EE 290N Class Project (Fall 1999)

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1 Introduction

The demand for high quality wireless communication services, and supporting a seamless transmission of information across wireless and backbone wired networks is increasing at an explosive rate. However, the inherent limited supply of bandwidth and the unpredictability of the propagation channel lead to the investigation of more spectrally efficient and reliable designs. An approach to achieve high spectral efficiency in the presence of channel distortion is to split up the channel bandwidth into small chunks of size much narrower than the coherence bandwidth. To elaborate, suppose $H(f)$ is the frequency response of a nonideal, band-limited channel with a bandwidth of W, and $\Phi_n(f)$ is the power spectral density of the additive Gaussian noise. Then dividing the bandwidth W into $N = W/\Delta f$ subbands of width Δf, where Δf is chosen sufficiently small such that $|H(f)|^2/\Phi_n(f)$ is approximately constant within each subband. With a constraint on the total transmit power, P_{Total}

$$\sum_{k=0}^{N-1} P_k \leq P_{Total}$$

where P_k is the power allocated to the k-th subband, the optimum power allocation is the well-known waterfilling solution. However, as the number of subband increases, the peak-to-average power (PAP) ratio of the transmitted and received signals increases as well. Large peaks require large linear ranges in the amplifiers to avoid nonlinear distortion and spectral spreading. Correspondingly, the power efficiency of the amplifiers becomes very low. Furthermore, the large PAP ratio also reduces the resolution of the digital-to-analog (D/A) and analog-to-digital (A/D) converters in the transmitter and receiver respectively. Therefore, to be more rigorous, we should constrain the peak power of the transmit and receive signals as well. In the next section, we will show how to formulate this optimization problem in maximizing the channel capacity with the total transmit power and peak power constraints into a *Geometric programming problem.*

2 Problem Formulation

Recall that the capacity of an ideal, band-limited, AWGN channel is

$$C = W \log_2 \left(1 + \frac{P_{Total}}{W N_0} \right)$$

where C is in bits/s. In the multicarrier system described above, the total channel capacity is

$$C = \Delta f \sum_{k=0}^{N-1} \log_2 \left[1 + \frac{P_k \left| H(k\Delta f) \right|^2}{\Phi_n(k\Delta f)} \right] \tag{2.1}$$

The transmitted signal at time t is

$$x(t) = \sum_{k=0}^{N-1} \sqrt{P_k} X_k e^{j2\pi k \Delta f t} \tag{2.2}$$

where $X_k = a_k + jb_k$ is the modulated symbols. For QPSk, $a_k, b_k \in \{-1, 1\}$. The real and imaginary parts of the transmitted signal are given by

$$x_R(t) = \sum_{k=0}^{N-1} \sqrt{P_k} \big[a_k \cos(j2\pi k \Delta f t) - b_k \sin(j2\pi k \Delta f t) \big] \tag{2.3}$$

and

$$x_I(t) = \sum_{k=0}^{N-1} \sqrt{P_k} \big[a_k \sin(j2\pi k \Delta f t) + b_k \cos(j2\pi k \Delta f t) \big] \tag{2.4}$$

The received signal at time t is

$$y(t) = \sum_{k=0}^{N-1} H(k\Delta f) \sqrt{P_k} X_k e^{j2\pi k \Delta f t} \tag{2.5}$$

The real and imaginary parts of the recieved signal are given by

$$\begin{aligned} y_R(t) = \sum_{k=0}^{N-1} \sqrt{P_k} \big[& H_R(k\Delta f)\, a_k \cos(j2\pi k \Delta f t) - H_R(k\Delta f)\, b_k \sin(j2\pi k \Delta f t) \\ & - H_I(k\Delta f)\, a_k \sin(j2\pi k \Delta f t) - H_I(k\Delta f)\, b_k \cos(j2\pi k \Delta f t) \big] \end{aligned} \tag{2.6}$$

and

$$\begin{aligned} y_I(t) = \sum_{k=0}^{N-1} \sqrt{P_k} \big[& H_R(k\Delta f)\, a_k \sin(j2\pi k \Delta f t) + H_R(k\Delta f)\, b_k \cos(j2\pi k \Delta f t) \\ & + H_I(k\Delta f)\, a_k \cos(j2\pi k \Delta f t) - H_I(k\Delta f)\, b_k \sin(j2\pi k \Delta f t) \big] \end{aligned} \tag{2.7}$$

Thus, the optimization problem is

$$
\begin{array}{lll}
\text{maximize} & \Delta f \sum_{k=0}^{N-1} \log_2 \left[1 + \frac{P_k\,|H(k\Delta f)|^2}{\Phi_n(k\Delta f)}\right] & \\
\text{subject to} & \sum_{k=0}^{N-1} P_k \le P_{Total} & \\
& x_R(t) \le P_{Peak} & \forall\, t \text{ and } a_k, b_k \in \{-1, 1\} \\
& x_I(t) \le P_{Peak} & \forall\, t \text{ and } a_k, b_k \in \{-1, 1\} \\
& y_R(t) \le P_{Peak} & \forall\, t \text{ and } a_k, b_k \in \{-1, 1\} \\
& y_I(t) \le P_{Peak} & \forall\, t \text{ and } a_k, b_k \in \{-1, 1\} \\
& \mathbf{P} \succeq \mathbf{0} &
\end{array}
\tag{2.8}
$$

which is equivalent to

$$
\begin{array}{lll}
\text{maximize} & \Pi_{k=0}^{N-1} \left[1 + \alpha_k P_k\right] & \\
\text{subject to} & \mathbf{1}^T \mathbf{P} \le P_{Total} & \\
& \mathbf{s}(\mathbf{a}, \mathbf{b}, t)^T \sqrt{\mathbf{P}} \le 1 & \forall\, t \text{ and } \mathbf{a}, \mathbf{b} \in \{-1, 1\}^N \\
& \mathbf{u}(\mathbf{a}, \mathbf{b}, t)^T \sqrt{\mathbf{P}} \le 1 & \forall\, t \text{ and } \mathbf{a}, \mathbf{b} \in \{-1, 1\}^N \\
& \mathbf{v}(\mathbf{a}, \mathbf{b}, t)^T \sqrt{\mathbf{P}} \le 1 & \forall\, t \text{ and } \mathbf{a}, \mathbf{b} \in \{-1, 1\}^N \\
& \mathbf{w}(\mathbf{a}, \mathbf{b}, t)^T \sqrt{\mathbf{P}} \le 1 & \forall\, t \text{ and } \mathbf{a}, \mathbf{b} \in \{-1, 1\}^N \\
& \mathbf{P} \succeq \mathbf{0} &
\end{array}
\tag{2.9}
$$

where

$$
\begin{aligned}
s_k(\mathbf{a}, \mathbf{b}, t) &= \big[a_k \cos(j2\pi k\Delta f t) - b_k \sin(j2\pi k\Delta f t)\big]/P_{Peak} \\
u_k(\mathbf{a}, \mathbf{b}, t) &= \big[a_k \sin(j2\pi k\Delta f t) + b_k \cos(j2\pi k\Delta f t)\big]/P_{Peak} \\
v_k(\mathbf{a}, \mathbf{b}, t) &= \big[H_R(k\Delta f)\, a_k \cos(j2\pi k\Delta f t) - H_R(k\Delta f)\, b_k \sin(j2\pi k\Delta f t)) \\
&\quad - H_I(k\Delta f)\, a_k \sin(j2\pi k\Delta f t) - H_I(k\Delta f)\, b_k \cos(j2\pi k\Delta f t)\big]/P_{Peak} \\
w_k(\mathbf{a}, \mathbf{b}, t) &= \big[H_R(k\Delta f)\, a_k \sin(j2\pi k\Delta f t) + H_R(k\Delta f)\, b_k \cos(j2\pi k\Delta f t) \\
&\quad + H_I(k\Delta f)\, a_k \cos(j2\pi k\Delta f t) - H_I(k\Delta f)\, b_k \sin(j2\pi k\Delta f t)\big]/P_{Peak} \\
\alpha_k &= \frac{|H(k\Delta f)|^2}{\Phi_n(k\Delta f)}
\end{aligned}
$$

Obviously, the optimization problem (2.9) is a *Geometric programming problem.*