# ee290N Project:
# Air Profile Optimization in the Fast Positive Force Transient Control

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December 15, 1999

## 1 Introduction

The fast positive force transient (FPFT) control problem for automotive engines was introduced in [1]. We proposed an approach for solving the FPFT problem, as well as a hybrid model for the vehicle. In particular this model describes the behavior of a four stroke internal combustion engine equipped with a drive-by-wire throttle system and a direct-injection fuel system.

In the FPFT region of operation (see [3], [2] for an overview on decomposing the engine control problem into regions of operation), the engine control unit has to react to a fast gas pedal motion that is interpreted as a request for a fast increase of the torque delivered by the engine while maintaining a reasonable level of comfort, specified in terms of vehicle acceleration and jerk. The goal is to control the evolution of the system from an initial condition characterized by the delivery of a torque $u_0$ to a final condition characterized by the delivery of a requested torque $u_R$ in minimum time subject to constraints on acceleration and jerk. The available control actions are on fuel injection, spark ignition and, since we are considering cars equipped with drive-by-wire electronics, throttle angle (which regulates the air entering the cylinders). Our approach to the hybrid control problem is to first introduce an auxiliary relaxed problem: a continuous time problem with jerk as input. This problem is solved optimally, using Pontryagin's Maximum Principle [4]. Then, the solution is mapped back in the hybrid domain obtaining feedback laws for the throttle angle, the spark advance and the fuel injection inputs.

In this mapping phase the control law is modified in order to take into account the constraints present in the hybrid model. In particular the hybrid model is such that the produced torque is a piecewise-constant function of time, whereas the optimal continuous time solution corresponds to jerk output being a piecewise-constant function. These two constraints only match when the system is at an equilibrium point, where both torque and jerk are constants (jerk is null), not during transients.

The modifications, that lead to the proposed hybrid control law, have the obvious effect of increasing the time required to steer the state to the final manifold, when compared to the continuous time optimal solution. This is not an error, rather a consequence of the relaxation performed at the beginning: the relaxed problem has by definition an optimal solution with a lower (non greater) value for the cost function. By comparing the costs attained by the proposed hybrid solution and those attained by the optimal continuous solution to the relaxed problem, one can bound the cost of the (unknown) optimal hybrid solution. In particular the difference of the two (proposed hybrid vs optimal continuous) is an upper bound for the suboptimality (distance from optimality) of the proposed hybrid solution.

Moreover, care must be taken in the mapping to guarantee the convergence (in finite time) of the closed loop system under the hybrid control law. This issue was investigated during the final project for EE291E in Spring 1999.

In the proposed mapping process, we developed a decentralized control scheme where the air dynamics control law and the injection and ignition control laws are designed independently. We use a variable structure control (VSC) algorithm to track robustly a desired profile for the air entering the cylinders.

The optimal torque profile for the continuos system is strongly discontinuous (see Figure 1), and such profiles are clearly not feasible for the continuous output of the air dynamics. The discontinuous optimal profile must then be approximated by a continuous one. In [1], the choice of the desired air profile is based on a simplyfied model for the system (rigid transmission) that results in a continuous torque profile that can be easily translated in a continuous air profile.

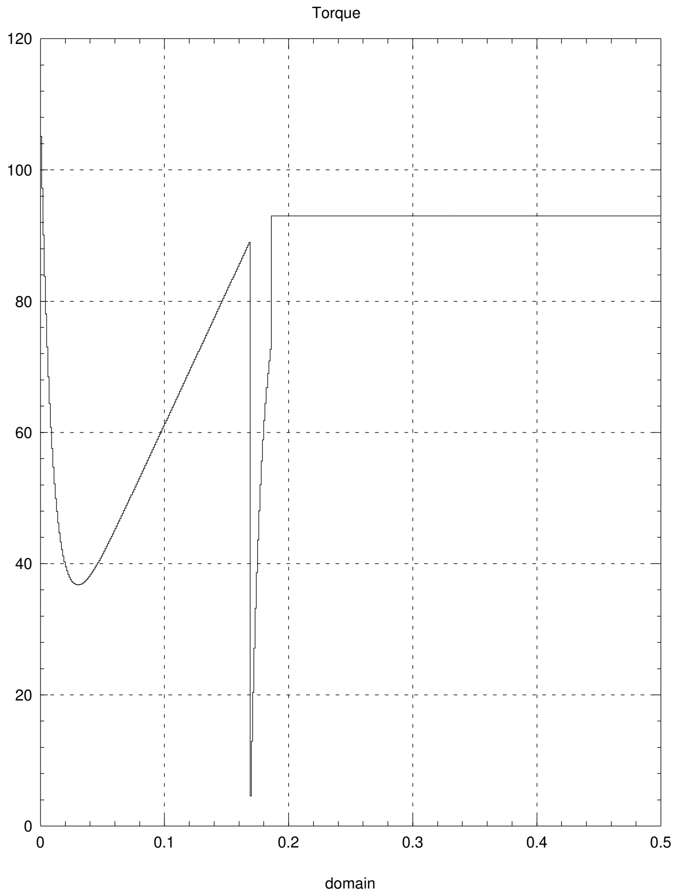


Figure 1: Optimal torque profile

## 2 Goals of the project

I intend to investigate the advantages and improvements that can be achieved by using optimization techniques to determine a reference air profile for the air dynamics.

# 3 Proposed strategy

In [1] the optimal torque profile is already translated into a piece-wise constant profile which is amenable to the hybrid system modeling the engine like in Figure 2.

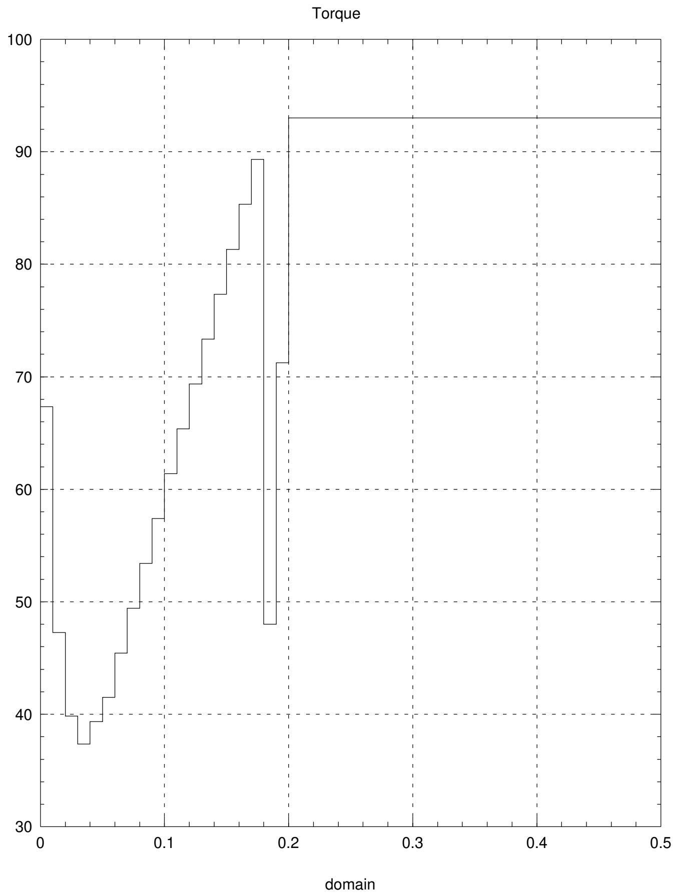


Figure 2: Piece-wise constant torque profile

## 3.1 Air dynamics model

The air dynamics is strongly non linear, it depends on the ratio between the intake manifold pressure and atmosphere pressure and on the engine revolution speed (at higher rpm's the engine pumps air out of the intake manifold faster than at lower rpm's).

During a fast force transient the engine speed does not change very much, so we can linearize the air dynamics around the initial engine speed. The raising edge of the manifold pressure is usually faster than the falling edge so we will use the slower dynamics to model the slew rate limitations. At a given engine speed there is a linear dependency of the value of the potential torque and the value of the pressure in the intake manifold, so we use the former as our state variable. The discrete time (DT) model is

$$T_{k+1} = A_d T_k + B_d \alpha_k \tag{1}$$

where $T_k$ is the value of the potential torque at time $t_k$ and $\alpha_k$ is the sample of the position of the throttle valve at time $t_k$.

Clearly, once the initial state ($T_1$) and the samples $\alpha_k|_{1\leq k<N}$ are assigned, the corresponding $T_k$'s are uniquely determined for $1 \leq k \leq N$. The correspondence is set by the state transition matrix in the following way

$$T = A_{dN}\ T_1 + B_{dN}\ \alpha \tag{2}$$

where $T = [T_1, ..., T_N]'$ and $\alpha = [\alpha_1, ..., \alpha_N]'$ are the samples of the input and output signals and matrices $A_{dN}$ and $B_{dN}$ are obtained by the induction basis

$$A_{dN}(1,1) = 1;\ B_{dN}(1,1) = zeros(1,N)$$

and the induction step

$$A_{dN}(t,1) = A_d\ A_{dN}(t-1,1);$$
$$B_{dN}(t,:) = [A_d\ B_{dN}(t-1, 1:t-2), B_d, A_d\ B_{dN}(t-1, t:N)]$$

## 3.2 Optimal Program

We want to minimize the maximum difference between the potential torque $T$ determined by the air dynamics output and the desired torque $\hat{T}$ determined assuming an infinitely fast air dynamics like in Figure 2. Formally

$$\begin{array}{rl} \min\limits_{\alpha} & \max\limits_{k} \|T_k - \hat{T}_k\| \\ \text{subject to} & \text{air dynamics} \\ & \alpha_k \in [0, \alpha_{\max}] \end{array}$$

where $\alpha_{\max}$ corresponds to a full-open throttle. Using a slack variable $t$ it can be rewritten as

$$\begin{array}{rl} \min\limits_{\alpha} & t \\ \text{subject to} & \text{air dynamics} \\ & \|T_k - \hat{T}_k\| \leq t \\ & \alpha_k \in [0, \alpha_{\max}] \end{array}$$

Using equation (2) and renaming variables we form an equivalent LP

$$\begin{array}{rl} \min\limits_{x} & c'x \\ \text{subject to} & Ax \leq B \\ & VLB \leq x \leq VUB \end{array}$$

where $x = [\alpha_1, ..., \alpha_N, t]$, $c' = [0, ..., 0, 1]$ so that $c'x = t$, matrices $A$ and $B$ are constructed using (2), the initial condition $T_1$ and the desired torque profile $\hat{T}$. Finally vectors $VLB$ and $VUB$ bound $\alpha_k$ betwen 0 and $\alpha_{\max}$ and do not put any bound on $t$.

## 3.3 LP solution

The solution to the LP in (3) provides the samples $\alpha$ and correspondingly a profile $T$ that is feasible for the discretized air dynamics. Since this discretized linear model is a conservative approximation for the non linear one, the profile $T$ is feasible for the original dynamics too. Figure 3 depicts the desired torque profile $\hat{T}$ (solid line), the solution to the LP $T$ (dashed line) and the rigid model approximation (dotted line).

Figure 3 corresponds to an initial torque $T_1 = 17$ Nm and Figure 4 corresponds to an initial torque $T_1 = 40$ Nm.

When we start from higher initial values of the torque the engine revolution speed is higher. Then the desired piece-wise constant profile follows the continuous time profile (Figure reffig:opttorquecont) more closely. In fact Figure 4 shows sharper and bigger steps at the beginning and at the end of the desired torque profile $\hat{T}$. However the air dynamics imposes a strong slew-rate limitation on the acheivable profiles $T$ and these steps cannot be closely tracked. Since we are using the max-norm, the same big error ($\simeq 22$ Nm) that we have at those steps is acheived at several points of the profile. As will be shown in the simulations, this large error in the torque may lead to undesired results.

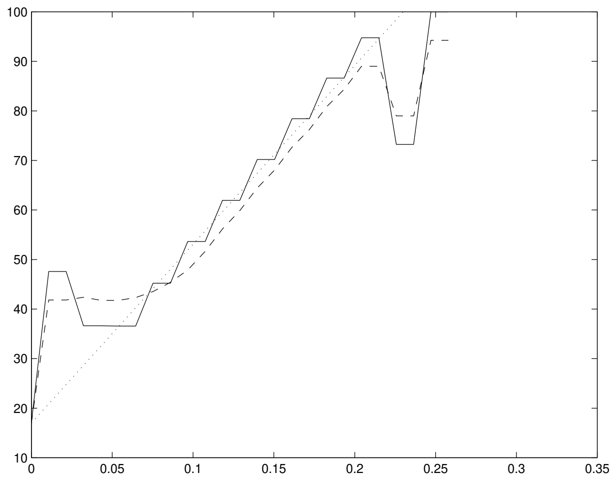


Figure 3: LP solution and rigid model approximation ($T_1 = 17$ Nm)

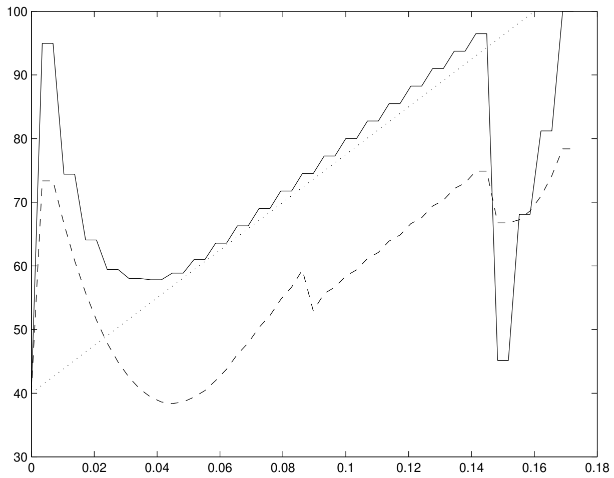


Figure 4: LP solution and rigid model approximation ($T_1 = 40$ Nm)

# 4 Simulation Results

The air profile obtained by the solution to the LP has been used to generate the reference (open loop trajectory planner) for the air dynamics controller (closed loop variable structure robust controller), in place of the simpler rigid transmission model. The results of the simulations are depicted in Figures 5 and 7 for the rigid model planner and in Figures 6 and 8 for the LP solution planner.

For initial condition $T_1 = 17$Nm, the transient time is reduced from about 349 msec to about 305 msec when using the LP solution.

For initial condition $T_1 = 40$Nm, the LP solution is so far (error $\simeq$ 22 Nm) from the desired profile that the control algorithm does not converge properly.

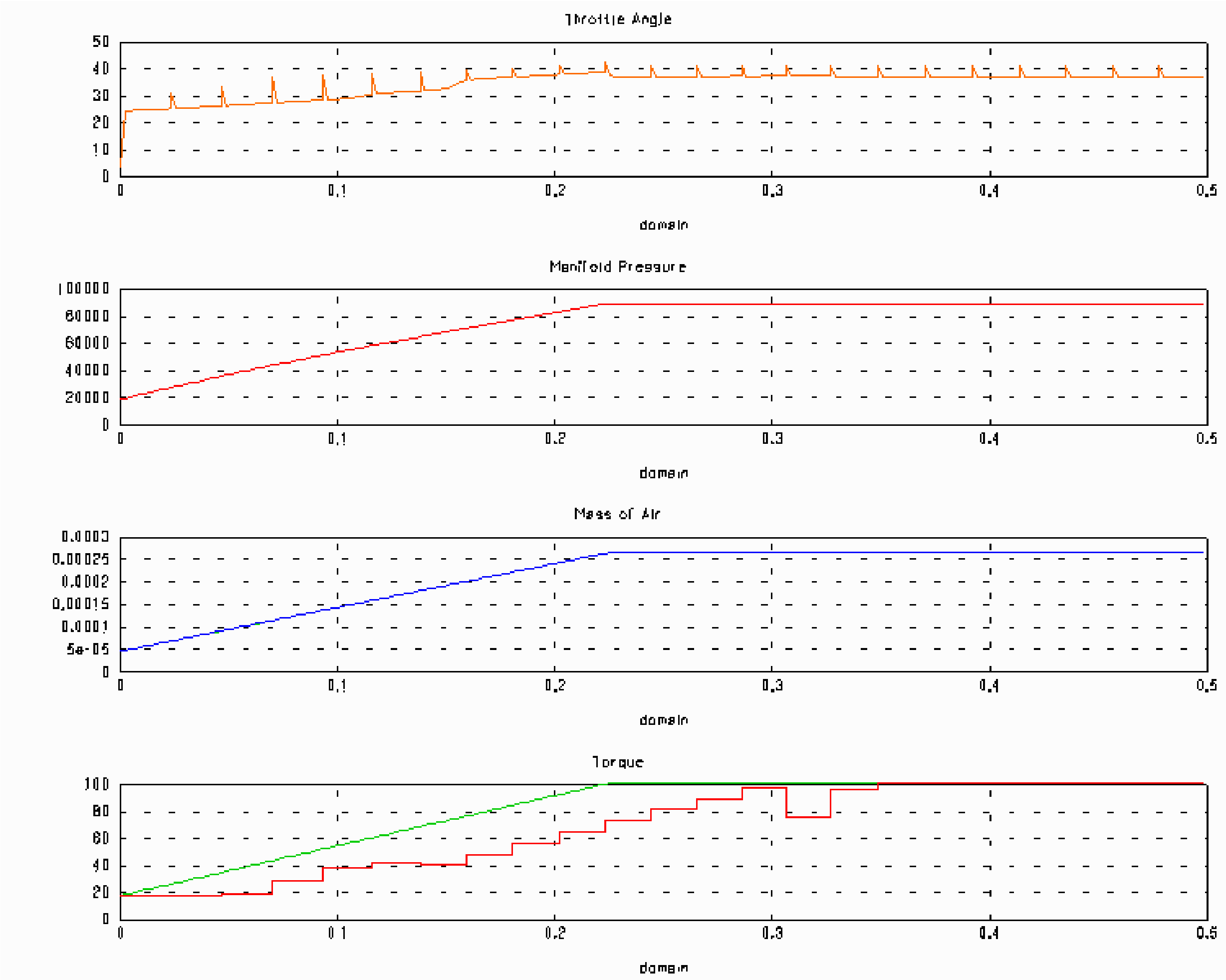


Figure 5: Closed loop trajectory using the rigid model ($T_1 = 17$ Nm)

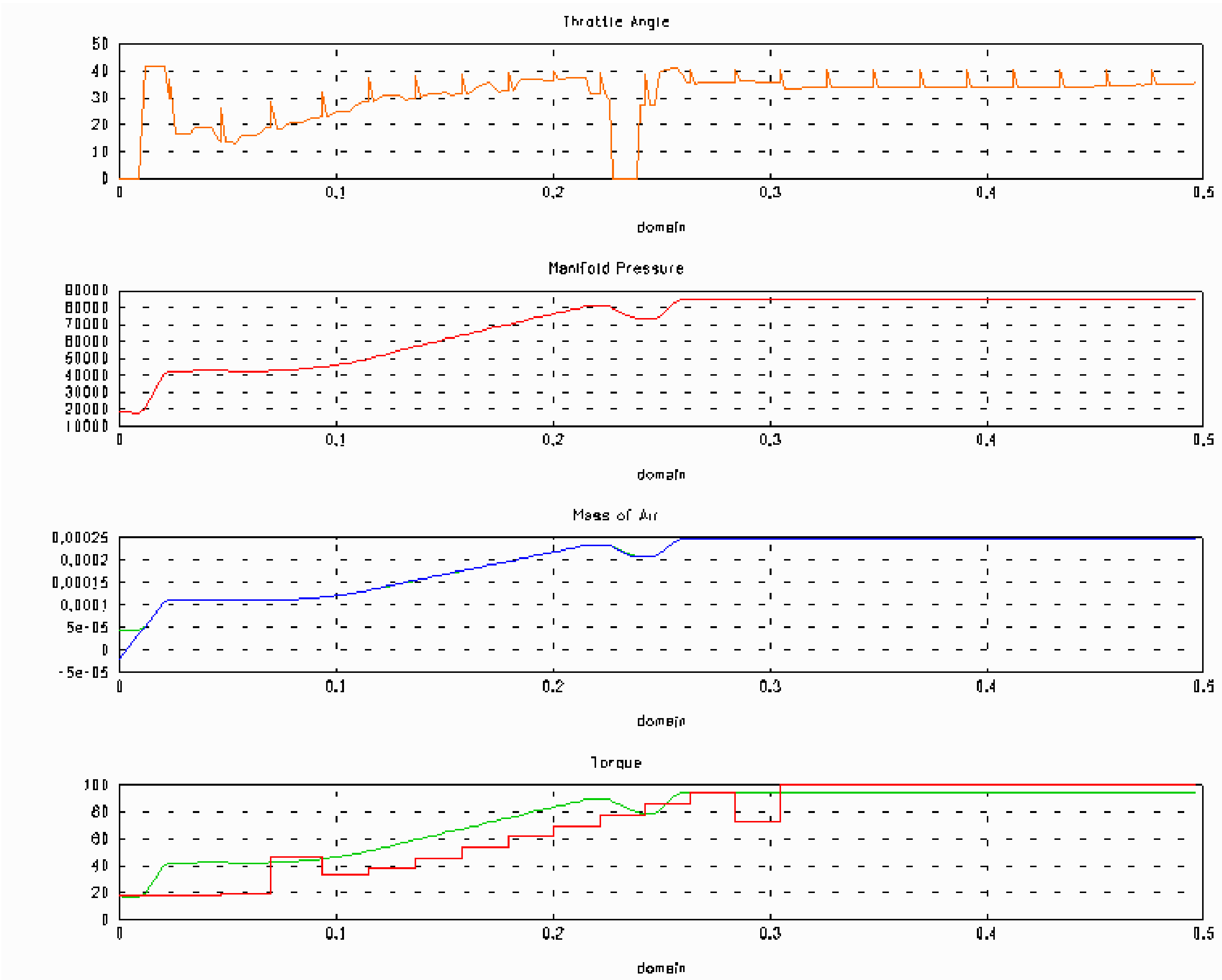


Figure 6: Closed loop trajectory using the LP solution ($T_1 = 17$ Nm)

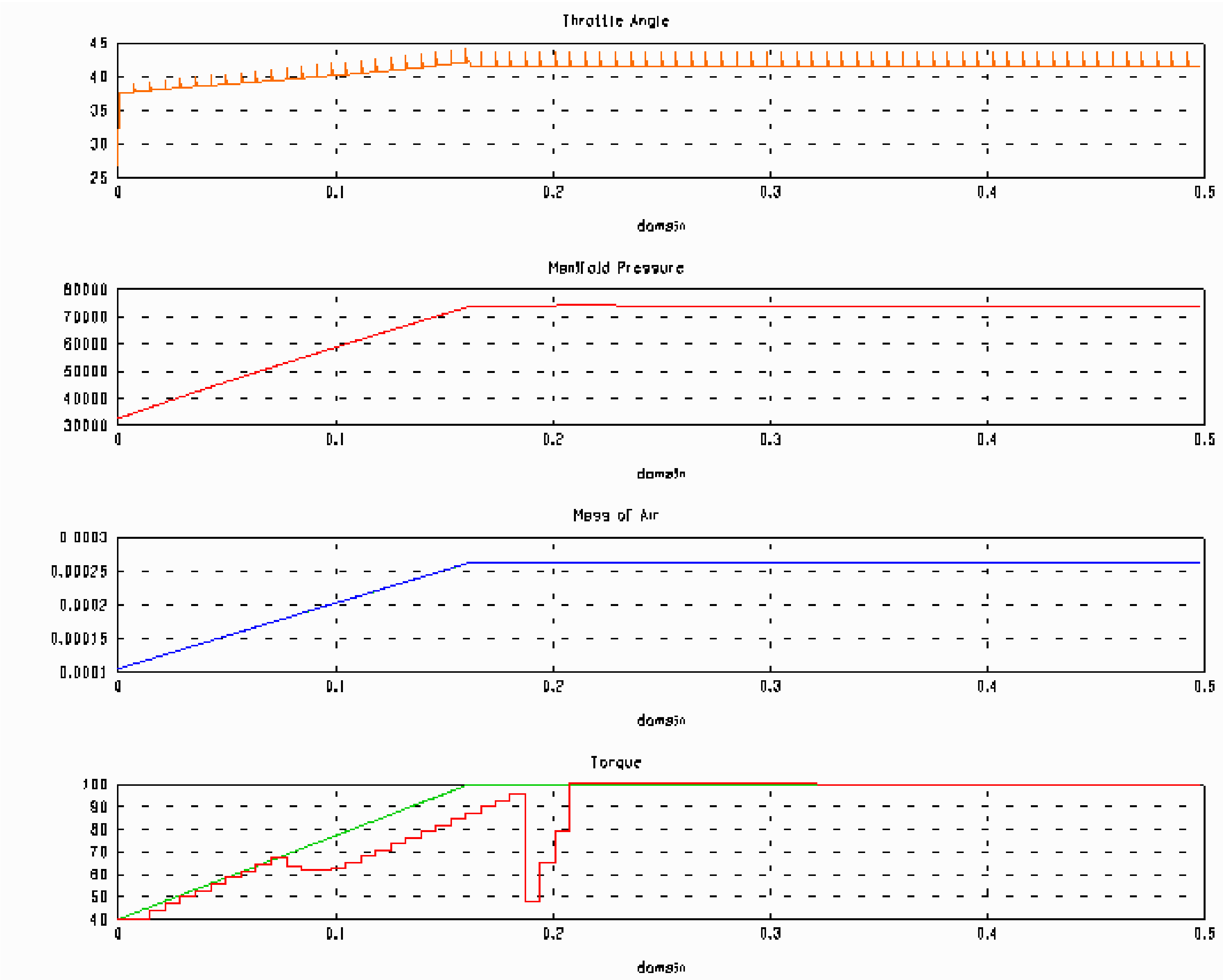


Figure 7: Closed loop trajectory using the rigid model ($T_1 = 40$ Nm)

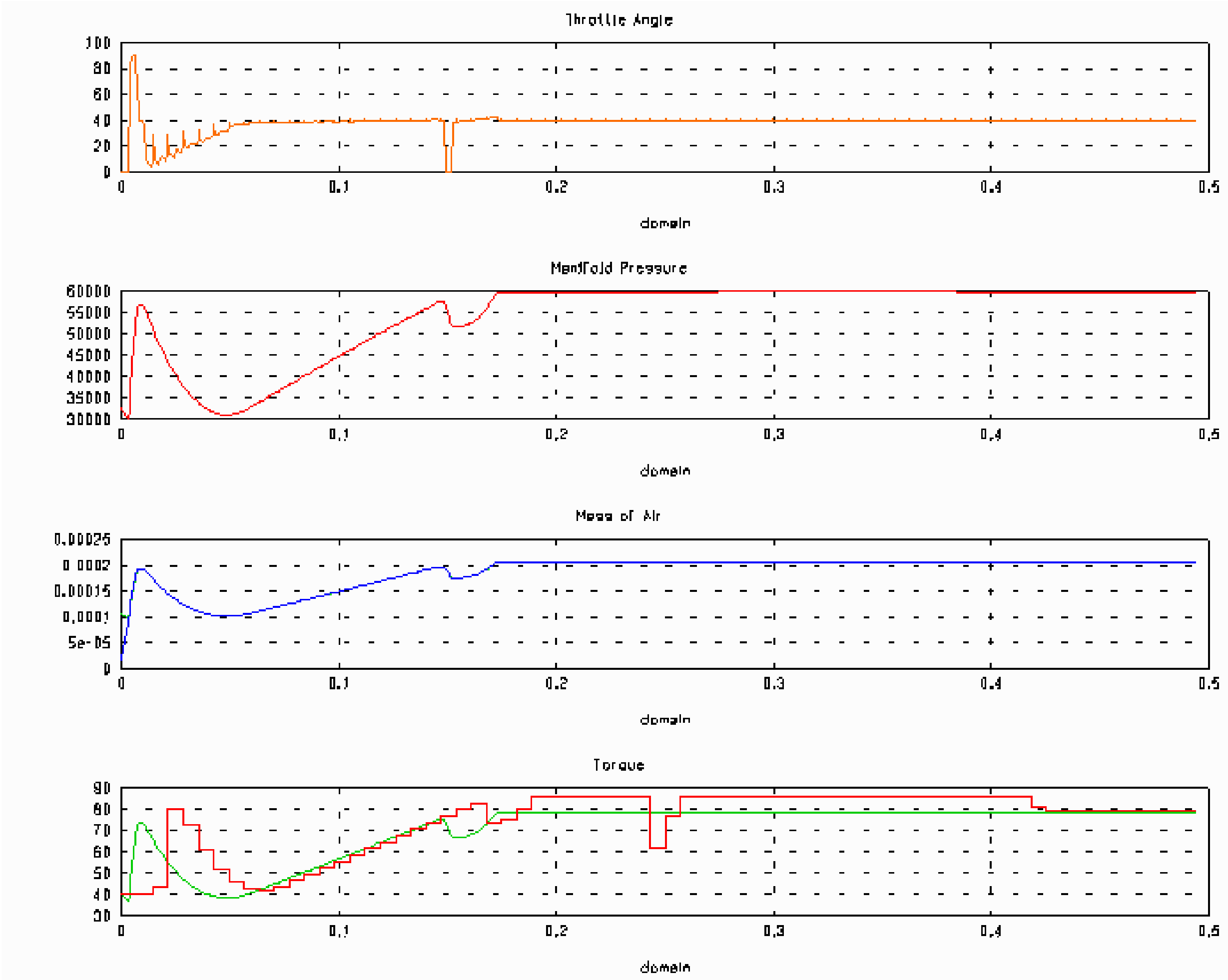


Figure 8: Closed loop trajectory using the LP solution ($T_1 = 40$ Nm)

# 5 Conclusions

The simple rigid transmission model is sufficient to guarantee convergence of the closed loop trajectories. However it is possible to improve performance in terms of transient time. The lack of performance in the rigid model solution is mainly due to the absence of the first peak in the torque profile. As it is well illustrated in the LP-based solutions, it is possible to acheive a good initial peak and thus improve on the transient time. However having chosen the max-norm of the error as our cost function, may result in a poor air reference which is significantly far from the desired profile.

The LP-based trajectory planner is able to improve about 44 msec over the rigid-model-based one for low revolution speed. On the other hand errors up to 22 Nm with respect to the desired profile may lead to poor performance of the closed loop controller.

A good optimization problem should acheive the first peak as closely as possible and then (as a secondary objective) try to track the rest of the profile as close as possible. The max-norm instead does not try to correct any error which is below the max so that if the first step is bigger than the slew-rate allows for, the rest of the samples may be affected by the same amount of error even if it were possible to track them perfectly.

## 5.1 Future Work

To acheive better performance also at high revolution speeds, it is necessary to change the cost function in the optimization problem.

One possible solution could be to compute the first sample separately, then use an LP to track the desired profile from the second sample to the second to last and having as initial condition (for the second sample) the result of the first step. Finally the last sample could be calculated starting from the previous step. This would allow to decouple the big errors in the initial and final peak samples from the error in the smooth part of the desired profile.

Alternatively one could use a different norm, (Least Squares for example) to force the correction also for errors in the smooth part of the desired profile.

# References


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