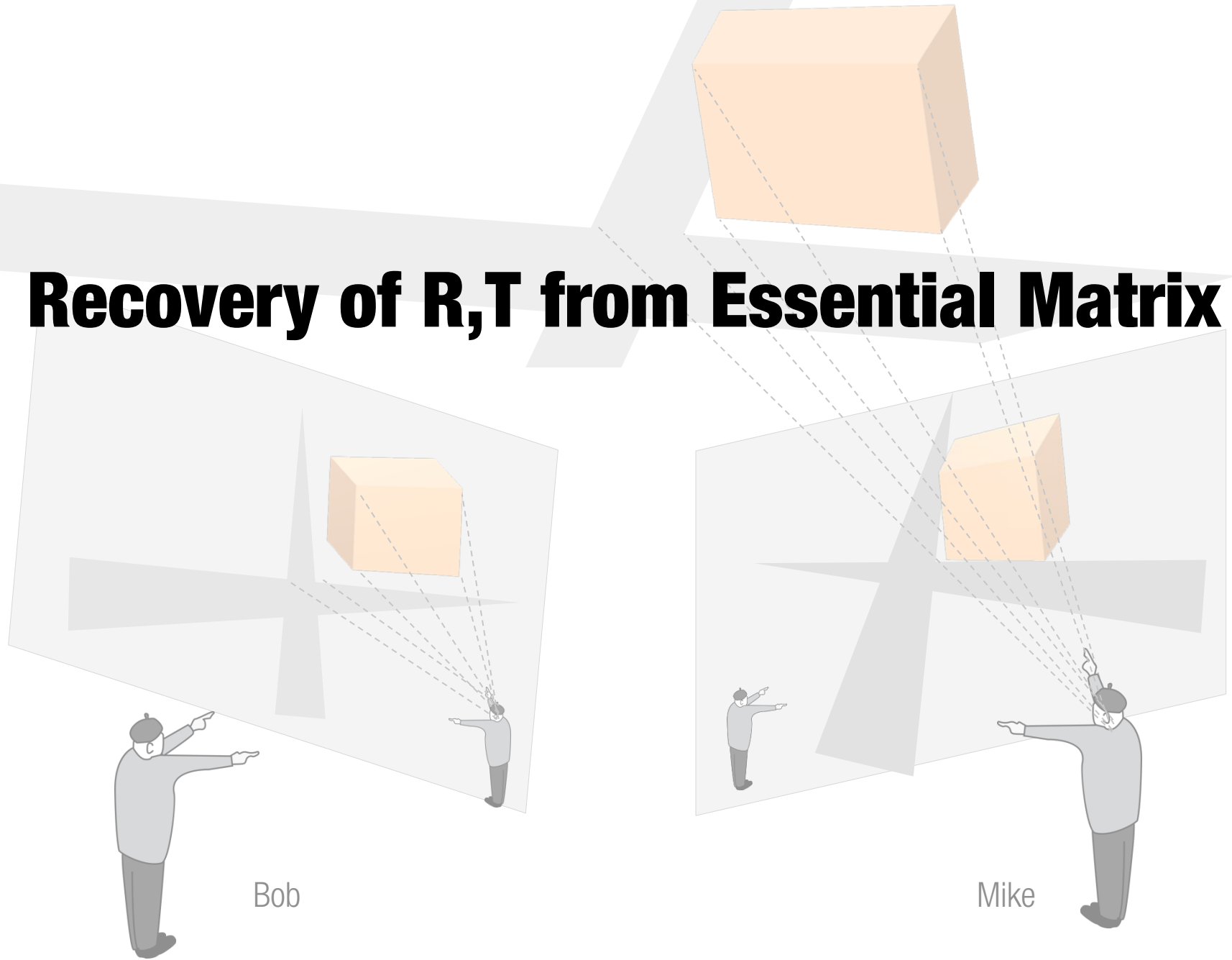
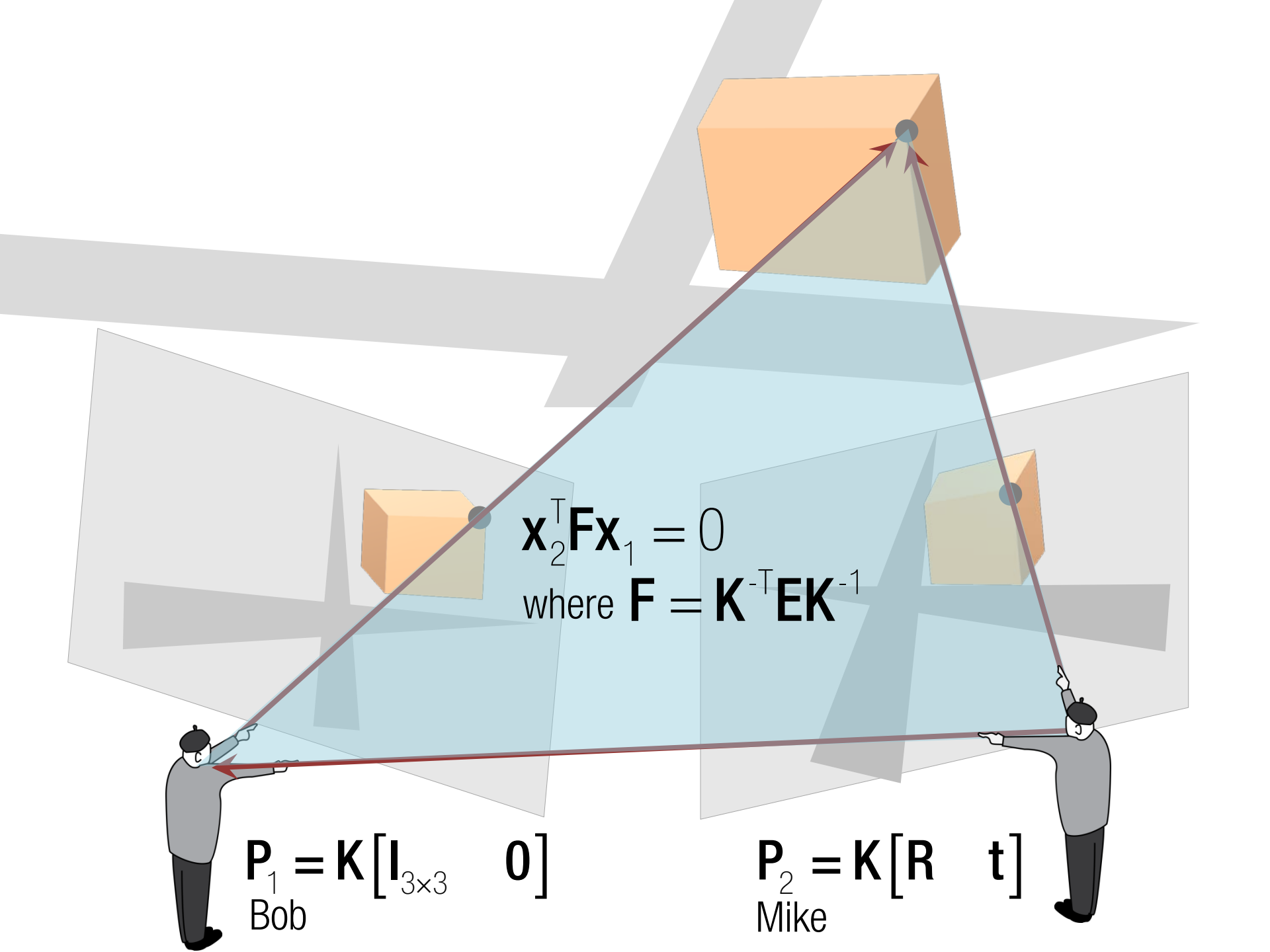


# Recovery of R,T from Essential Matrix



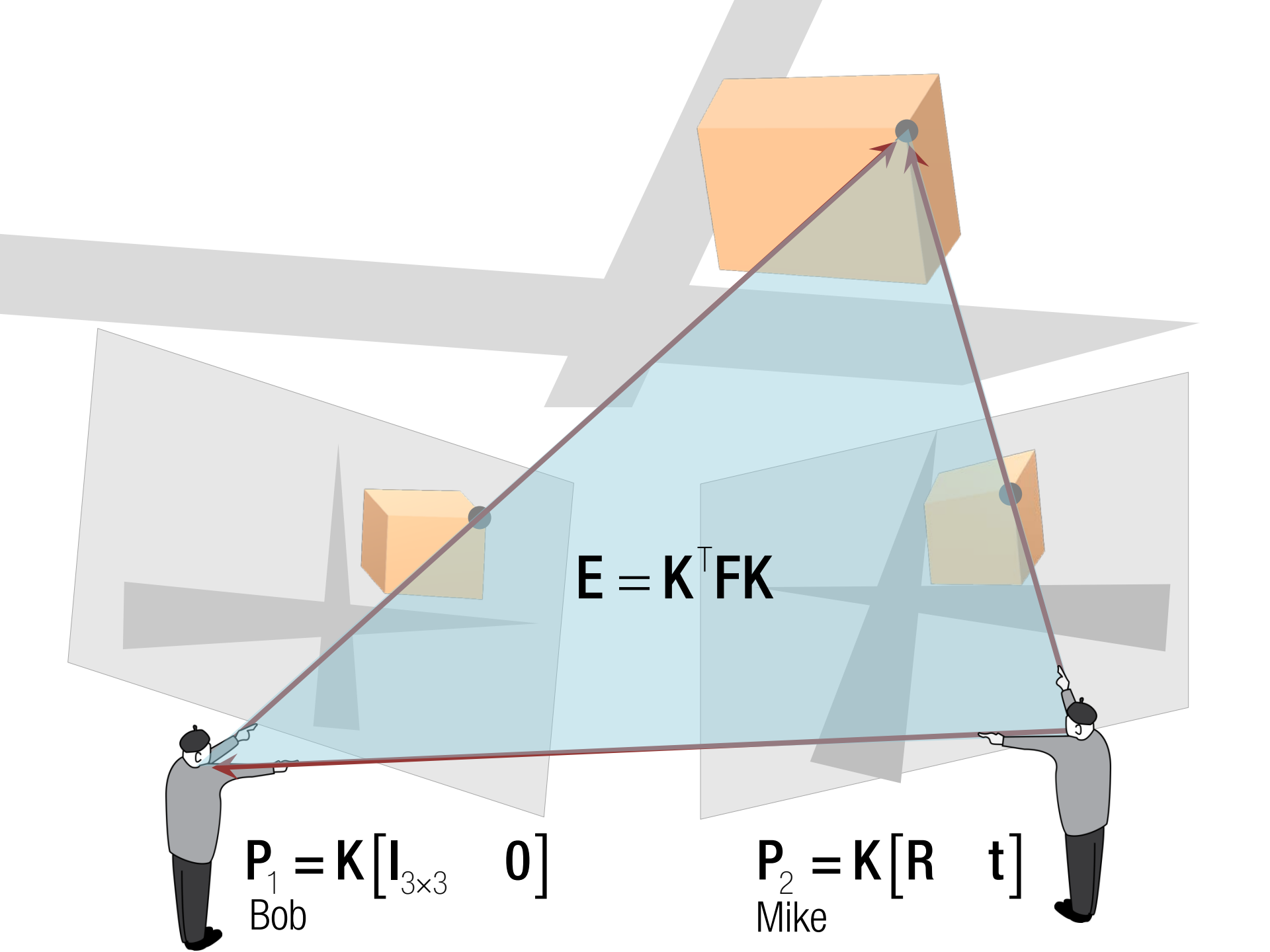


$$\mathbf{x}_2^T \mathbf{F} \mathbf{x}_1 = 0$$

where  $\mathbf{F} = \mathbf{K}^{-T} \mathbf{E} \mathbf{K}^{-1}$

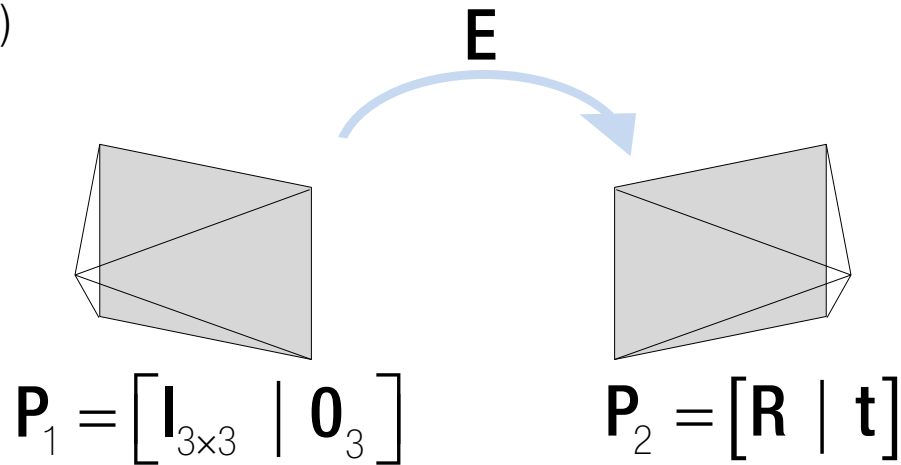
Bob  $\mathbf{P}_1 = \mathbf{K} \begin{bmatrix} \mathbf{I}_{3 \times 3} & \mathbf{0} \end{bmatrix}$

Mike  $\mathbf{P}_2 = \mathbf{K} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}$



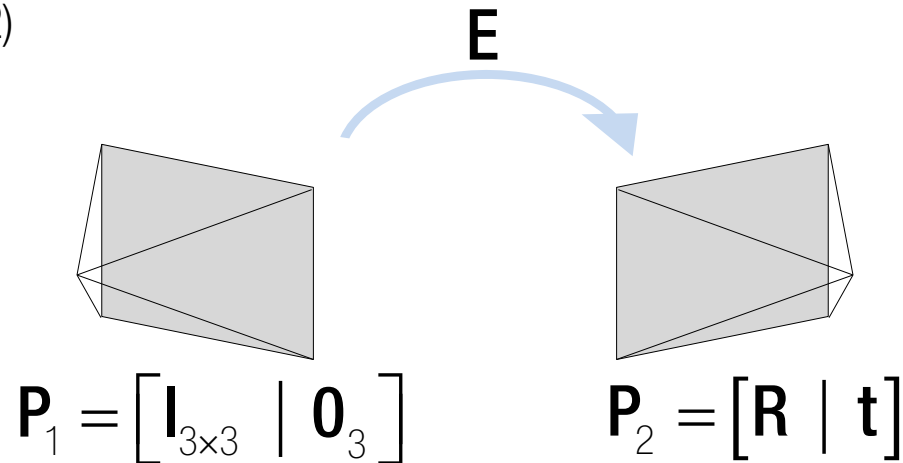
$\mathbf{E} = \begin{bmatrix} \mathbf{t} \end{bmatrix}_{\times} \mathbf{R}$  How to decompose the essential matrix to rotation and translation?

Essential matrix (rank 2)



$\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$  How to decompose the essential matrix to rotation and translation?

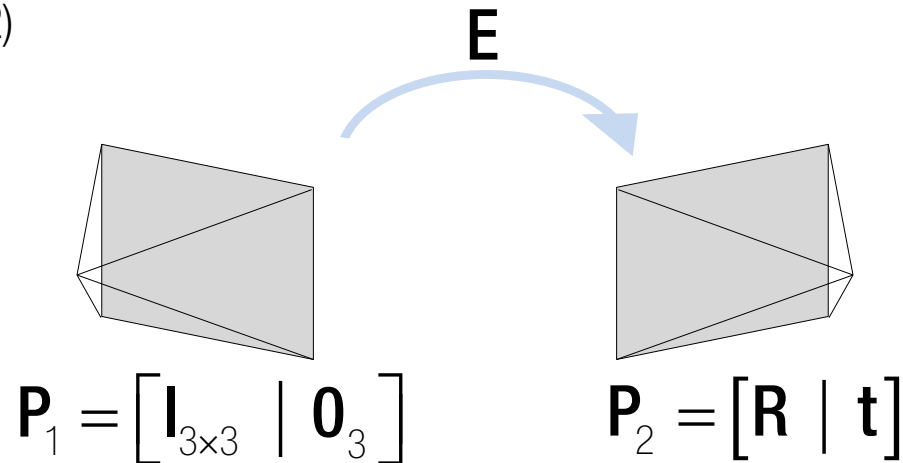
Essential matrix (rank 2)



$\mathbf{t}$  : Epipole in image 2 because  $\mathbf{P}_2 \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} = [\mathbf{R} \mid \mathbf{t}] \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} = \mathbf{t}$

$\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$  How to decompose the essential matrix to rotation and translation?

Essential matrix (rank 2)

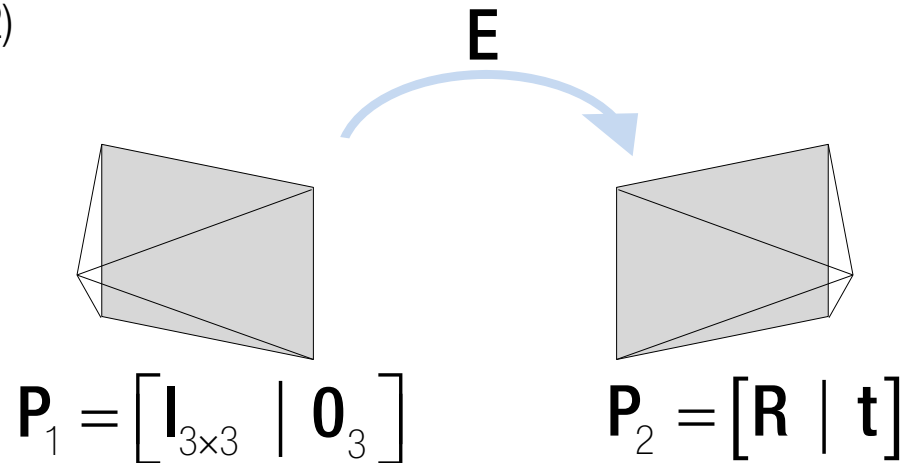


$\mathbf{t}$  : Epipole in image 2 because  $\mathbf{P}_2 \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} = [\mathbf{R} \mid \mathbf{t}] \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} = \mathbf{t}$

$\mathbf{t}^{\top} \mathbf{E} = \mathbf{0}$  : Left nullspace of the essential matrix is the epipole in image 2.

$\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$  How to decompose the essential matrix to rotation and translation?

Essential matrix (rank 2)



$\mathbf{t}$  : Epipole in image 2 because  $\mathbf{P}_2 \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} = [\mathbf{R} \mid \mathbf{t}] \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} = \mathbf{t}$

$\mathbf{t}^T \mathbf{E} = \mathbf{0}$  : Left nullspace of the essential matrix is the epipole in image 2.

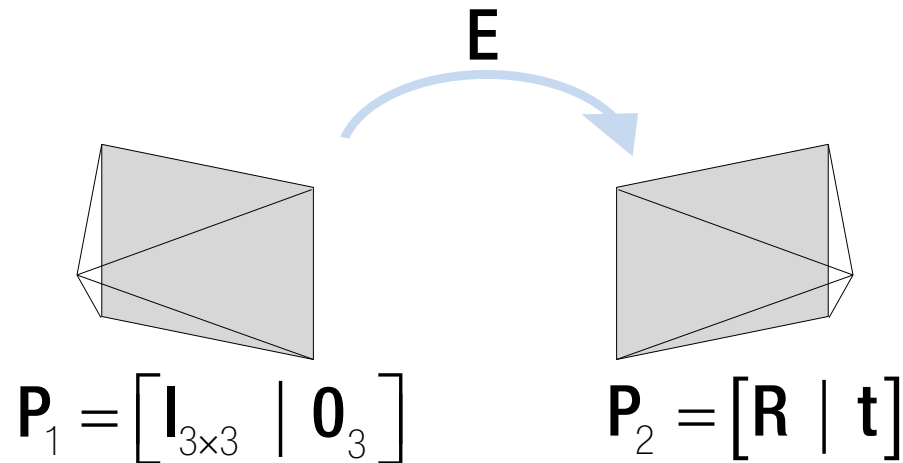
$\rightarrow \mathbf{t} = \mathbf{u}_3$ , or  $-\mathbf{u}_3$  where  $\mathbf{U} = [\mathbf{u}_1 \quad \mathbf{u}_2 \quad \mathbf{u}_3]$  and  $\mathbf{E} = \mathbf{U} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{V}^T$

Singular value decomposition (SVD)



$\mathbf{E} = \begin{bmatrix} \mathbf{t} \end{bmatrix}_{\times} \mathbf{R}$  How to decompose the essential matrix to rotation and translation?

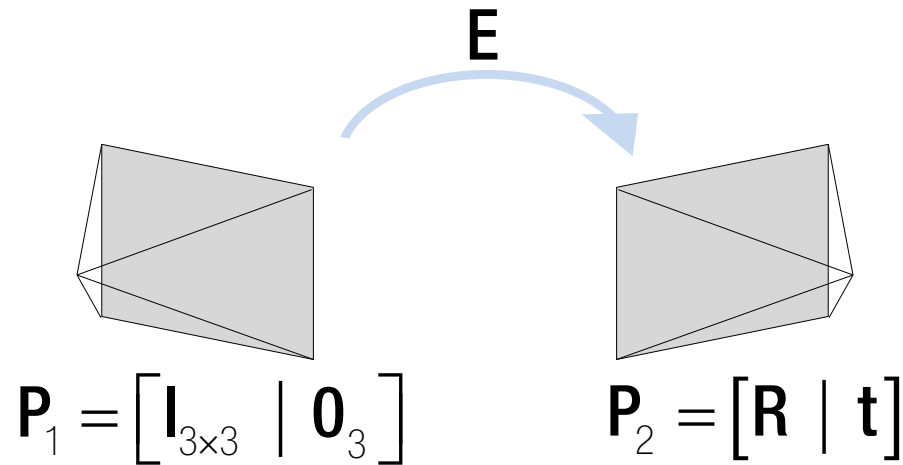
Essential matrix



$$\mathbf{E} = \mathbf{U} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{V}^T$$

$\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$  How to decompose the essential matrix to rotation and translation?

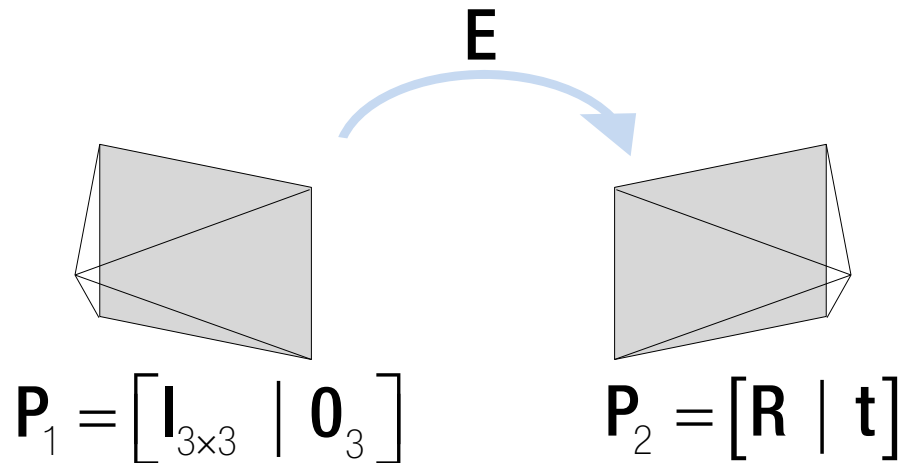
Essential matrix



$$\mathbf{E} = \mathbf{U} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{V}^T = [\mathbf{t}]_{\times} \mathbf{R}$$

$\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$  How to decompose the essential matrix to rotation and translation?

Essential matrix

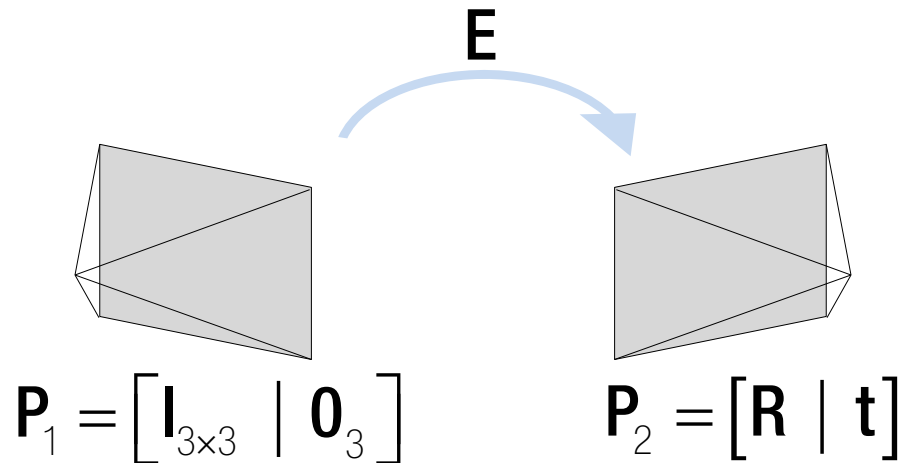


$$\mathbf{E} = \mathbf{U} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{V}^T = [\mathbf{t}]_{\times} \mathbf{R} = \left( \mathbf{U} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{U}^T \right) (\mathbf{U} \mathbf{Y} \mathbf{V}^T)$$

where  $\mathbf{U} = [\mathbf{u}_1 \quad \mathbf{u}_2 \quad \mathbf{t}]$

$\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$  How to decompose the essential matrix to rotation and translation?

Essential matrix



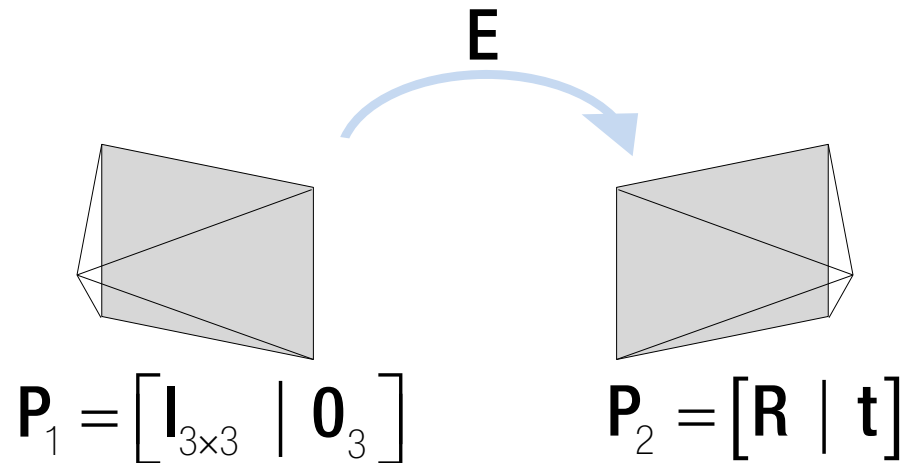
$$\mathbf{E} = \mathbf{U} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{V}^T = [\mathbf{t}]_{\times} \mathbf{R} = \left( \mathbf{U} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{U}^T \right) (\mathbf{U} \mathbf{Y} \mathbf{V}^T)$$

How do we set  $\mathbf{Y}$ ?

where  $\mathbf{U} = [\mathbf{u}_1 \quad \mathbf{u}_2 \quad \mathbf{t}]$

$\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$  How to decompose the essential matrix to rotation and translation?

Essential matrix



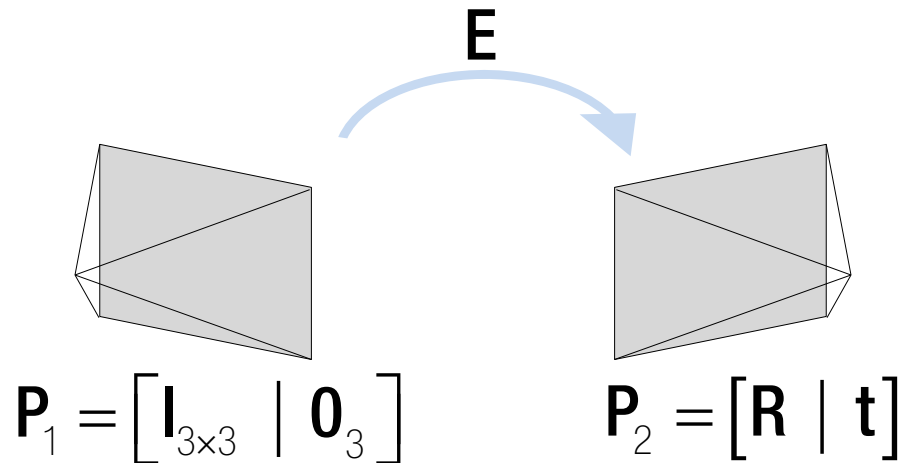
$$\mathbf{E} = \mathbf{U} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{V}^T = [\mathbf{t}]_{\times} \mathbf{R} = \mathbf{U} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{Y} \mathbf{V}^T$$

How do we set  $\mathbf{Y}$ ?

$$\therefore \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{Y}$$

$\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$  How to decompose the essential matrix to rotation and translation?

Essential matrix



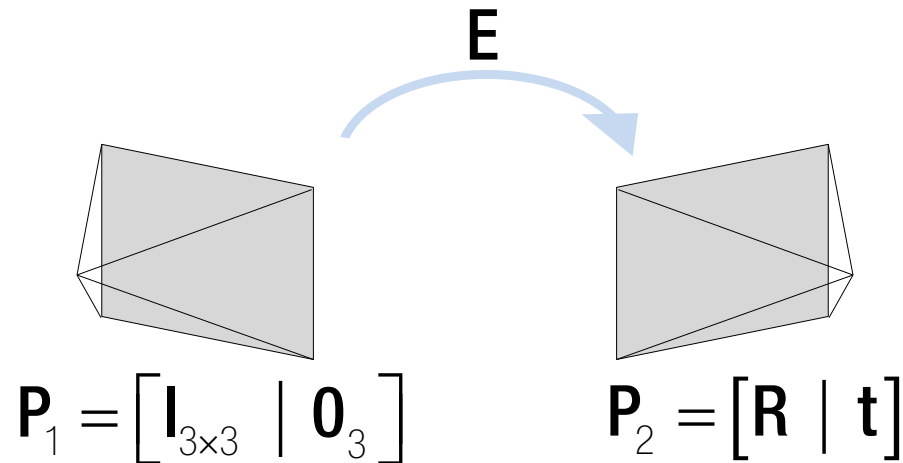
$$\mathbf{E} = \mathbf{U} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{V}^T = [\mathbf{t}]_{\times} \mathbf{R} = \mathbf{U} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{Y} \mathbf{V}^T$$

How do we set  $\mathbf{Y}$ ?

$$\therefore \mathbf{Y} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{ or } \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T$$

$\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$  How to decompose the essential matrix to rotation and translation?

Essential matrix

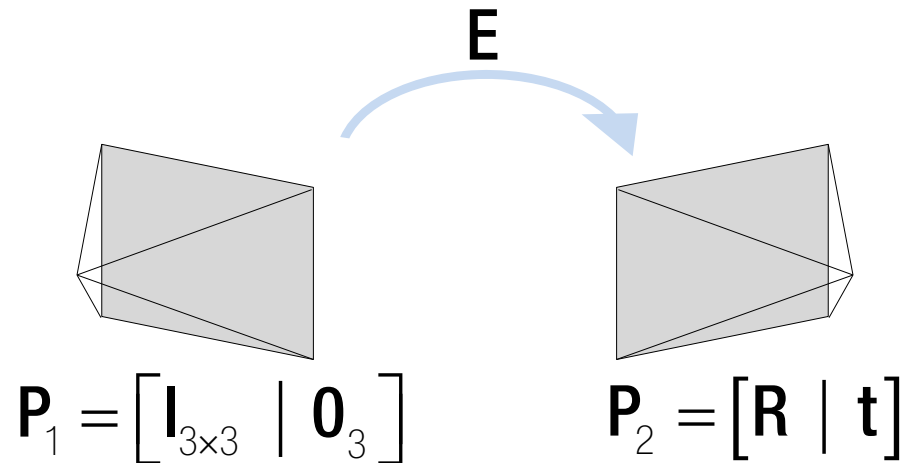


$$\mathbf{t} = \mathbf{u}_3, \text{ or } -\mathbf{u}_3 \quad \mathbf{R} = \mathbf{U} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{V}^T, \text{ or } \mathbf{U} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \mathbf{V}^T$$

where  $\mathbf{E} = \mathbf{U} \mathbf{D} \mathbf{V}^T$      $\mathbf{U} = [\mathbf{u}_1 \quad \mathbf{u}_2 \quad \mathbf{t}]$

$\mathbf{E} = \begin{bmatrix} \mathbf{t} \\ \times \end{bmatrix} \mathbf{R}$  How to decompose the essential matrix to rotation and translation?

Essential matrix



Four configurations:

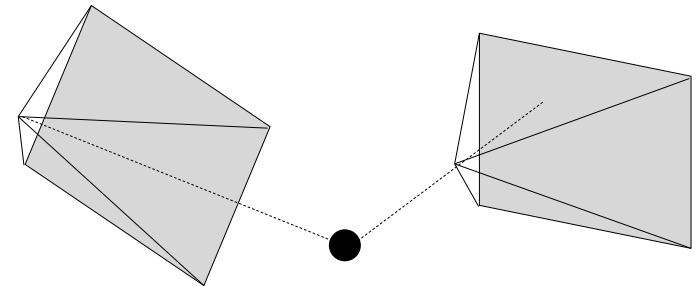
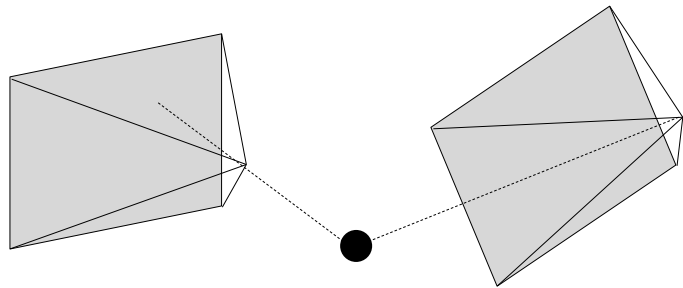
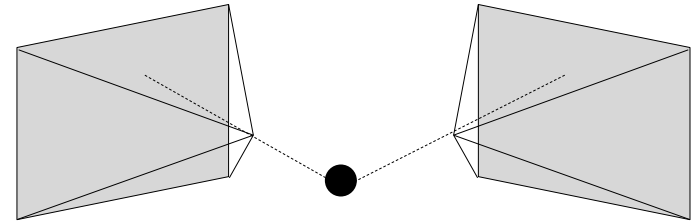
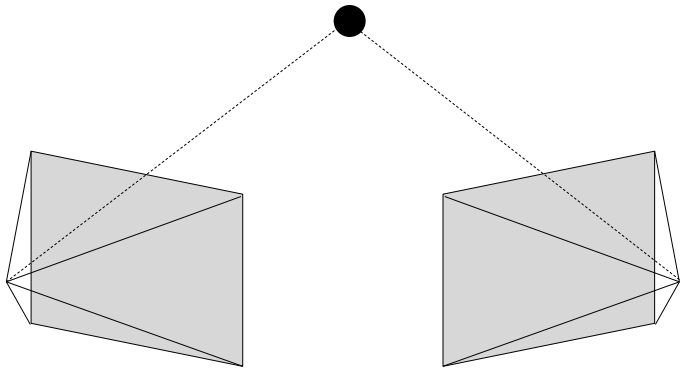
$$\mathbf{P}_2 = \begin{bmatrix} \mathbf{U}\mathbf{Y}\mathbf{V}^T & | & \mathbf{u}_3 \end{bmatrix}, \text{ or } \begin{bmatrix} \mathbf{U}\mathbf{Y}^T\mathbf{V}^T & | & \mathbf{u}_3 \end{bmatrix}, \text{ or } \begin{bmatrix} \mathbf{U}\mathbf{Y}\mathbf{V}^T & | & -\mathbf{u}_3 \end{bmatrix}, \text{ or } \begin{bmatrix} \mathbf{U}\mathbf{Y}^T\mathbf{V}^T & | & -\mathbf{u}_3 \end{bmatrix}$$



$\mathbf{E} = \begin{bmatrix} \mathbf{t} \end{bmatrix}_{\times} \mathbf{R}$  How to decompose the essential matrix to rotation and translation?

Essential matrix

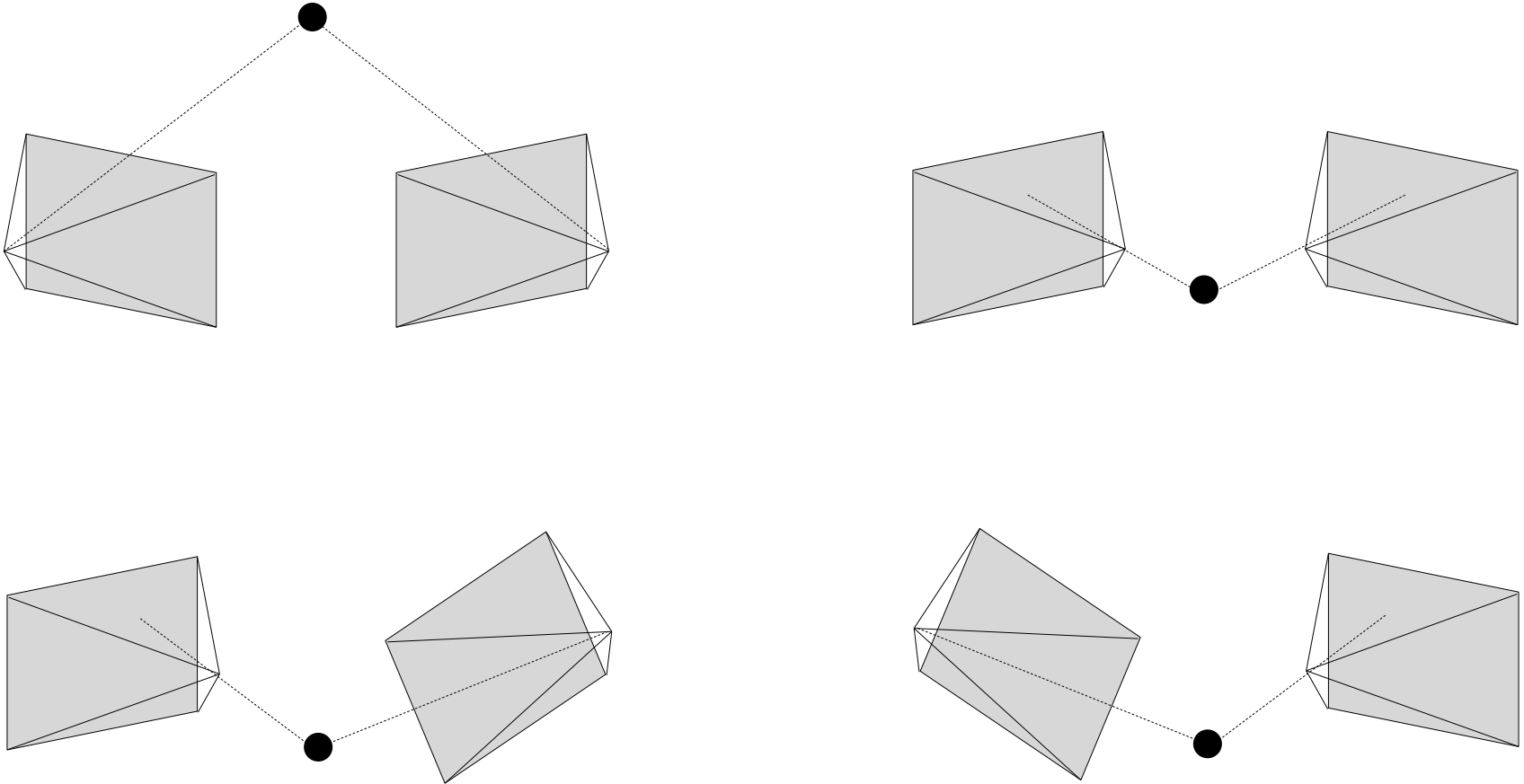
Four configurations:



$\mathbf{E} = \begin{bmatrix} \mathbf{t} \end{bmatrix}_{\times} \mathbf{R}$  How to decompose the essential matrix to rotation and translation?

Essential matrix

Four configurations: can be resolved by point triangulation.



## 2.2 Camera Pose Extraction

**Goal** Given  $\mathbf{E}$ , enumerate four camera pose configurations,  $(\mathbf{C}_1, \mathbf{R}_1)$ ,  $(\mathbf{C}_2, \mathbf{R}_2)$ ,  $(\mathbf{C}_3, \mathbf{R}_3)$ , and  $(\mathbf{C}_4, \mathbf{R}_4)$  where  $\mathbf{C} \in \mathbb{R}^3$  is the camera center and  $\mathbf{R} \in SO(3)$  is the rotation matrix, i.e.,  $\mathbf{P} = \mathbf{KR} \begin{bmatrix} \mathbf{I}_{3 \times 3} & -\mathbf{C} \end{bmatrix}$ :

`[Cset Rset] = ExtractCameraPose(E)`

(INPUT)  $\mathbf{E}$ : essential matrix

(OUTPUT)  $\mathbf{Cset}$  and  $\mathbf{Rset}$ : four configurations of camera centers and rotations, i.e.,  $\mathbf{Cset}\{i\} = \mathbf{C}_i$  and  $\mathbf{Rset}\{i\} = \mathbf{R}_i$ .

There are four camera pose configurations given an essential matrix. Let  $\mathbf{E} = \mathbf{UDV}^T$  and  $\mathbf{W} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ . The four configurations are enumerated below:

1.  $\mathbf{C}_1 = \mathbf{U}(:, 3)$  and  $\mathbf{R}_1 = \mathbf{UWV}^T$
2.  $\mathbf{C}_2 = -\mathbf{U}(:, 3)$  and  $\mathbf{R}_2 = \mathbf{UWV}^T$
3.  $\mathbf{C}_3 = \mathbf{U}(:, 3)$  and  $\mathbf{R}_3 = \mathbf{UW}^T\mathbf{V}^T$
4.  $\mathbf{C}_4 = -\mathbf{U}(:, 3)$  and  $\mathbf{R}_4 = \mathbf{UW}^T\mathbf{V}^T$ .

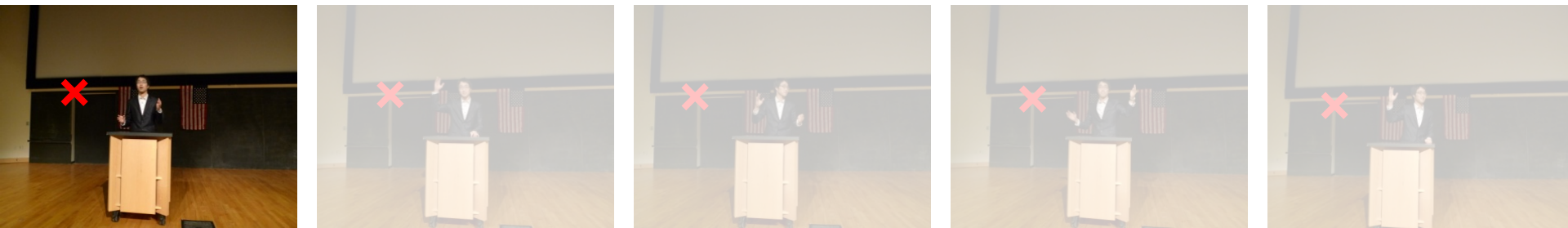
Note that the determinant of a rotation matrix is one. If  $\det(\mathbf{R}) = -1$ , the camera pose must be corrected, i.e.,  $\mathbf{C} \leftarrow -\mathbf{C}$  and  $\mathbf{R} \leftarrow -\mathbf{R}$ .

# Point Triangulation

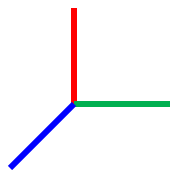


✘ 2D correspondences

# Point Triangulation



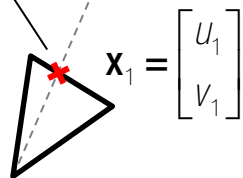
✗ 2D correspondences



3D point

$$\mathbf{x} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

2D projection



$$\mathbf{x}_1 = \begin{bmatrix} u_1 \\ v_1 \end{bmatrix}$$

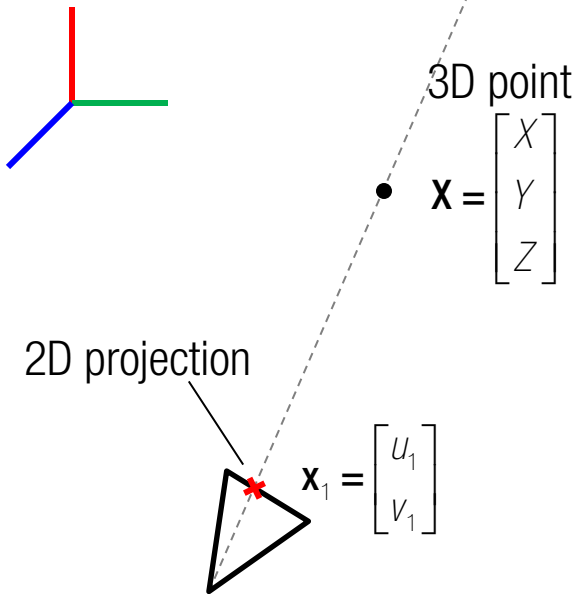
3D camera pose

$$\mathbf{P}_1 \in \mathbb{R}^{3 \times 4}$$

# Point Triangulation



✗ 2D correspondences



3D point  
 $\mathbf{x} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$

2D projection

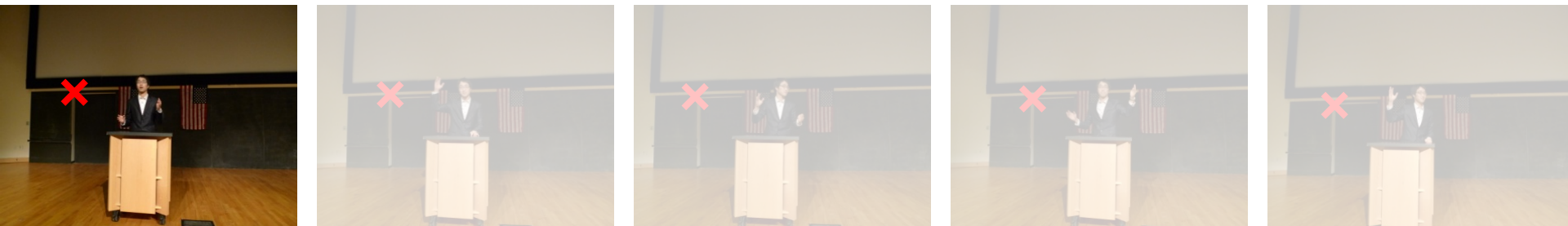
$\mathbf{x}_1 = \begin{bmatrix} u_1 \\ v_1 \end{bmatrix}$

3D camera pose

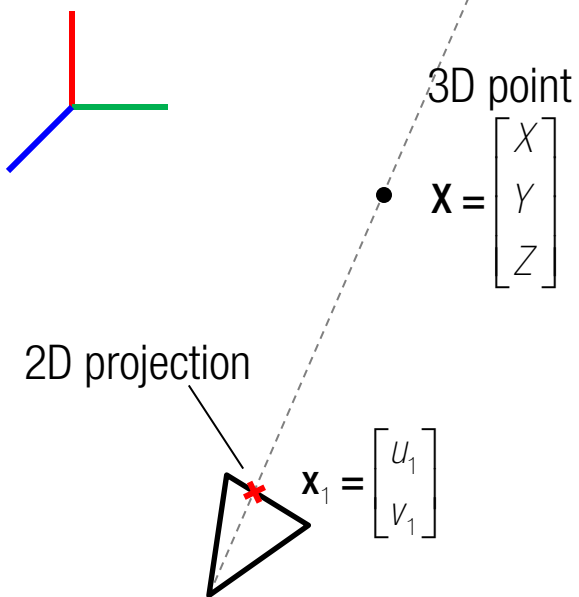
$\mathbf{P}_1 \in \mathbb{R}^{3 \times 4}$

$$\lambda \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix} = \mathbf{P}_1 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

# Point Triangulation



✗ 2D correspondences



3D point  
 $\mathbf{x} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$

2D projection

$\mathbf{x}_1 = \begin{bmatrix} u_1 \\ v_1 \end{bmatrix}$

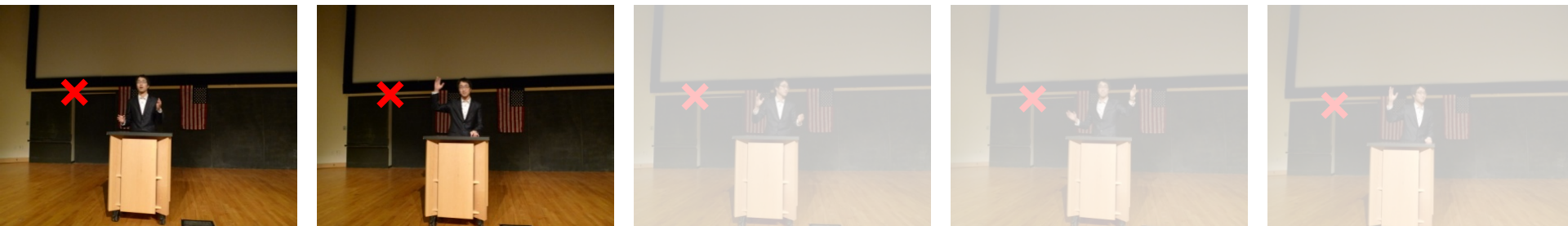
3D camera pose

$\mathbf{P}_1 \in \mathbb{R}^{3 \times 4}$

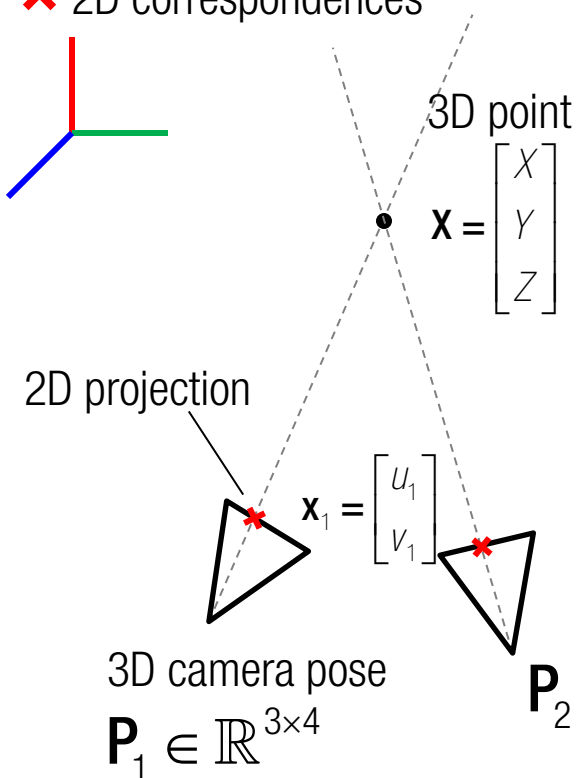
$$\lambda \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix} = \mathbf{P}_1 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix}_{\times} \mathbf{P}_1 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

Cross product between two parallel vectors equals to zero.

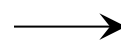
# Point Triangulation



✗ 2D correspondences



$$\lambda \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix} = \mathbf{P}_1 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

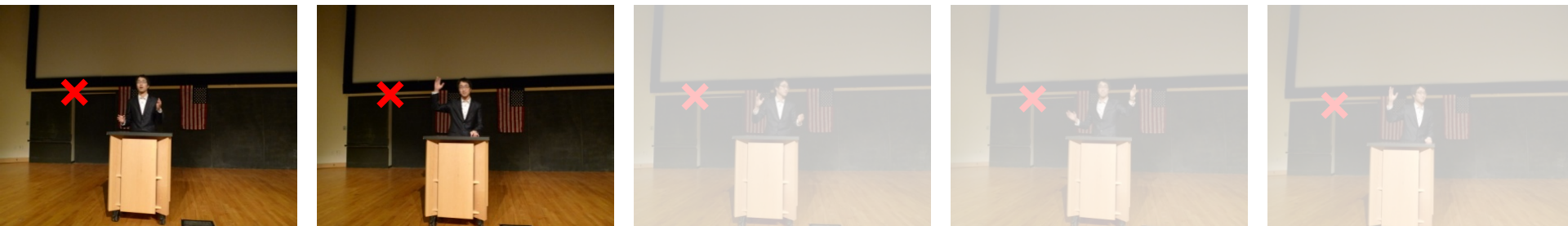


$$\begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix}_x \mathbf{P}_1 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = 0$$

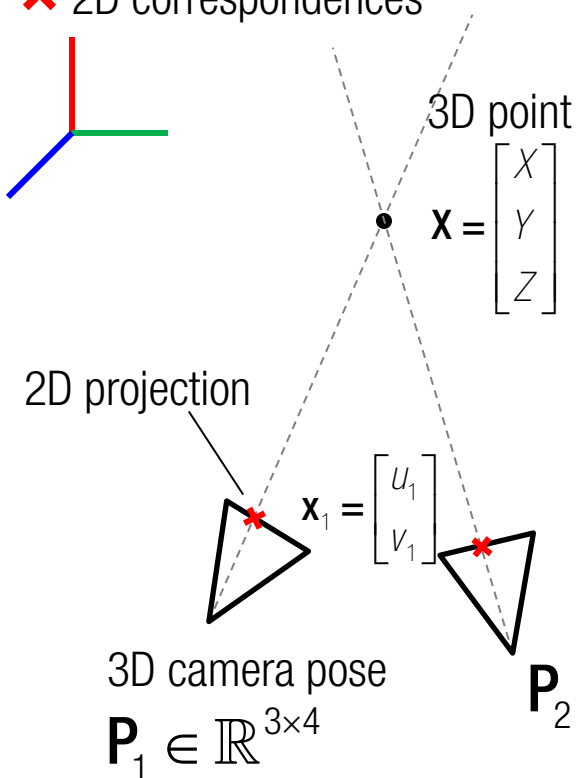
$$\begin{bmatrix} \mathbf{x}_2 \\ 1 \end{bmatrix}_x \mathbf{P}_2 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = 0$$



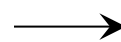
# Point Triangulation



✗ 2D correspondences



$$\lambda \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix} = \mathbf{P}_1 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$



$$\begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix}_x \mathbf{P}_1 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = 0$$

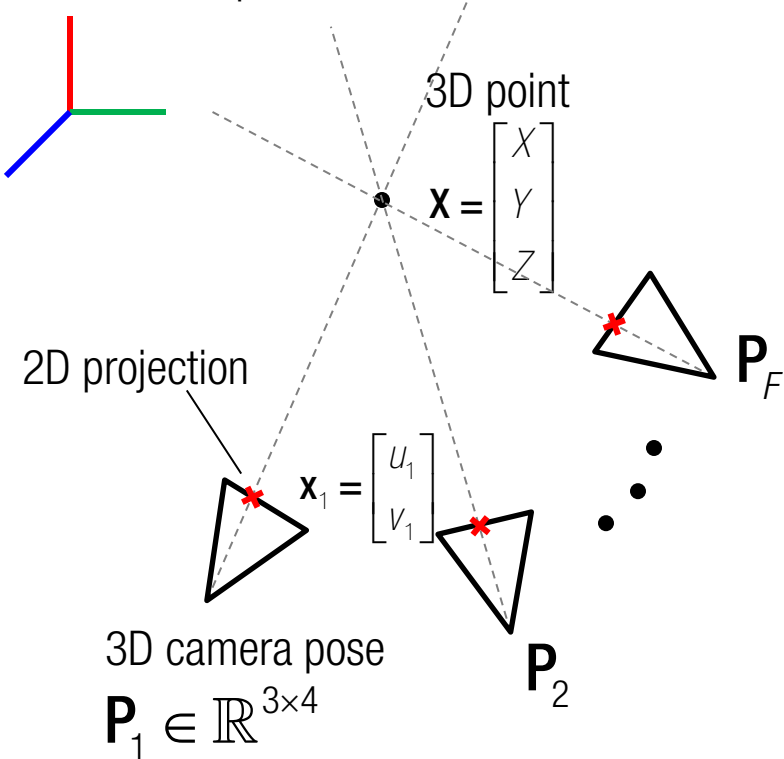
$$\begin{bmatrix} \mathbf{x}_2 \\ 1 \end{bmatrix}_x \mathbf{P}_2 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix}_x \mathbf{P}_1 \\ \begin{bmatrix} \mathbf{x}_2 \\ 1 \end{bmatrix}_x \mathbf{P}_2 \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = 0$$

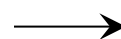
# Point Triangulation



✗ 2D correspondences



$$\lambda \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix} = \mathbf{P}_1 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$



$$\begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix}_x \mathbf{P}_1 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = 0$$

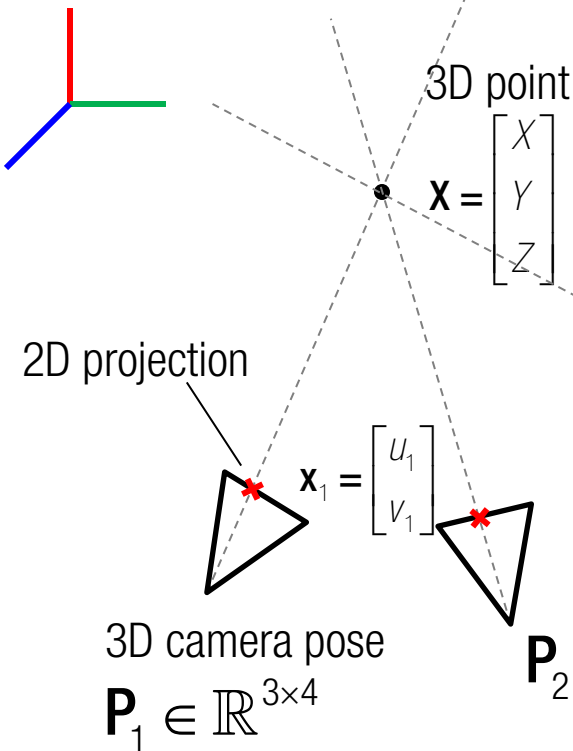
$$\begin{bmatrix} \mathbf{x}_2 \\ 1 \end{bmatrix}_x \mathbf{P}_2 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix}_x \mathbf{P}_1 \\ \begin{bmatrix} \mathbf{x}_2 \\ 1 \end{bmatrix}_x \mathbf{P}_2 \\ \vdots \\ \begin{bmatrix} \mathbf{x}_F \\ 1 \end{bmatrix}_x \mathbf{P}_F \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = 0$$

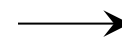
# Point Triangulation



✗ 2D correspondences



$$\lambda \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix} = \mathbf{P}_1 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$



$$\begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix}_x \mathbf{P}_1 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} \mathbf{x}_2 \\ 1 \end{bmatrix}_x \mathbf{P}_2 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = 0$$

$$3F \begin{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix}_x \mathbf{P}_1 \\ \vdots \\ \begin{bmatrix} \mathbf{x}_F \\ 1 \end{bmatrix}_x \mathbf{P}_F \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = 0$$

$$\text{rank} \left( \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}_x \mathbf{P} \right) = 2$$

Least squares if  $F \geq 2$

# Point Triangulation



$$P_1 = K_1 \begin{bmatrix} I_{3 \times 3} & 0_3 \end{bmatrix}$$

$$P_2 = K_2 \begin{bmatrix} I_{3 \times 3} & -C \end{bmatrix} \quad C = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

```
% Intrinsic parameter
K1 = [2329.558 0 1141.452; 0 2329.558 927.052; 0 0 1];
K2 = [2329.558 0 1241.731; 0 2329.558 927.052; 0 0 1];
```

```
% Camera matrices
P1 = K1 * [eye(3) zeros(3,1)];
C = [1;0;0];
P2 = K2 * [eye(3) -C];
```

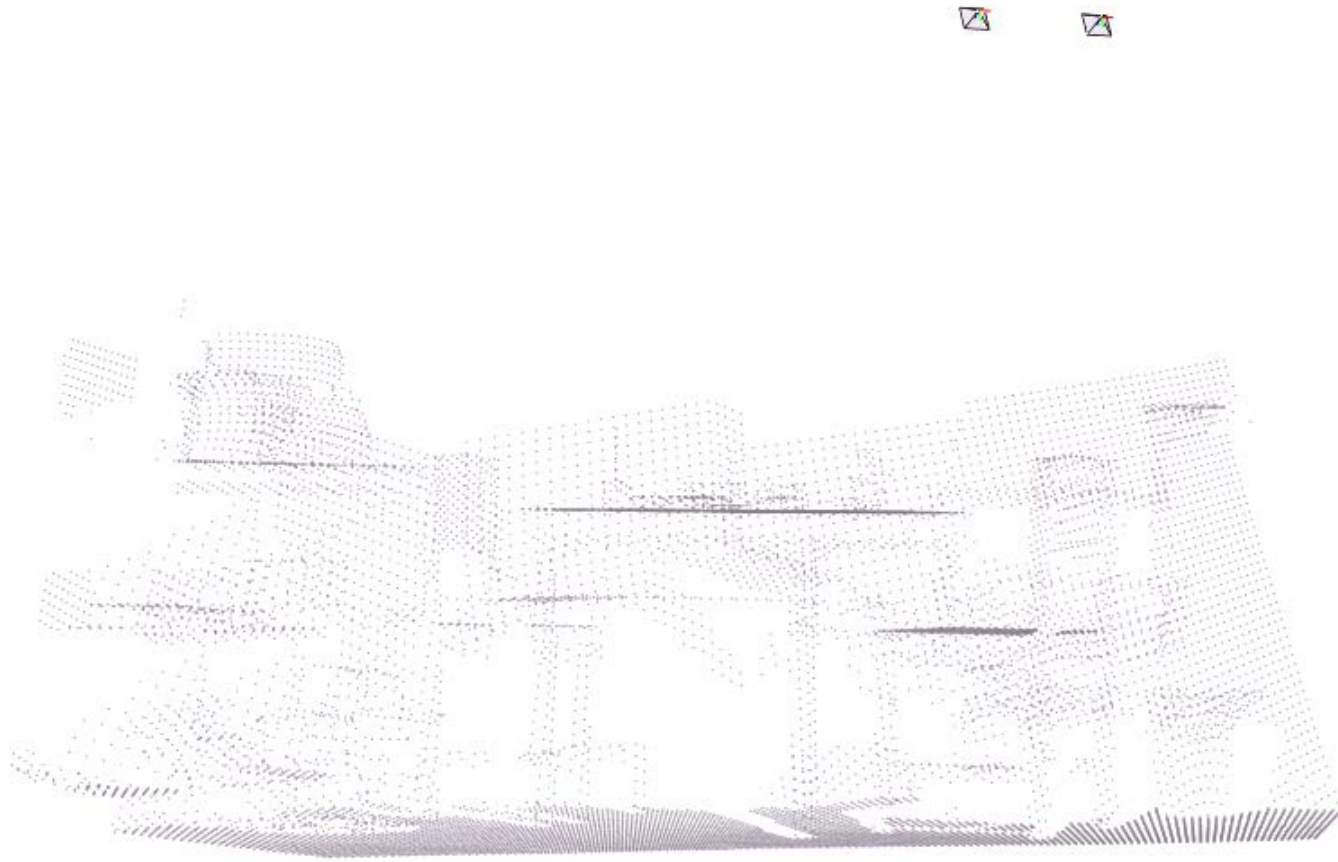
```
% Correspondences
x1 = [1382;986;1];
x2 = [1144;986;1];
skew1 = Vec2Skew(x1);
skew2 = Vec2Skew(x2);
```

```
% Solve
A = [skew1*P1; skew2*P2];
[u,d,v] = svd(A);
X = v(:,end)/v(end,end);
```

```
function skew = Vec2Skew(v)
skew = [0 -v(3) v(2); v(3) 0 -v(1); -v(2) v(1) 0];
```

```
X =
    0.7111
    0.1743
    6.8865
    1.0000
```

# Point Triangulation





### 3.1 Linear Triangulation

**Goal** Given two camera poses,  $(\mathbf{C}_1, \mathbf{R}_1)$  and  $(\mathbf{C}_2, \mathbf{R}_2)$ , and correspondences  $\mathbf{x}_1 \leftrightarrow \mathbf{x}_2$ , triangulate 3D points using linear least squares:

$\mathbf{X} = \text{LinearTriangulation}(\mathbf{K}, \mathbf{C}_1, \mathbf{R}_1, \mathbf{C}_2, \mathbf{R}_2, \mathbf{x}_1, \mathbf{x}_2)$

(INPUT)  $\mathbf{C}_1$  and  $\mathbf{R}_1$ : the first camera pose

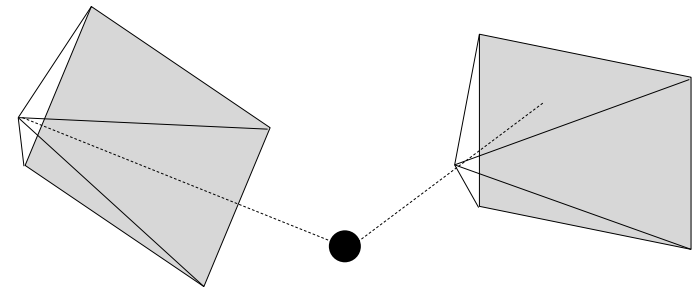
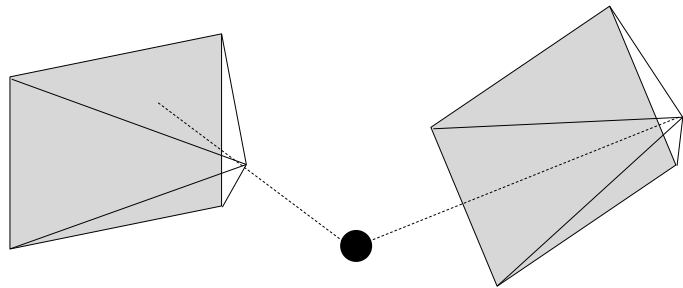
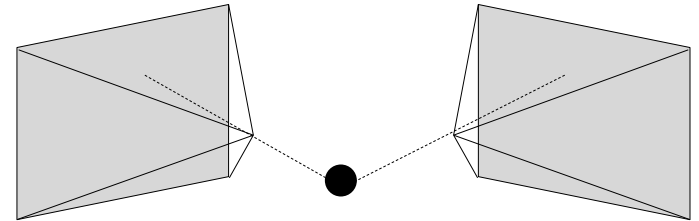
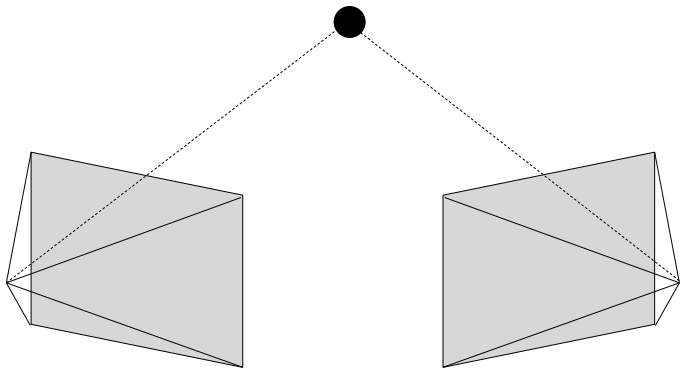
(INPUT)  $\mathbf{C}_2$  and  $\mathbf{R}_2$ : the second camera pose

(INPUT)  $\mathbf{x}_1$  and  $\mathbf{x}_2$ : two  $N \times 2$  matrices whose row represents correspondence between the first and second images where  $N$  is the number of correspondences.

(OUTPUT)  $\mathbf{X}$ :  $N \times 3$  matrix whose row represents 3D triangulated point.

# Camera pose disambiguation via point triangulation

Four configurations:





## 3.2 Camera Pose Disambiguation

**Goal** Given four camera pose configuration and their triangulated points, find the unique camera pose by checking the *cheirality* condition—the reconstructed points must be in front of the cameras:

[C R X0] = DisambiguateCameraPose(Cset, Rset, Xset)

(INPUT) Cset and Rset: four configurations of camera centers and rotations

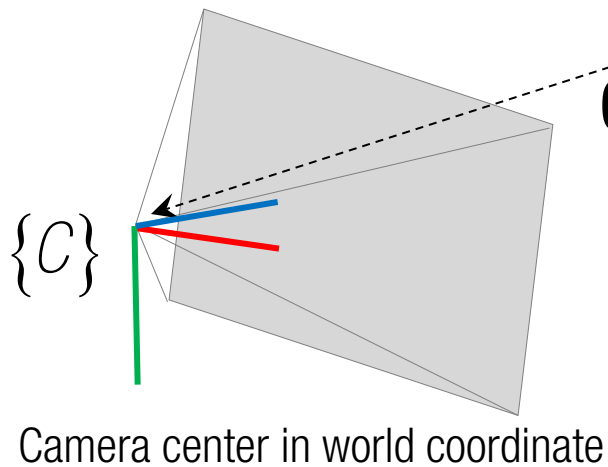
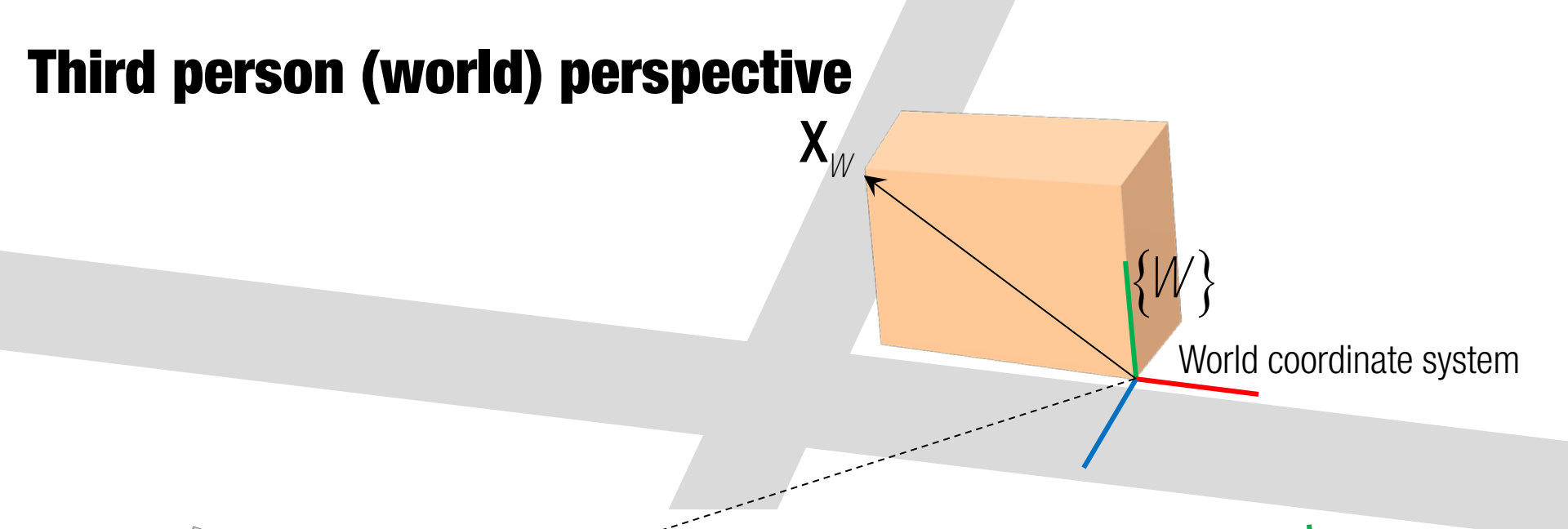
(INPUT) Xset: four sets of triangulated points from four camera pose configurations

(OUTPUT) C and R: the correct camera pose

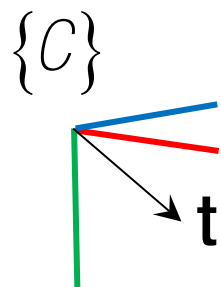
(OUTPUT) X0: the 3D triangulated points from the correct camera pose

The sign of the  $Z$  element in the camera coordinate system indicates the location of the 3D point with respect to the camera, i.e., a 3D point  $\mathbf{X}$  is in front of a camera if  $(\mathbf{C}, \mathbf{R})$  if  $\mathbf{r}_3(\mathbf{X} - \mathbf{C}) > 0$  where  $\mathbf{r}_3$  is the third row of  $\mathbf{R}$ . Not all triangulated points satisfy this condition due to the presence of correspondence noise. The best camera configuration,  $(\mathbf{C}, \mathbf{R}, \mathbf{X})$  is the one that produces the maximum number of points satisfying the cheirality condition.

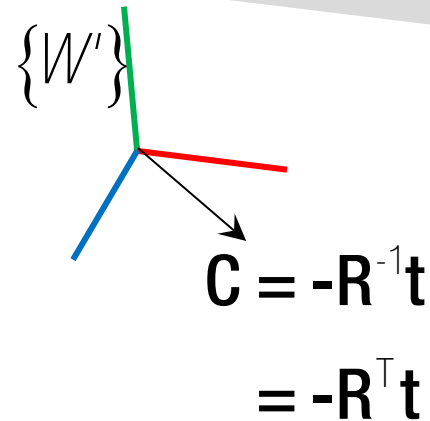
# Third person (world) perspective



$$\mathbf{C} = -\mathbf{R}^{-1}\mathbf{t}$$



$$\mathbf{R}^{-1} = \mathbf{R}^T$$



$$\begin{aligned} \mathbf{P} &= \mathbf{K}[\mathbf{R} \mid \mathbf{t}] \\ &= \mathbf{K}[\mathbf{R} \mid -\mathbf{R}\mathbf{C}] \\ &= \mathbf{K}\mathbf{R}[\mathbf{I}_{3 \times 3} \mid \underline{-\mathbf{C}}] \end{aligned}$$

Camera center seen from world coordinate system