Lecture 8 — September 25

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8.1 Lecture Outline

- determinininistic networks
- binary expansion model

8.2 Most General Channel Model

The channel models we will study satisify the following general properties,

- node i transmits x_i at time t
- node j receives y_j at time t
- the channel is memoryless

The last property allows the interaction between node j and its m_j incident nodes $i_1, ..., i_{m_j}$ to be fully characterized by the a probability distribution $p_j(y_j|x_1, ..., x_{i_{m_j}})$ which is independent of time.

Next, we will look at two special cases of the general channel model: the wireline and the wireless.

8.2.1 Wireline Model

Consider the unit link capacity wireless model, node i transmits,

$$x_i = [x_{i,j_1}, x_{i,j_2}, ..., x_{i,j_{m_i}}]^T, \;\; x_i \in \mathbb{F}, \quad (i, j_k) \in E \tag{8.1}$$

Node j recives, y_j, which can be decomposed into symbols for each link,

$$y_i = [y_{i_1,j}, y_{i_2,j}, ..., y_{i_{m_j},j}]^T, \quad (i_k, j) \in E \tag{8.2}$$

Since the recieved symbols can be decomposed and there is no additional noise,

$$y_{ij} = x_{ij} \;\; \forall (i, j) \in E. \tag{8.3}$$

From these descriptions, we can see that wireline model is simple and statisifes the following properties,

- orthogonal inputs
- no interaction between signals at the receiver
- noiseless

8.2.2 Wireless Model

In the complex, wireless gaussian model, node i transmits x_i where $x_i \in \mathbb{C}$. Node j receives a noisy, superposition of signals from its j_m incident nodes.

$$y_j = \sum_{i=1}^{m_j} g_{ij} x_i + w_j, \quad (i,j) \in E \tag{8.4}$$

where $g_{ij} \in \mathbb{C}$, and $w_j \sim \mathcal{CN}(0, \sigma^2)$. From the channel model description, it can be seen that the wireless model differs from the wireline model in satisfying the following properties instead

- broadcast medium (at transmitter)
- superposition of signals at receiver
- additive noise at receiver

8.3 Deterministic Channel Model

To bridge the gap between the two models, we consider deterministic models of the form,

$$y_j = h_j(x_1, ..., x_{j_m}) \tag{8.5}$$

where h_j is a deterministic function mapping from $\mathcal{X}^n$ to $\mathcal{Y}$. Our goal is to eventually prove a Ford Fulkerson result for our deterministic models. From 8.4, we can see that linearity is a property of the wireless channel model. In order to capture this important property, we consider linear deterministic channel models with the binary expansion model as special case.

8.3.1 Binary Expansion Model

Consider a (real) point to point link,

$$y = gx + w \tag{8.6}$$

where $w \sim \mathcal{N}(0, 1)$ and the signal power is normalized to be one, i.e $E[x^2] = 1$. Therefore, $SNR = g^2$. Consider the binary expansion of x,

$$x = 0.b_1 b_2 b_3 ... \tag{8.7}$$

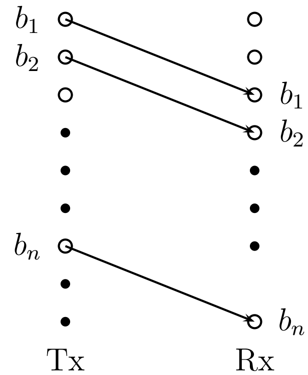

Figure 8.1. Binary Expansion Model for Point to Point Channel

Assume that the fade can be expressed as $g = 2^n$ for some $n \in \mathbb{N}_+$. It follows that the decimal point in the binary expansion of x is shifted to the right by n digits.

$$gx = b_1 b_2 ... b_{n-1} b_n . b_{n+1} b_{n+2} ... \tag{8.8}$$

Since the noise has unit power, the bits after the decimal expansion $b_{n+1}, b_{n+2}, ...$ will be corrupted and only the first n bits will be recieved reliably as shown in 8.1. This model suggest that the capacity of the channel is n.

The AWGN capacity is given by the well known formula,

$$C_{AWGN} = \frac{1}{2}\log(1 + SNR) \tag{8.9}$$

Since $SNR = g^2$, the capacity is,

$$\begin{aligned} C_{AWGN} &= \frac{1}{2}\log(1 + 2^{2n}) && (8.10) \\ &\approx n && (8.11) \end{aligned}$$

The approximation is accurate for n large (here, $n > 2$ is sufficiently large). Therefore, if SNR is moderate to high, then we can ignore the probabilistic aspects of the channel and use the binary expansion model.

Broadcast Channel

The classical broadcast channel consists of one transmitter and two recievers (one stronger and another weaker). Consider the deterministic model where the transmitter Tx wants to transmit 4 bits (per second) as shown in 8.2. The strong receiver Rx a is able to receive all 4 bits, while the weaker receiver Rx b is only able to receive the two most significant bits. This deterministic model for the broadcast channel has some resemblance to the wireline model. Typically, superposition coding is used at the transmitter. There is a common message sent

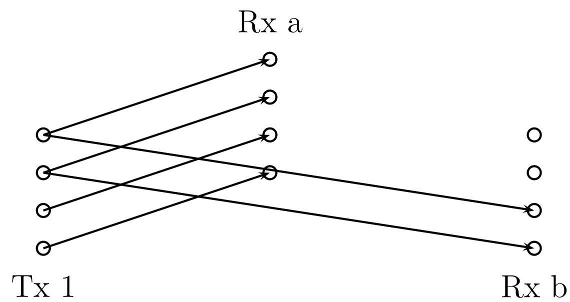

Figure 8.2. Deterministic for Broadcast Channel

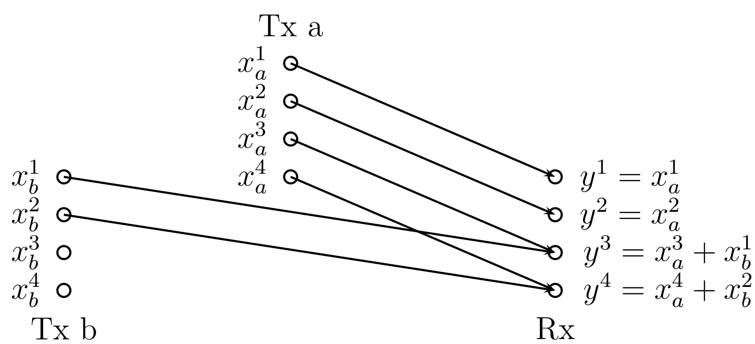

Figure 8.3. Deterministic for Multiple Access Channel

to both the users and an additional message sent to the stronger user. This is captured by the fact that both users receive the first two bits, while the stronger user receives an additional two bits of information.

Multiple Access Channel

In the multiple access channel we consider, two users (Tx a and Tx b) are trying to communicate to a single destination (Rx). Each user can communicate at most 4 bits . First, the receiver gets the first two bits of the stronger user. Then, he receives a superposition of signals from both users as shown in 8.3. We assume that the sum is a modulo sum and that a collision type effect happens so the transmitted signals cannot be decoded in this case.

Deterministic Model Notation

Let q denote the number of bits above noise level in the channel. For the point to point channel shown in 8.1, the transmitted and recieved signals can be written in vector form,

$$\mathbf{x} = [x^1, ..., x^q]^T \tag{8.12}$$
$$\mathbf{y} = [y^1, ..., y^q]^T \tag{8.13}$$

where $x_i, y_i \in \{0, 1\}$. Therefore, x_1 is the most significant bit. Let S denote the shfit matrix,

$$S = \begin{bmatrix} \mathbf{0}^T & 0 \\ \mathbf{I}_{q-1} & \mathbf{0} \end{bmatrix}$$

Let n model the channel strenght. For example, the fade in the point to point channel is expressed as $g = 2^n$. Using these notations, the deterministic point to point channel model can written as

$$\mathbf{y} = S^{q-n}\mathbf{x} \tag{8.14}$$

This model captures linearity of the system. From the above equation, it can be seen that the receiver gets the first n components of $\mathbf{x}$. In other words, the receiver gets the n most signficant bits from the transmitter.

The broadcast channel model can be written as,

$$\mathbf{y_a} = S^{q-n^a}\mathbf{x} \tag{8.15}$$
$$\mathbf{y_b} = S^{q-n^b}\mathbf{x} \tag{8.16}$$

Similiary, the multiple access channel model can be written as,

$$\mathbf{y} = S^{q-n_a}\mathbf{x_a} + S^{q-n_b}\mathbf{x_b} \tag{8.17}$$

Relay Channel

Consider the single relay channel where each node can transmit or receive up to 3 bits of information. The capacity of the deterministic relay channel is 3 bits/sec. The source can transmit 3 bits b_1, b_2, b_3. The relay recieves all three bits, while the receiver recieves only b_1, b_2 on its bottom two links. The relay can then forward b_1 to the receiver on the top link. This is shown in 8.4.

Another scheme to achieve capacity is amplify and forward. Therefore, the relay simply sends b_1 on its top link, b_2 on its second link, and b_3 on its last link. The receiver gets

$$y^1 = b_1 \tag{8.18}$$
$$y^2 = b_1 + b_2 \tag{8.19}$$
$$y^3 = b_3 + b_2 \tag{8.20}$$

From this set of linear equations, b_1, b_2, b_3 can be solved.

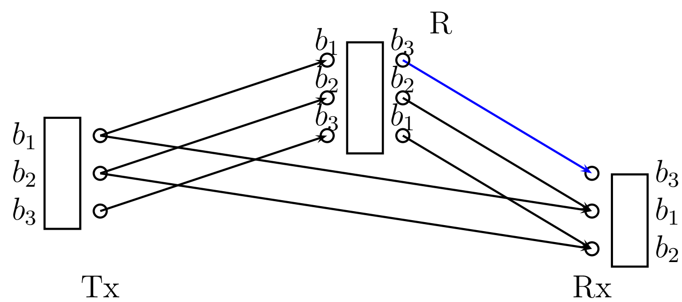

Figure 8.4. Deterministic Model for Relay Channel