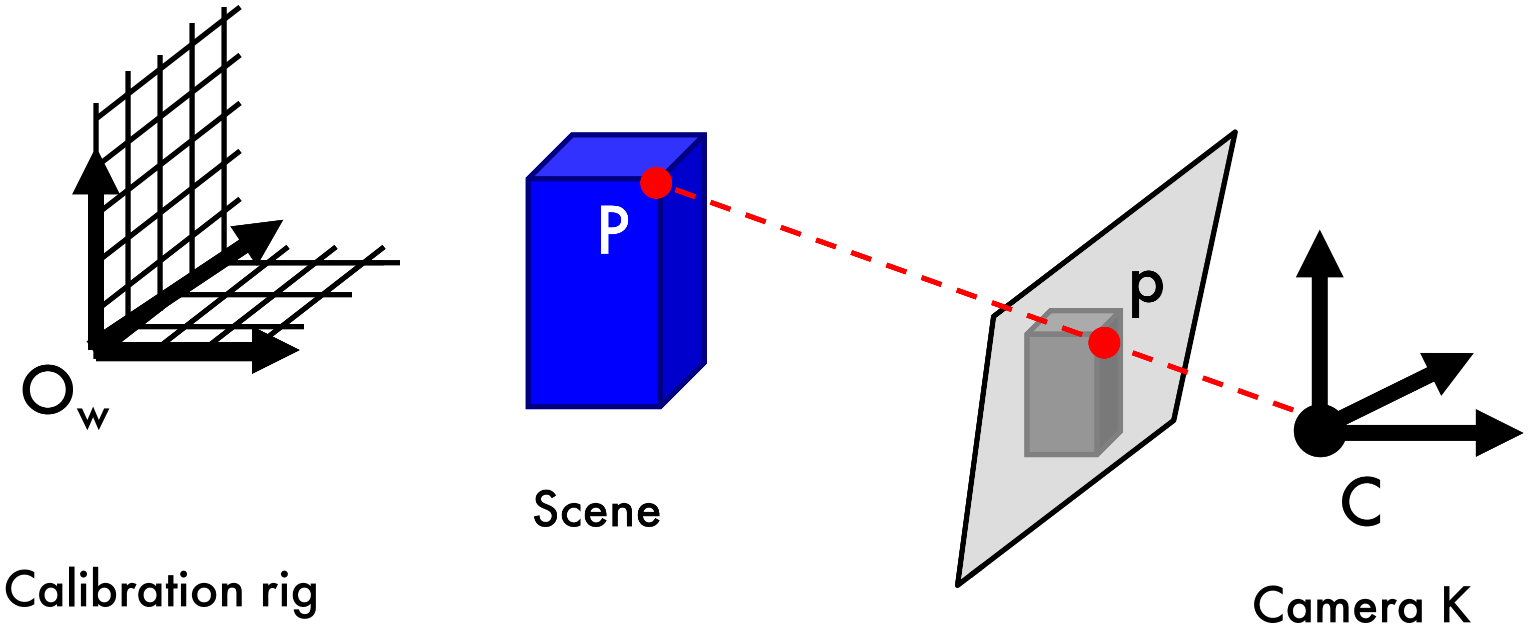
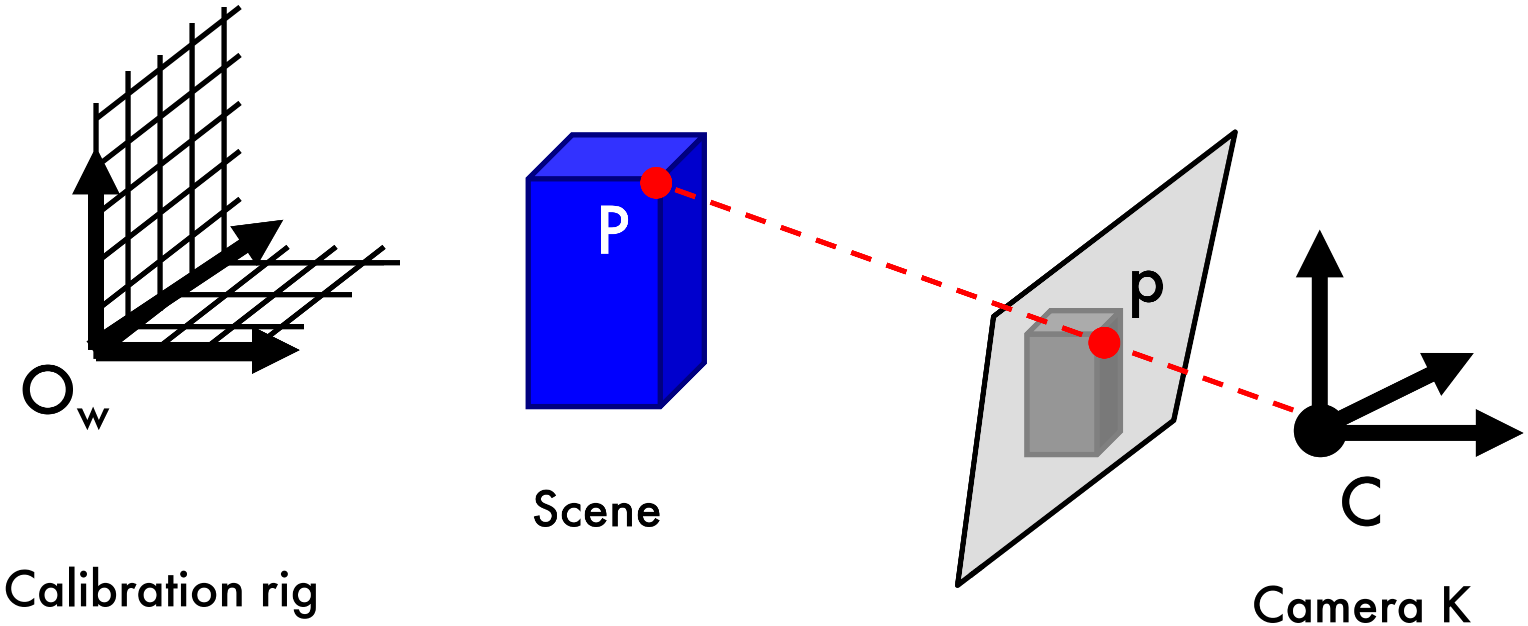
Recovering structure from a single view



From calibration rig	→	location/pose of the rig, K
From points and lines at infinity + orthogonal lines and planes	→	structure of the scene, K
Knowledge about scene (point correspondences, geometry of lines & planes, etc…		

Recovering structure from a single view



Why is it so difficult?

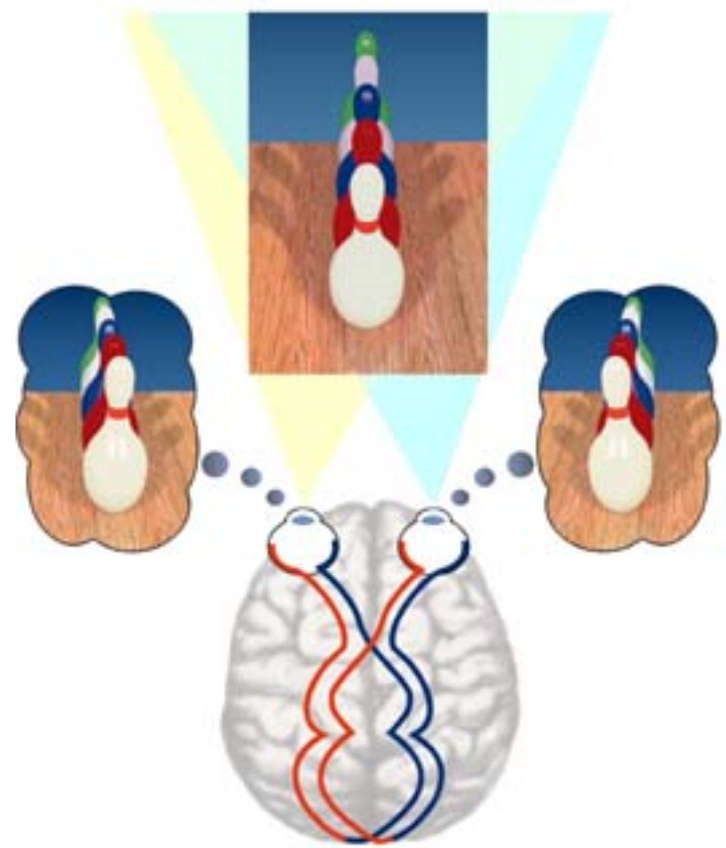
Intrinsic ambiguity of the mapping from 3D to image (2D)

Recovering structure from a single view

Intrinsic ambiguity of the mapping from 3D to image (2D)

Courtesy slide S. Lazebnik

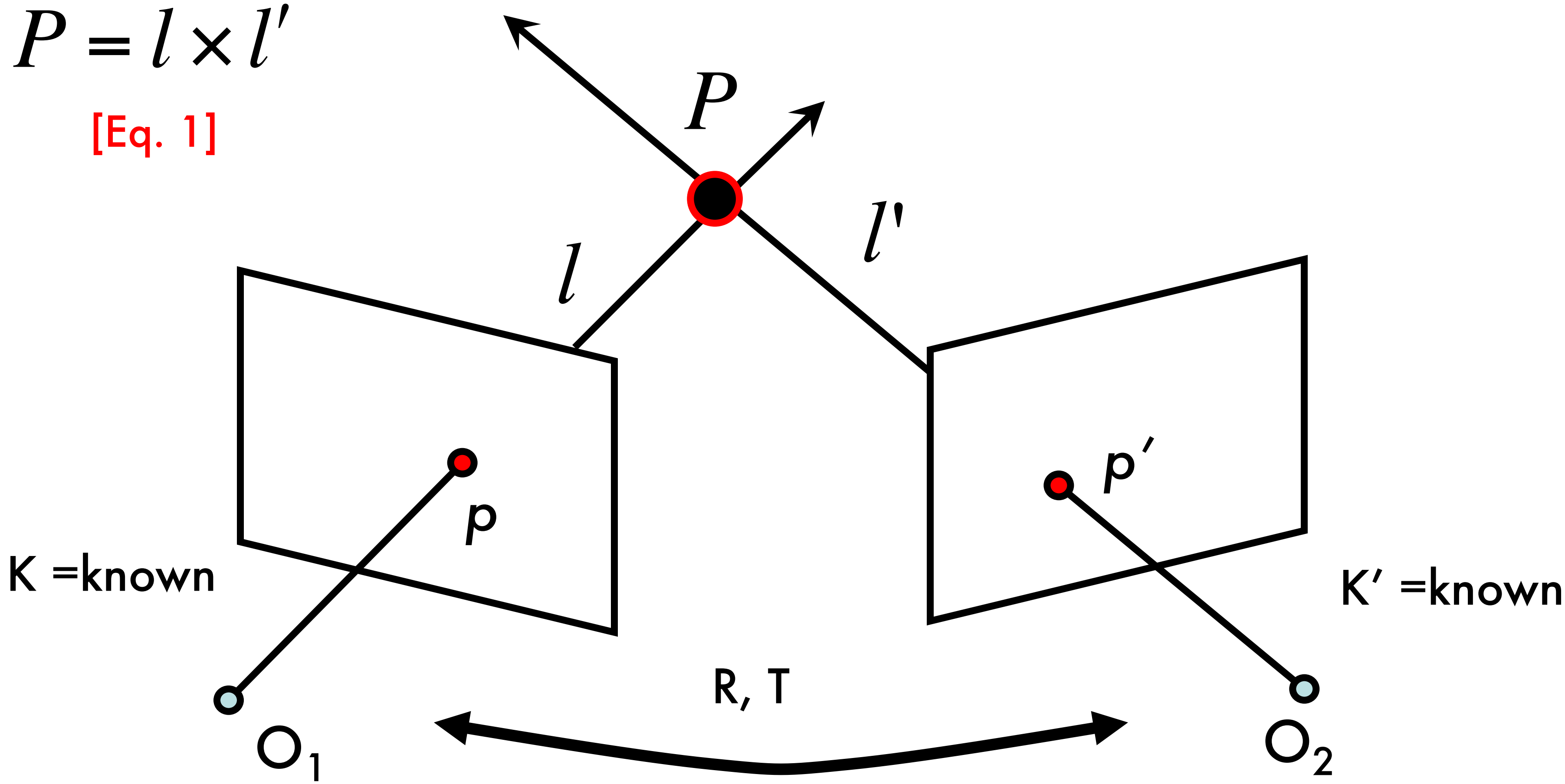
Two eyes help!



Two eyes help!

$$P = l \times l'$$

[Eq. 1]

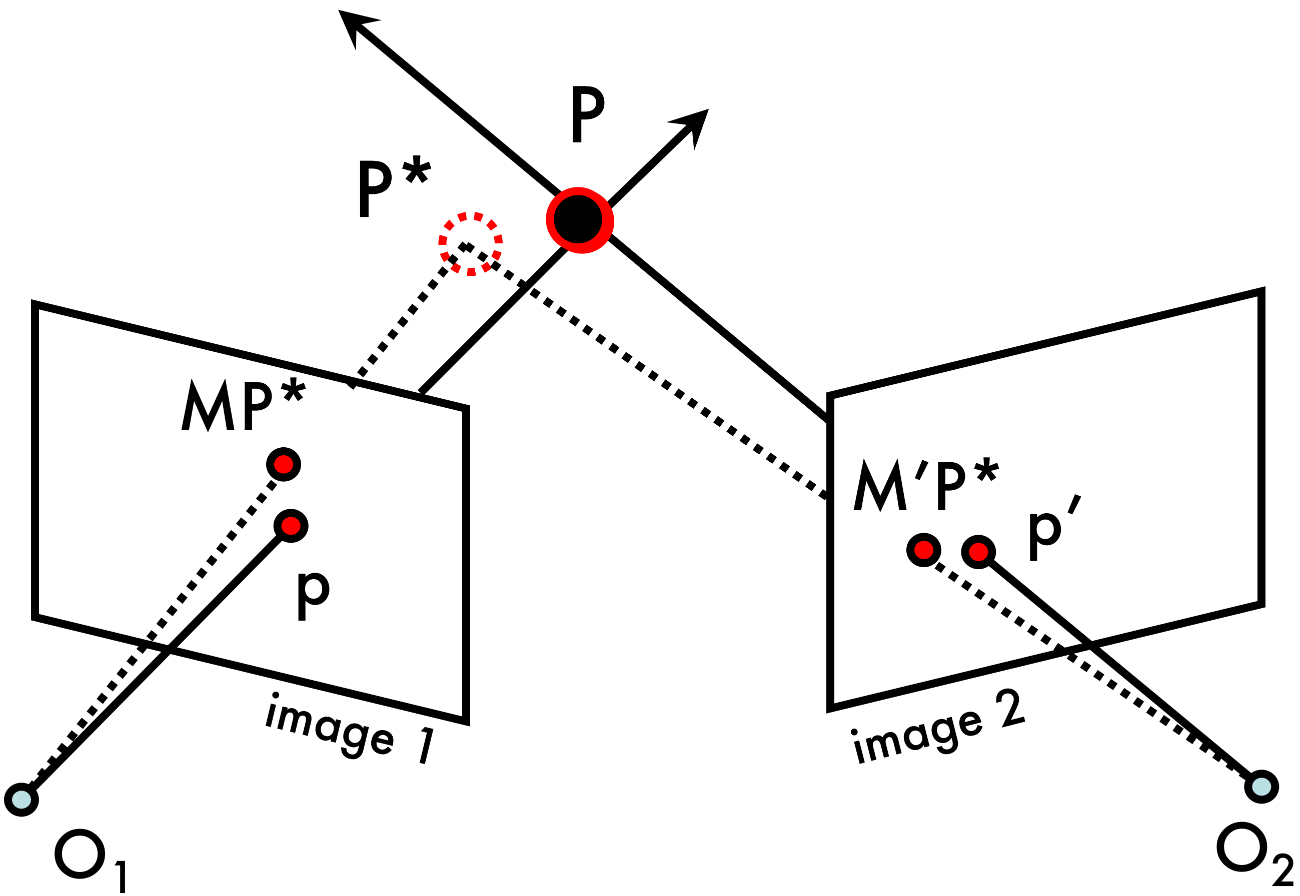


This is called triangulation

Triangulation

- Find P* that minimizes

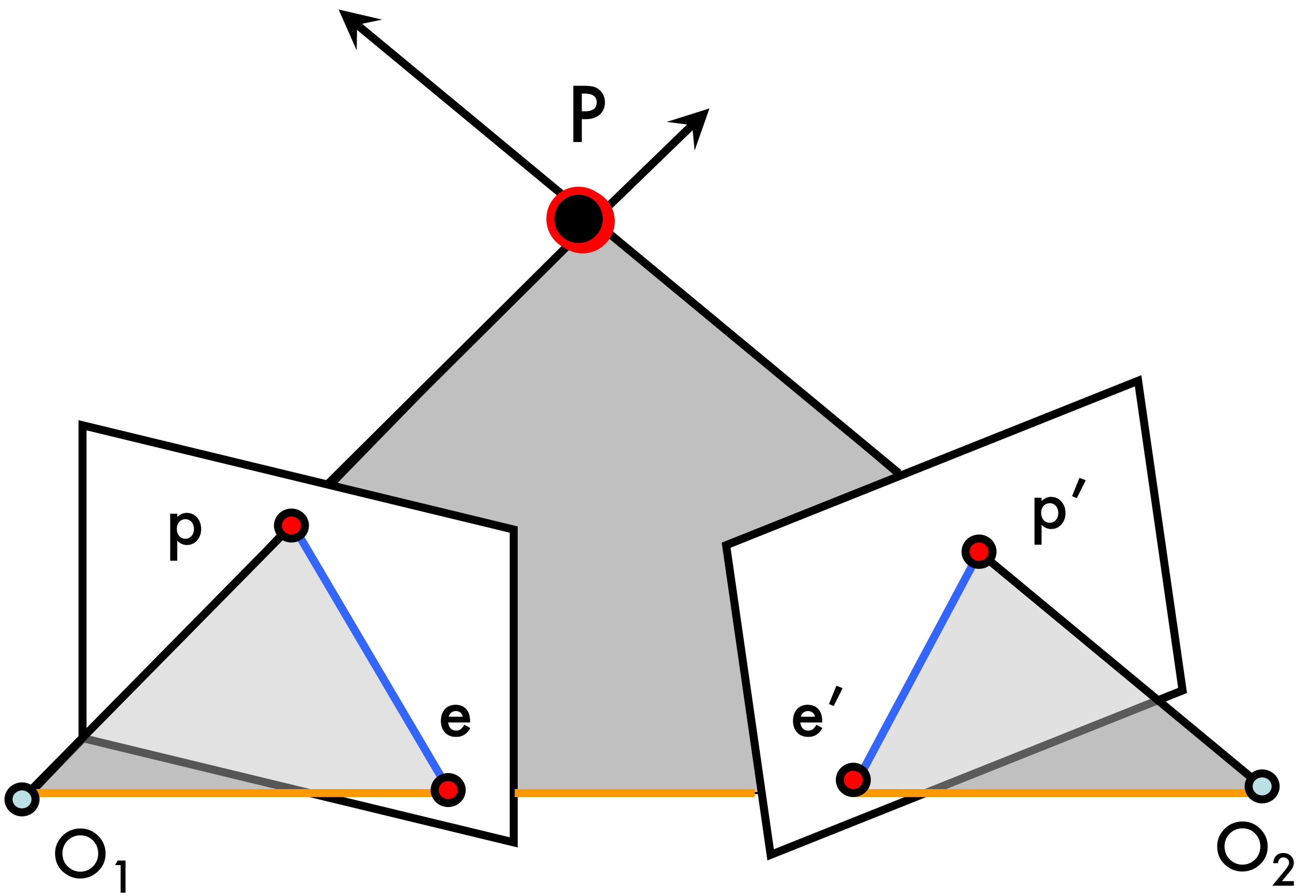
$$d(p, M\ P^*) + d(p', M' P^*)$$ [Eq. 2]



Multi (stereo)-view geometry

- **Camera geometry:** Given corresponding points in two images, find camera matrices, position and pose.
- **Scene geometry:** Find coordinates of 3D point from its projection into 2 or multiple images.
- **Correspondence:** Given a point **p** in one image, how can I find the corresponding point **p′** in another one?

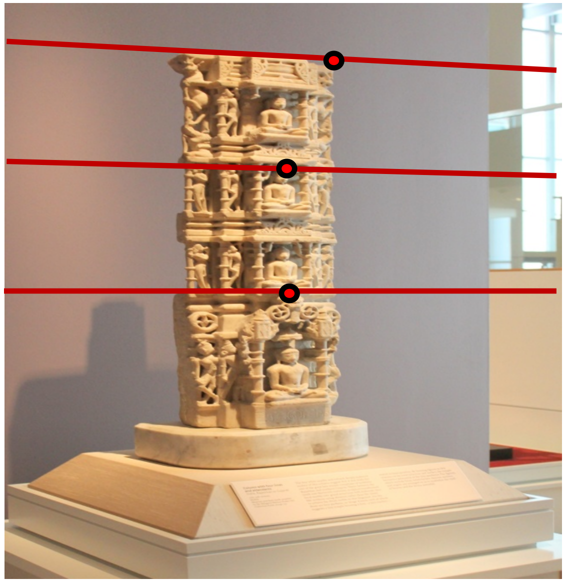
Epipolar geometry

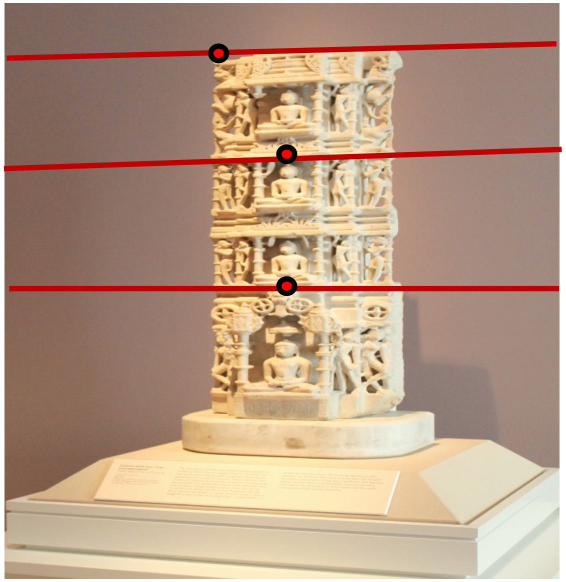


- Epipolar Plane
- Baseline
- Epipolar Lines
- Epipoles e, e′

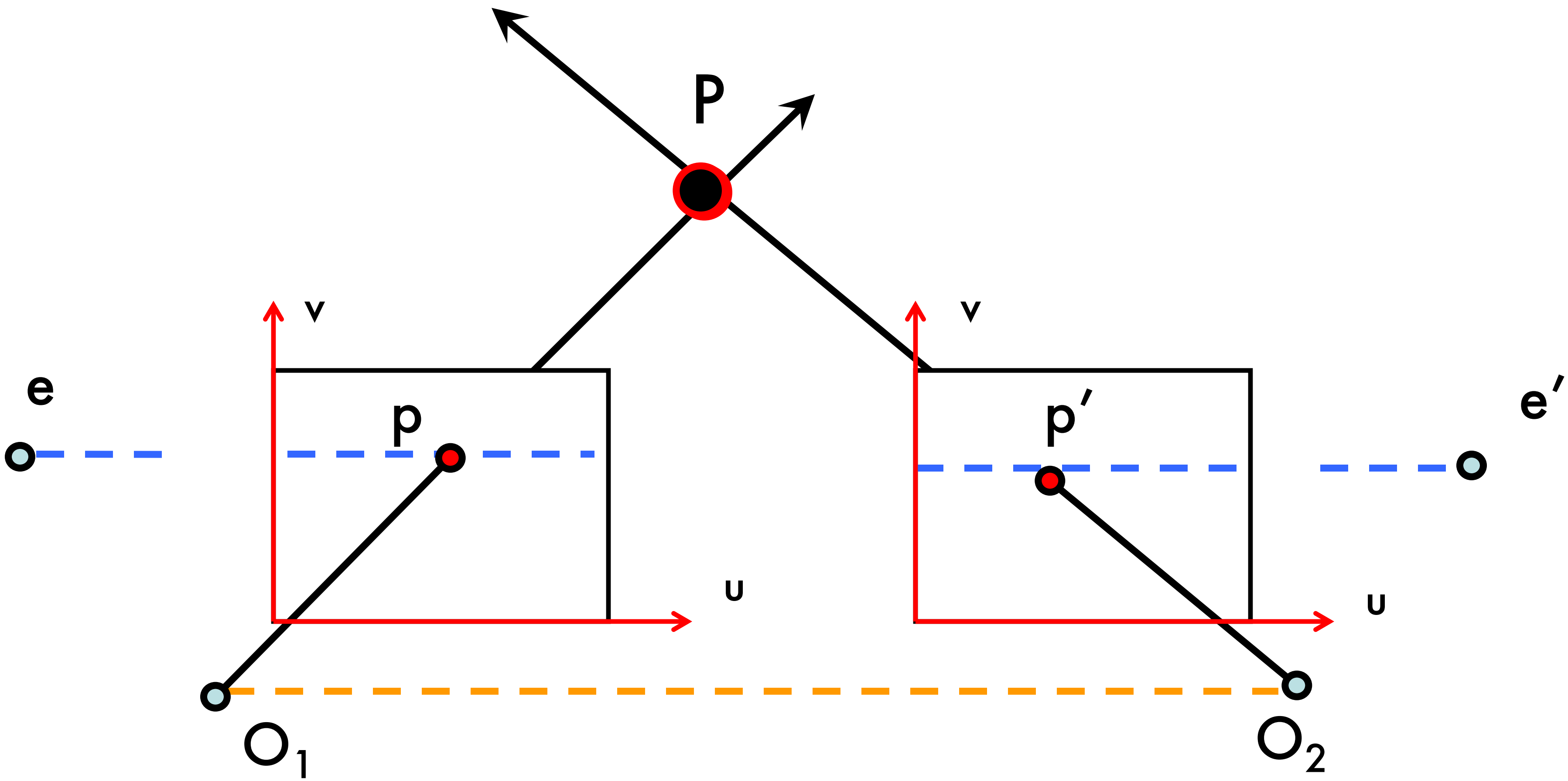
  = intersections of baseline with image planes
  = projections of the other camera center

Example of epipolar lines





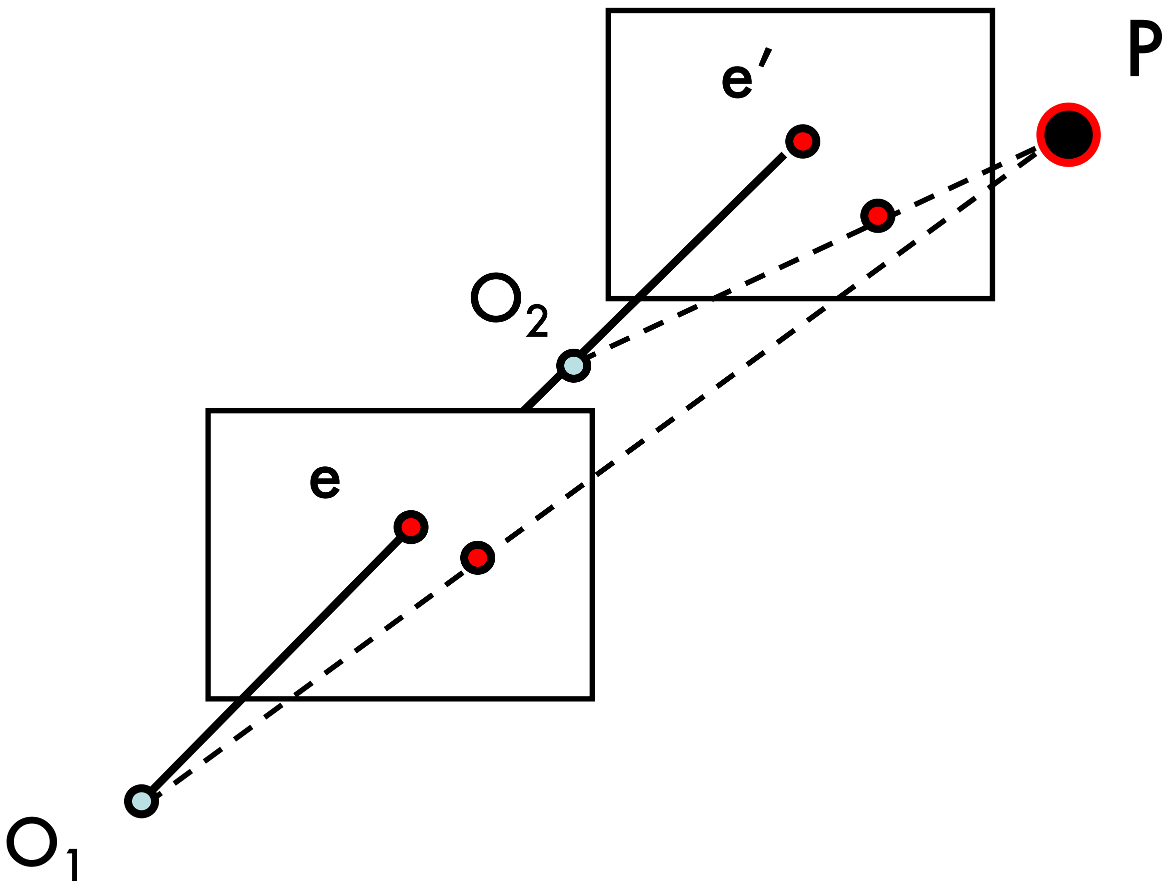
Example: Parallel image planes

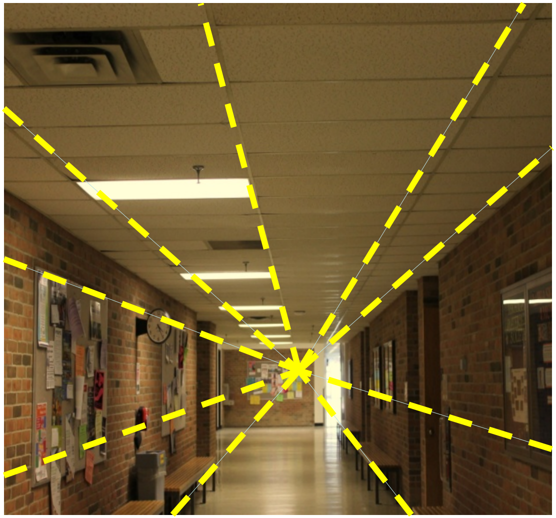


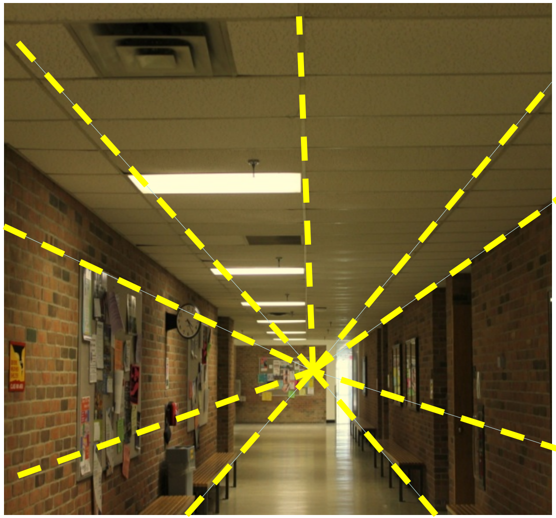
- Baseline intersects the image plane at infinity
- Epipoles are at infinity
- Epipolar lines are parallel to u axis

Example: Parallel Image Planes

Example: Forward translation





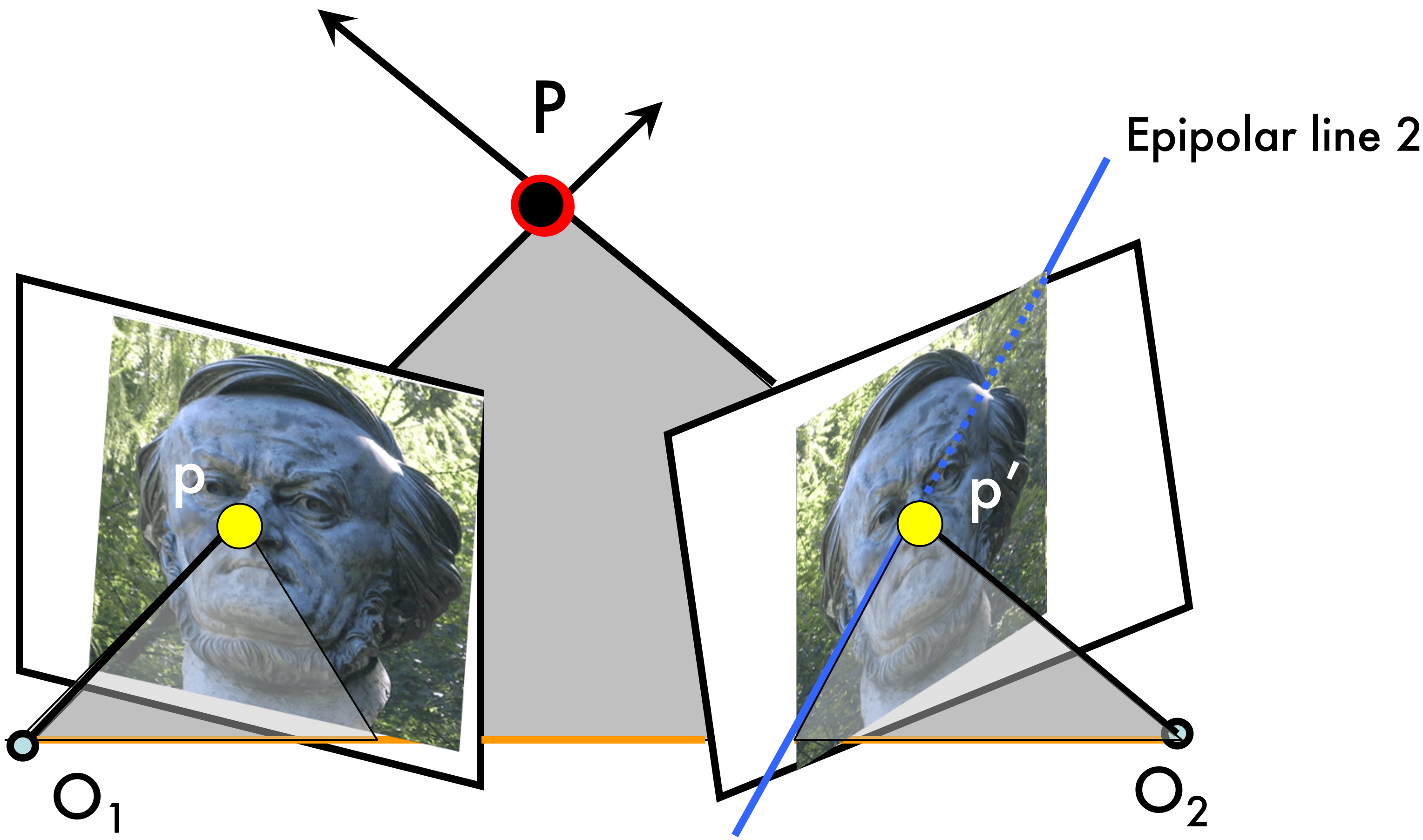


- The epipoles have same position in both images
- Epipole called FOE (focus of expansion)

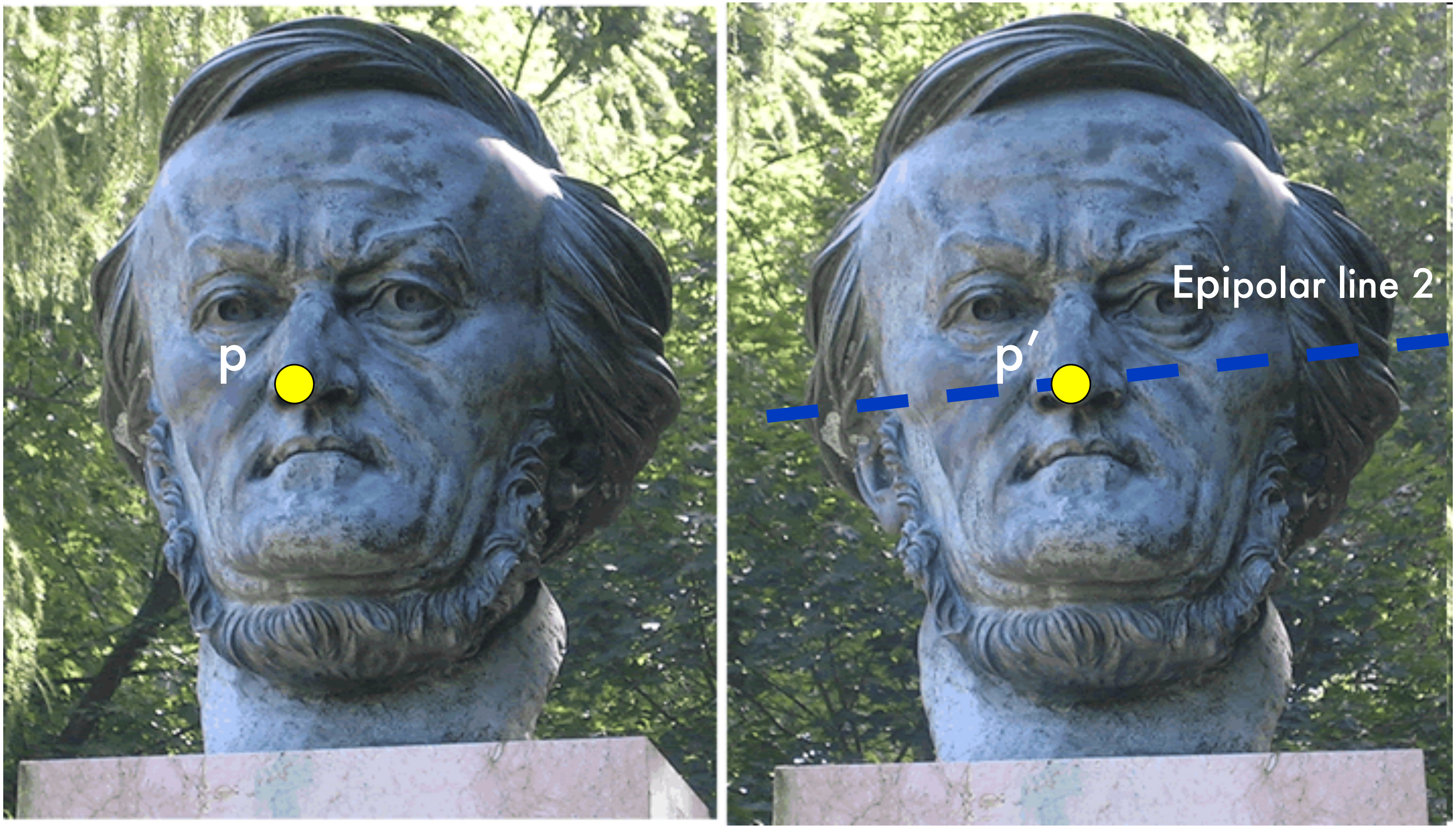
Epipolar Constraint

- Two views of the same object
- Given a point on left image, how can I find the corresponding point on right image?

Epipolar geometry



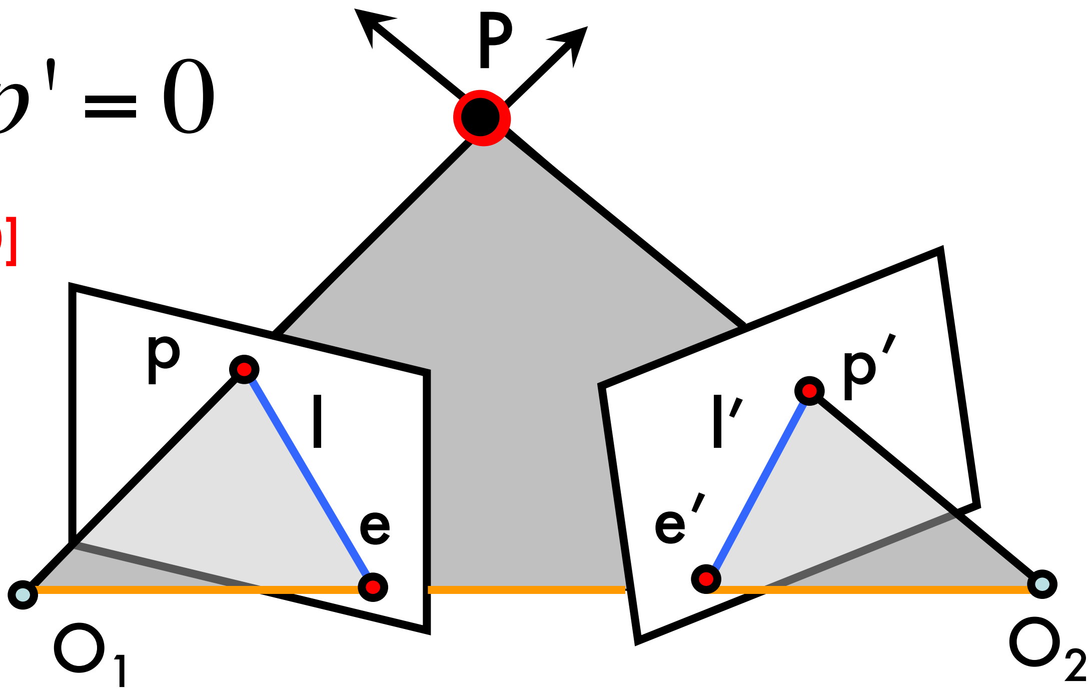
Epipolar Constraint



Epipolar Constraint

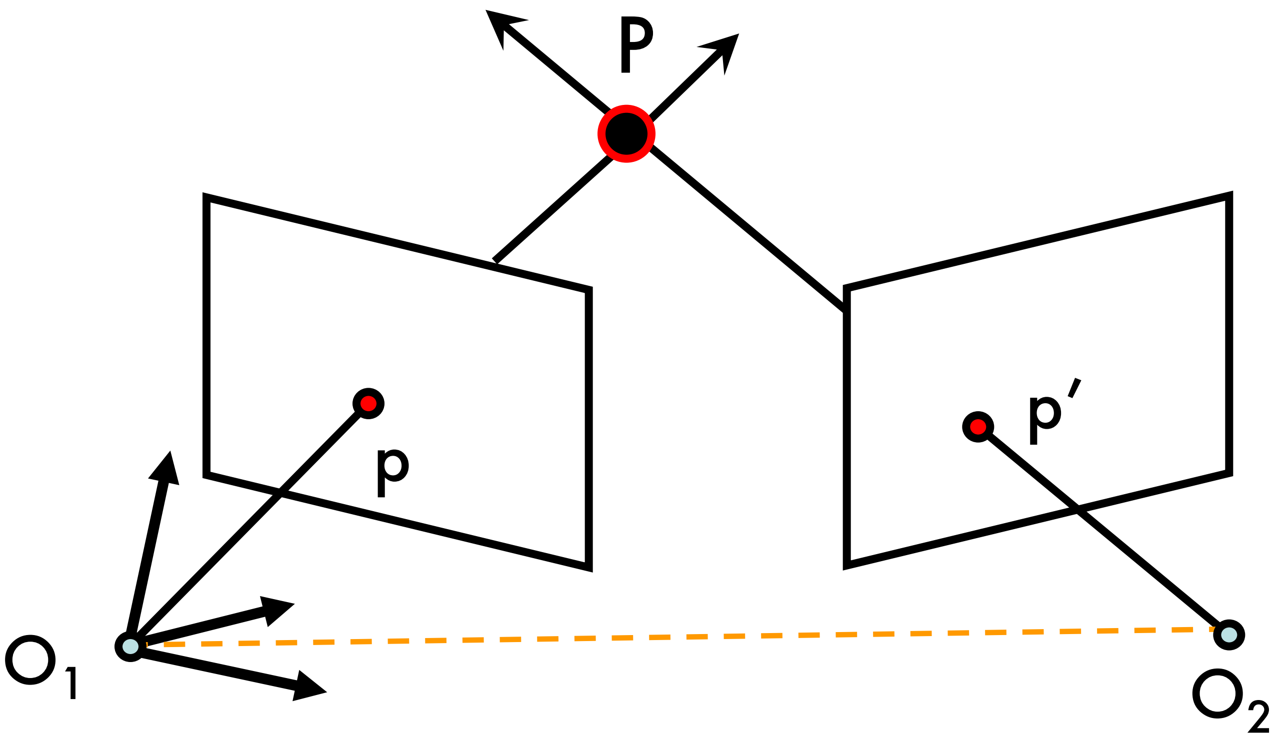
$$p^T \cdot E\, p' = 0$$

[Eq. 10]



- l = E p′ is the epipolar line associated with p′
- l′ = E^T p is the epipolar line associated with p
- E e′ = 0 and E^T e = 0
- E is 3x3 matrix; 5 DOF
- E is singular (rank two)

Epipolar Constraint



[Eq. 13] $$p^T F p' = 0$$

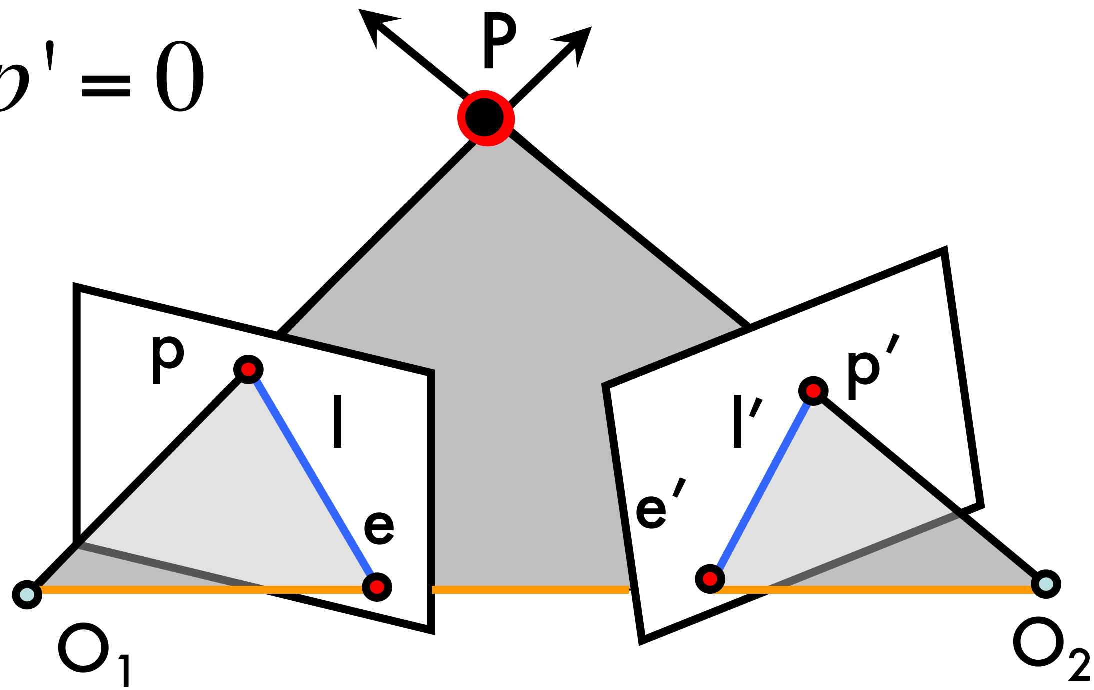
$$F = K^{-T} \cdot [T_\times] \cdot R\, K'^{-1}$$ [Eq. 14]

F = Fundamental Matrix

(Faugeras and Luong, 1992)

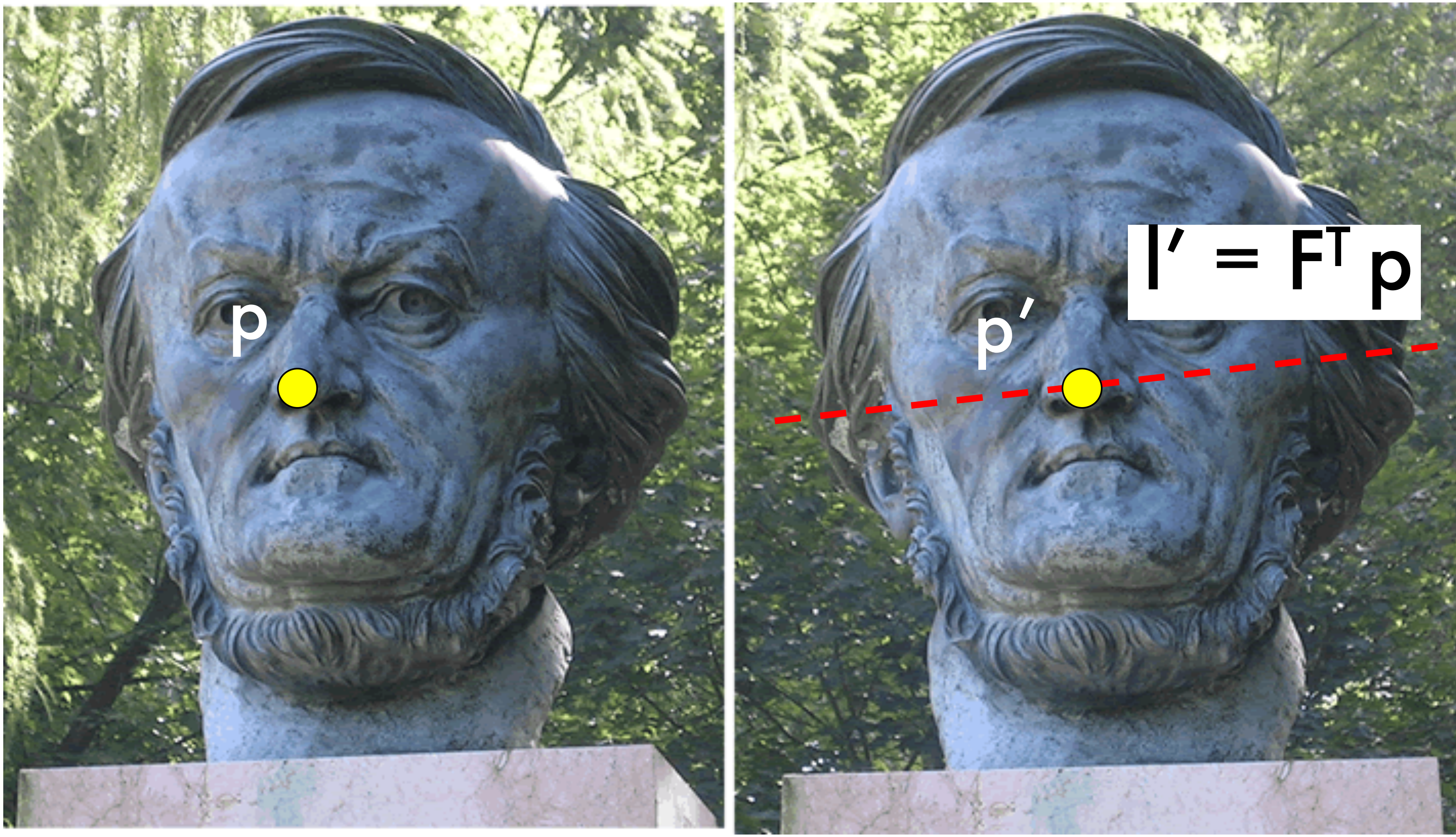
Epipolar Constraint

$$p^T \cdot F\ p' = 0$$



- l = F p′ is the epipolar line associated with p′
- l′= F^T p is the epipolar line associated with p
- F e′ = 0 and F^T e = 0
- F is 3x3 matrix; 7 DOF
- F is singular (rank two)

Why F is useful?



- Suppose F is known
- No additional information about the scene and camera is given
- Given a point on left image, we can compute the corresponding epipolar line in the second imag

Why F is useful?

- F captures information about the epipolar geometry of 2 views + camera parameters

- MORE IMPORTANTLY: F gives constraints on how the scene changes under view point transformation (without reconstructing the scene!)

- Powerful tool in:
    - 3D reconstruction
    - Multi-view object/scene matching