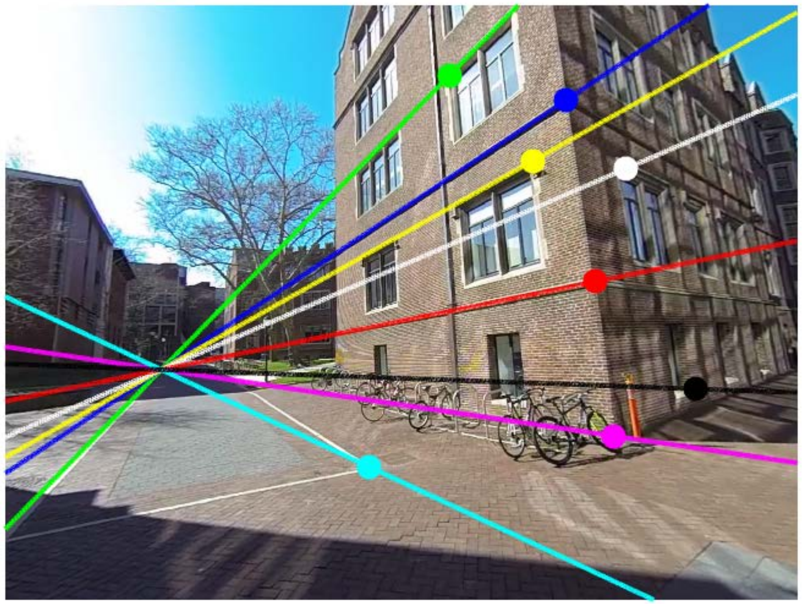
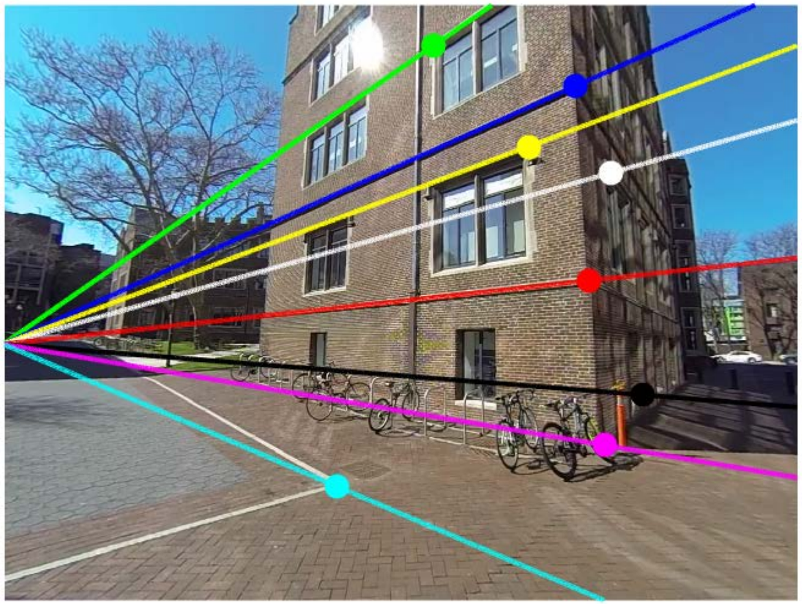
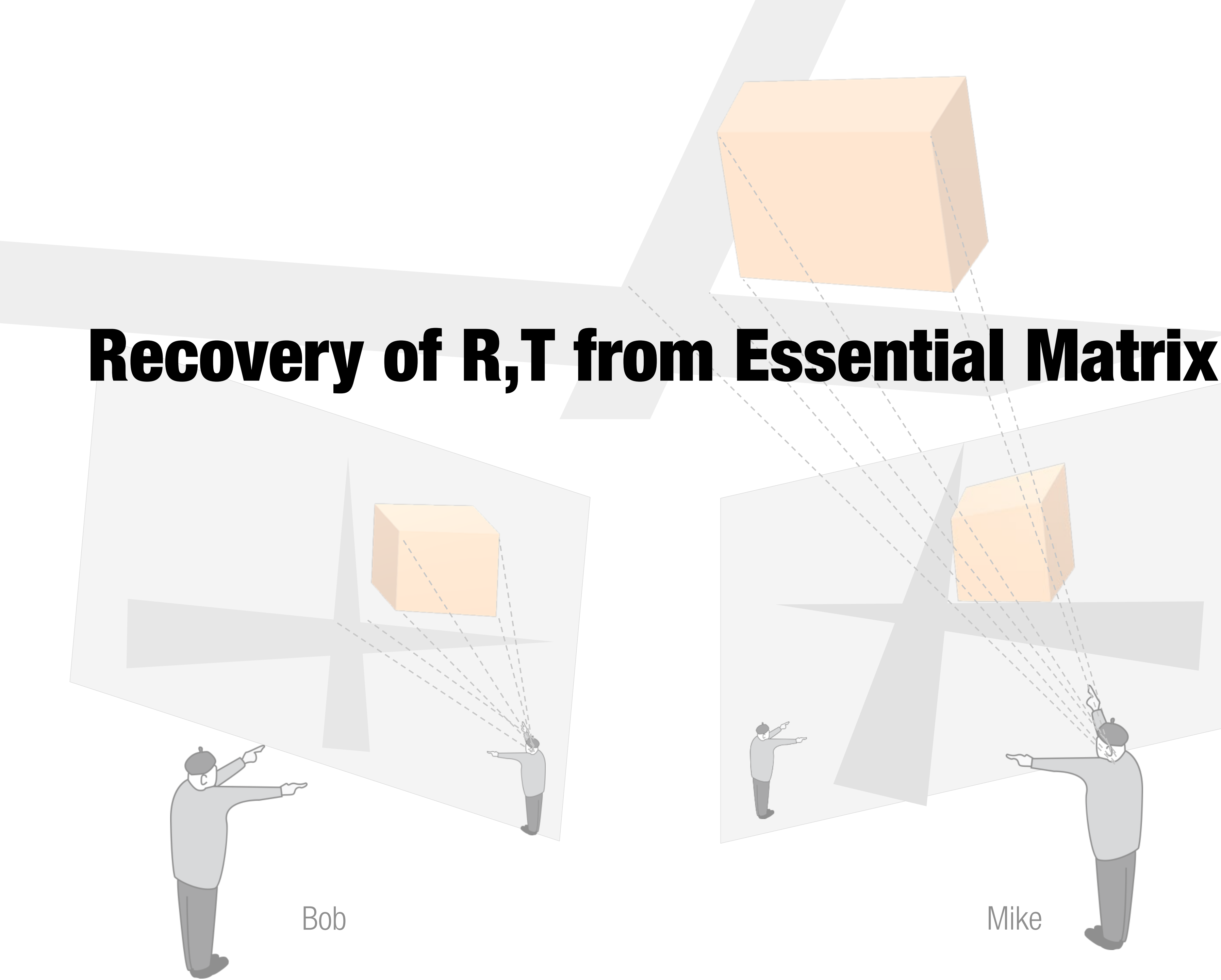
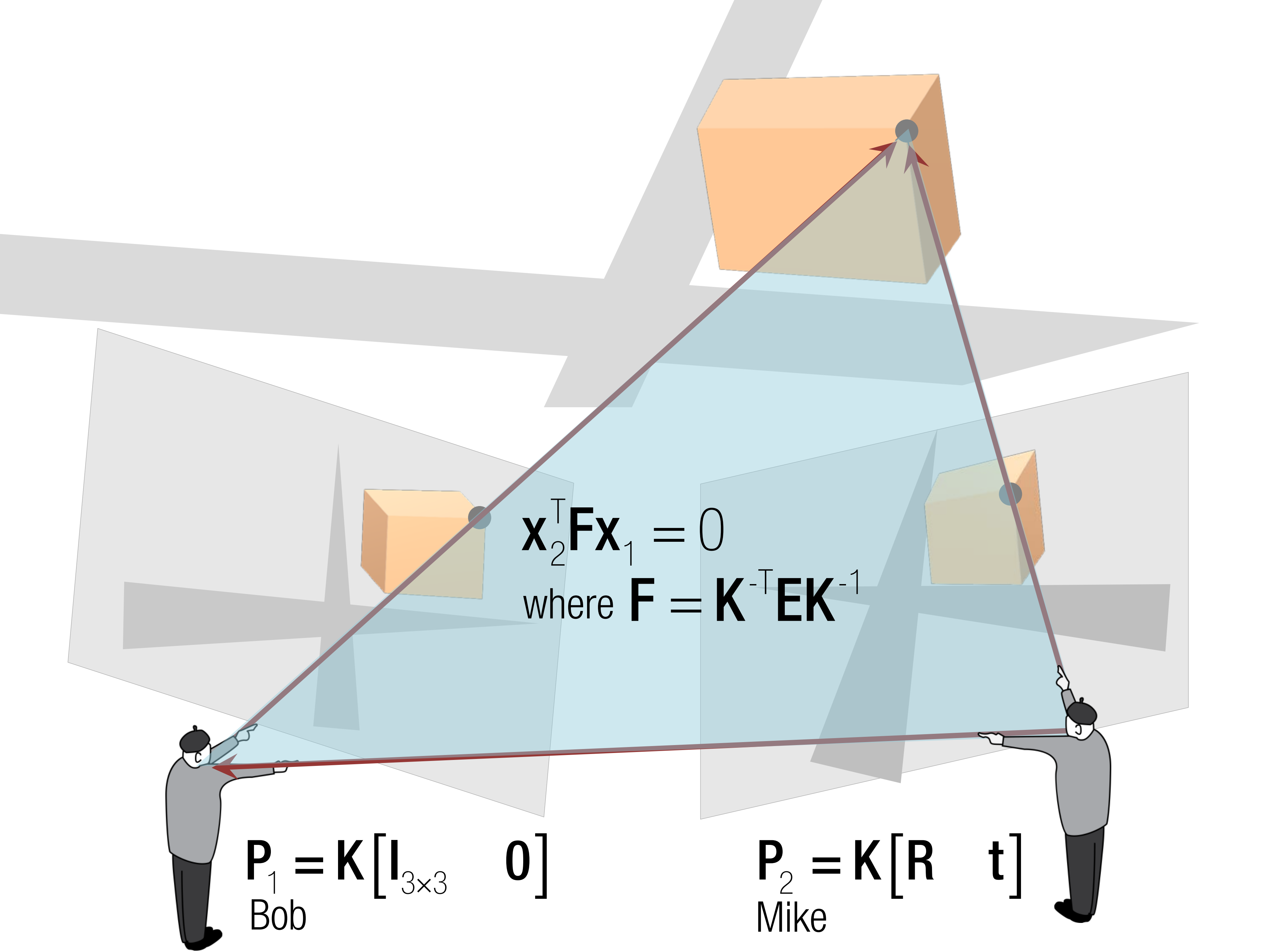


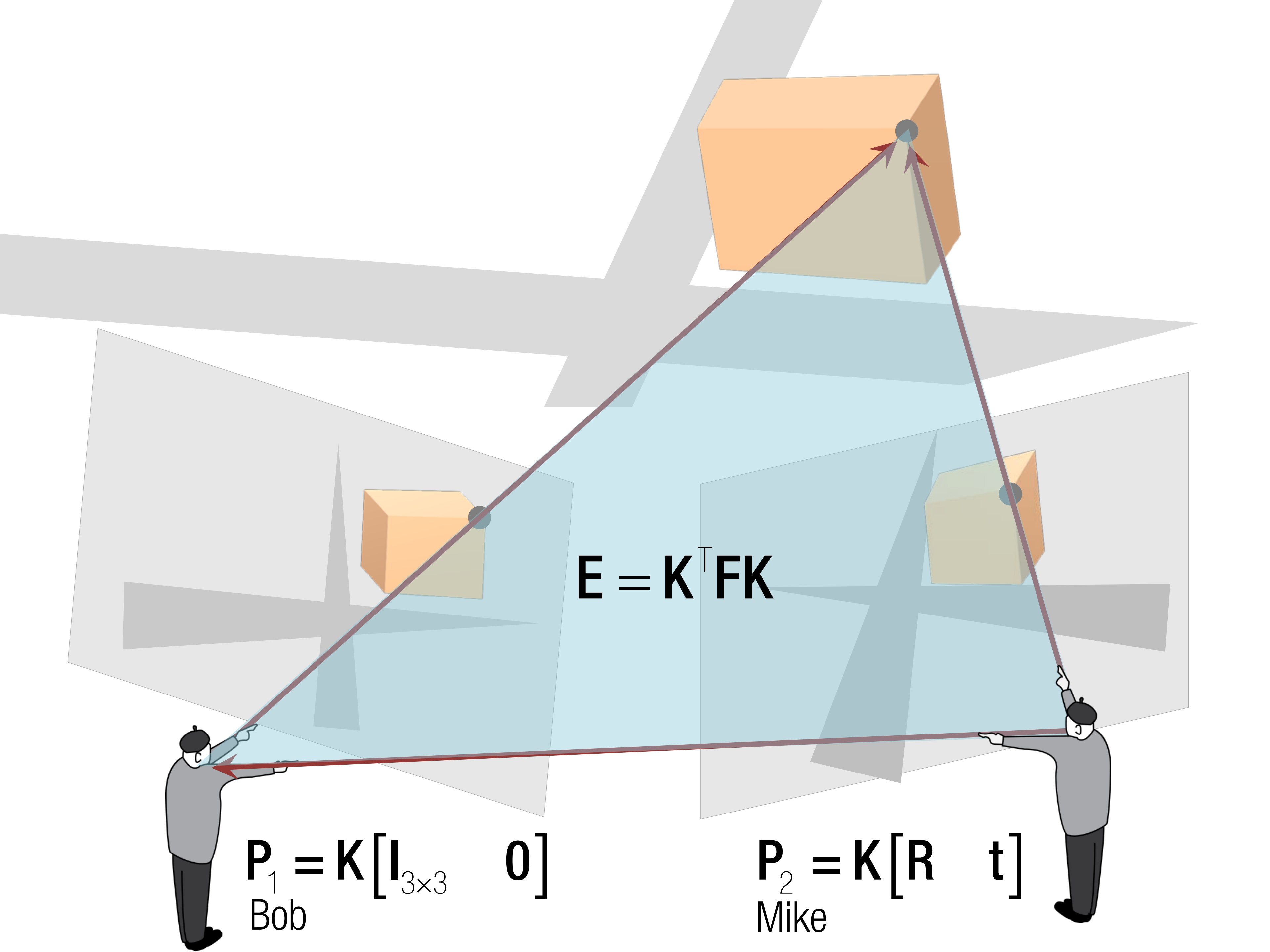


Recovery of R,T from Essential Matrix





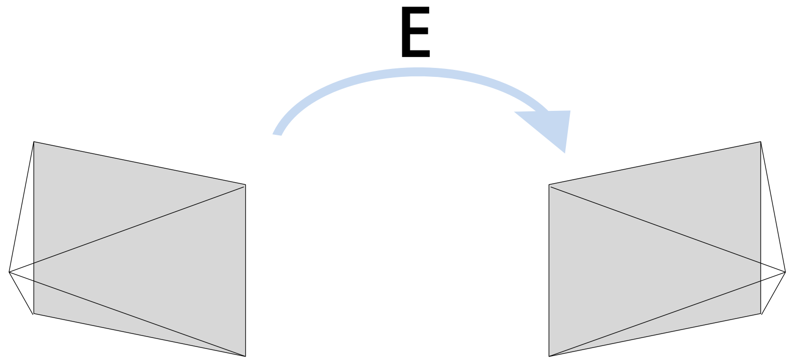
$x_2^T F x_1 = 0$
where $F = K^{-T} E K^{-1}$
$P_1 = K[I_{3\times3} \quad 0]$
Bob
$P_2 = K[R \quad t]$
Mike



$E = K^{\top}FK$
$P_1 = K[I_{3\times3} \quad 0]$
Bob
$P_2 = K[R \quad t]$
Mike

$$\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$$ How to decompose the essential matrix to rotation and translation?

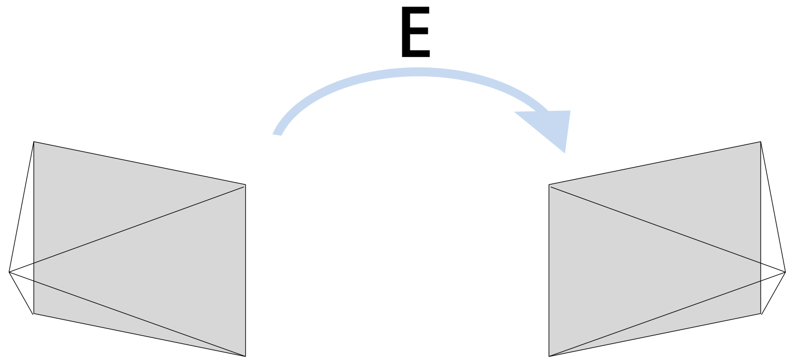
Essential matrix (rank 2)



$$\mathbf{P}_1 = \left[\mathbf{I}_{3\times3} \mid \mathbf{0}_3 \right] \qquad \mathbf{P}_2 = \left[\mathbf{R} \mid \mathbf{t} \right]$$

$$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R}$$ How to decompose the essential matrix to rotation and translation?

Essential matrix (rank 2)

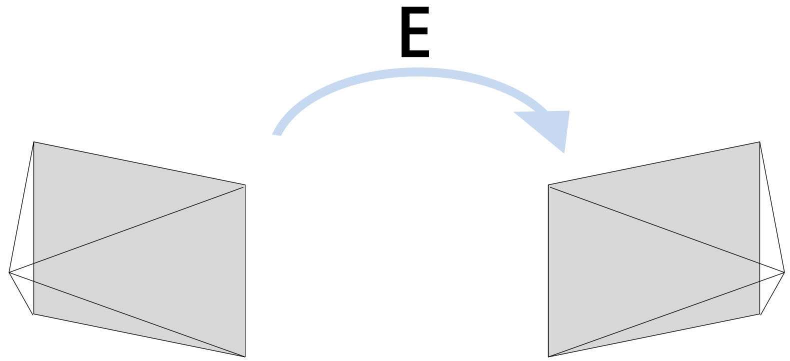


$$\mathbf{P}_1 = \begin{bmatrix} \mathbf{I}_{3\times 3} & | & \mathbf{0}_3 \end{bmatrix} \qquad \mathbf{P}_2 = \begin{bmatrix} \mathbf{R} & | & \mathbf{t} \end{bmatrix}$$

$\mathbf{t}$: Epipole in image 2 because $\mathbf{P}_2 \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} = [\mathbf{R} \mid \mathbf{t}] \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} = \mathbf{t}$

$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R}$ How to decompose the essential matrix to rotation and translation?

Essential matrix (rank 2)



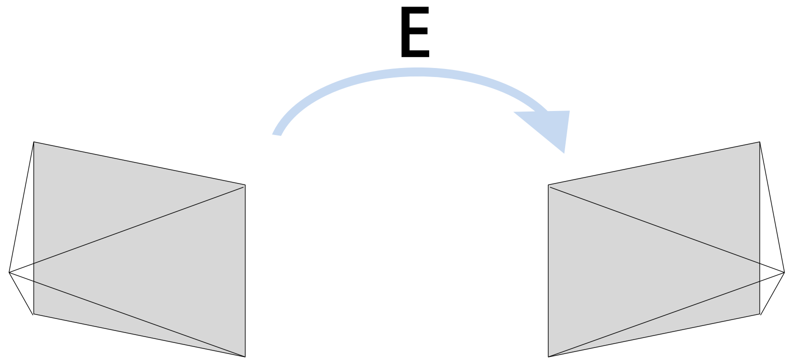
$$\mathbf{P}_1 = \left[\mathbf{I}_{3\times 3} \mid \mathbf{0}_3\right] \qquad \mathbf{P}_2 = [\mathbf{R} \mid \mathbf{t}]$$

$\mathbf{t}$: Epipole in image 2 because $\mathbf{P}_2 \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} = [\mathbf{R} \mid \mathbf{t}] \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} = \mathbf{t}$

$\mathbf{t}^\top \mathbf{E} = \mathbf{0}$: Left nullspace of the essential matrix is the epipole in image 2.

$\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$ How to decompose the essential matrix to rotation and translation?

Essential matrix (rank 2)



$$\mathbf{P}_1 = \left[\mathbf{I}_{3\times3} \mid \mathbf{0}_3 \right] \qquad \mathbf{P}_2 = \left[\mathbf{R} \mid \mathbf{t} \right]$$

$\mathbf{t}$: Epipole in image 2 because $\mathbf{P}_2 \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} = [\mathbf{R} \mid \mathbf{t}] \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} = \mathbf{t}$

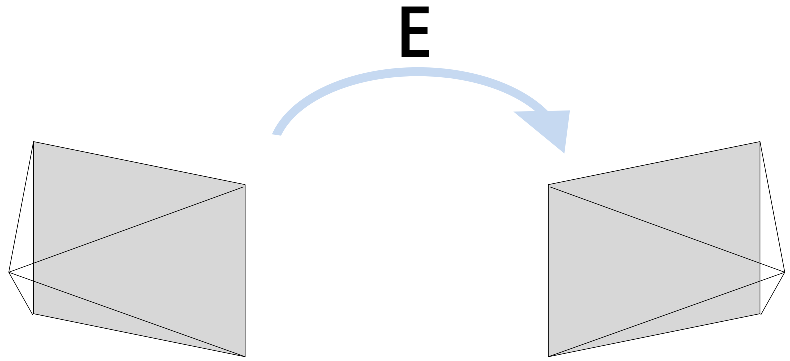
$\mathbf{t}^{\top}\mathbf{E} = \mathbf{0}$: Left nullspace of the essential matrix is the epipole in image 2.

$\longrightarrow$ $\mathbf{t} = \mathbf{u}_3$, or $-\mathbf{u}_3$ where $\mathbf{U} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix}$ and $\mathbf{E} = \mathbf{U} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{V}^{\top}$

Singular value decomposition (SVD)

$$\underbrace{\mathbf{E} = [\mathbf{t}]_\times \mathbf{R}}_{\text{Essential matrix}}$$

How to decompose the essential matrix to rotation and translation?



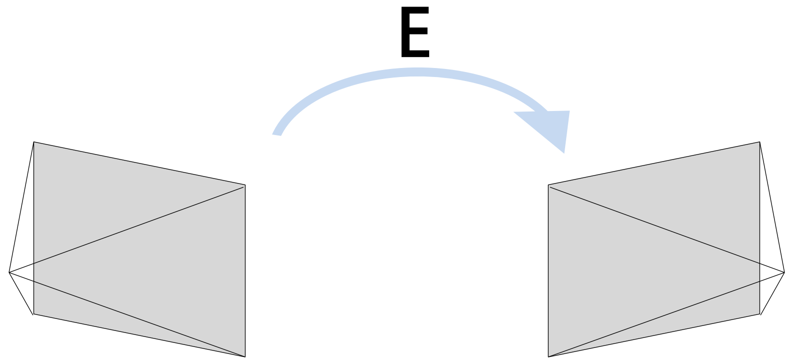
$$\mathbf{P}_1 = \begin{bmatrix} \mathbf{I}_{3\times 3} & | & \mathbf{0}_3 \end{bmatrix} \qquad \mathbf{P}_2 = \begin{bmatrix} \mathbf{R} & | & \mathbf{t} \end{bmatrix}$$

$$\mathbf{E} = \mathbf{U} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{V}^\mathsf{T}$$

$$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R}$$

Essential matrix

How to decompose the essential matrix to rotation and translation?

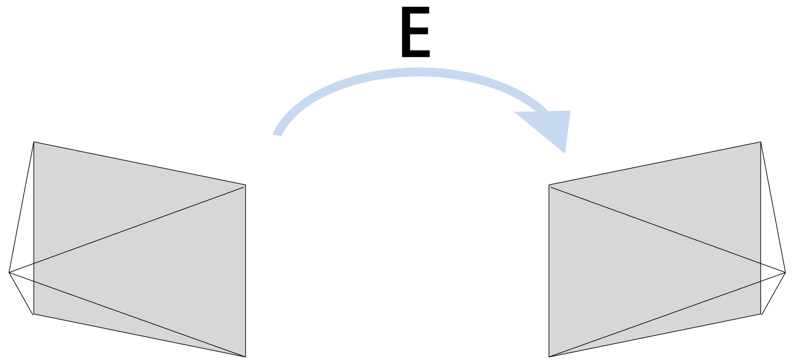


$$\mathbf{P}_1 = \left[\mathbf{I}_{3\times3} \mid \mathbf{0}_3\right] \qquad \mathbf{P}_2 = [\mathbf{R} \mid \mathbf{t}]$$

$$\mathbf{E} = \mathbf{U}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{V}^\top = [\mathbf{t}]_\times \mathbf{R}$$

$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R}$ How to decompose the essential matrix to rotation and translation?

Essential matrix



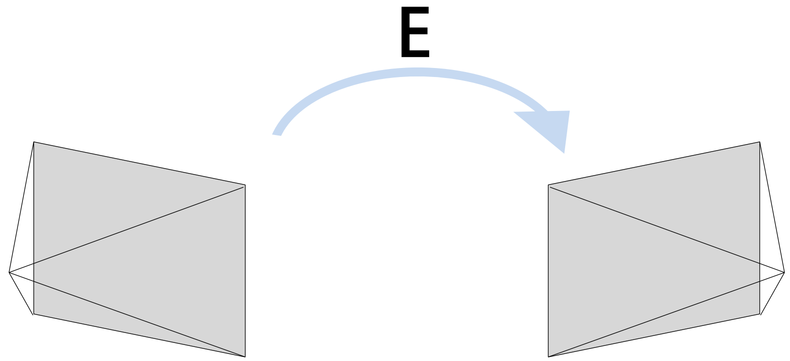
$\mathbf{P}_1 = \left[\mathbf{I}_{3\times3} \mid \mathbf{0}_3\right]$ $\mathbf{P}_2 = [\mathbf{R} \mid \mathbf{t}]$

$$\mathbf{E} = \mathbf{U}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}\mathbf{V}^\top = [\mathbf{t}]_\times \mathbf{R} = \left(\mathbf{U}\begin{bmatrix}0 & 1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\mathbf{U}^\top\right)\left(\mathbf{UYV}^\top\right)$$

where $\mathbf{U} = \begin{bmatrix}\mathbf{u}_1 & \mathbf{u}_2 & \mathbf{t}\end{bmatrix}$

$$\underbrace{\mathbf{E} = [\mathbf{t}]_\times \mathbf{R}}_{\text{Essential matrix}}$$

How to decompose the essential matrix to rotation and translation?



$$\mathbf{P}_1 = [\mathbf{I}_{3\times3} \mid \mathbf{0}_3] \qquad \mathbf{P}_2 = [\mathbf{R} \mid \mathbf{t}]$$

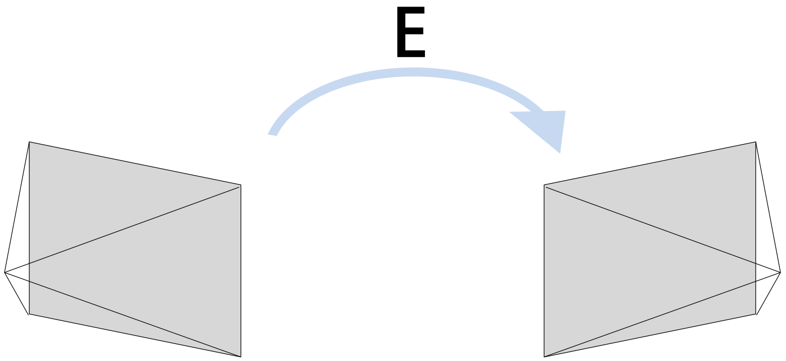
$$\mathbf{E} = \mathbf{U}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{V}^\mathsf{T} = [\mathbf{t}]_\times \mathbf{R} = \left(\mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{U}^\mathsf{T}\right)\left(\mathbf{U}\mathbf{Y}\mathbf{V}^\mathsf{T}\right)$$

How do we set $\mathbf{Y}$?

where $\mathbf{U} = [\mathbf{u}_1 \quad \mathbf{u}_2 \quad \mathbf{t}]$

$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R}$ How to decompose the essential matrix to rotation and translation?

Essential matrix



$\mathbf{P}_1 = \begin{bmatrix} \mathbf{I}_{3\times3} & | & \mathbf{0}_3 \end{bmatrix}$ $\mathbf{P}_2 = \begin{bmatrix} \mathbf{R} & | & \mathbf{t} \end{bmatrix}$

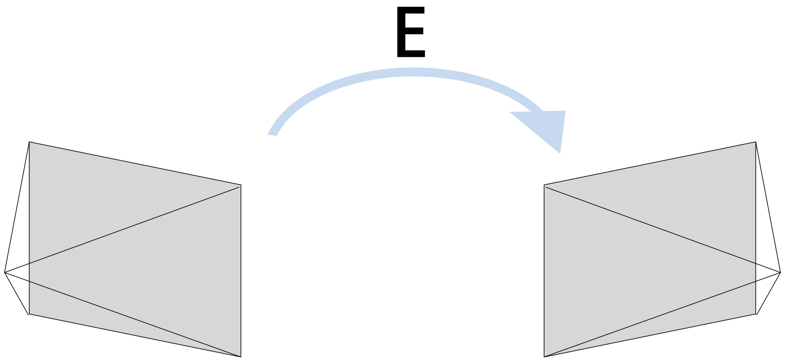
$$\mathbf{E} = \mathbf{U}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{V}^\top = [\mathbf{t}]_\times \mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{Y}\mathbf{V}^\top$$

How do we set $\mathbf{Y}$?

$$\therefore \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{Y}$$

$\underline{\mathbf{E} = [\mathbf{t}]_\times \mathbf{R}}$ How to decompose the essential matrix to rotation and translation?

Essential matrix



$$\mathbf{P}_1 = [\mathbf{I}_{3\times3} \mid \mathbf{0}_3] \qquad \mathbf{P}_2 = [\mathbf{R} \mid \mathbf{t}]$$

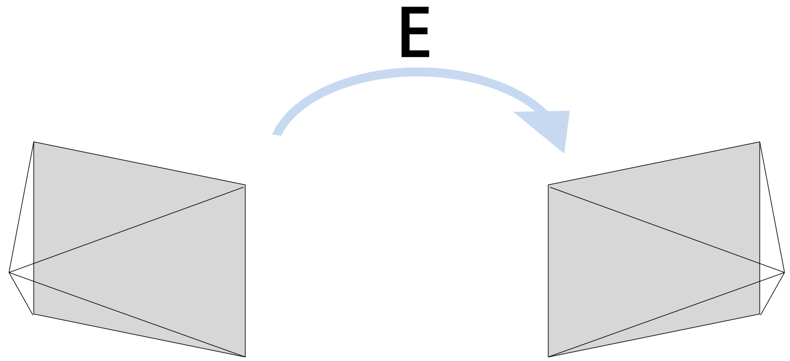
$$\mathbf{E} = \mathbf{U}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{V}^\mathsf{T} = [\mathbf{t}]_\times \mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{Y}\mathbf{V}^\mathsf{T}$$

How do we set $\mathbf{Y}$?

$$\therefore \mathbf{Y} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{ or } \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^\mathsf{T}$$

$\mathbf{E} = [\mathbf{t}]_{\times}\mathbf{R}$ How to decompose the essential matrix to rotation and translation?

Essential matrix

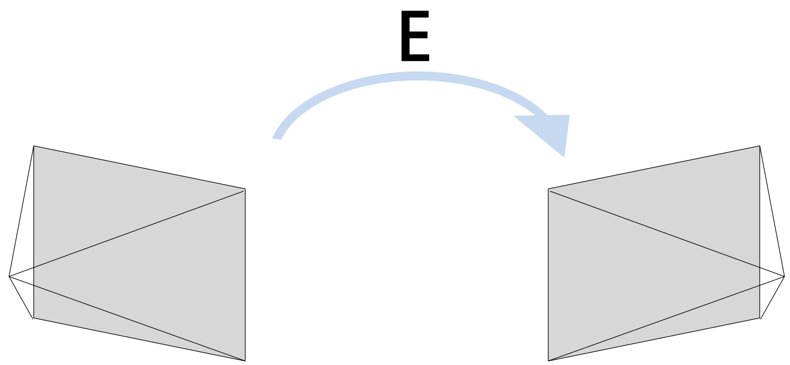


$\mathbf{P}_1 = [\mathbf{I}_{3\times 3} \mid \mathbf{0}_3]$ $\qquad$ $\mathbf{P}_2 = [\mathbf{R} \mid \mathbf{t}]$

$$\mathbf{t} = \mathbf{u}_3 \text{ , or } -\mathbf{u}_3 \qquad \mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\mathbf{V}^{\mathsf{T}} \text{, or } \mathbf{U}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{\mathsf{T}}\mathbf{V}^{\mathsf{T}}$$

where $\mathbf{E} = \mathbf{U}\mathbf{D}\mathbf{V}^{\mathsf{T}} \qquad \mathbf{U} = [\mathbf{u}_1 \quad \mathbf{u}_2 \quad \mathbf{t}]$

$$\underbrace{\mathbf{E} = [\mathbf{t}]_\times \mathbf{R}}_{\text{Essential matrix}}$$ How to decompose the essential matrix to rotation and translation?



$$\mathbf{P}_1 = \begin{bmatrix} \mathbf{I}_{3\times 3} & | & \mathbf{0}_3 \end{bmatrix} \qquad \mathbf{P}_2 = \begin{bmatrix} \mathbf{R} & | & \mathbf{t} \end{bmatrix}$$

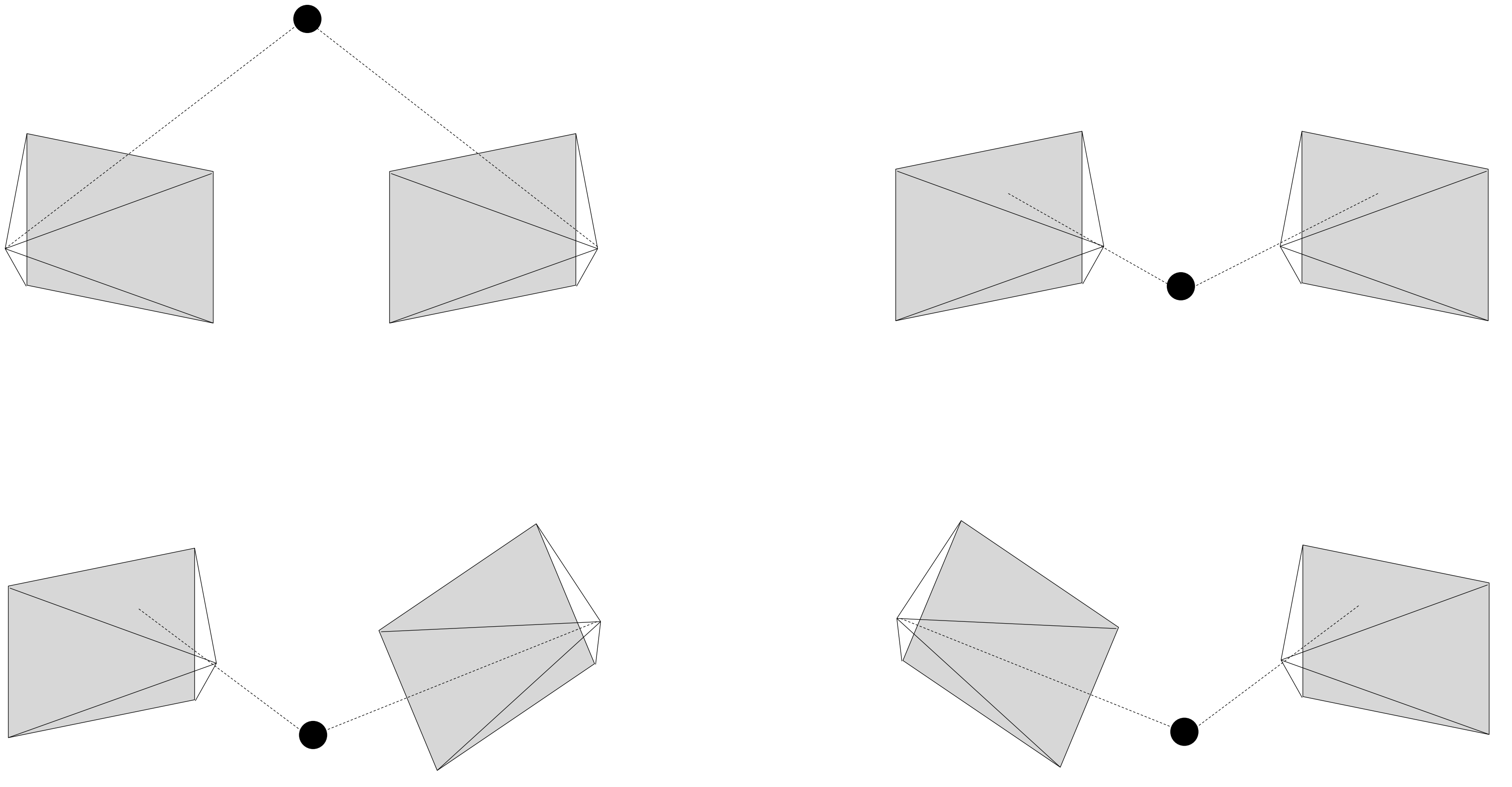
Four configurations:

$$\mathbf{P}_2 = \begin{bmatrix} \mathbf{UYV}^\top & | & \mathbf{u}_3 \end{bmatrix}, \text{ or } \begin{bmatrix} \mathbf{UY}^\top\mathbf{V}^\top & | & \mathbf{u}_3 \end{bmatrix}, \text{ or } \begin{bmatrix} \mathbf{UYV}^\top & | & \mathbf{-u}_3 \end{bmatrix}, \text{ or } \begin{bmatrix} \mathbf{UY}^\top\mathbf{V}^\top & | & \mathbf{-u}_3 \end{bmatrix}$$

$$\underbrace{\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}}_{\text{Essential matrix}}$$

How to decompose the essential matrix to rotation and translation?

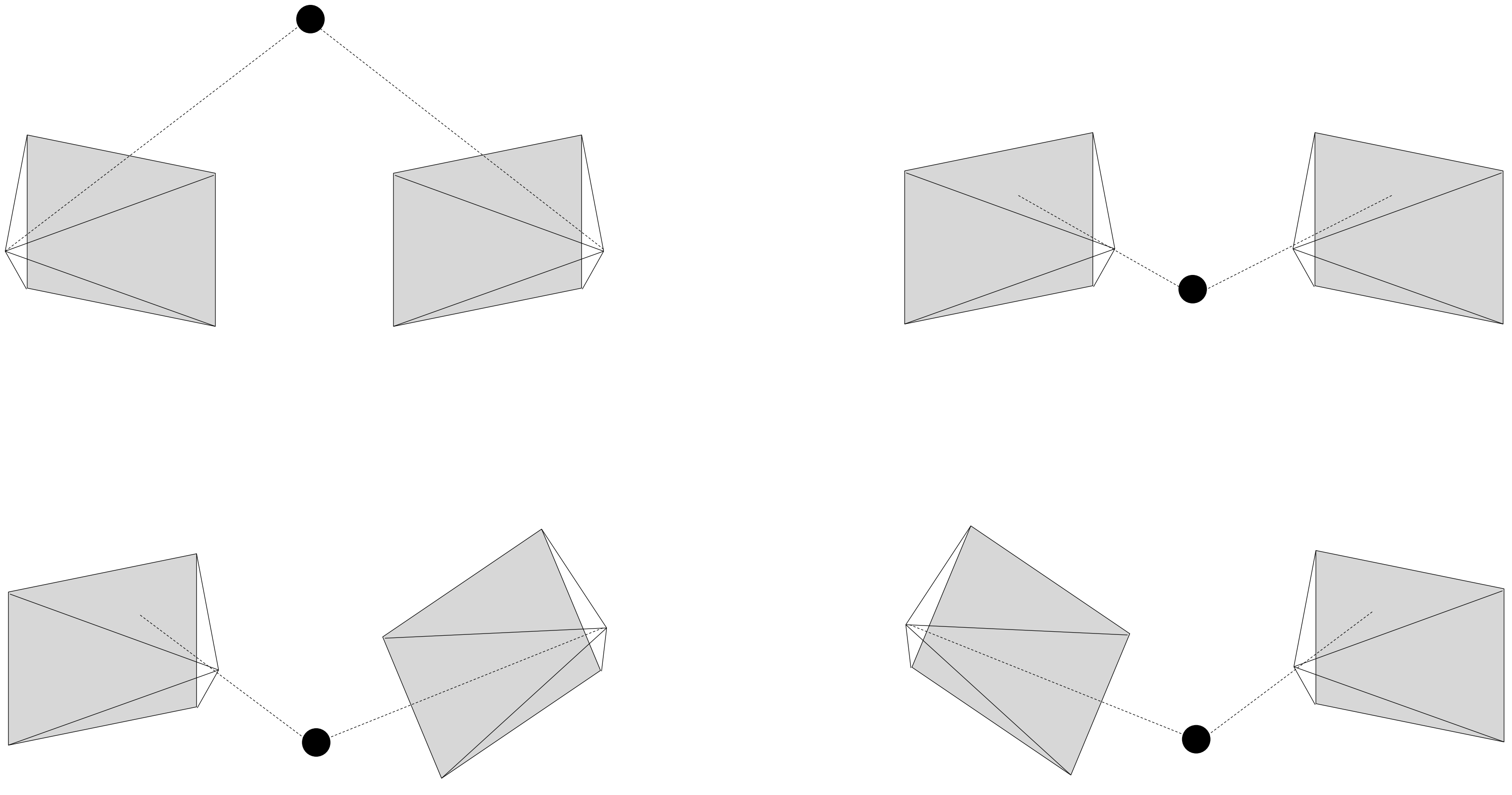
Four configurations:



$$\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$$ How to decompose the essential matrix to rotation and translation?

Essential matrix

Four configurations: can be resolved by point triangulation.



Four possible reconstructions from E

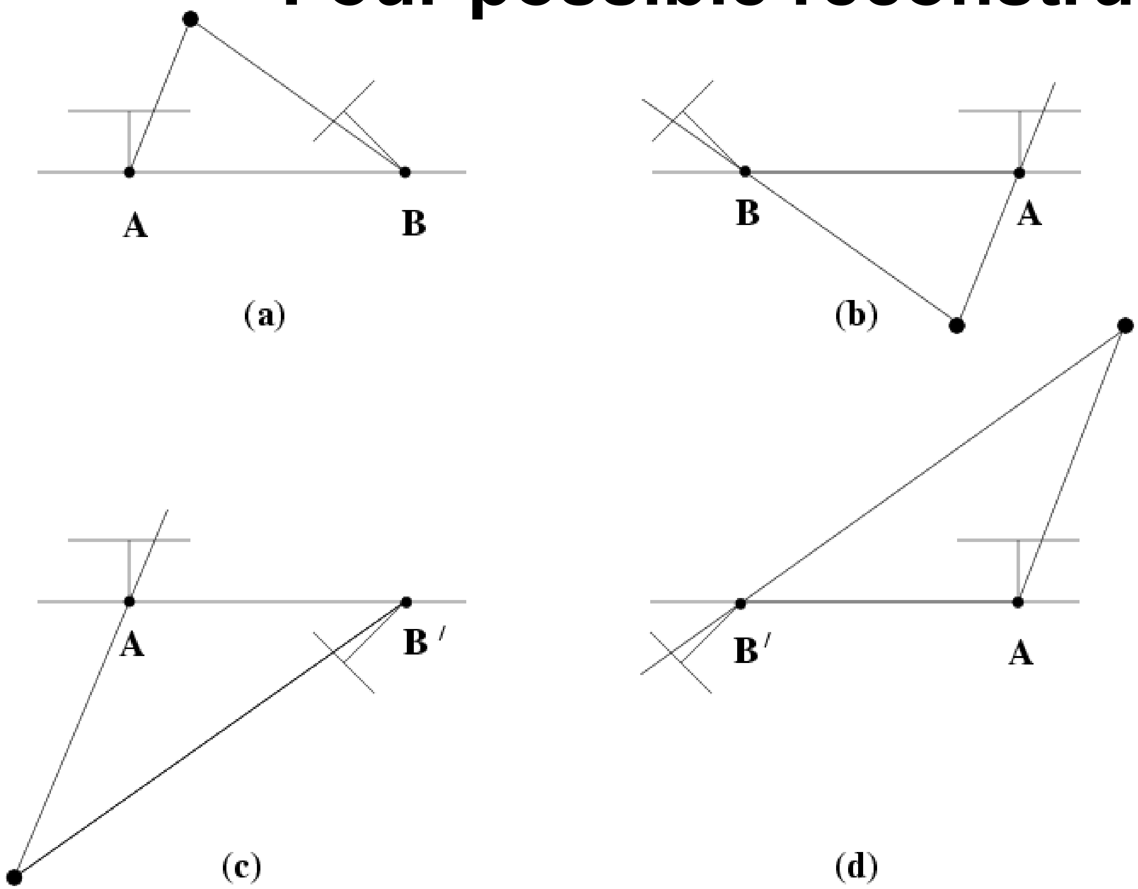


Fig. 9.12. **The four possible solutions for calibrated reconstruction from E.** *Between the left and right sides there is a baseline reversal. Between the top and bottom rows camera B rotates 180° about the baseline. Note, only in (a) is the reconstructed point in front of both cameras.*

Result 9.19. *For a given essential matrix* $\mathtt{E} = \mathtt{U}\,\mathrm{diag}(1,1,0)\mathtt{V}^\mathsf{T}$, *and first camera matrix* $\mathtt{P} = [\mathtt{I} \mid \mathbf{0}]$, *there are four possible choices for the second camera matrix* $\mathtt{P}'$, *namely*

$$\mathtt{P}' = [\mathtt{U}\mathtt{W}\mathtt{V}^\mathsf{T} \mid +\mathbf{u}_3] \text{ or } [\mathtt{U}\mathtt{W}\mathtt{V}^\mathsf{T} \mid -\mathbf{u}_3] \text{ or } [\mathtt{U}\mathtt{W}^\mathsf{T}\mathtt{V}^\mathsf{T} \mid +\mathbf{u}_3] \text{ or } [\mathtt{U}\mathtt{W}^\mathsf{T}\mathtt{V}^\mathsf{T} \mid -\mathbf{u}_3].$$

(only one solution where points is in front of both cameras)

2.2 Camera Pose Extraction

Goal Given $\mathbf{E}$, enumerate four camera pose configurations, $(\mathbf{C}_1, \mathbf{R}_1)$, $(\mathbf{C}_2, \mathbf{R}_2)$, $(\mathbf{C}_3, \mathbf{R}_3)$, and $(\mathbf{C}_4, \mathbf{R}_4)$ where $\mathbf{C} \in \mathbb{R}^3$ is the camera center and $\mathbf{R} \in SO(3)$ is the rotation matrix, i.e., $\mathbf{P} = \mathbf{KR}\begin{bmatrix} \mathbf{I}_{3\times3} & -\mathbf{C} \end{bmatrix}$:

```
[Cset Rset] = ExtractCameraPose(E)
```

(INPUT) `E`: essential matrix

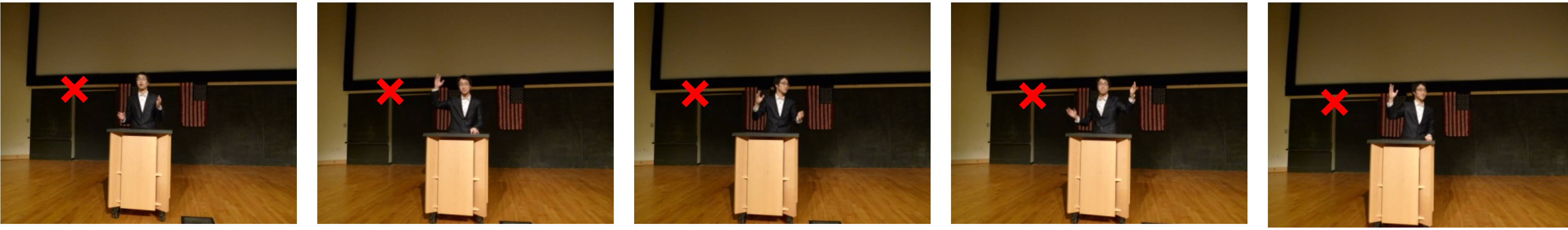
(OUTPUT) `Cset` and `Rset`: four configurations of camera centers and rotations, i.e., `Cset{i}`$=\mathbf{C}_i$ and `Rset{i}`$=\mathbf{R}_i$.

There are four camera pose configurations given an essential matrix. Let $\mathbf{E} = \mathbf{UDV}^\mathsf{T}$ and $\mathbf{W} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. The four configurations are enumerated below:

1. $\mathbf{C}_1 = \mathbf{U}(:,3)$ and $\mathbf{R}_1 = \mathbf{UWV}^\mathsf{T}$
2. $\mathbf{C}_2 = -\mathbf{U}(:,3)$ and $\mathbf{R}_2 = \mathbf{UWV}^\mathsf{T}$
3. $\mathbf{C}_3 = \mathbf{U}(:,3)$ and $\mathbf{R}_3 = \mathbf{UW}^\mathsf{T}\mathbf{V}^\mathsf{T}$
4. $\mathbf{C}_4 = -\mathbf{U}(:,3)$ and $\mathbf{R}_4 = \mathbf{UW}^\mathsf{T}\mathbf{V}^\mathsf{T}$.

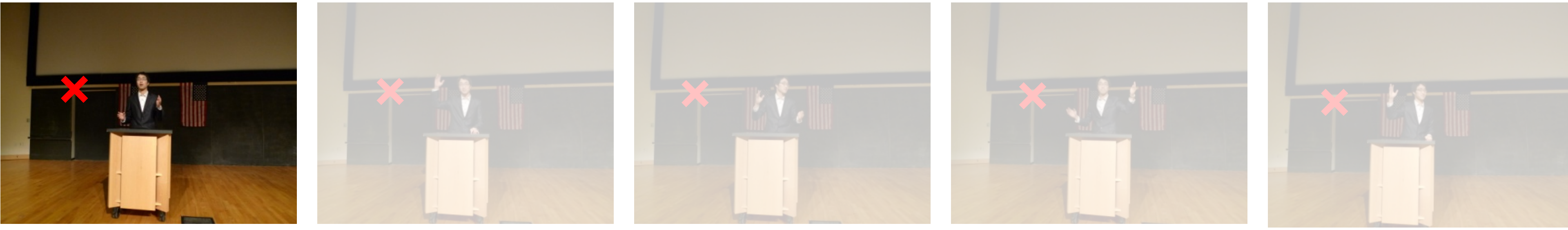
Note that the determinant of a rotation matrix is one. If $\det(\mathbf{R}) = -1$, the camera pose must be corrected, i.e., $\mathbf{C} \leftarrow -\mathbf{C}$ and $\mathbf{R} \leftarrow -\mathbf{R}$.

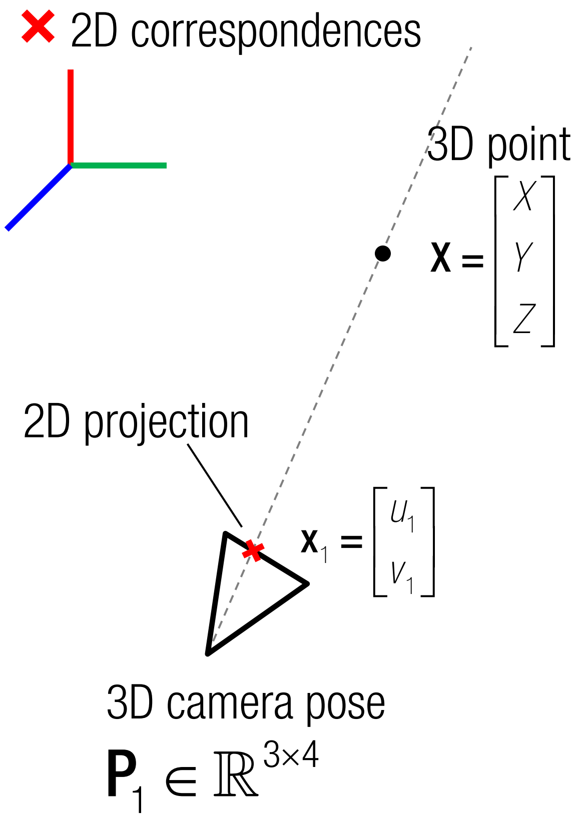
Point Triangulation



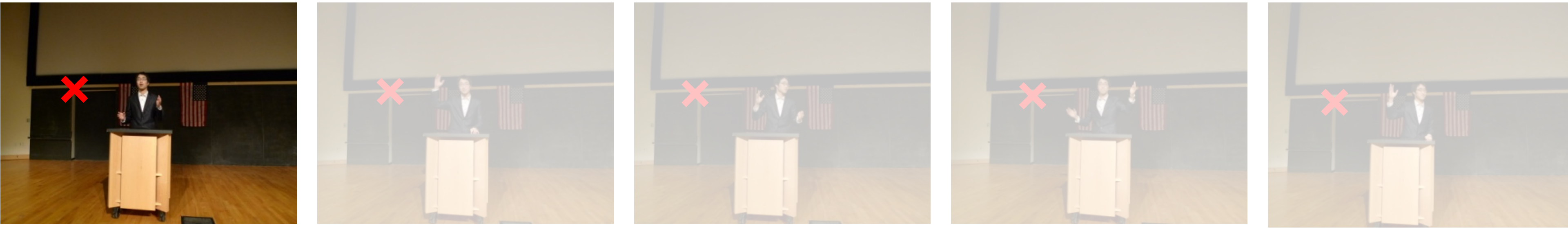
✕ 2D correspondences

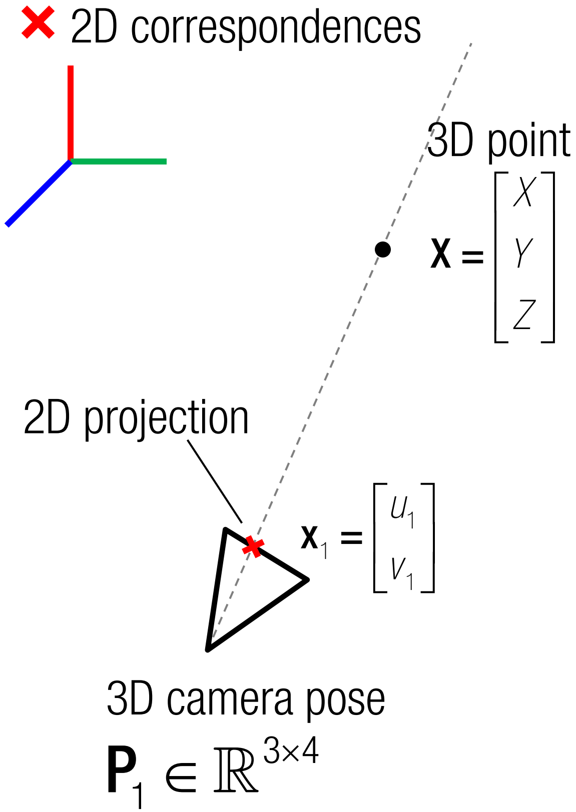
Point Triangulation





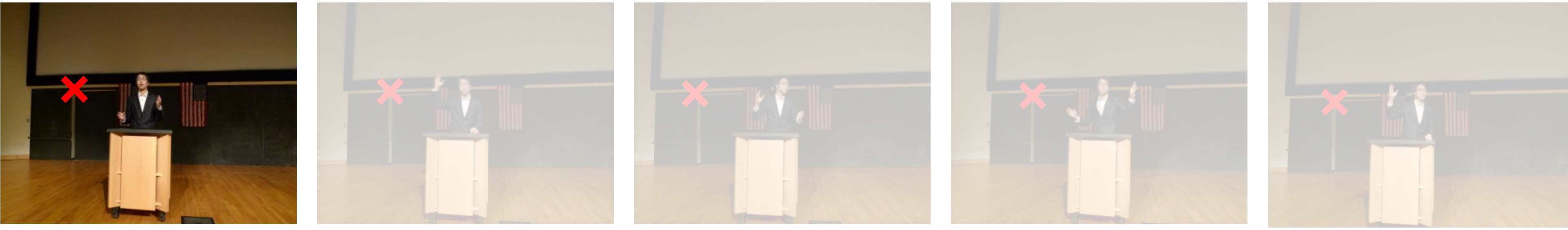
Point Triangulation

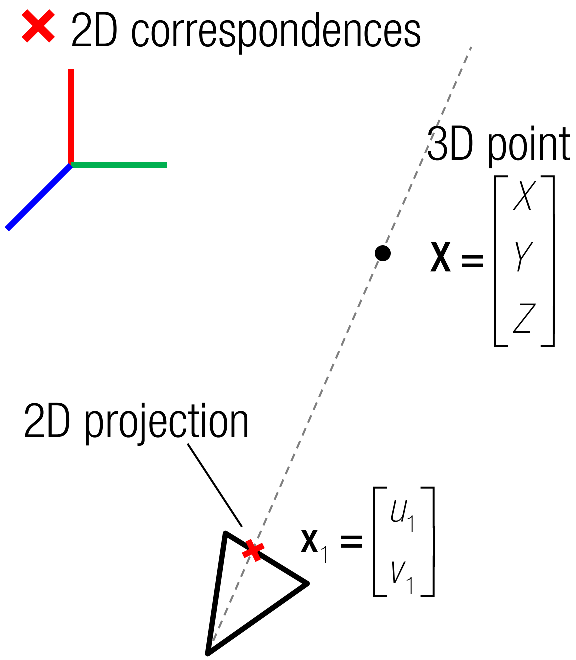




$$\lambda \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix} = \mathbf{P}_1 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

Point Triangulation





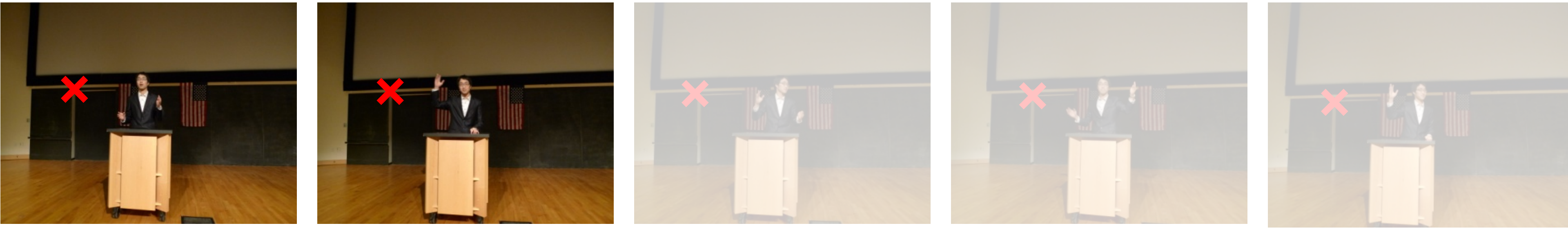
3D camera pose

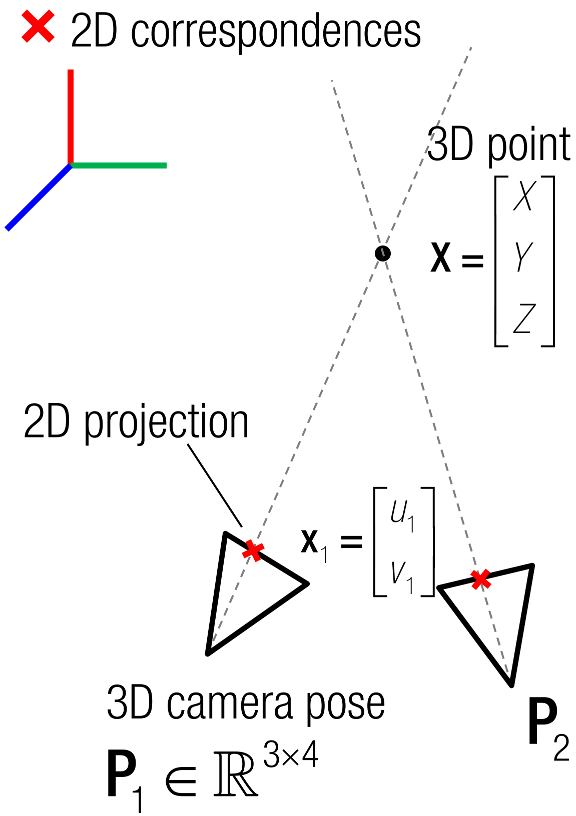
$$\mathbf{P}_1 \in \mathbb{R}^{3\times 4}$$

$$\lambda \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix} = \mathbf{P}_1 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix}_\times \mathbf{P}_1 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

Cross product between two parallel vectors equals to zero.

Point Triangulation

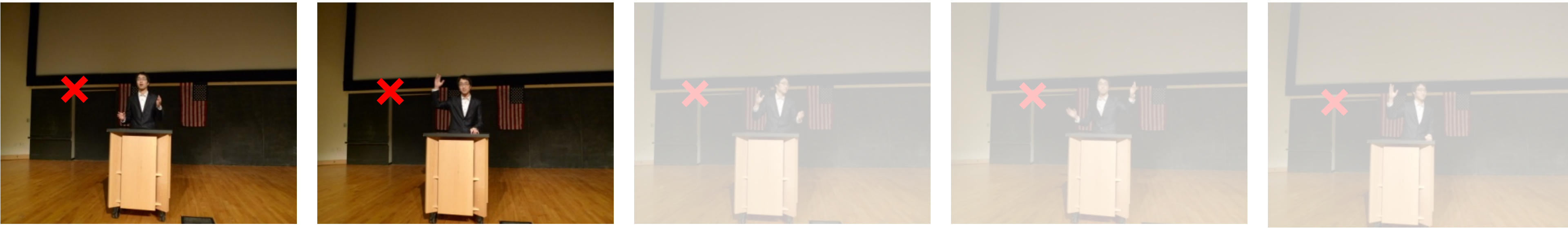


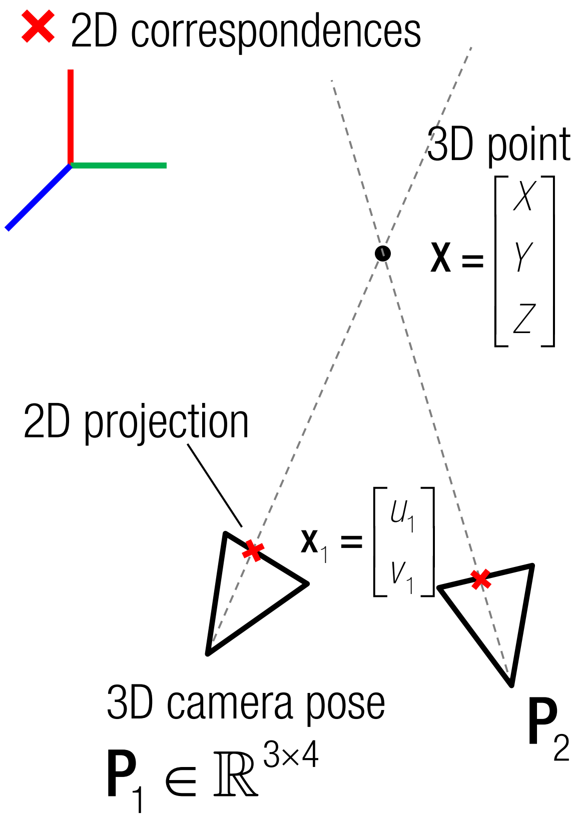


$$\lambda \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix} = \mathbf{P}_1 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix}_\times \mathbf{P}_1 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

$$\begin{bmatrix} \mathbf{x}_2 \\ 1 \end{bmatrix}_\times \mathbf{P}_2 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

Point Triangulation



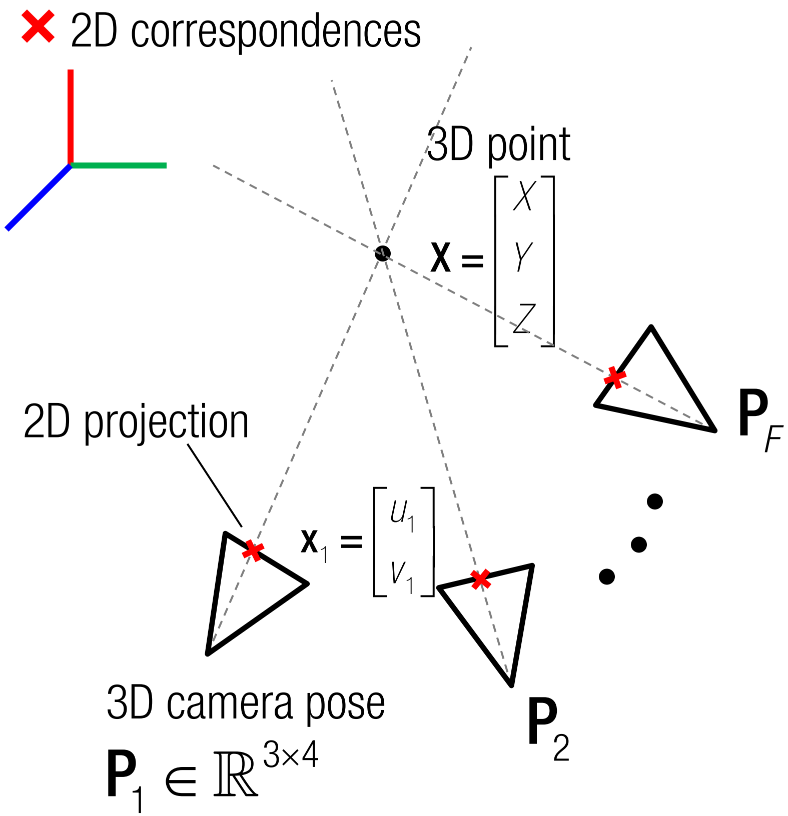


$$\lambda \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix} = \mathbf{P}_1 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix}_\times \mathbf{P}_1 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

$$\begin{bmatrix} \mathbf{x}_2 \\ 1 \end{bmatrix}_\times \mathbf{P}_2 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

$$\begin{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix}_\times \mathbf{P}_1 \\ \begin{bmatrix} \mathbf{x}_2 \\ 1 \end{bmatrix}_\times \mathbf{P}_2 \\ \\ \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

Point Triangulation

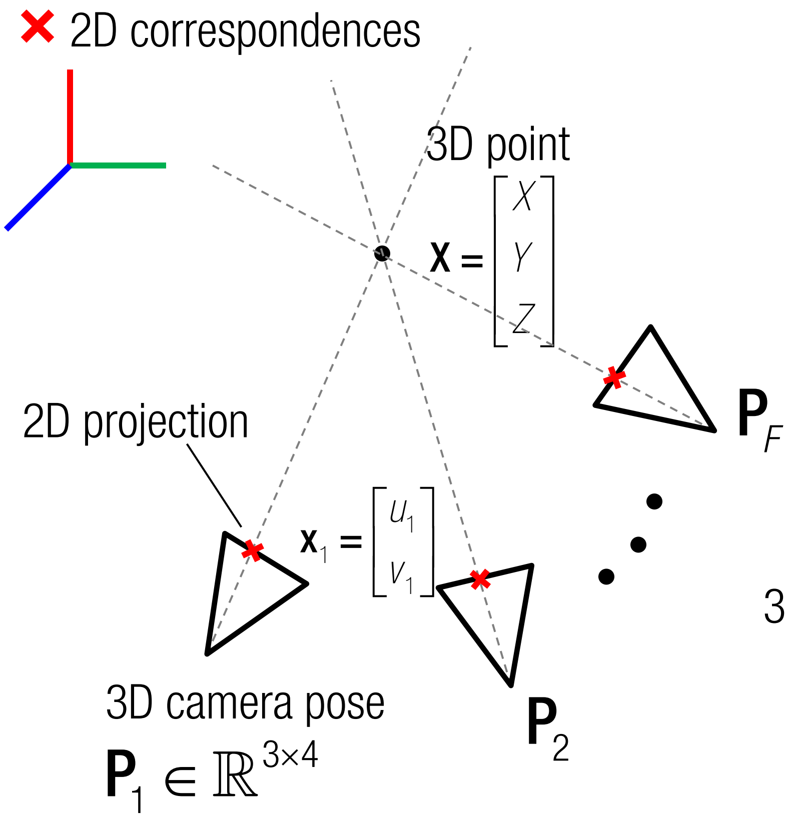


$$\lambda \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix} = \mathbf{P}_1 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix}_\times \mathbf{P}_1 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

$$\begin{bmatrix} \mathbf{x}_2 \\ 1 \end{bmatrix}_\times \mathbf{P}_2 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

$$\begin{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix}_\times \mathbf{P}_1 \\ \begin{bmatrix} \mathbf{x}_2 \\ 1 \end{bmatrix}_\times \mathbf{P}_2 \\ \vdots \\ \begin{bmatrix} \mathbf{x}_F \\ 1 \end{bmatrix}_\times \mathbf{P}_F \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

Point Triangulation



$$\lambda \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix} = \mathbf{P}_1 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix}_\times \mathbf{P}_1 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

$$\begin{bmatrix} \mathbf{x}_2 \\ 1 \end{bmatrix}_\times \mathbf{P}_2 \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

$$\underbrace{\begin{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix}_\times \mathbf{P}_1 \\ \begin{bmatrix} \mathbf{x}_2 \\ 1 \end{bmatrix}_\times \mathbf{P}_2 \\ \vdots \\ \begin{bmatrix} \mathbf{x}_F \\ 1 \end{bmatrix}_\times \mathbf{P}_F \end{bmatrix}}_{3F \times 4} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

$$\text{rank}\left(\begin{bmatrix} \mathbf{x} \\ 1 \end{bmatrix}_\times \mathbf{P}\right) = 2$$

Least squares if $F \geq 2$

Point Triangulation

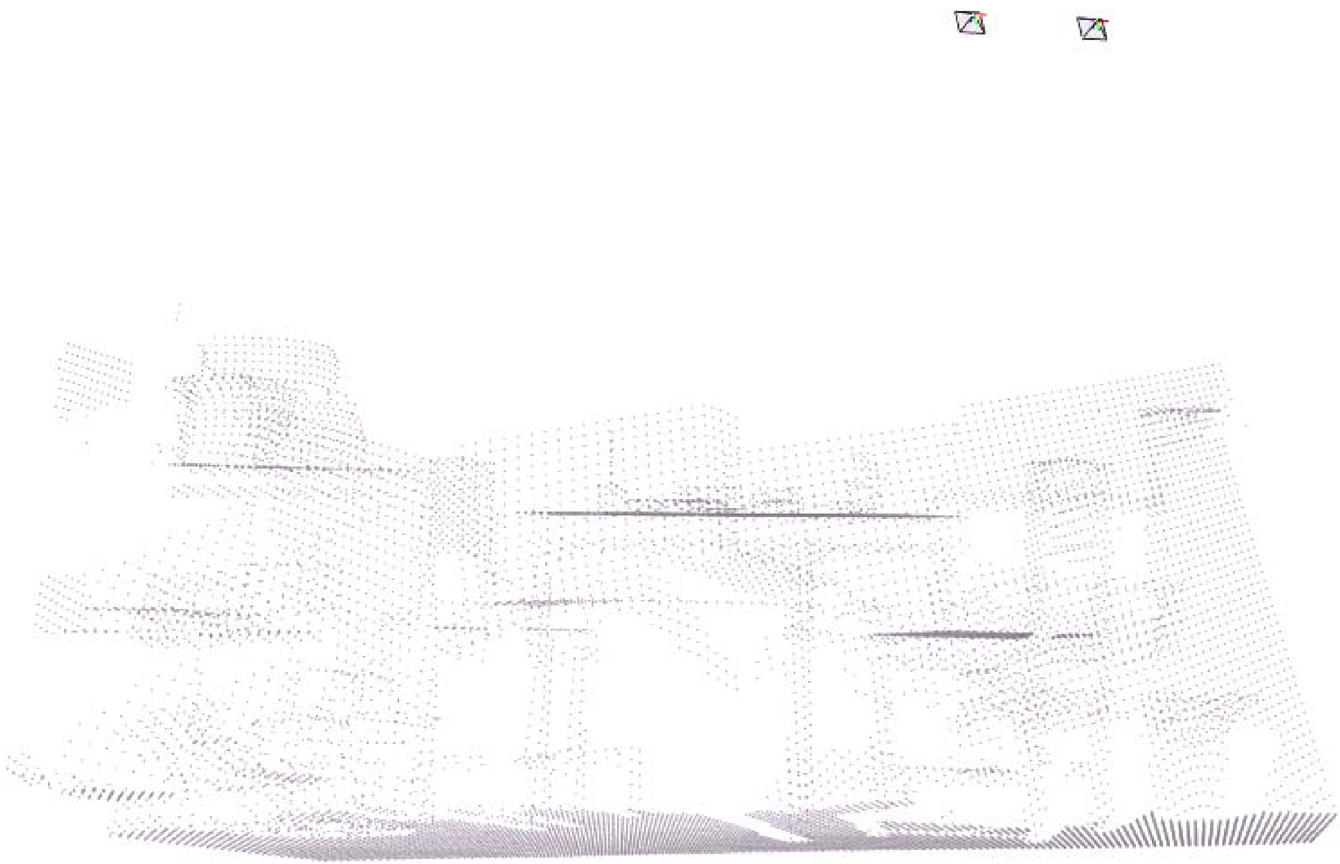
$$P_1 = K_1 \begin{bmatrix} I_{3\times3} & 0_3 \end{bmatrix}$$

$$P_2 = K_2 \begin{bmatrix} I_{3\times3} & -C \end{bmatrix} \qquad C = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

```
% Intrinsic parameter
K1 = [2329.558 0 1141.452; 0 2329.558 927.052; 0 0 1];
K2 = [2329.558 0 1241.731; 0 2329.558 927.052; 0 0 1];

% Camera matrices
P1 = K1 * [eye(3) zeros(3,1)];
C = [1;0;0];
P2 = K2 * [eye(3) -C];

% Correspondences
x1 = [1382;986;1];
x2 = [1144;986;1];
skew1 = Vec2Skew(x1);
skew2 = Vec2Skew(x2);

% Solve
A = [skew1*P1; skew2*P2];
[u,d,v] = svd(A);
X = v(:,end)/v(end,end);

function skew = Vec2Skew(v)
skew  = [0 -v(3) v(2); v(3) 0 -v(1); -v(2) v(1) 0];
```

```
X =
    0.7111
    0.1743
    6.8865
    1.0000
```

Point Triangulation



3.1 Linear Triangulation

Goal Given two camera poses, $(\mathbf{C}_1, \mathbf{R}_1)$ and $(\mathbf{C}_2, \mathbf{R}_2)$, and correspondences $\mathbf{x}_1 \leftrightarrow \mathbf{x}_2$, triangulate 3D points using linear least squares:

```
X = LinearTriangulation(K, C1, R1, C2, R2, x1, x2)
```

(INPUT) `C1` and `R1`: the first camera pose

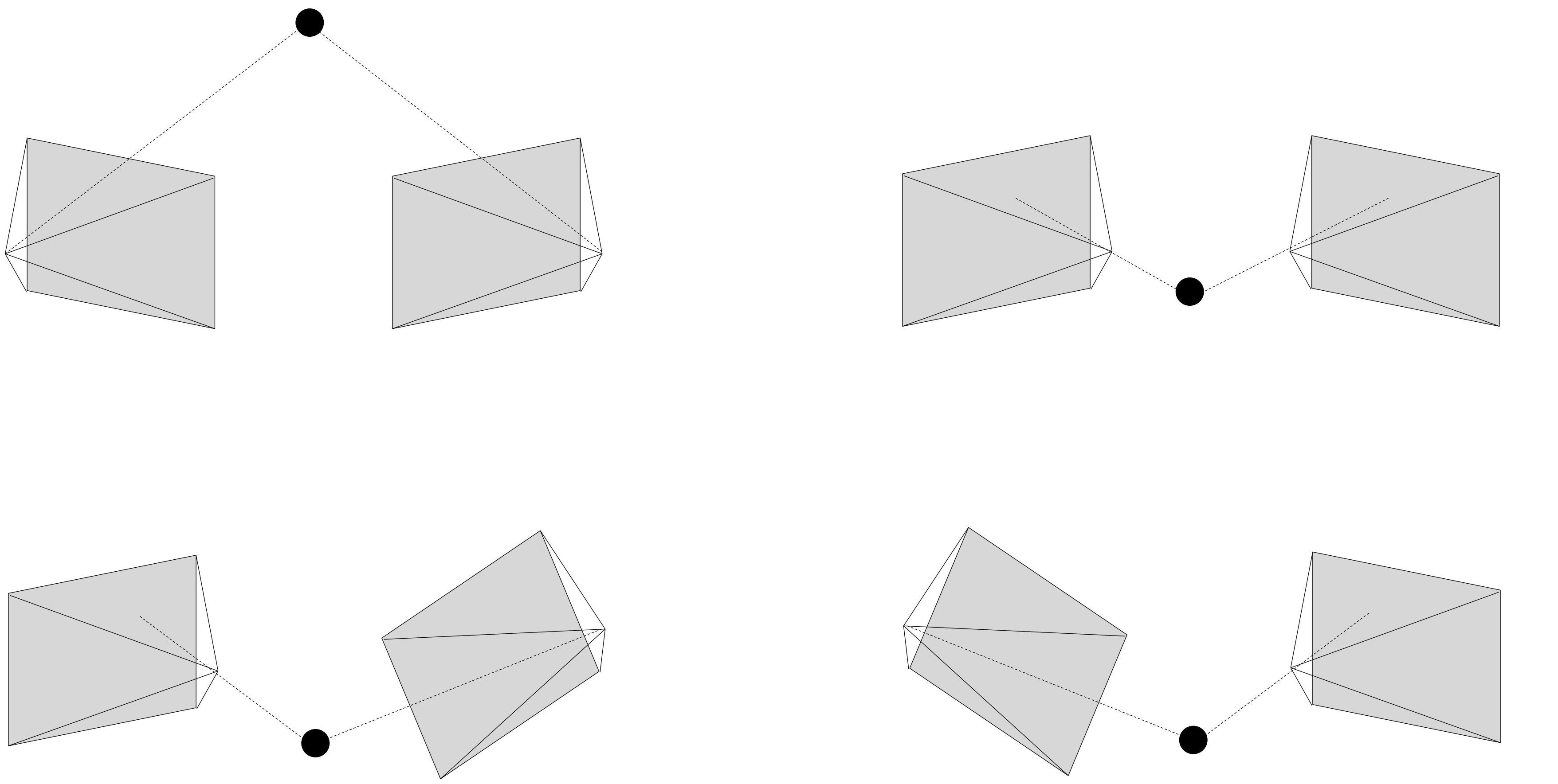
(INPUT) `C2` and `R2`: the second camera pose

(INPUT) `x1` and `x2`: two $N \times 2$ matrices whose row represents correspondence between the first and second images where N is the number of correspondences.

(OUTPUT) `X`: $N \times 3$ matrix whose row represents 3D triangulated point.

Camera pose disambiguation via point triangulation

Four configurations:



3.2 Camera Pose Disambiguation

Goal Given four camera pose configuration and their triangulated points, find the unique camera pose by checking the *cheirality* condition—the reconstructed points must be in front of the cameras:

```
[C R X0] = DisambiguateCameraPose(Cset, Rset, Xset)
```

(INPUT) `Cset` and `Rset`: four configurations of camera centers and rotations

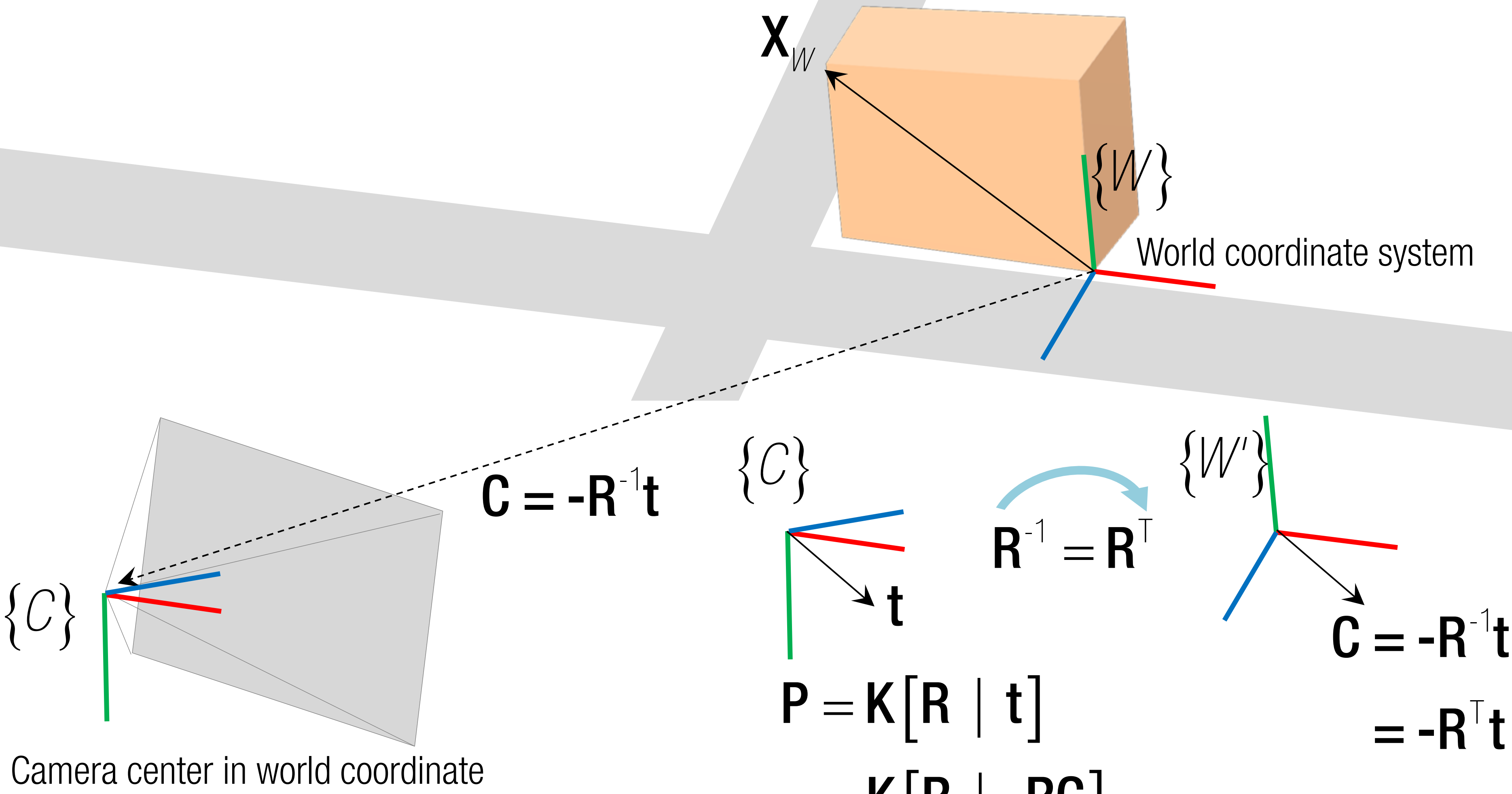
(INPUT) `Xset`: four sets of triangulated points from four camera pose configurations

(OUTPUT) `C` and `R`: the correct camera pose

(OUTPUT) `X0`: the 3D triangulated points from the correct camera pose

The sign of the Z element in the camera coordinate system indicates the location of the 3D point with respect to the camera, i.e., a 3D point $\mathbf{X}$ is in front of a camera if $(\mathbf{C}, \mathbf{R})$ if $\mathbf{r}_3(\mathbf{X} - \mathbf{C}) > 0$ where $\mathbf{r}_3$ is the third row of $\mathbf{R}$. Not all triangulated points satisfy this condition due to the presence of correspondence noise. The best camera configuration, $(\mathbf{C}, \mathbf{R}, \mathbf{X})$ is the one that produces the maximum number of points satisfying the cheirality condition.

Third person (world) perspective



$$\mathbf{P} = \mathbf{K}\left[\mathbf{R} \mid \mathbf{t}\right]$$
$$= \mathbf{K}\left[\mathbf{R} \mid -\mathbf{R}\mathbf{C}\right]$$
$$= \mathbf{K}\mathbf{R}\left[\mathbf{I}_{3\times3} \mid -\underline{\mathbf{C}}\right]$$

Camera center seen from world coordinate system