^{min} \text{ for } i = 1, \ldots, K \end{aligned}$$

The equalities can be replaced by inequalities since we are interested in the minimum. Therefore, the geometric program in the standard format becomes,

$$\begin{aligned}
\min \quad & t \\
& \sum_{j=1}^{K} \alpha_j \left(\mathbf{s}_i^t \mathbf{s}_j\right)^2 p_j + \sigma^2 + \alpha_i(1+t_i)p_i \leq 0 \quad \text{for } i = 1, \ldots, K \\
& p_i - P_i^{max} \leq 0 \text{ for } i = 1, \ldots, K \\
& P_i^{min} - p_i \leq 0 \text{ for } i = 1, \ldots, K
\end{aligned} \tag{5}$$

The above geometric program has an objective function of 1 dimension and has $3K+1$ constraints with K original variables and $K+1$ extra variables. (This can be translated into a convex form by taking $f(x) = log(x)$ for the variables.)

3 Robust Power allocation

In practise, each user will have completely knowledge of its own signature sequence $\mathbf{s}_i$. The channel tracking system tracks α_i, but only to a certain accuracy. Similarly, there is an error in the known noise variance σ. The signature sequence of other users is also known to some accuracy, due to limited feedback and resources available to transmit this information. So, the uncertainty for user i can be summarized by,

$$\begin{aligned}
& \left|\alpha_j^{(i)} - \alpha_i^o\right| \leq \delta_j^{(i)} \\
& \left|\left(\mathbf{s}_j - \mathbf{s}_j^o\right)_l\right| \leq \epsilon_{jl} \text{ for } j = 1, \ldots, K, j \neq i \\
& \left|\sigma - \sigma^0\right| \leq \xi
\end{aligned}$$

Now, the aim would be to maximize the product of the SIRs in the worst possible case, that is,

$$\begin{aligned}
& \max_{p} \min_{\epsilon, \delta, \xi} \prod_{i=1}^{K} SIR_i \\
& SIR_i = \frac{\alpha_i \left(\mathbf{s}_i^t \mathbf{s}_i\right)^2 p_i}{\sum_{j \neq i} \alpha_j \left(\mathbf{s}_i^t \mathbf{s}_j\right)^2 p_j + \sigma^2} \\
& \left|\alpha_j^{(i)} - \alpha_j^o\right| \leq \delta_j^{(i)} \text{ for } j = 1, \ldots, K \\
& \left|\left(\mathbf{s}_j - \mathbf{s}_j^o\right)_l\right| \leq \epsilon_{jl} \text{ for } j = 1, \ldots, K, j \neq i \\
& \left|\sigma - \sigma^0\right| \leq \xi
\end{aligned}$$

The numerator and the denominator of the SIR for user i have variables that are independently varying. So, the minimum with respect to the errors will be when the numerator is minimized and the denominator is maximized separately. The minimum in the numerator occurs at $\alpha_i = \alpha_i^o - \delta_i^{(i)}$. Now, we need to maximize the interference from the other users under the uncertain data available. Since the correlation from the signature sequence occurs only as as squared term, we need to maximize the path gain, $\alpha_j = \alpha_j^{(i)} + \delta_j^{(i)}$ and the noise variance at $\sigma = \sigma^o + \xi$. In the case of $\mathbf{s}_i^t\mathbf{s}_j$, this is maximized by choosing $\mathbf{s}_j^{(i)} = \mathbf{s}_j^o + \epsilon_j^{(i)*}$, where $\epsilon_j^{(i)*} \in \mathbb{R}^N$ is given by,

$$\left(\epsilon_j^{*(i)}\right)_l = \epsilon_{jl}\mathrm{sgn}\left(\mathbf{s}_{il}\right)\mathrm{sgn}\left(\mathbf{s}_i^t\mathbf{s}_j^o\right)$$

Therefore, the robust problem reduces to,

$$\begin{array}{ll}\min & t \\ & \sum_{j=1}^{K}(\alpha_j^o + \delta_j^{(i)})\left(\mathbf{s}_i^t\mathbf{s}_j^o + \mathbf{s}_i^t\epsilon_j^{*(i)}\right)^2 p_j + (\sigma^o+\xi)^2 - (\alpha_i^o - \delta_i^{(i)})(1 + \frac{\alpha_i^o - \delta_i^{(i)}}{\alpha_i^o + \delta_i^{(i)}}t_i)p_i \leq 0 \\ & p_i - P_i^{max} \leq 0 \text{ for } i = 1, \dots, K \\ & P_i^{min} - p_i \leq 0 \text{ for } i = 1, \dots, K\end{array} \qquad (6)$$

The robust problem happens to be another geometric program. Notice that given the known parameters, $\epsilon_j^{*(i)}$ can be solved for all i and the problem is similar to the previous one, with different parameters fed in for the path gains and signature sequences. Therefore, its complexity is not much different.

4 Simulation Results

In the simulation, we considered a system with $K = 3$ users and path gains of [10.50.25]. The optimal power allocation for the original problem is solved. Then, the errors are introduced as a percentage of the corresponding values (that is, $x = x^o(1 + \delta_x)$). Then, the robust version of the program was run. The results are shown in Figure 1. Notice that the user with highest path gain, which happens to be the user with the lowest power is the one to be affected the most. His alloted power has a steep rise as the error in the parameters increases. The simulation results are plotted over a wide range of noise variance. Also notice that the noise variance where communication is impossible to be 'fair' shifts to the left as the errors increases.

In the figure, user 1 is plotted in 'blue', user 2 in 'red' and user 3 in 'green'. The error percentages increase through 0,0.001,0.01,0.1 in the solid, dotted, starred solid and triangulated solid lines respectively.

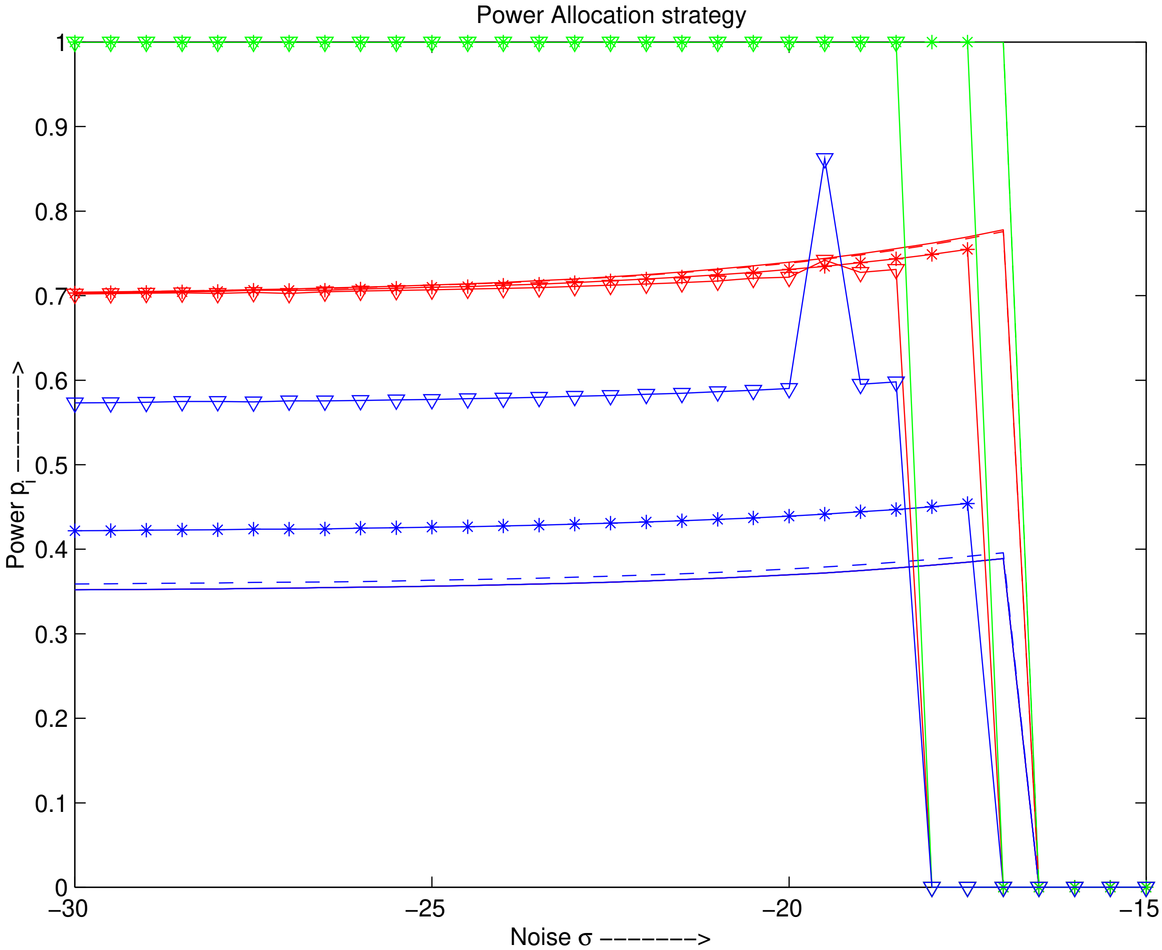

Figure 1: Power Allocation stratigies for various error percentages

5 Future Work

The scheme analyzed is maximizing the product SIR. I think there is a link with this and maximizing sum capacity at high SNR. This needs to be explored and a better understanding of this might lead to "fairer" schemes. The current implementation is very slow and takes a long time even for small number of users. May be a separate C code or a nice looking algorithm specially rigged for this will help in real time implementations. I have had a chance to look at the robust linear MMSE receiver. This NP hard at the first sight. This needs more work and might yield better insights into the problem. Also, we need to extend this scheme to a more general framework of errors in parameters.