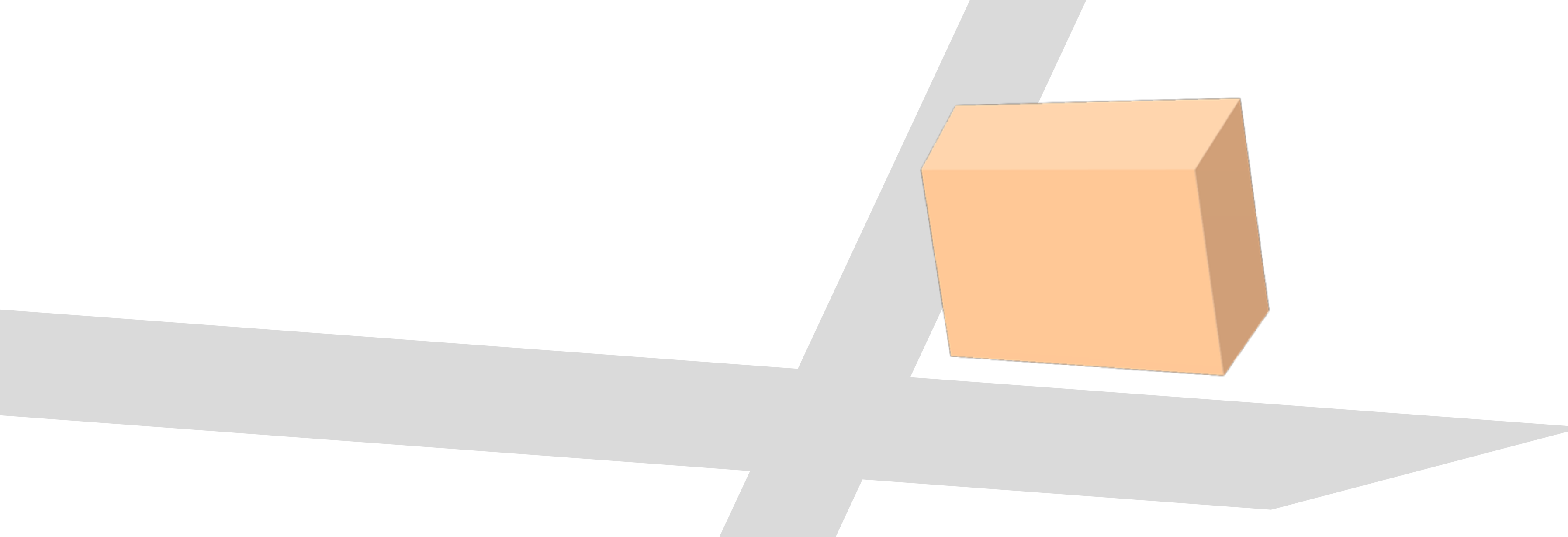
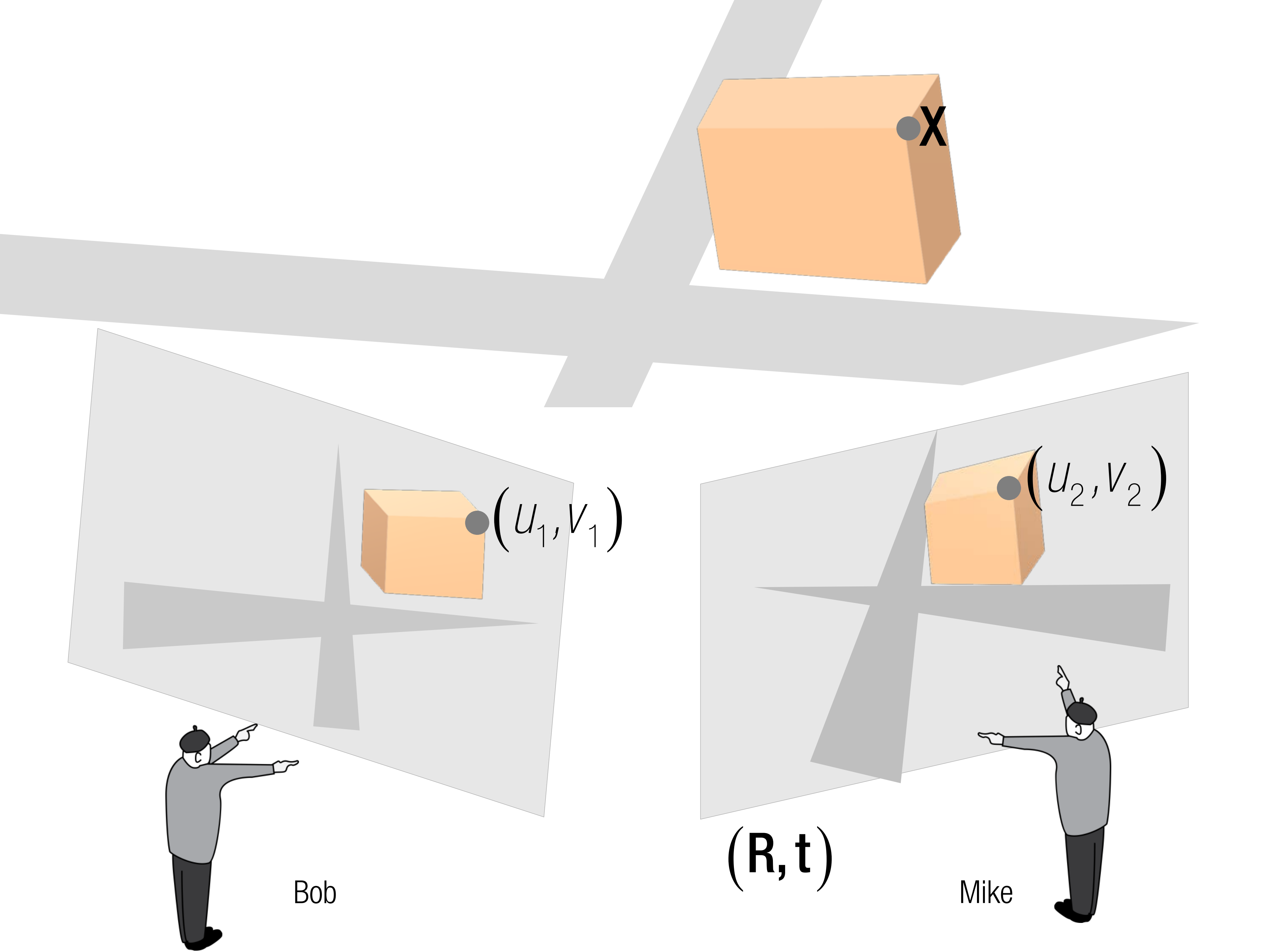
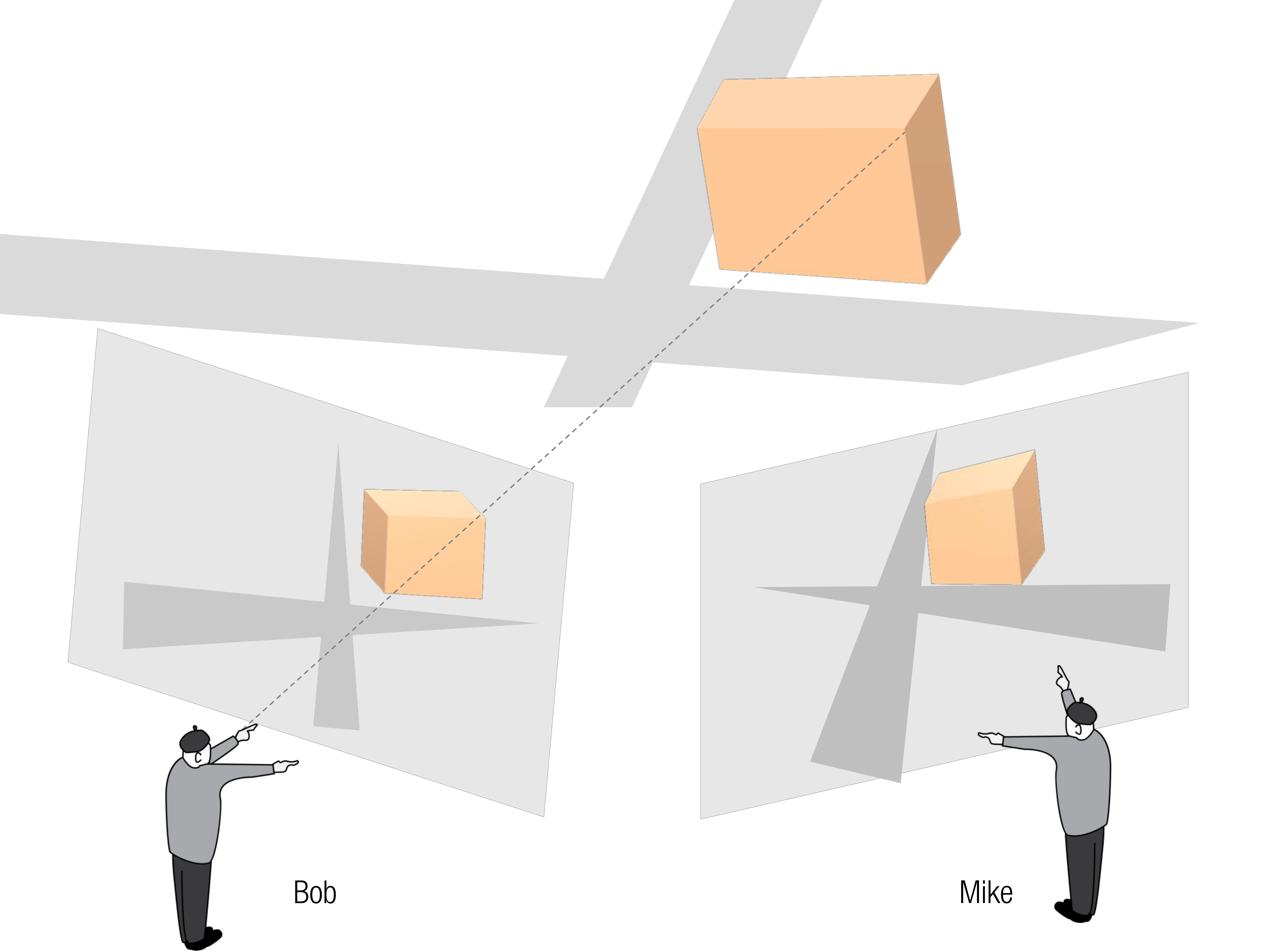
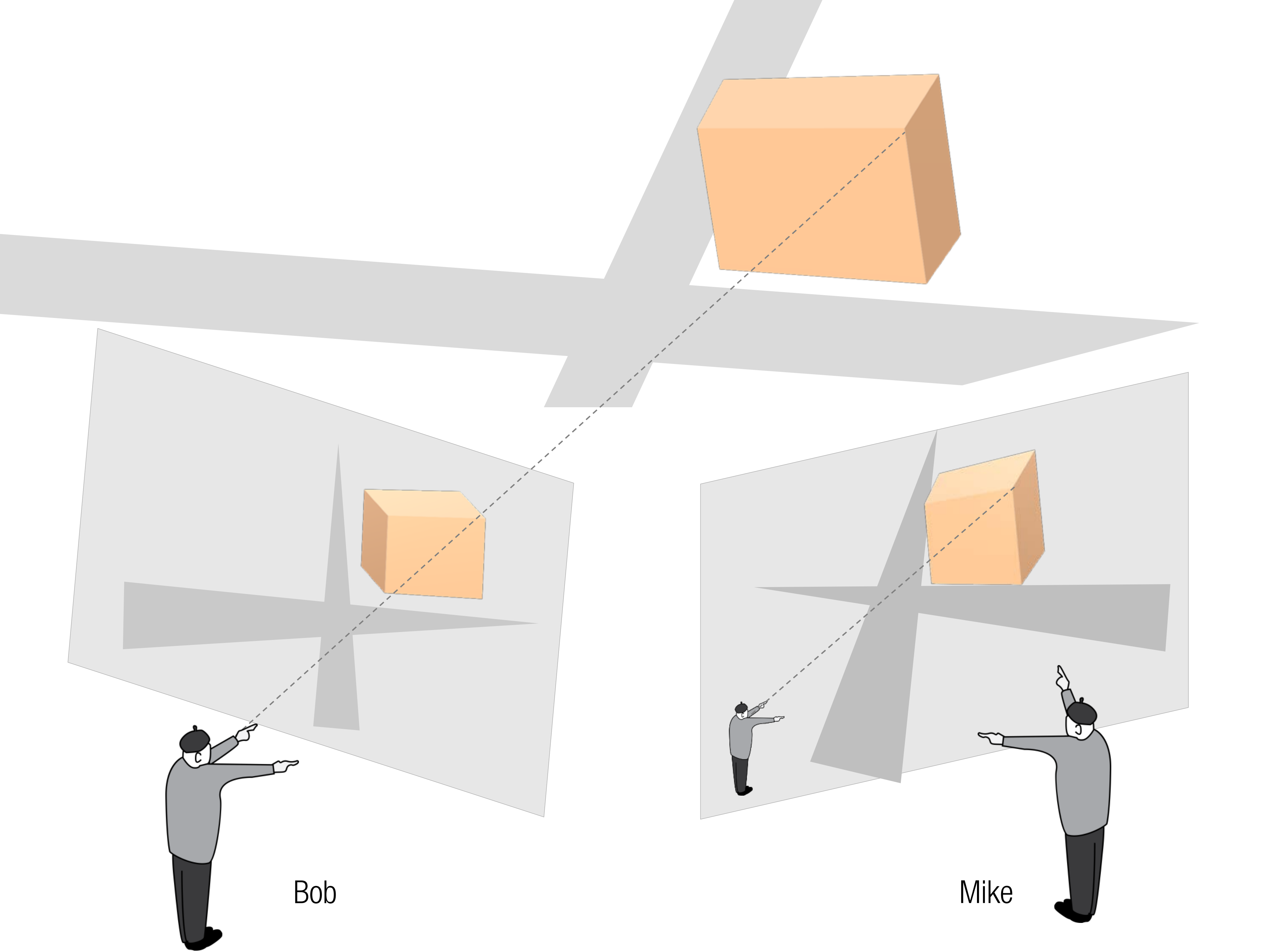
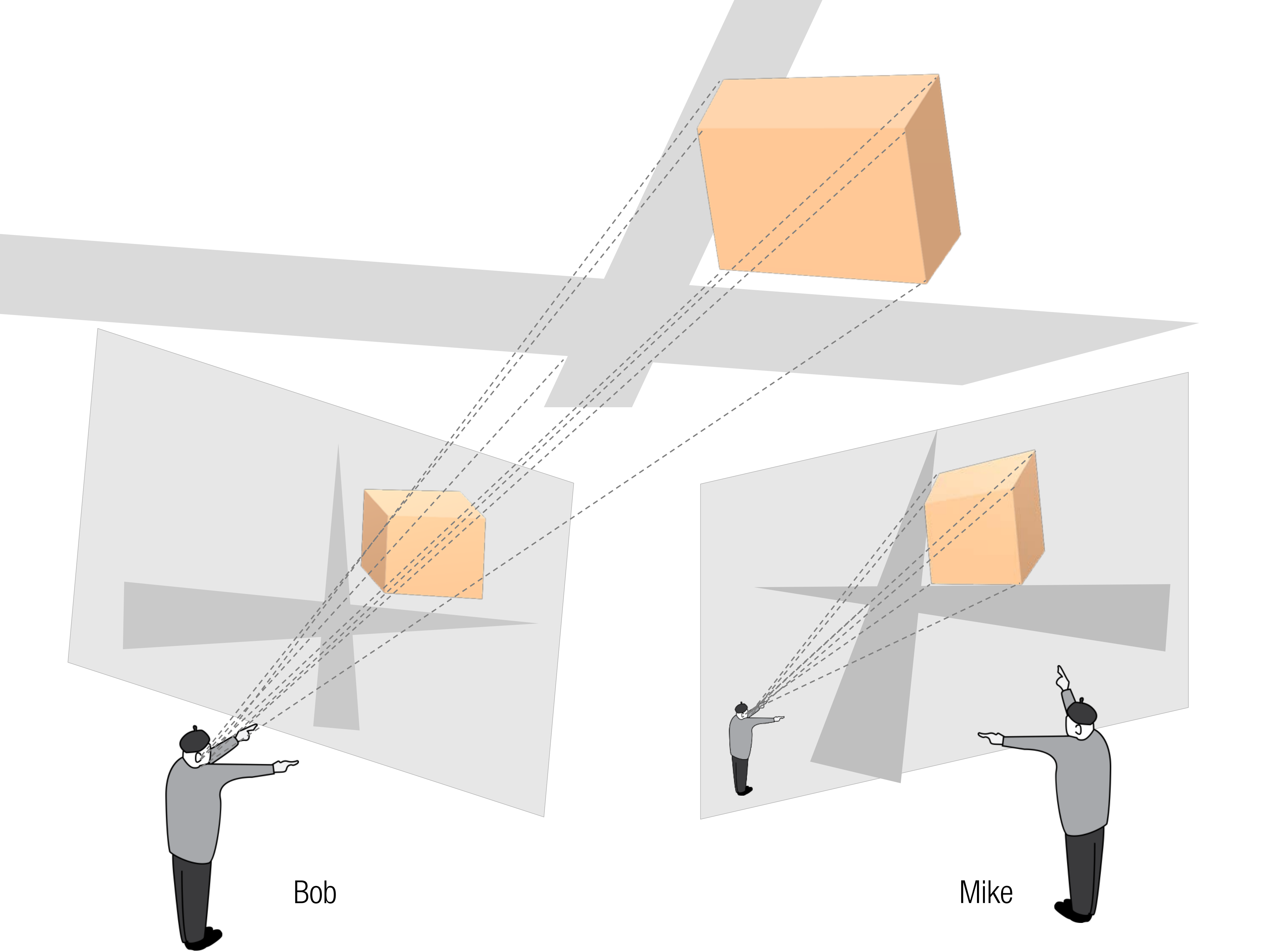
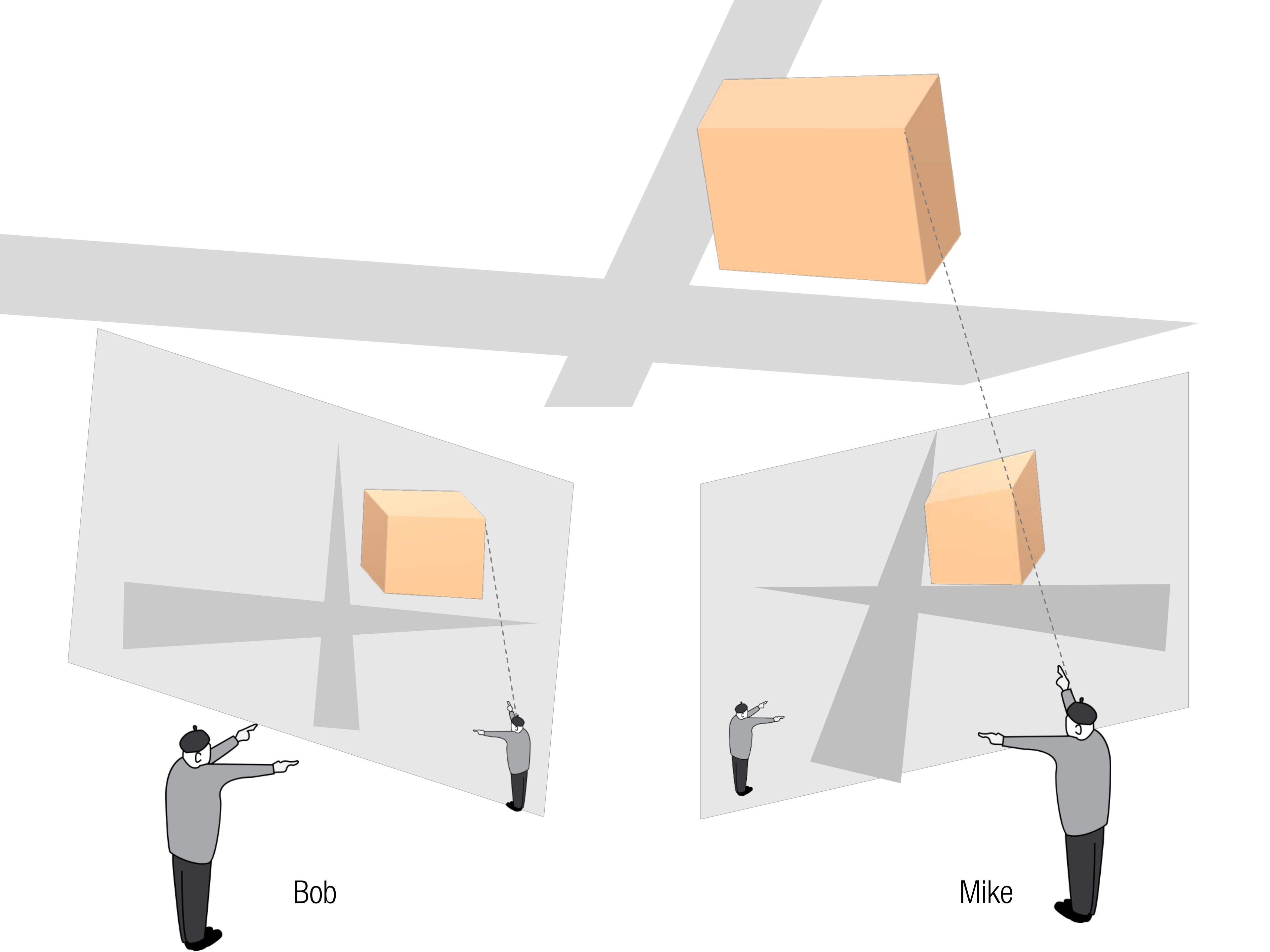


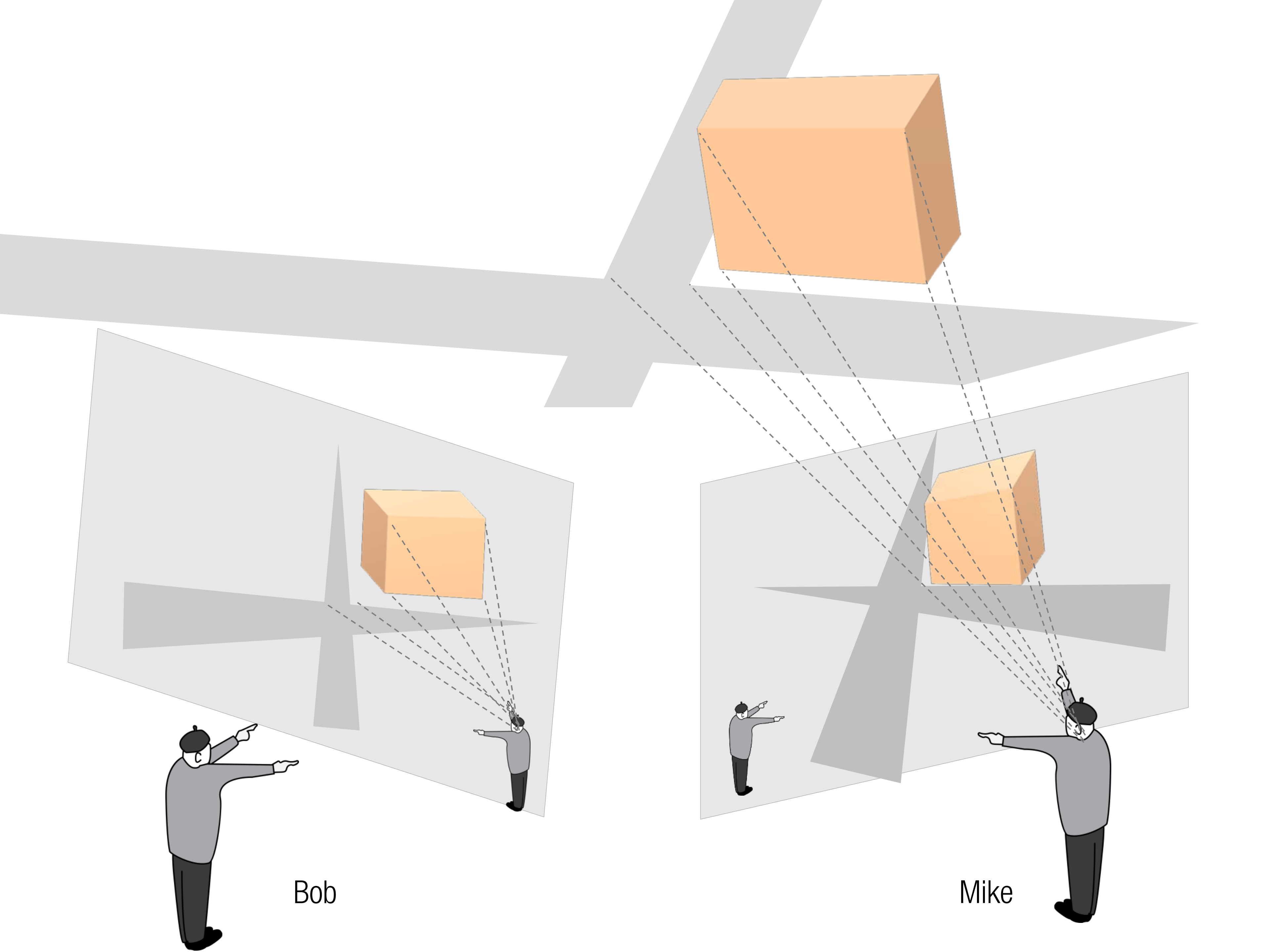

X
(u_1, v_1)
(u_2, v_2)
$(\mathbf{R}, \mathbf{t})$
Bob
Mike

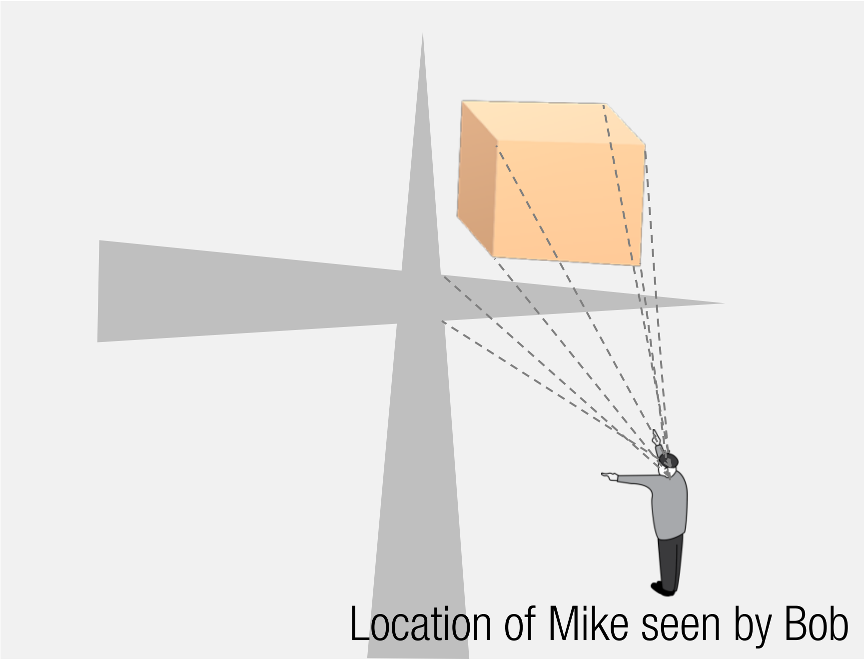

Bob
Mike


Bob
Mike

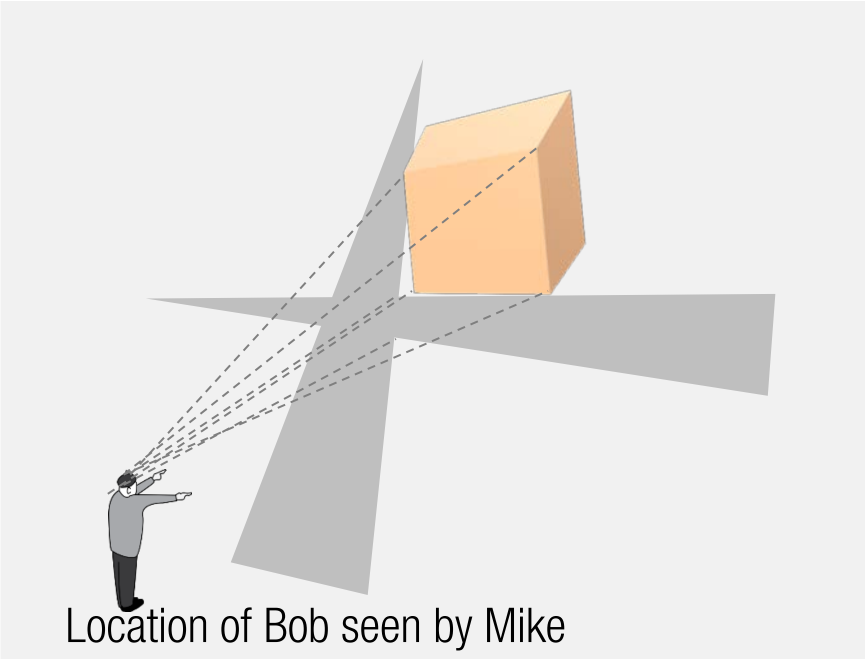

Bob
Mike


Bob
Mike

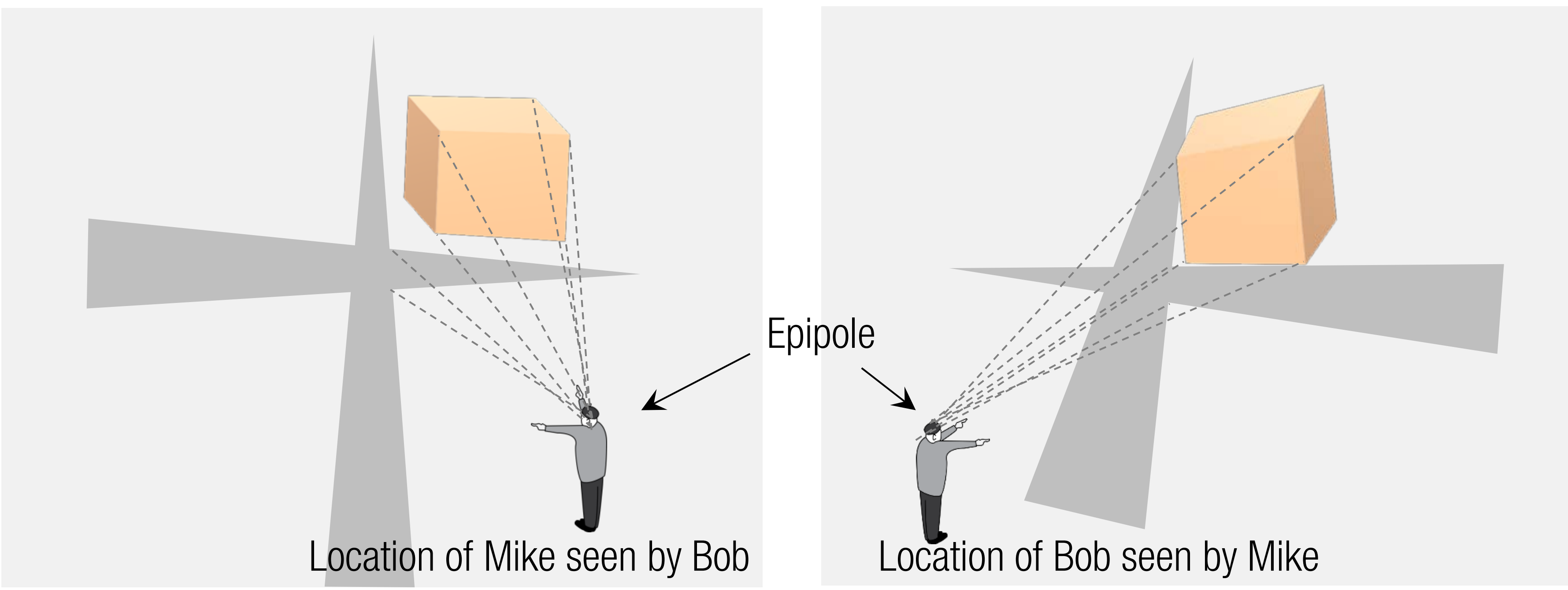

Bob
Mike



Bob's view

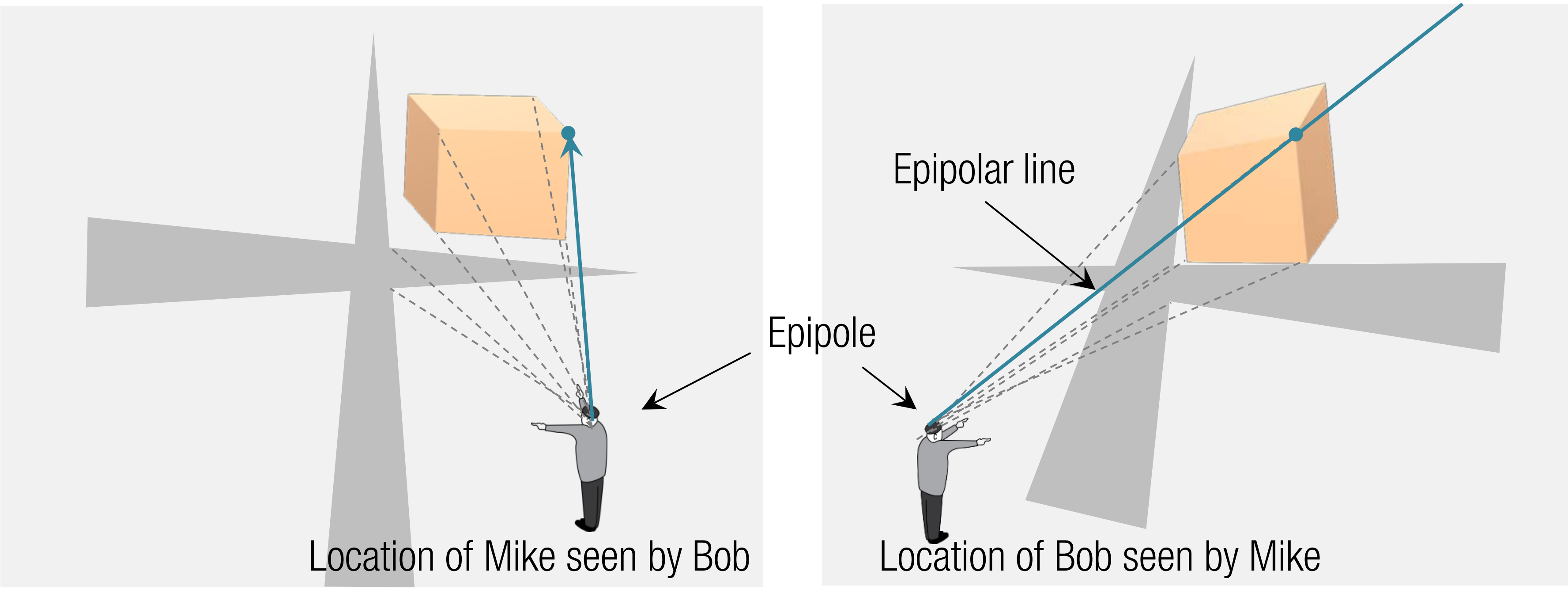


Mike's view



Bob's view

Mike's view



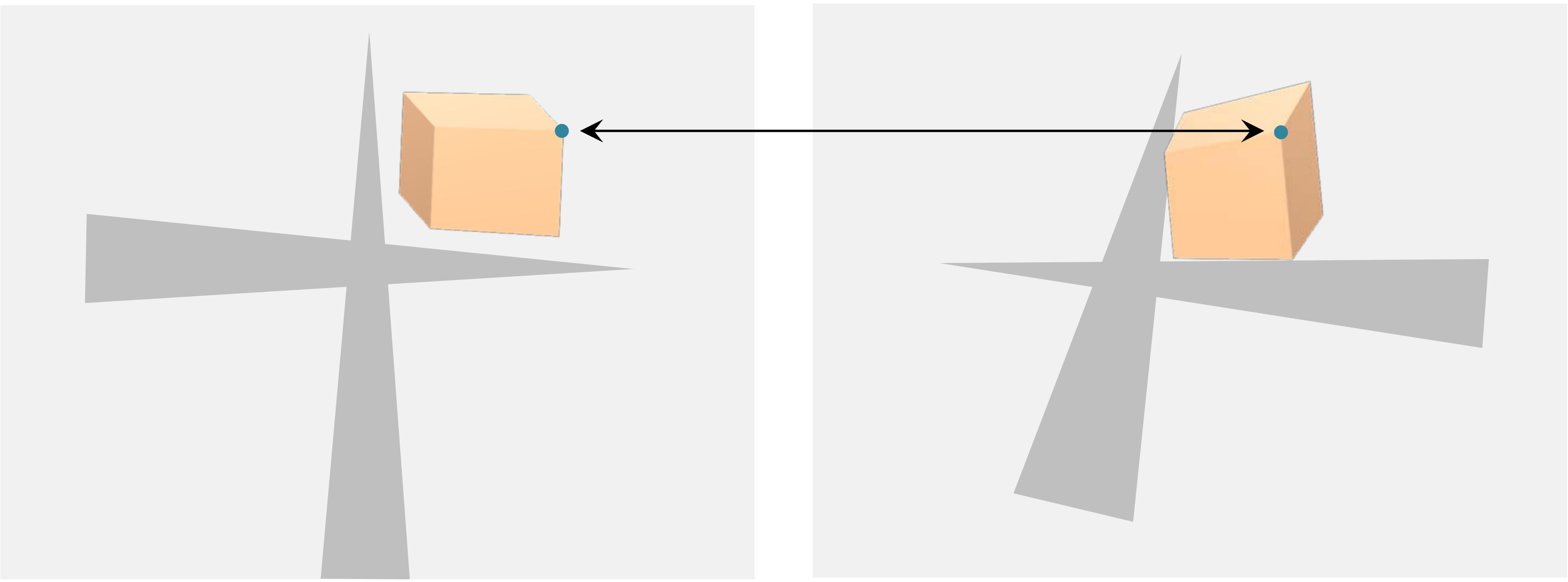
Bob’s view

Mike’s view

Observation:

Given a point in Bob’s view, there exists a conjugate line passing the corresponding point in Mike’s view.

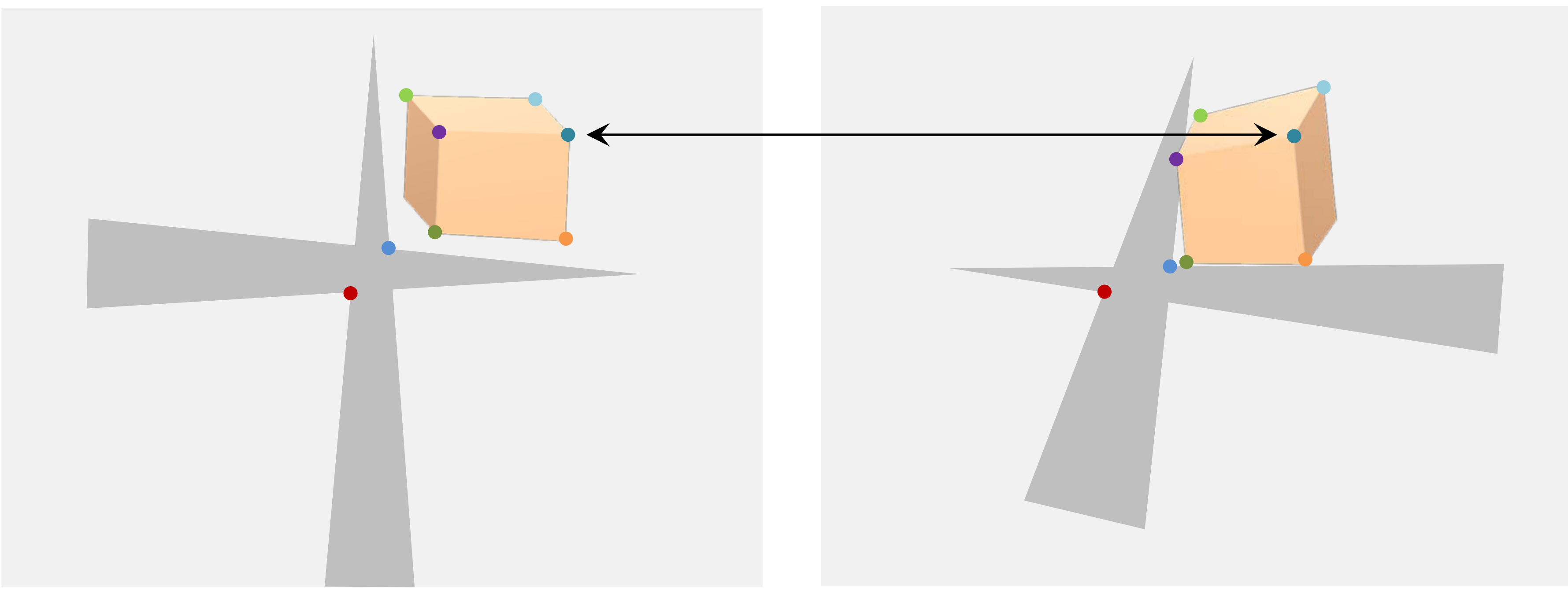
Point correspondence



Bob's view

Mike's view

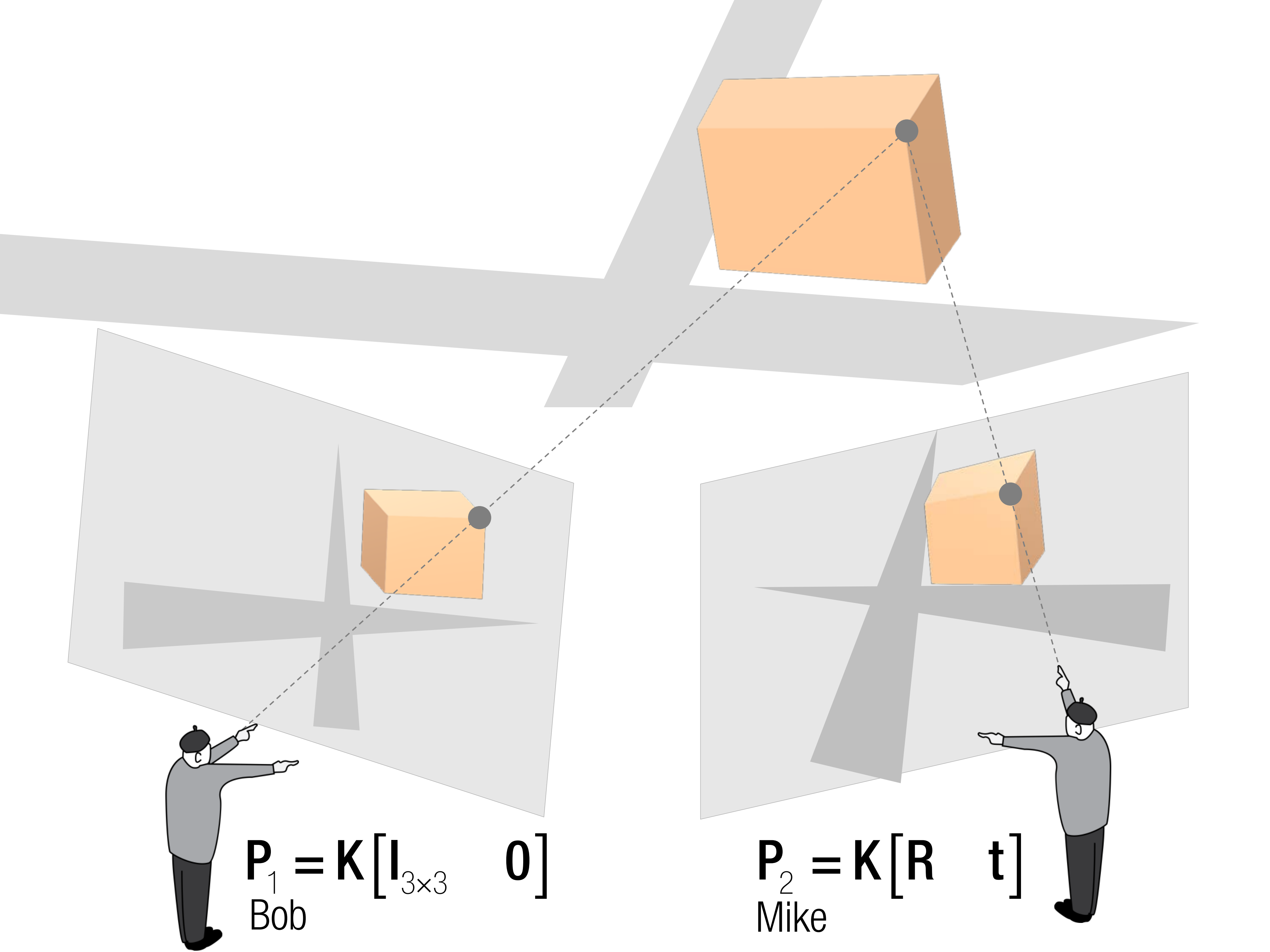
Point correspondence

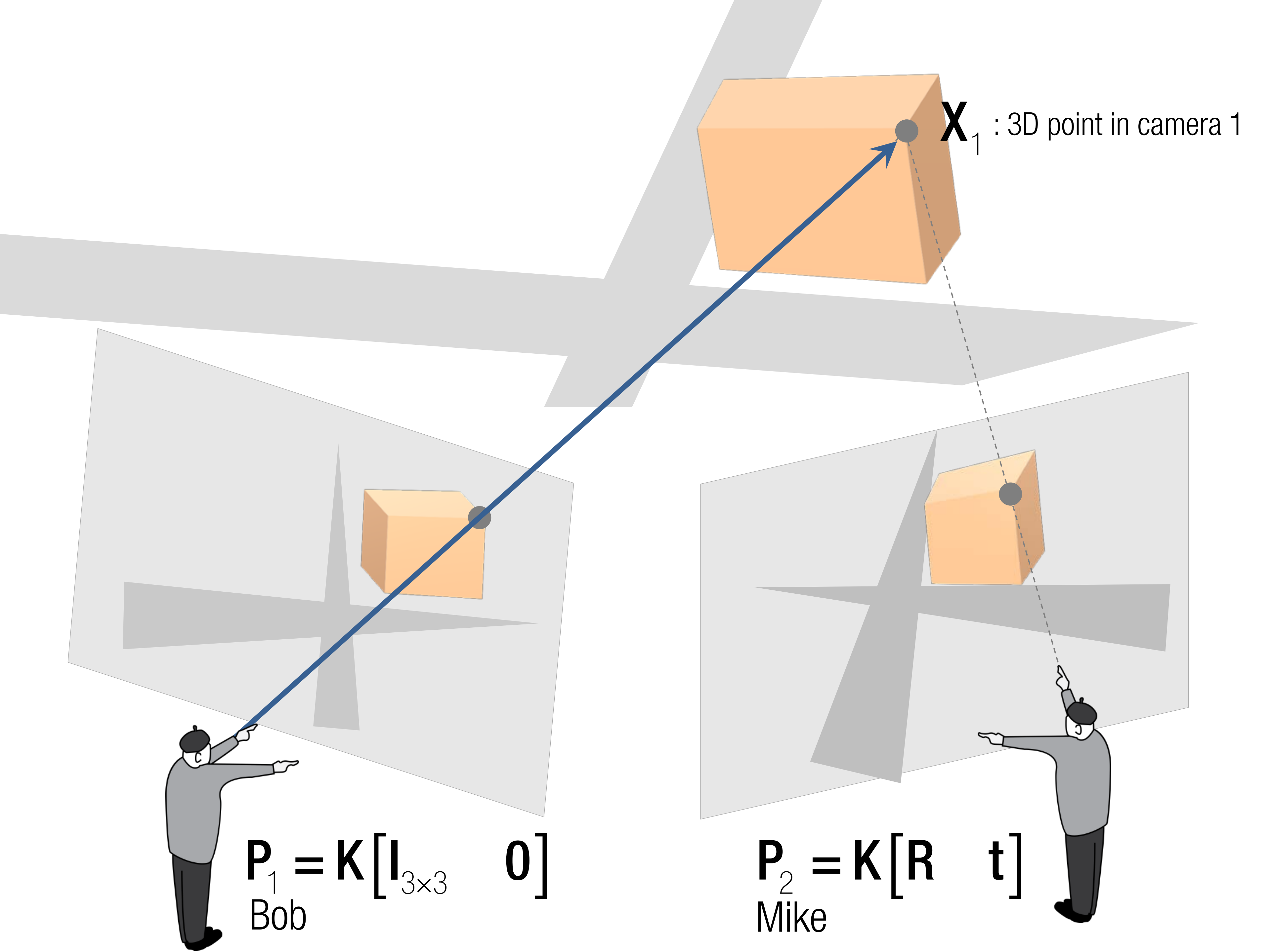


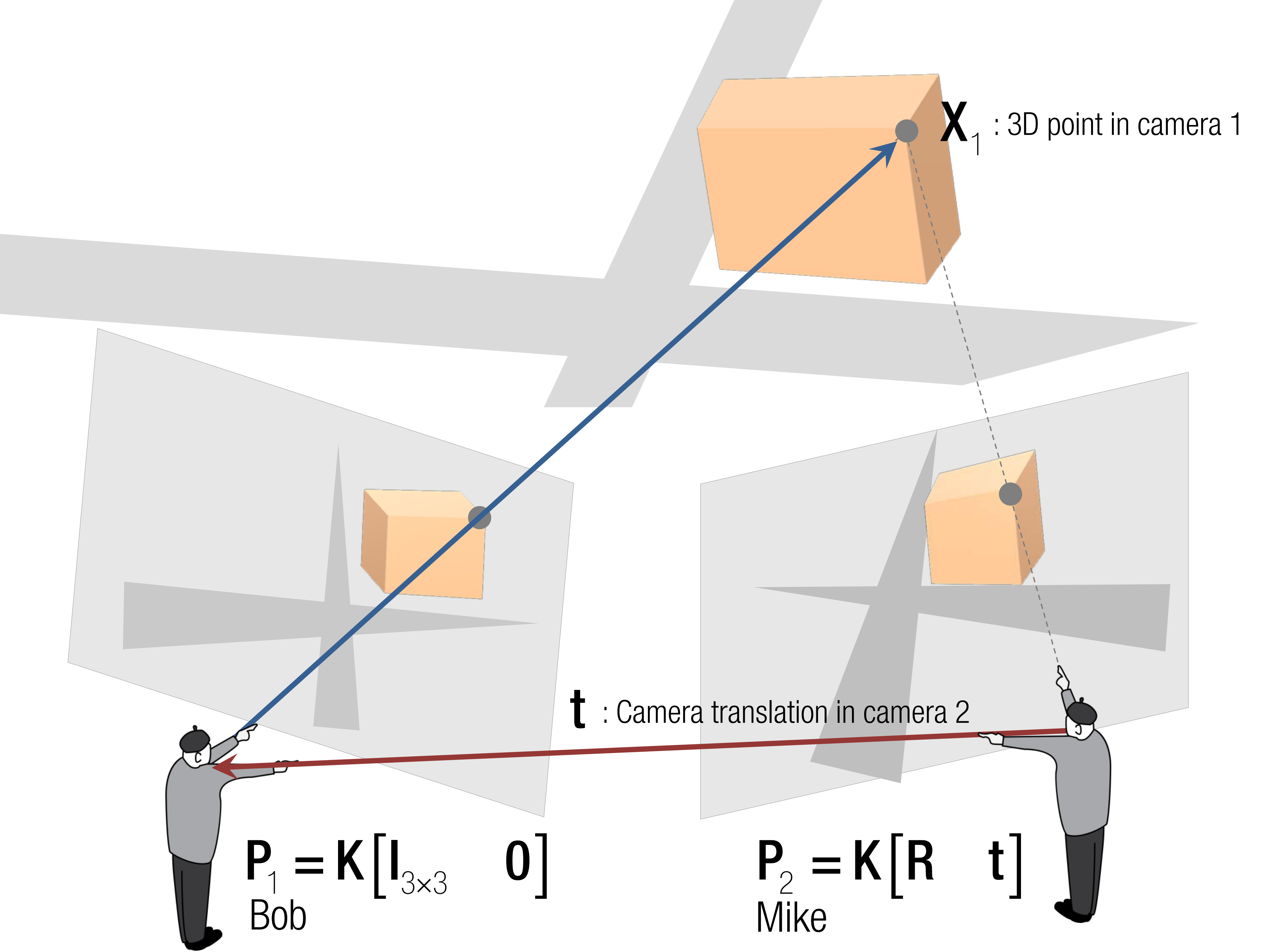
Bob’s view

Mike’s view

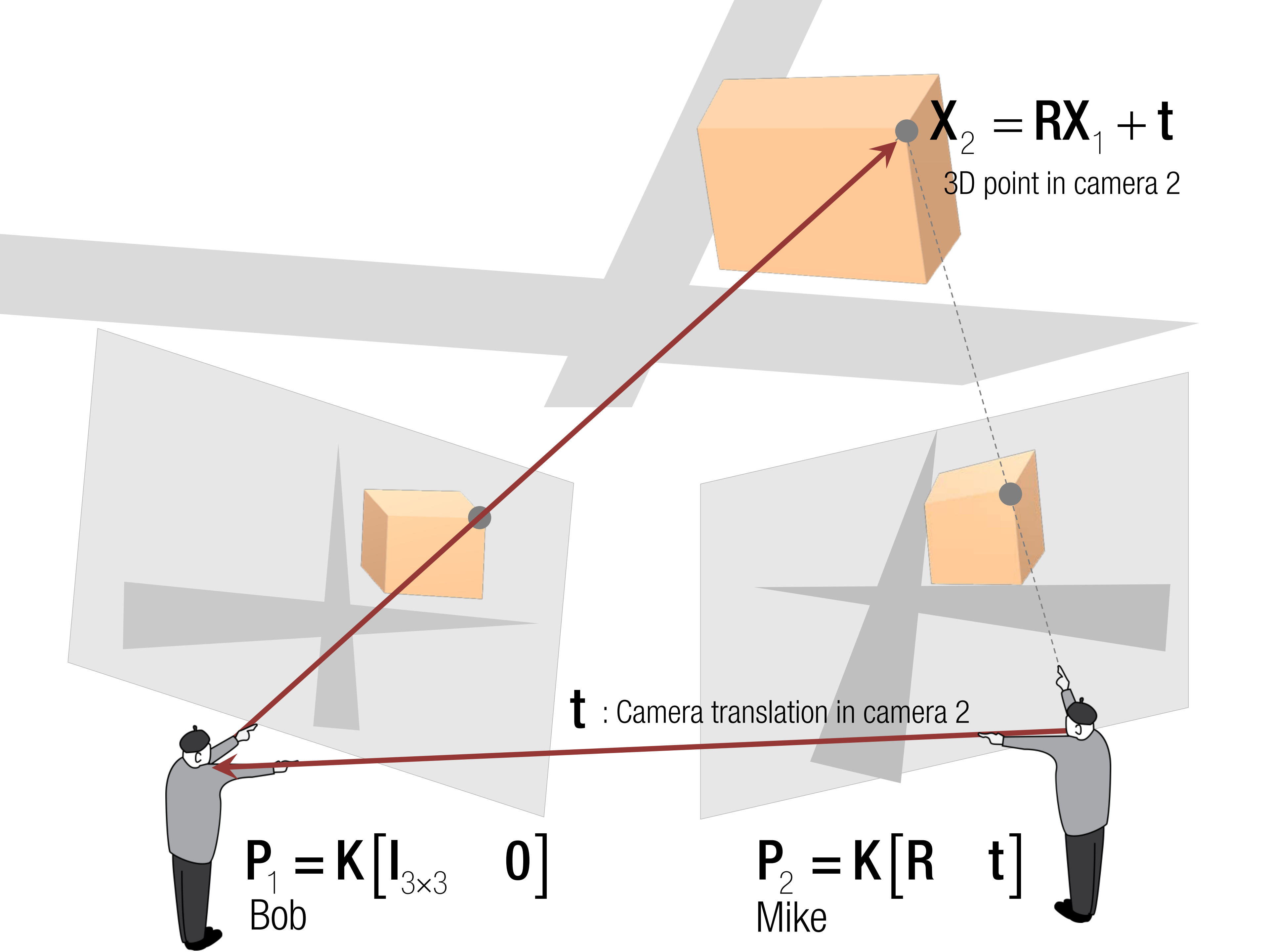
8 correspondences

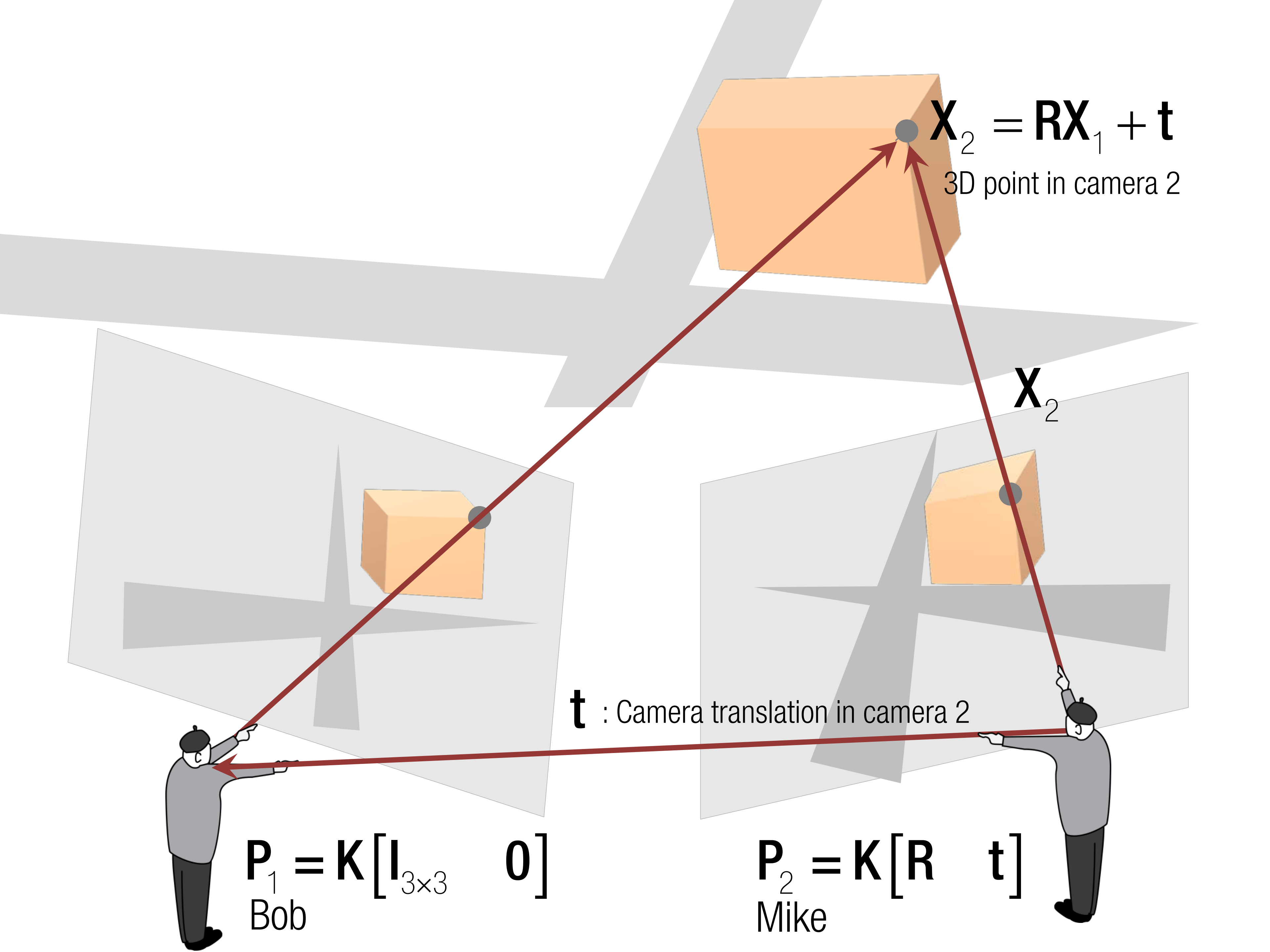

$P_1 = K\begin{bmatrix} I_{3\times3} & 0 \end{bmatrix}$
Bob
$P_2 = K\begin{bmatrix} R & t \end{bmatrix}$
Mike


$\mathbf{X}_1$: 3D point in camera 1
$\mathbf{P}_1 = \mathbf{K}\begin{bmatrix}\mathbf{I}_{3\times3} & \mathbf{0}\end{bmatrix}$
Bob
$\mathbf{P}_2 = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$
Mike

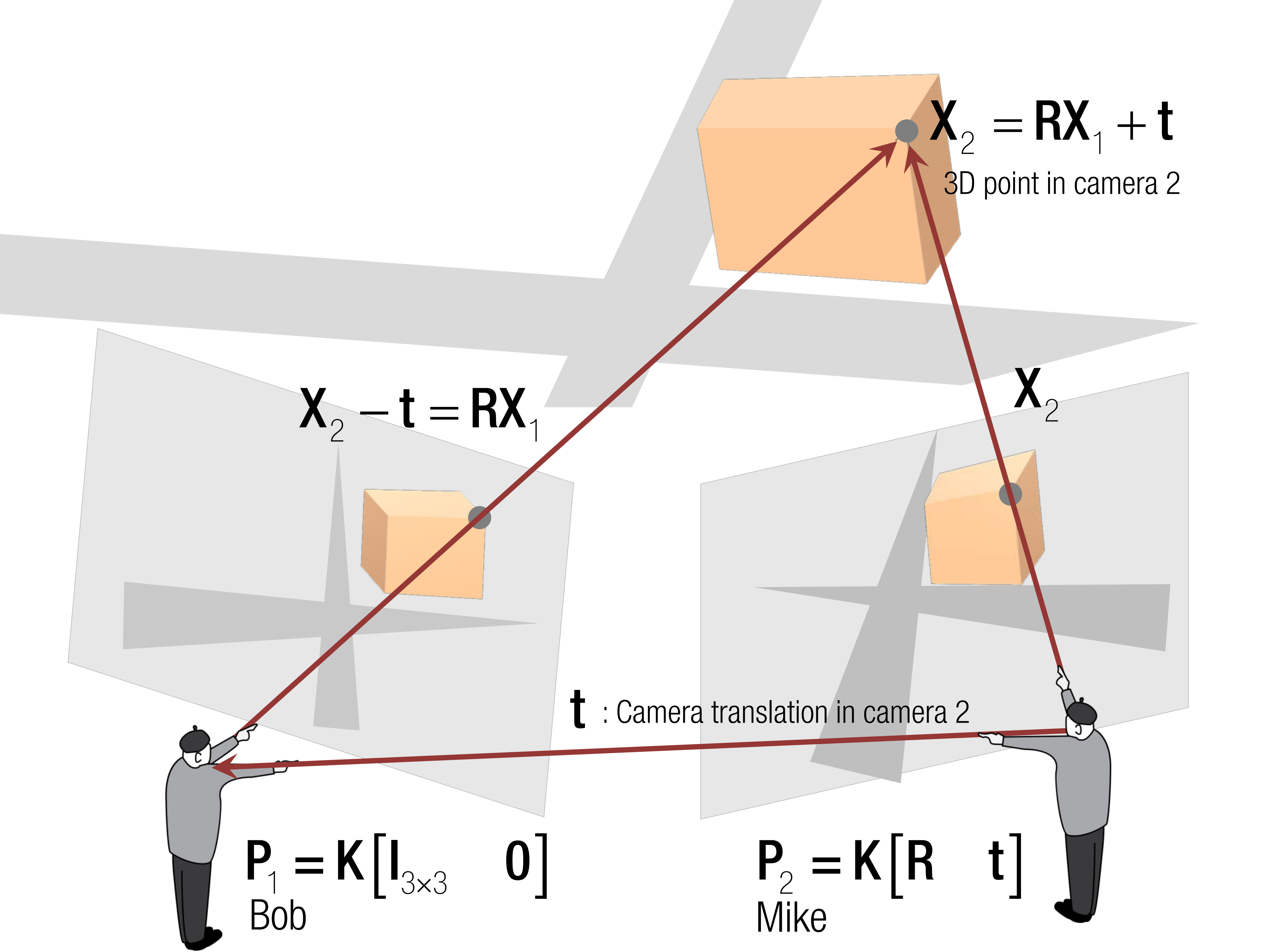


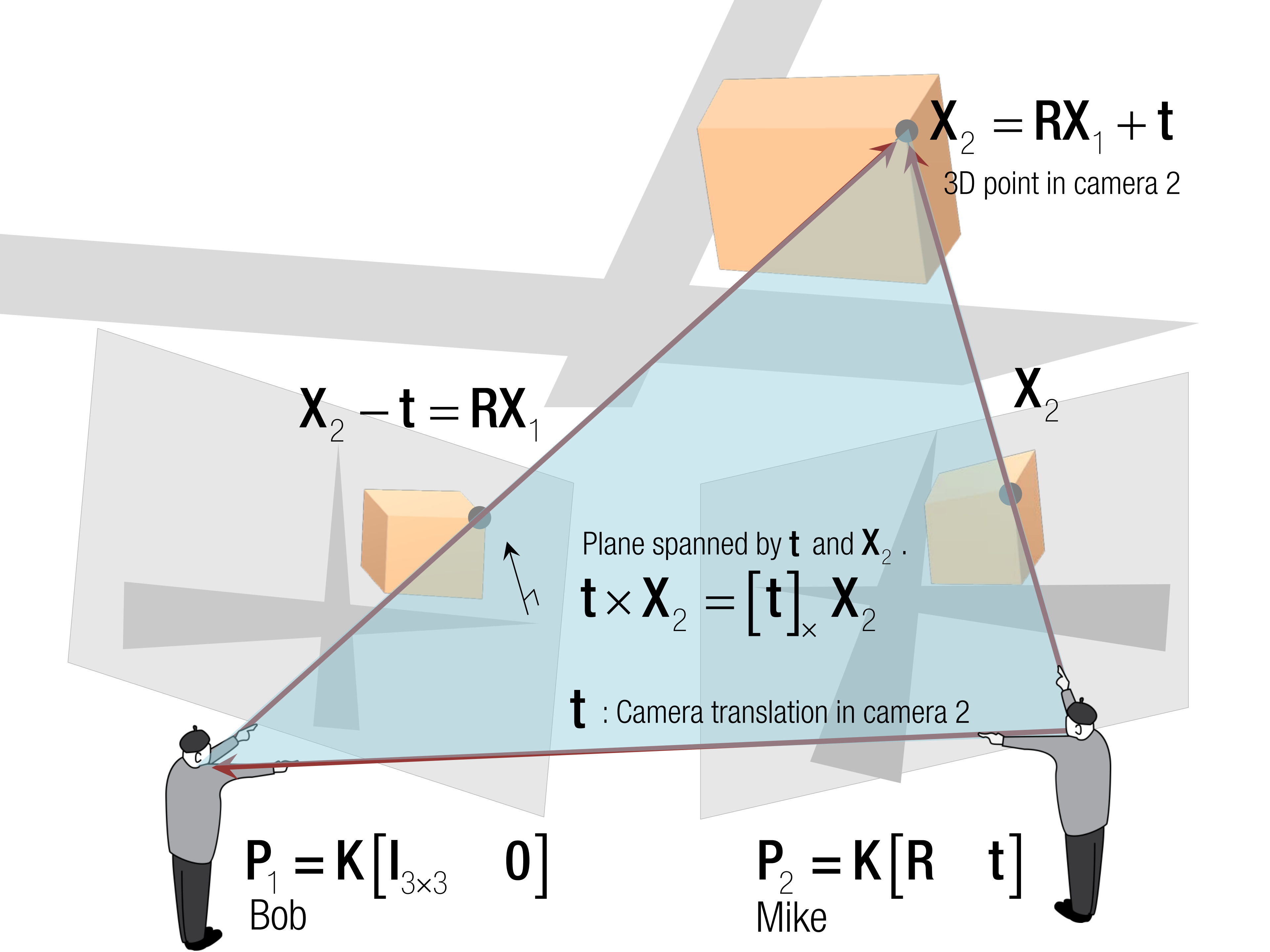
$\mathbf{X}_1$: 3D point in camera 1
$\mathbf{t}$: Camera translation in camera 2
$\mathbf{P}_1 = \mathbf{K}\left[\mathbf{I}_{3\times3} \quad \mathbf{0}\right]$
Bob
$\mathbf{P}_2 = \mathbf{K}\left[\mathbf{R} \quad \mathbf{t}\right]$
Mike


$\mathbf{X}_2 = \mathbf{R}\mathbf{X}_1 + \mathbf{t}$
3D point in camera 2
$\mathbf{t}$: Camera translation in camera 2
$\mathbf{P}_1 = \mathbf{K}\begin{bmatrix}\mathbf{I}_{3\times3} & \mathbf{0}\end{bmatrix}$
Bob
$\mathbf{P}_2 = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$
Mike

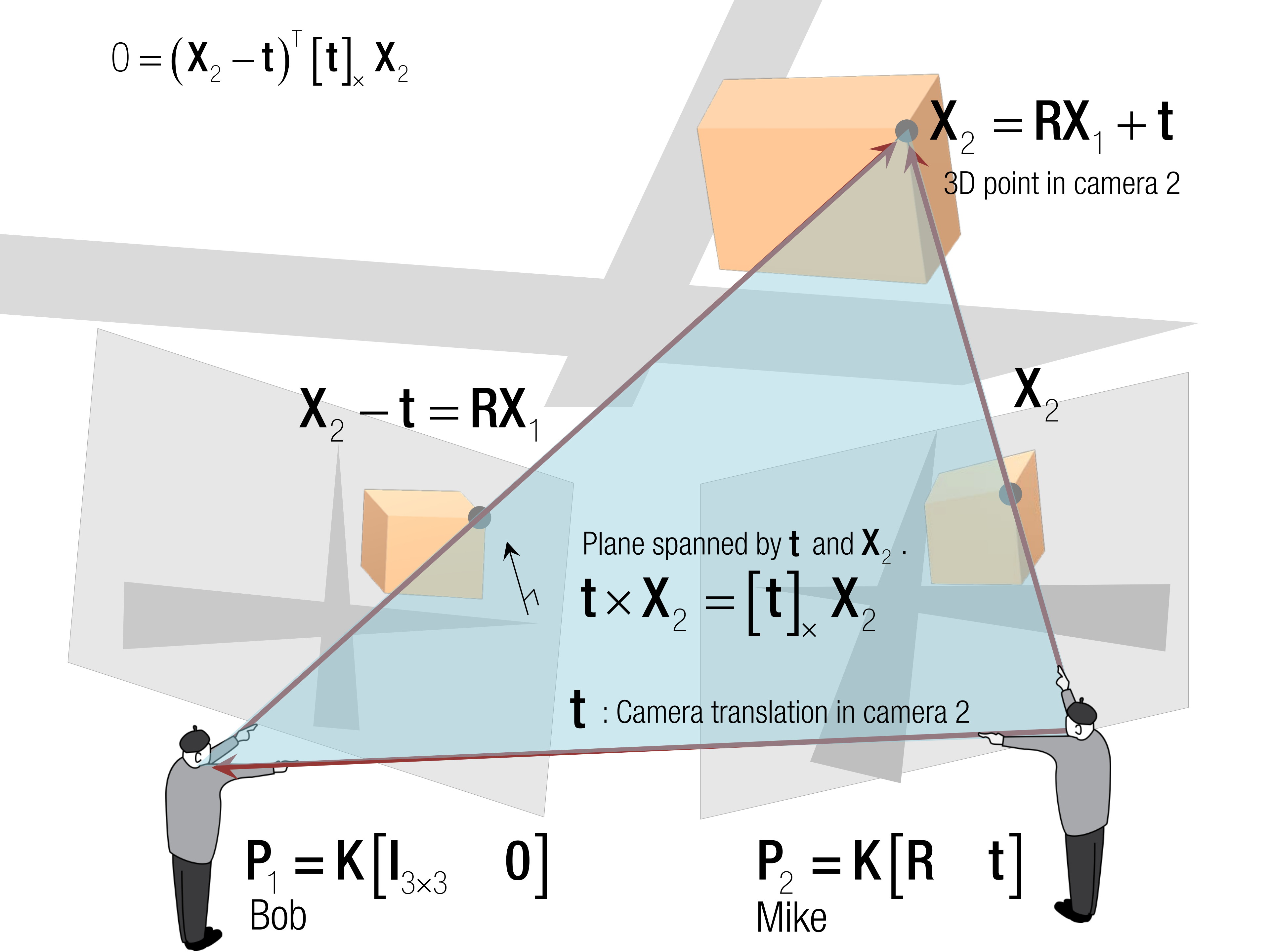


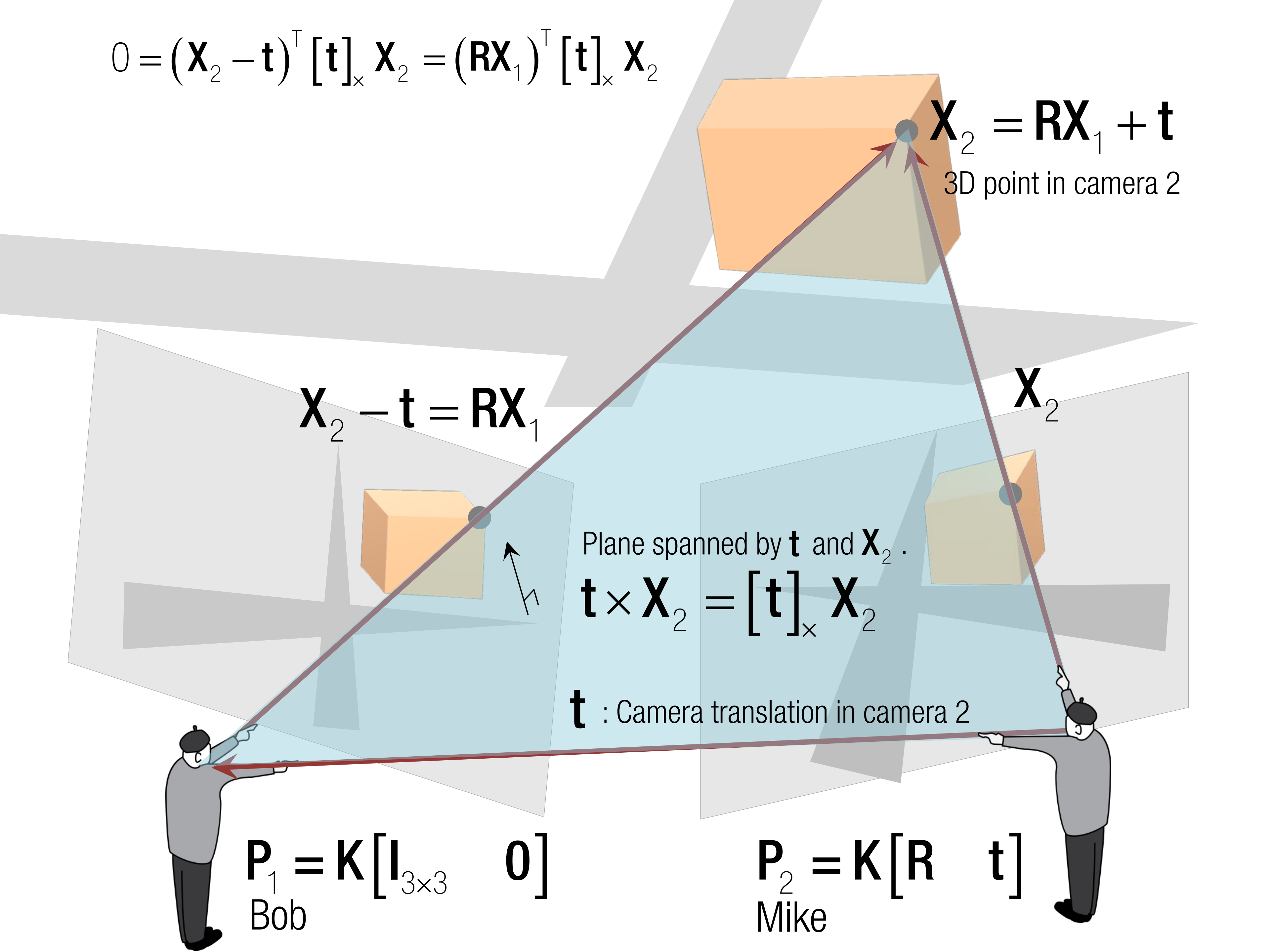
$\mathbf{X}_2 = \mathbf{R}\mathbf{X}_1 + \mathbf{t}$
3D point in camera 2
$\mathbf{X}_2$
$\mathbf{t}$: Camera translation in camera 2
$\mathbf{P}_1 = \mathbf{K}\begin{bmatrix}\mathbf{I}_{3\times3} & \mathbf{0}\end{bmatrix}$
Bob
$\mathbf{P}_2 = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$
Mike


$\mathbf{X}_2 = \mathbf{R}\mathbf{X}_1 + \mathbf{t}$
3D point in camera 2
$\mathbf{X}_2 - \mathbf{t} = \mathbf{R}\mathbf{X}_1$
$\mathbf{X}_2$
$\mathbf{t}$: Camera translation in camera 2
$\mathbf{P}_1 = \mathbf{K}\begin{bmatrix}\mathbf{I}_{3\times3} & \mathbf{0}\end{bmatrix}$
Bob
$\mathbf{P}_2 = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$
Mike

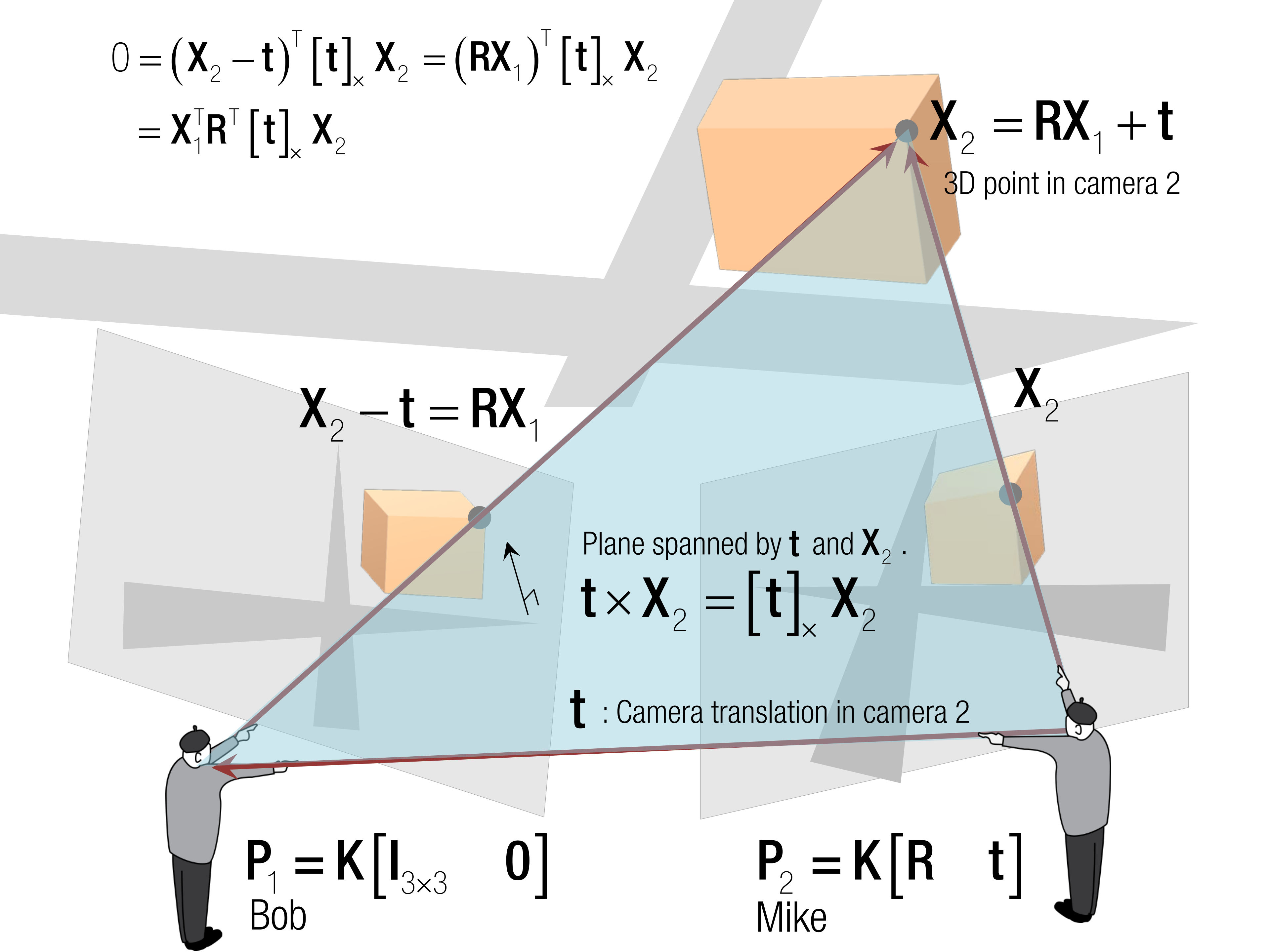


$\mathbf{X}_2 = \mathbf{R}\mathbf{X}_1 + \mathbf{t}$
3D point in camera 2
$\mathbf{X}_2 - \mathbf{t} = \mathbf{R}\mathbf{X}_1$
$\mathbf{X}_2$
Plane spanned by $\mathbf{t}$ and $\mathbf{X}_2$.
$\mathbf{t} \times \mathbf{X}_2 = [\mathbf{t}]_\times \mathbf{X}_2$
$\mathbf{t}$: Camera translation in camera 2
$\mathbf{P}_1 = \mathbf{K}[\mathbf{I}_{3\times 3} \quad \mathbf{0}]$
Bob
$\mathbf{P}_2 = \mathbf{K}[\mathbf{R} \quad \mathbf{t}]$
Mike


$0 = (\mathbf{X}_2 - \mathbf{t})^{\top} [\mathbf{t}]_{\times} \mathbf{X}_2$
$\mathbf{X}_2 = \mathbf{R}\mathbf{X}_1 + \mathbf{t}$
3D point in camera 2
$\mathbf{X}_2 - \mathbf{t} = \mathbf{R}\mathbf{X}_1$
$\mathbf{X}_2$
Plane spanned by $\mathbf{t}$ and $\mathbf{X}_2$.
$\mathbf{t} \times \mathbf{X}_2 = [\mathbf{t}]_{\times} \mathbf{X}_2$
$\mathbf{t}$: Camera translation in camera 2
$\mathbf{P}_1 = \mathbf{K}\begin{bmatrix} \mathbf{I}_{3\times 3} & \mathbf{0} \end{bmatrix}$
Bob
$\mathbf{P}_2 = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}$
Mike



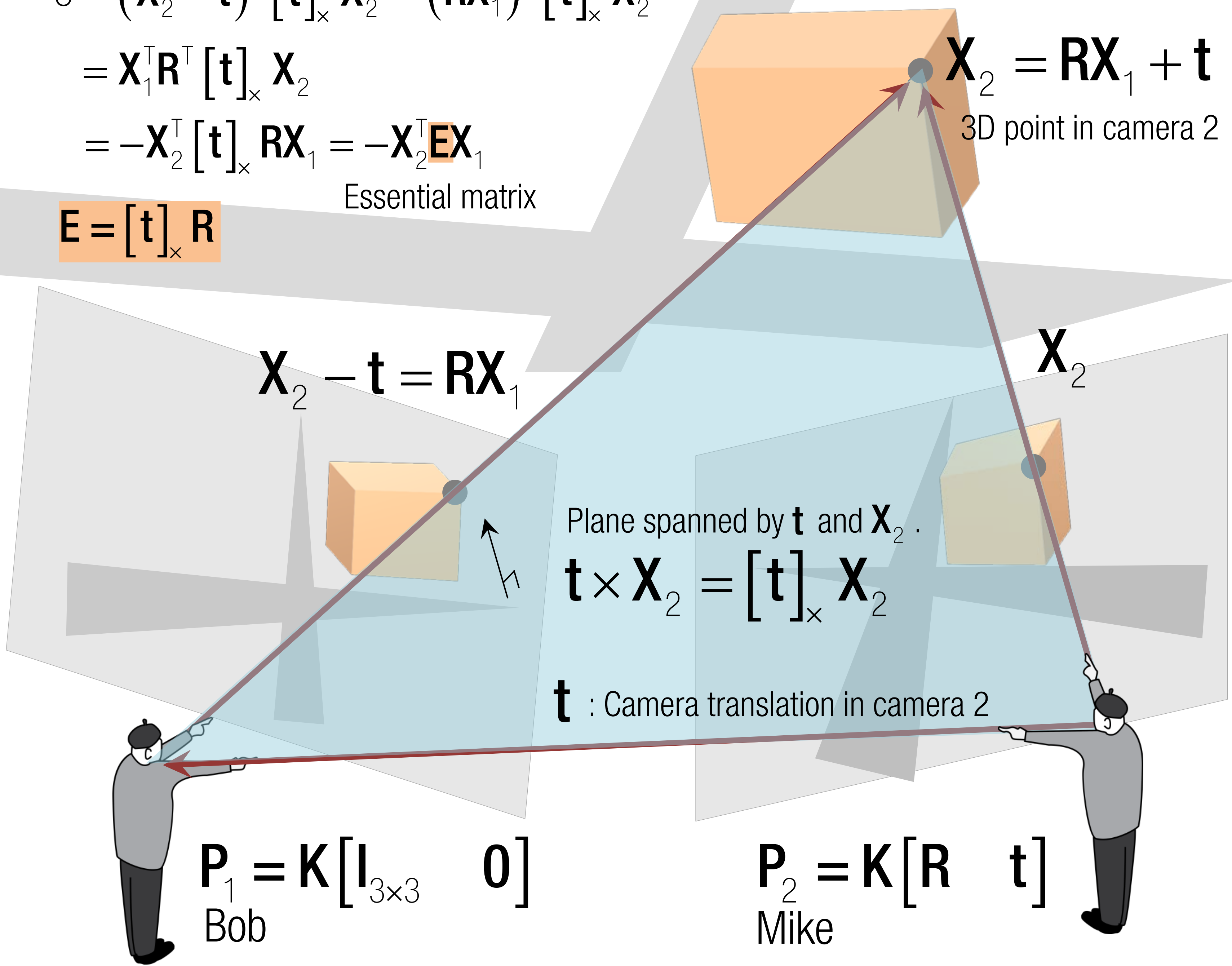
$0 = (\mathbf{X}_2 - \mathbf{t})^\top [\mathbf{t}]_\times \mathbf{X}_2 = (\mathbf{R}\mathbf{X}_1)^\top [\mathbf{t}]_\times \mathbf{X}_2$
$\mathbf{X}_2 = \mathbf{R}\mathbf{X}_1 + \mathbf{t}$
3D point in camera 2
$\mathbf{X}_2 - \mathbf{t} = \mathbf{R}\mathbf{X}_1$
$\mathbf{X}_2$
Plane spanned by $\mathbf{t}$ and $\mathbf{X}_2$.
$\mathbf{t} \times \mathbf{X}_2 = [\mathbf{t}]_\times \mathbf{X}_2$
$\mathbf{t}$: Camera translation in camera 2
$\mathbf{P}_1 = \mathbf{K}\begin{bmatrix}\mathbf{I}_{3\times3} & \mathbf{0}\end{bmatrix}$
Bob
$\mathbf{P}_2 = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$
Mike

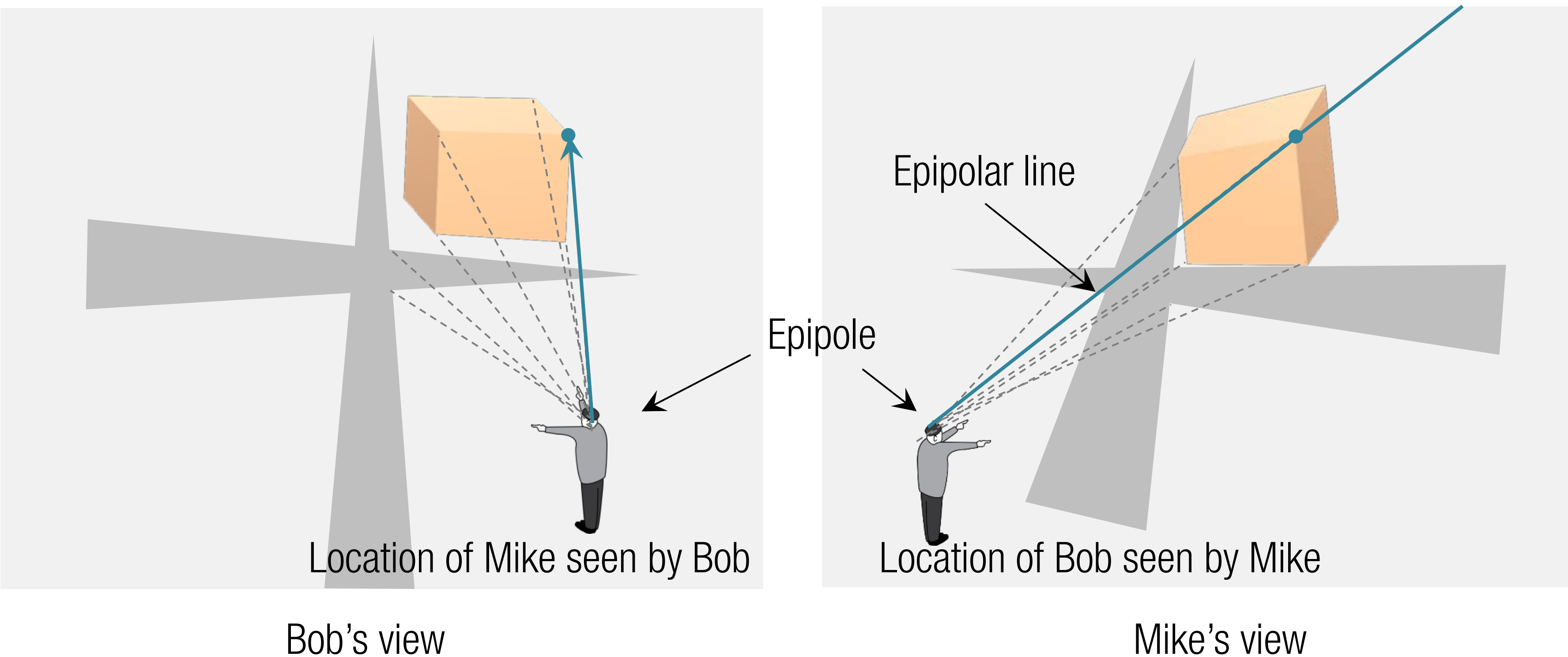

$0 = (\mathbf{X}_2 - \mathbf{t})^\top [\mathbf{t}]_\times \mathbf{X}_2 = (\mathbf{R}\mathbf{X}_1)^\top [\mathbf{t}]_\times \mathbf{X}_2$
$= \mathbf{X}_1^\top \mathbf{R}^\top [\mathbf{t}]_\times \mathbf{X}_2$
$\mathbf{X}_2 = \mathbf{R}\mathbf{X}_1 + \mathbf{t}$
3D point in camera 2
$\mathbf{X}_2 - \mathbf{t} = \mathbf{R}\mathbf{X}_1$
$\mathbf{X}_2$
Plane spanned by $\mathbf{t}$ and $\mathbf{X}_2$.
$\mathbf{t} \times \mathbf{X}_2 = [\mathbf{t}]_\times \mathbf{X}_2$
$\mathbf{t}$: Camera translation in camera 2
$\mathbf{P}_1 = \mathbf{K}[\mathbf{I}_{3\times 3} \quad \mathbf{0}]$
Bob
$\mathbf{P}_2 = \mathbf{K}[\mathbf{R} \quad \mathbf{t}]$
Mike

$$
\begin{aligned}
0 &= (\mathbf{X}_2 - \mathbf{t})^{\mathsf{T}} [\mathbf{t}]_\times \mathbf{X}_2 = (\mathbf{R}\mathbf{X}_1)^{\mathsf{T}} [\mathbf{t}]_\times \mathbf{X}_2 \\
&= \mathbf{X}_1^{\mathsf{T}} \mathbf{R}^{\mathsf{T}} [\mathbf{t}]_\times \mathbf{X}_2 \\
&= -\mathbf{X}_2^{\mathsf{T}} [\mathbf{t}]_\times \mathbf{R}\mathbf{X}_1 = -\mathbf{X}_2^{\mathsf{T}} \mathbf{E} \mathbf{X}_1
\end{aligned}
$$

Essential matrix

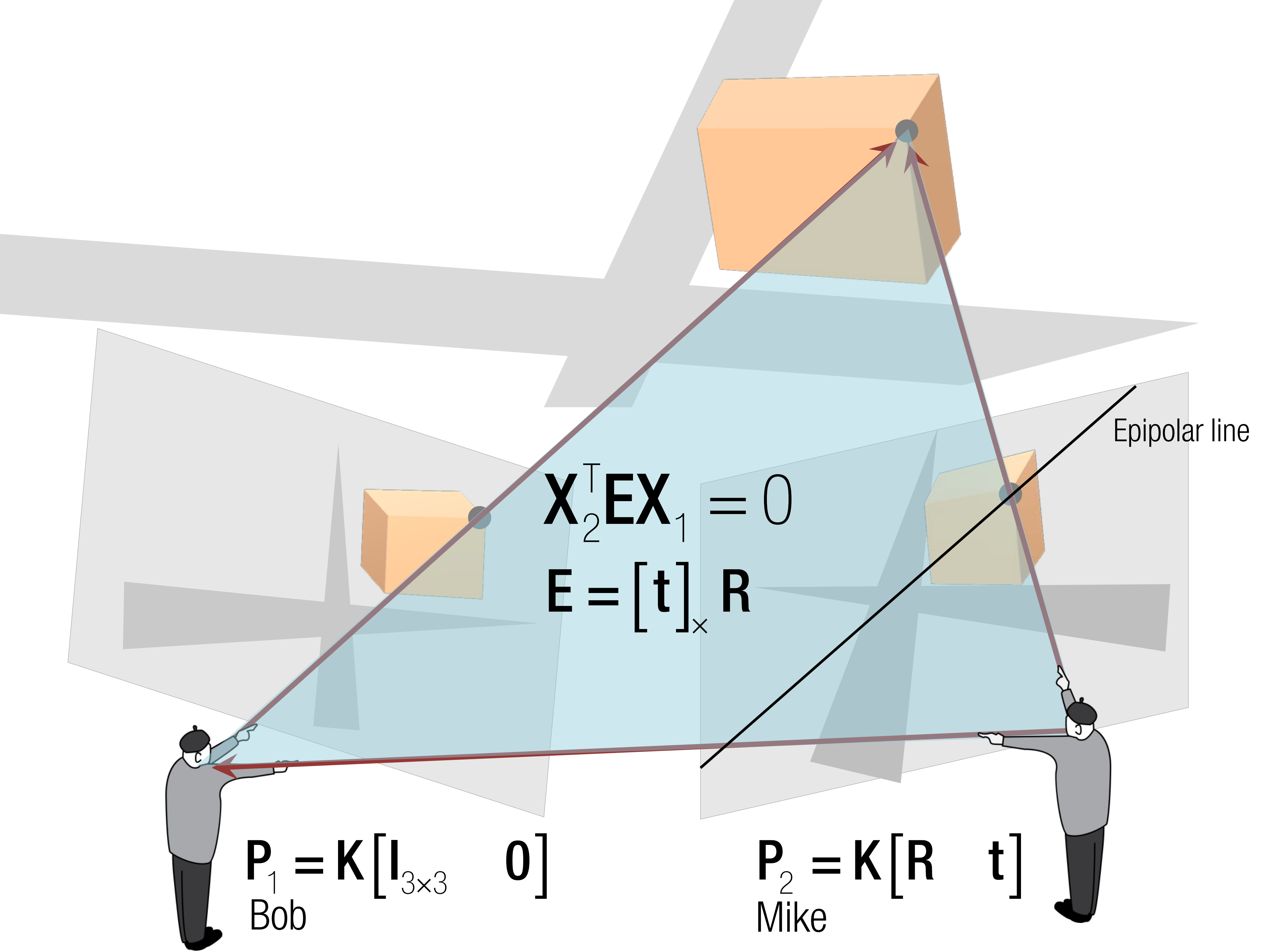
$$\boxed{\mathbf{E} = [\mathbf{t}]_\times \mathbf{R}}$$



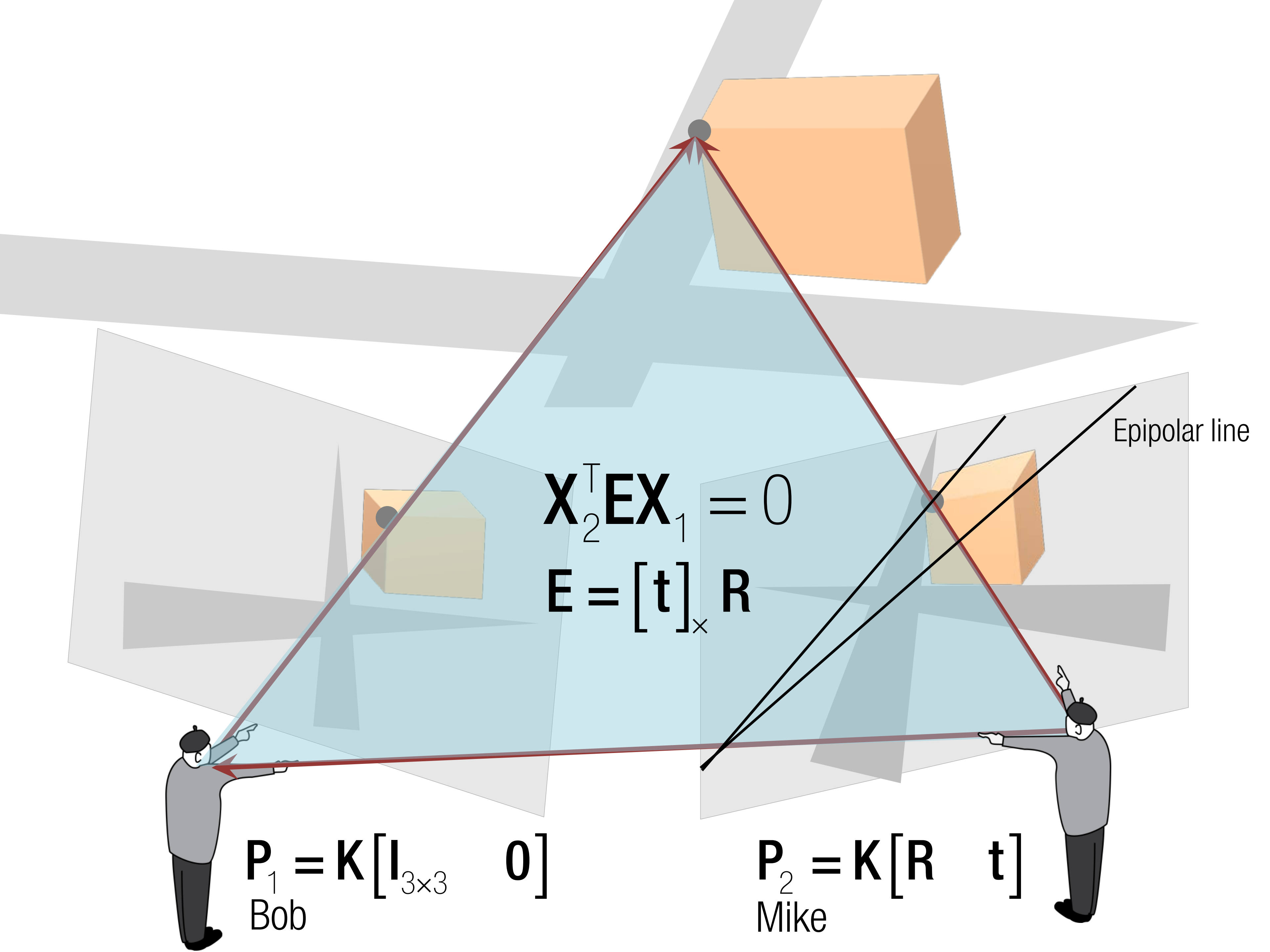


Observation:

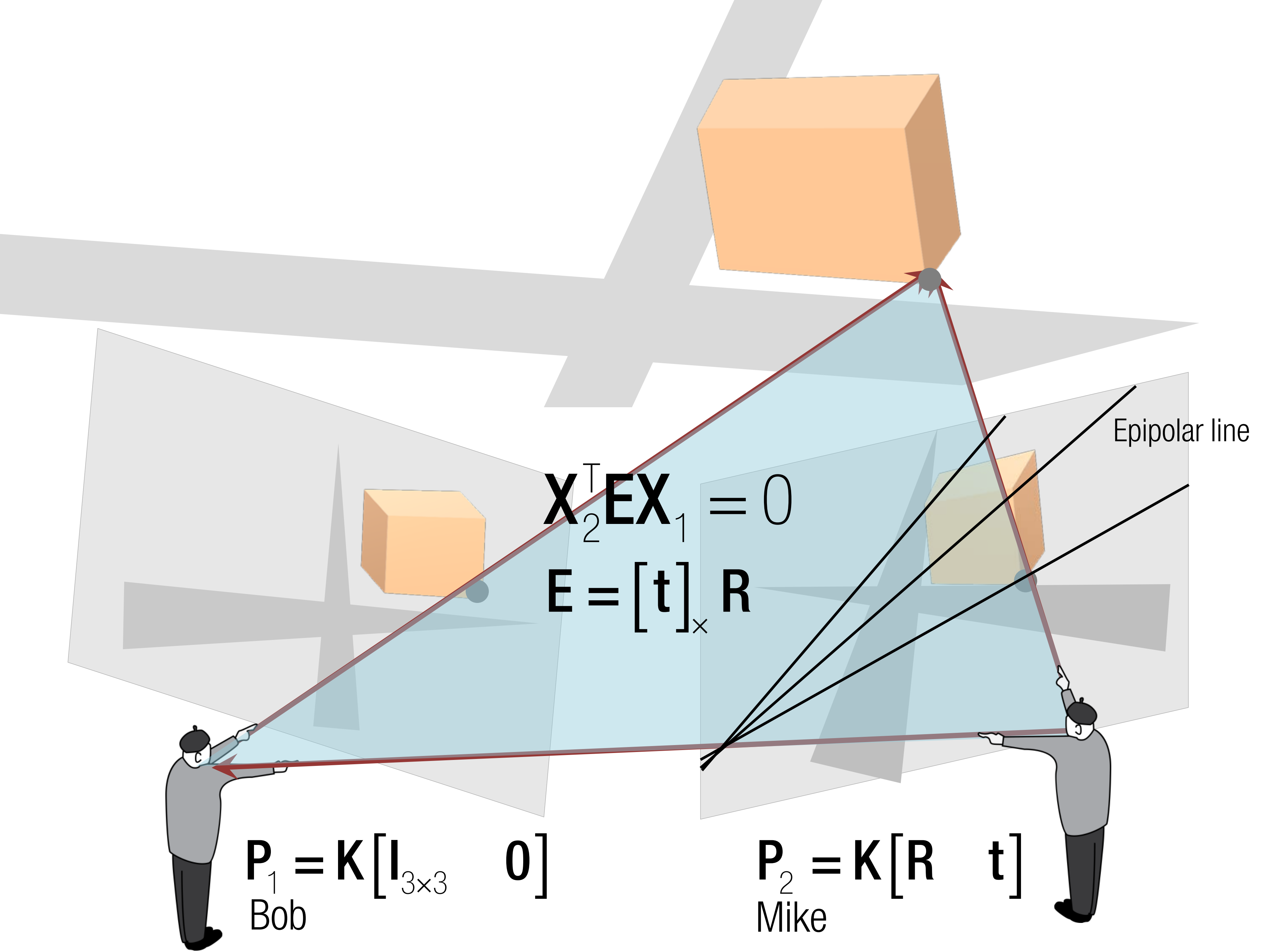
Given a point in Bob’s view, there exists a conjugate line passing the corresponding point in Mike’s view.

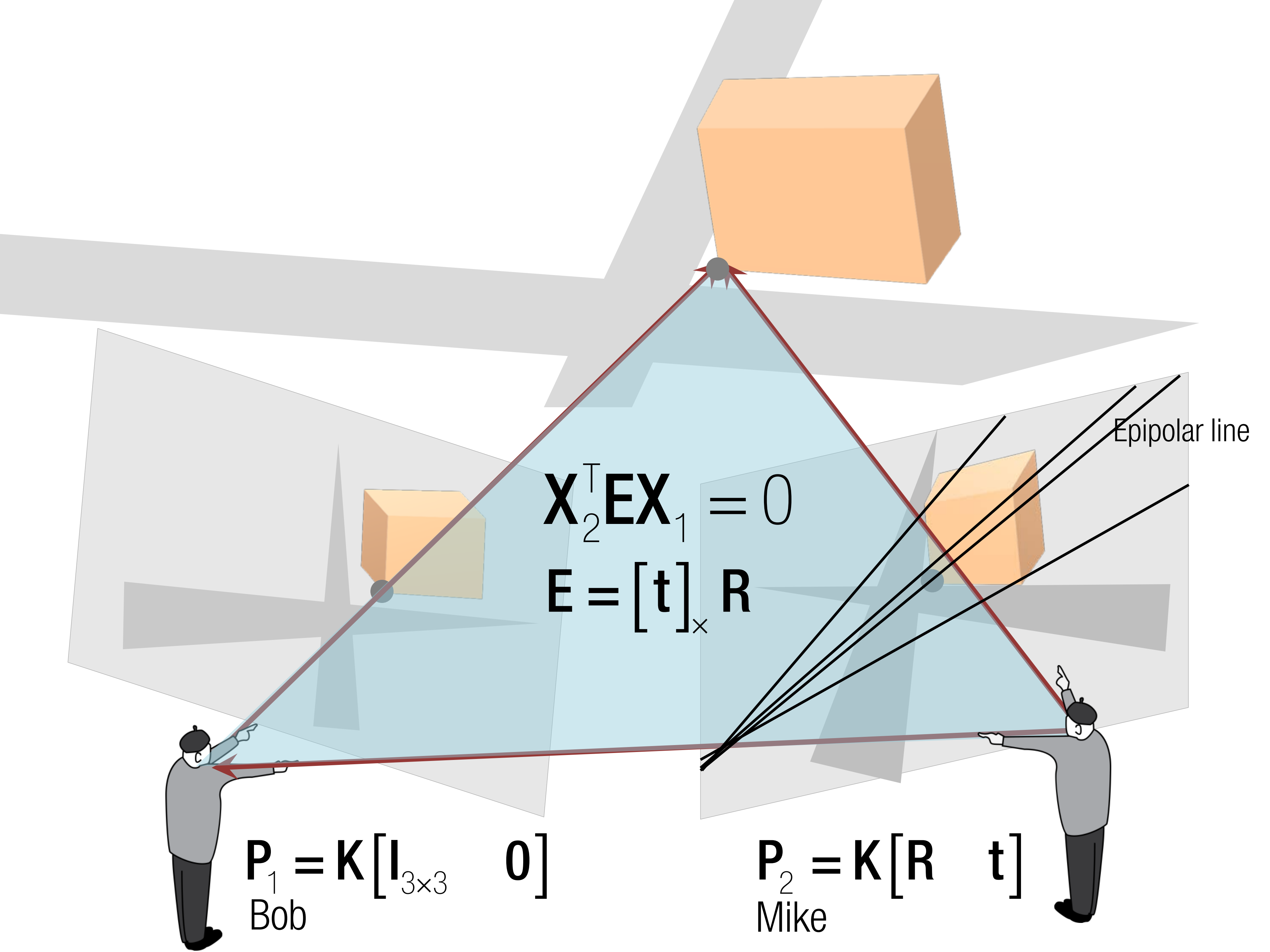


Epipolar line
$X_2^T E X_1 = 0$
$E = [t]_\times R$
$P_1 = K[I_{3\times3} \quad 0]$
Bob
$P_2 = K[R \quad t]$
Mike

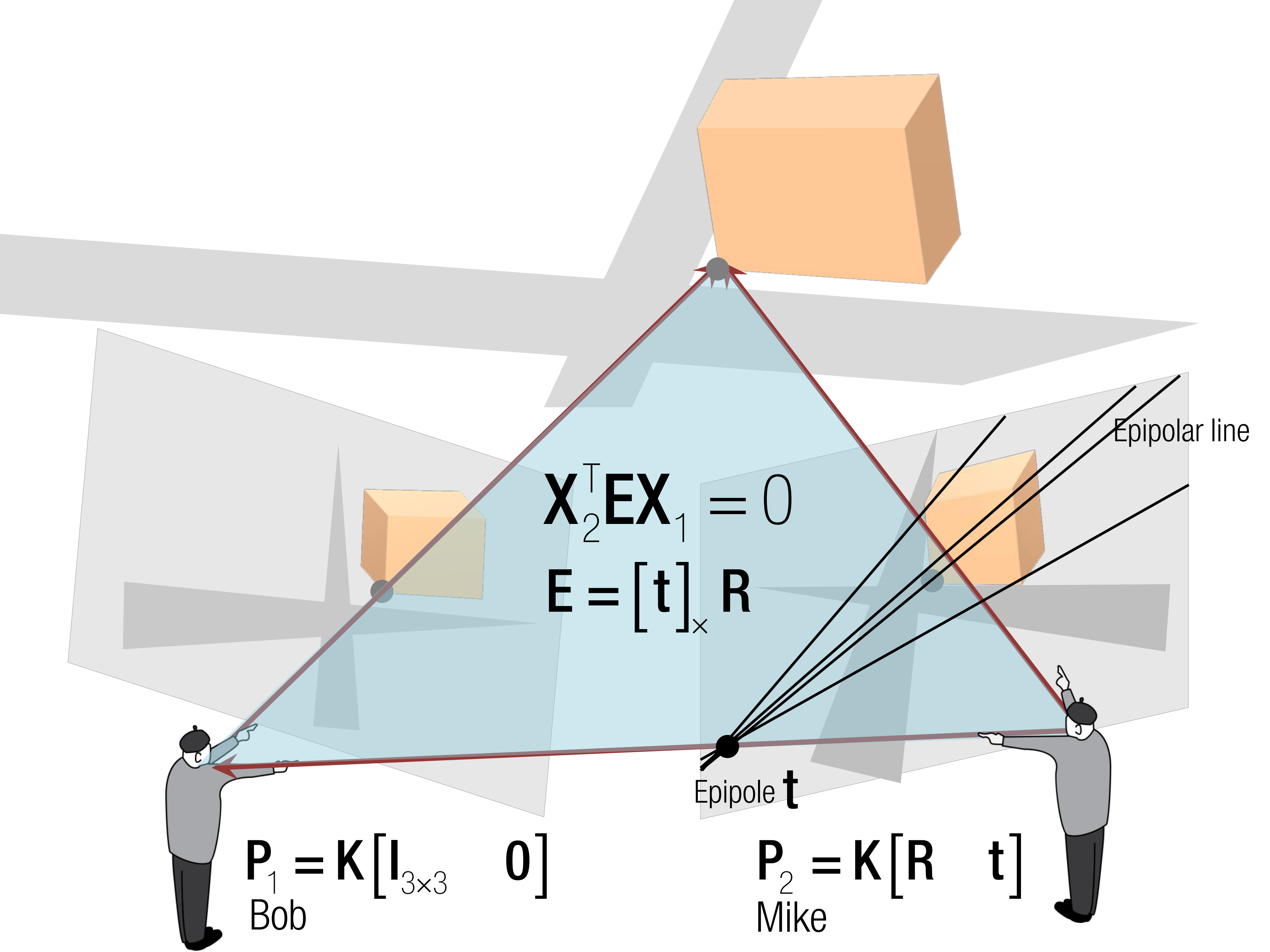


Epipolar line
$\mathbf{X}_2^{\top}\mathbf{E}\mathbf{X}_1 = 0$
$\mathbf{E} = [\mathbf{t}]_{\times}\mathbf{R}$
$\mathbf{P}_1 = \mathbf{K}[\mathbf{I}_{3\times3} \quad \mathbf{0}]$
Bob
$\mathbf{P}_2 = \mathbf{K}[\mathbf{R} \quad \mathbf{t}]$
Mike


Epipolar line
$X_2^T E X_1 = 0$
$E = [t]_\times R$
$P_1 = K[I_{3\times3} \quad 0]$
Bob
$P_2 = K[R \quad t]$
Mike

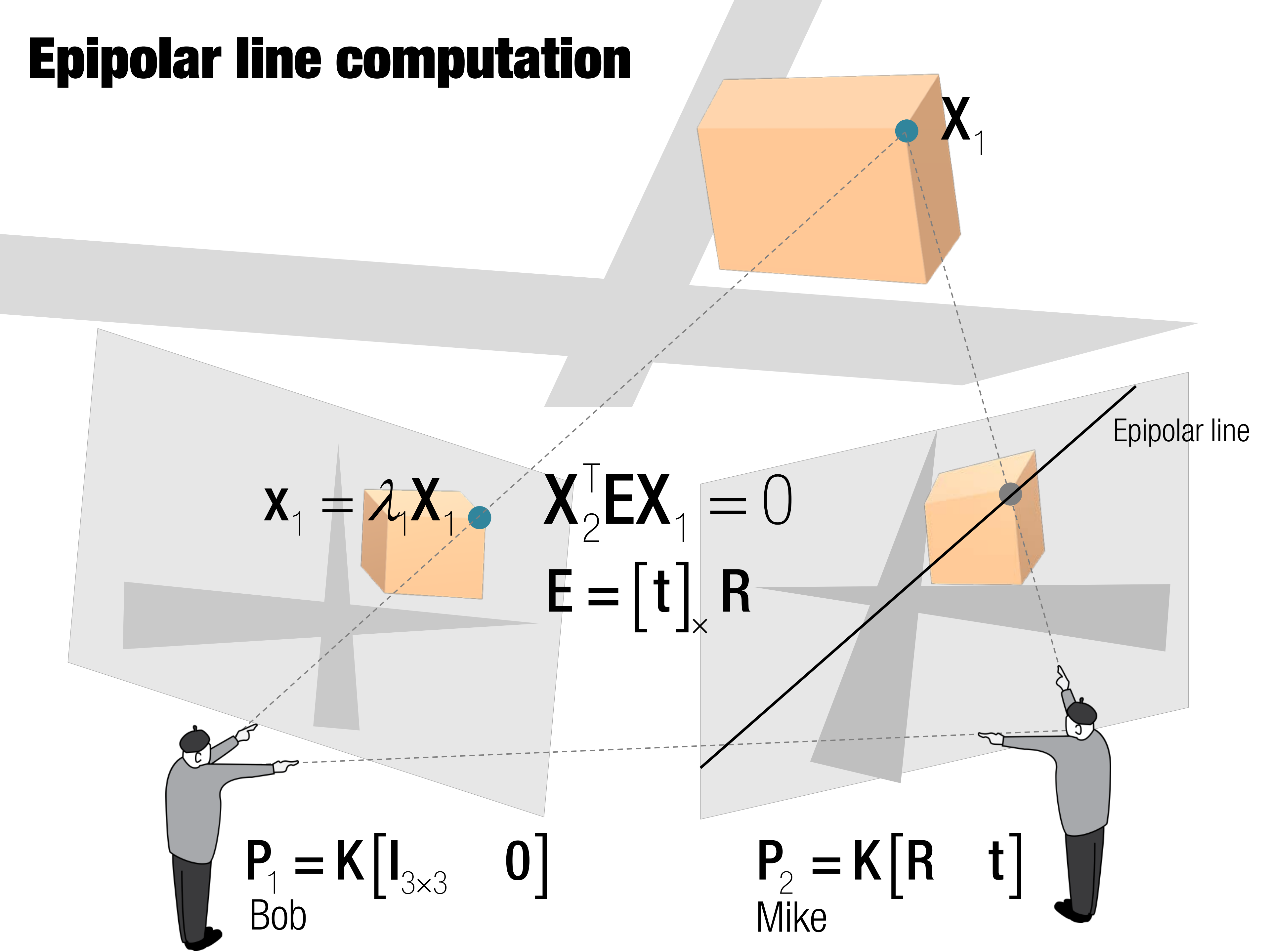


Epipolar line
$X_2^{\top} E X_1 = 0$
$E = [t]_{\times} R$
$P_1 = K[I_{3\times 3} \quad 0]$
Bob
$P_2 = K[R \quad t]$
Mike



$X_2^{\mathsf{T}} E X_1 = 0$
$E = [t]_\times R$
Epipolar line
Epipole t
$P_1 = K[I_{3\times3} \quad 0]$
Bob
$P_2 = K[R \quad t]$
Mike

Epipolar line computation



$$\mathbf{x}_1 = \lambda_1 \mathbf{X}_1$$

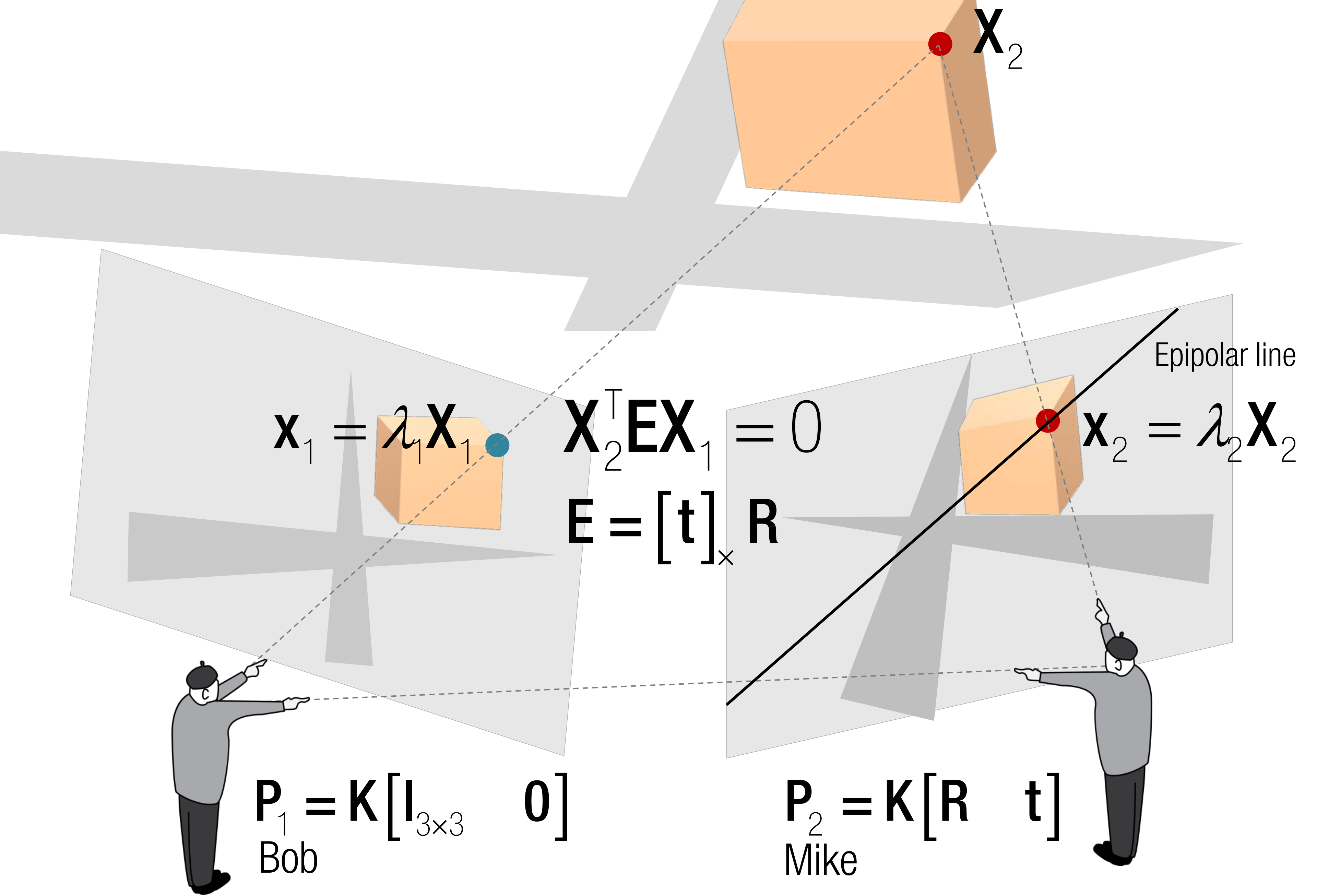
$$\mathbf{X}_2^{\mathsf{T}} \mathbf{E} \mathbf{X}_1 = 0$$

$$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R}$$

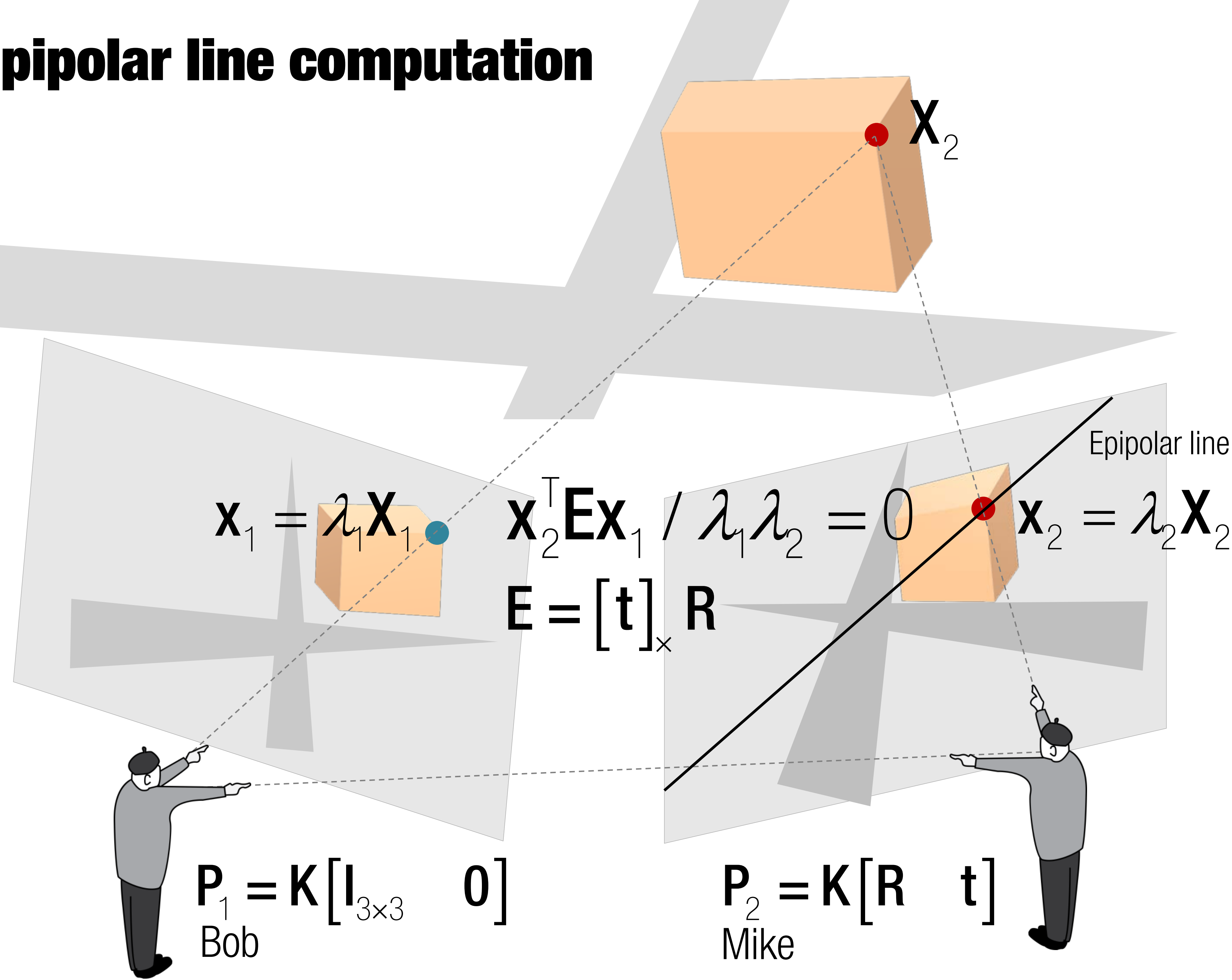
$$\mathbf{P}_1 = \mathbf{K}\begin{bmatrix}\mathbf{I}_{3\times3} & \mathbf{0}\end{bmatrix}$$

$$\mathbf{P}_2 = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$$

Epipolar line computation



Epipolar line computation



$$\mathbf{x}_1 = \lambda_1 \mathbf{X}_1$$

$$\mathbf{x}_2^{\mathsf{T}} \mathbf{E} \mathbf{x}_1 \,/\, \lambda_1 \lambda_2 = 0$$

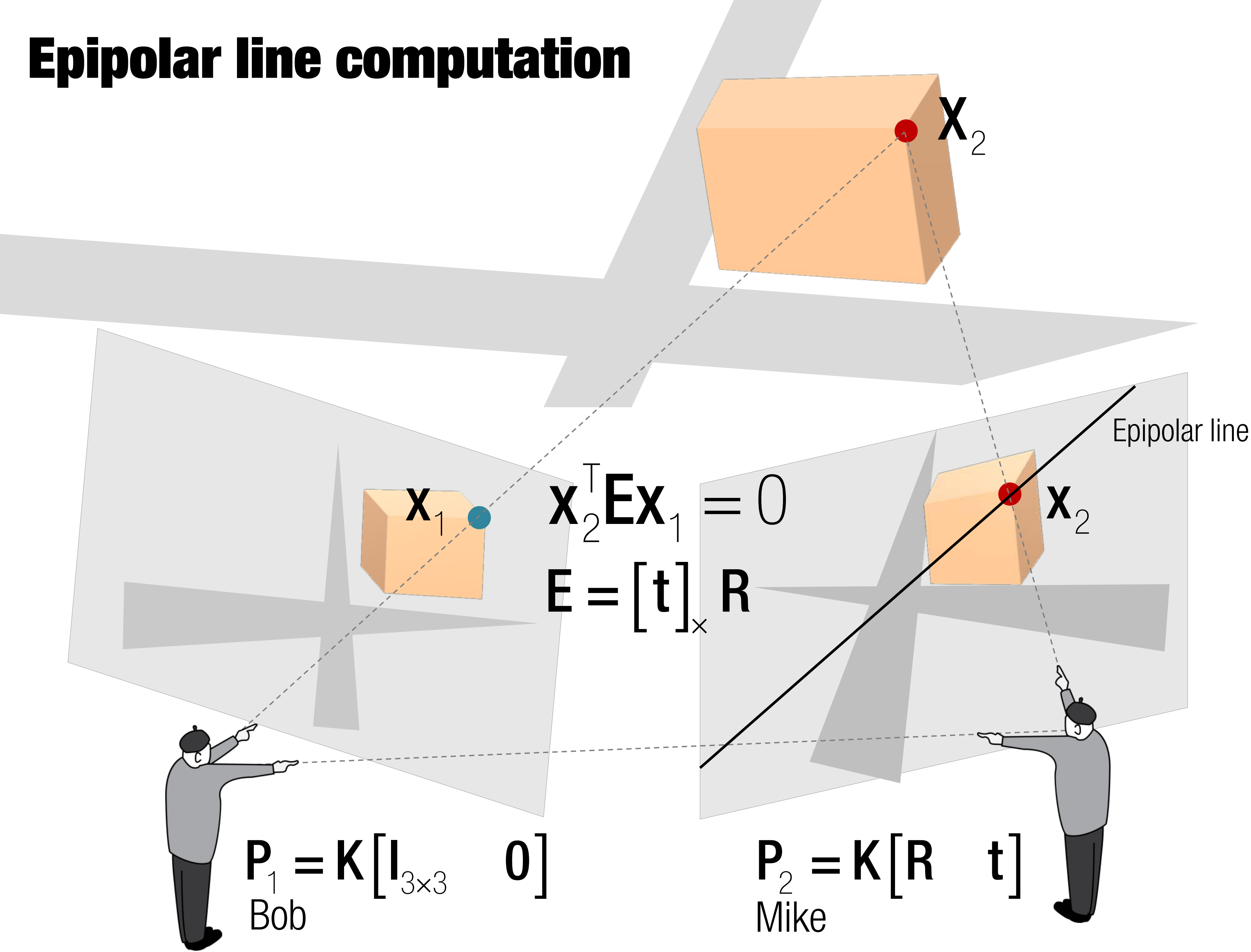
$$\mathbf{x}_2 = \lambda_2 \mathbf{X}_2$$

$$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R}$$

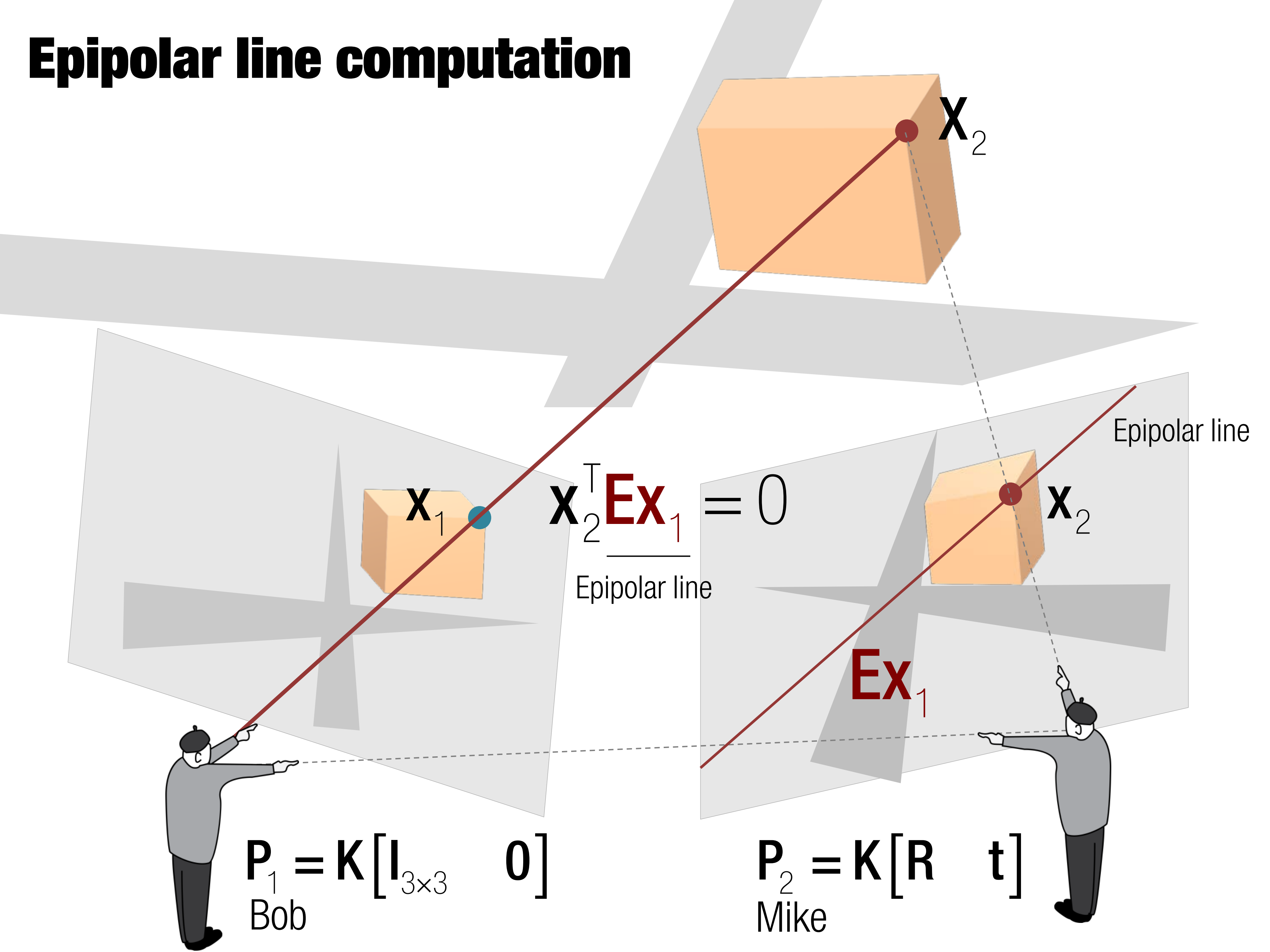
$$\mathbf{P}_1 = \mathbf{K}\begin{bmatrix}\mathbf{I}_{3\times3} & \mathbf{0}\end{bmatrix}$$

$$\mathbf{P}_2 = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$$

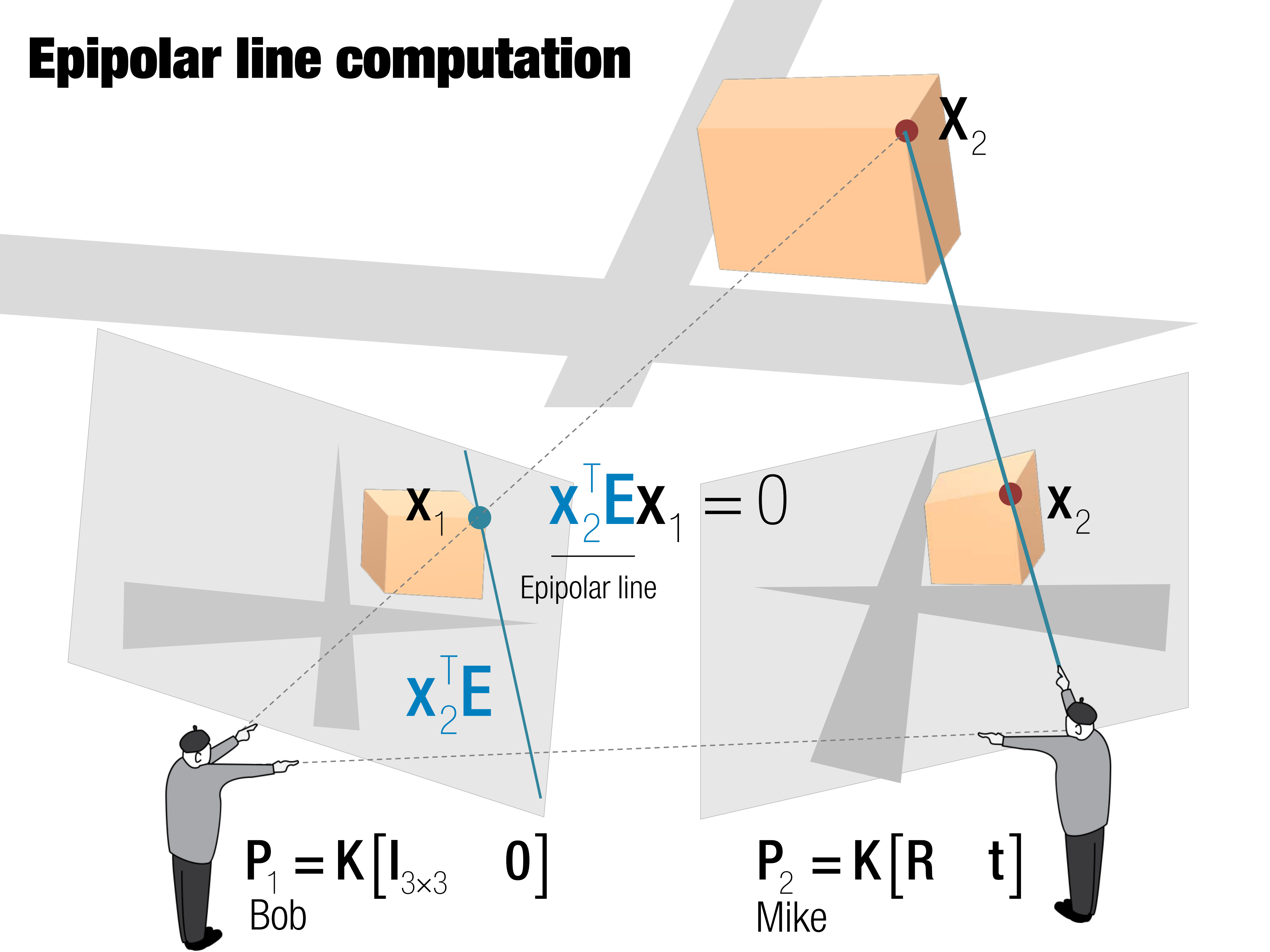
Epipolar line computation



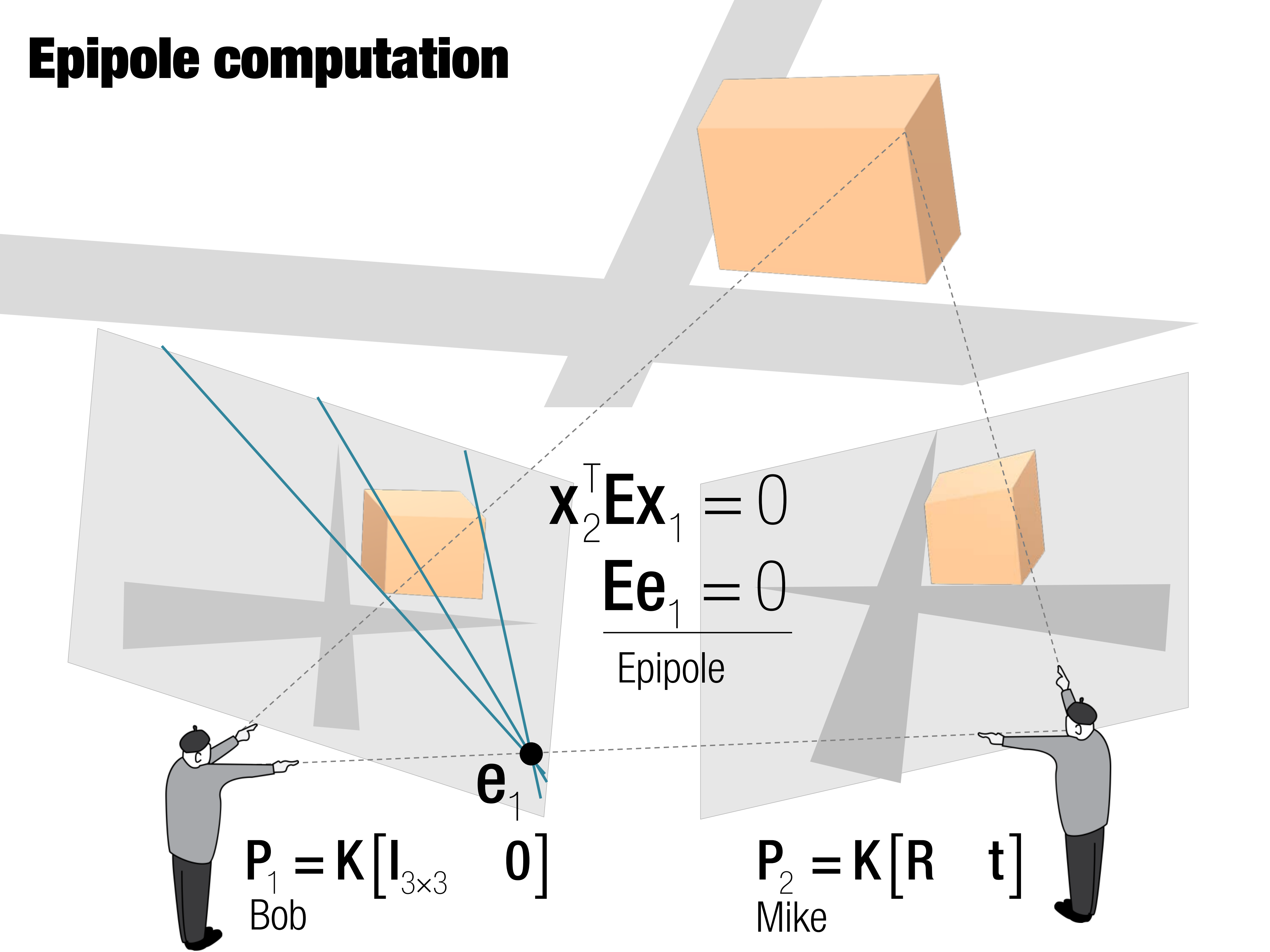
Epipolar line computation



Epipolar line computation



Epipole computation



Epipole computation