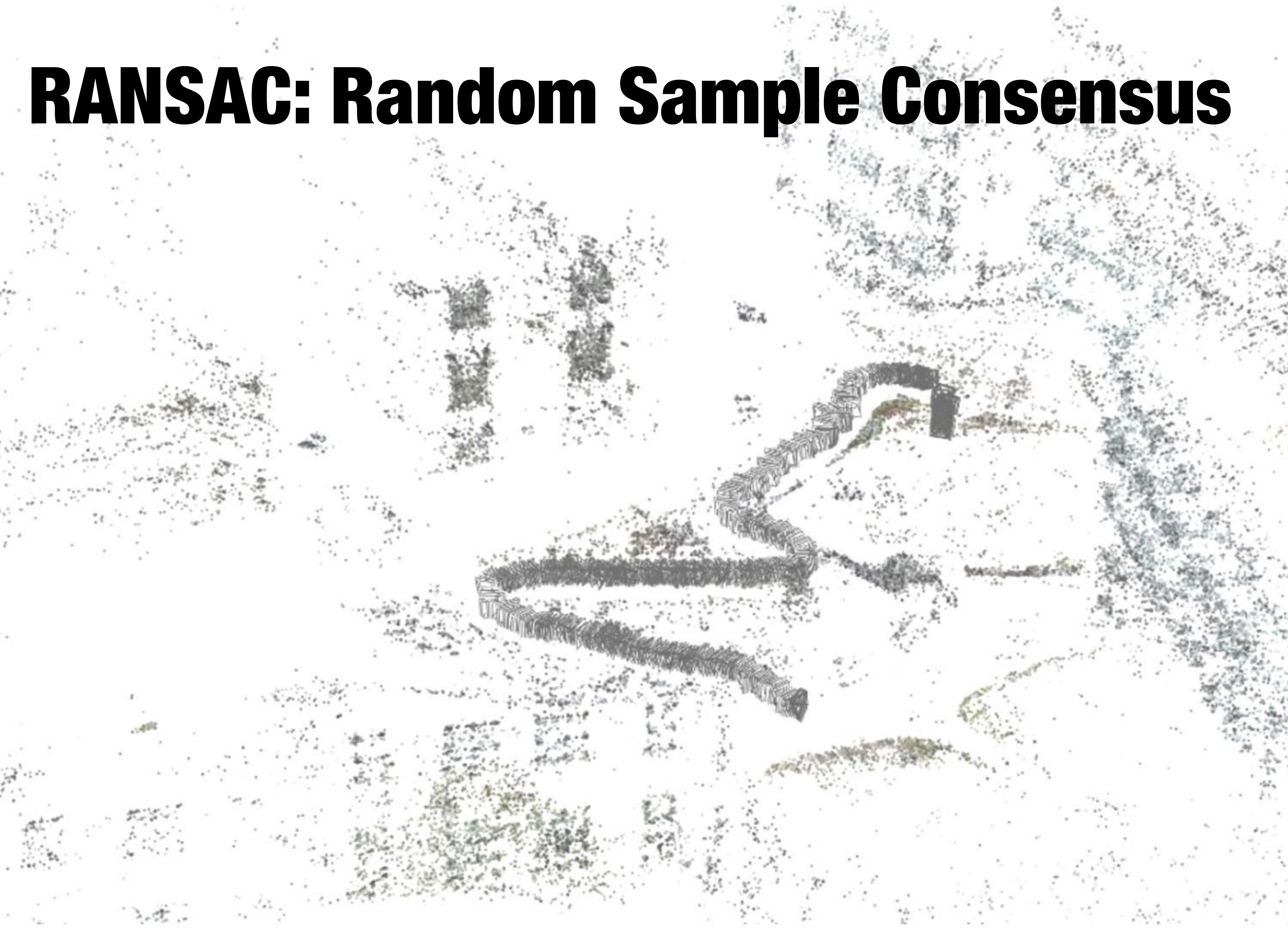
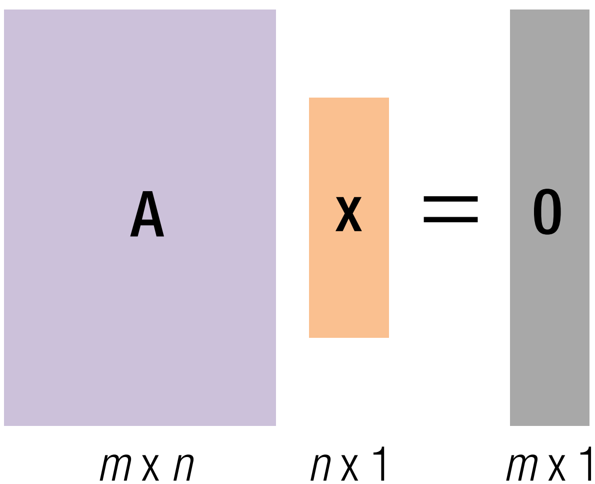
RANSAC: Random Sample Consensus



Linear Homogeneous Equations



Linear least square solve produces a trivial solution:

$$\mathbf{x} = \left(\mathbf{A}^{T}\mathbf{A}\right)^{-1}\mathbf{A}^{T}\mathbf{b} \rightarrow \mathbf{x} = \mathbf{0}$$

An additional constraint on $\mathbf{x}$ to avoid the trivial solution: $\|\mathbf{x}\| = 1$

1) $\mathbf{rank(A)} = r < n-1$: infinite number of solutions

$$\mathbf{x} = \lambda_{r+1}\mathbf{V}_{r+1} + \cdots + \lambda_n \mathbf{V}_n \quad \text{where} \quad \sum_{i=r+1}^{n} \lambda_i^2 = 1$$

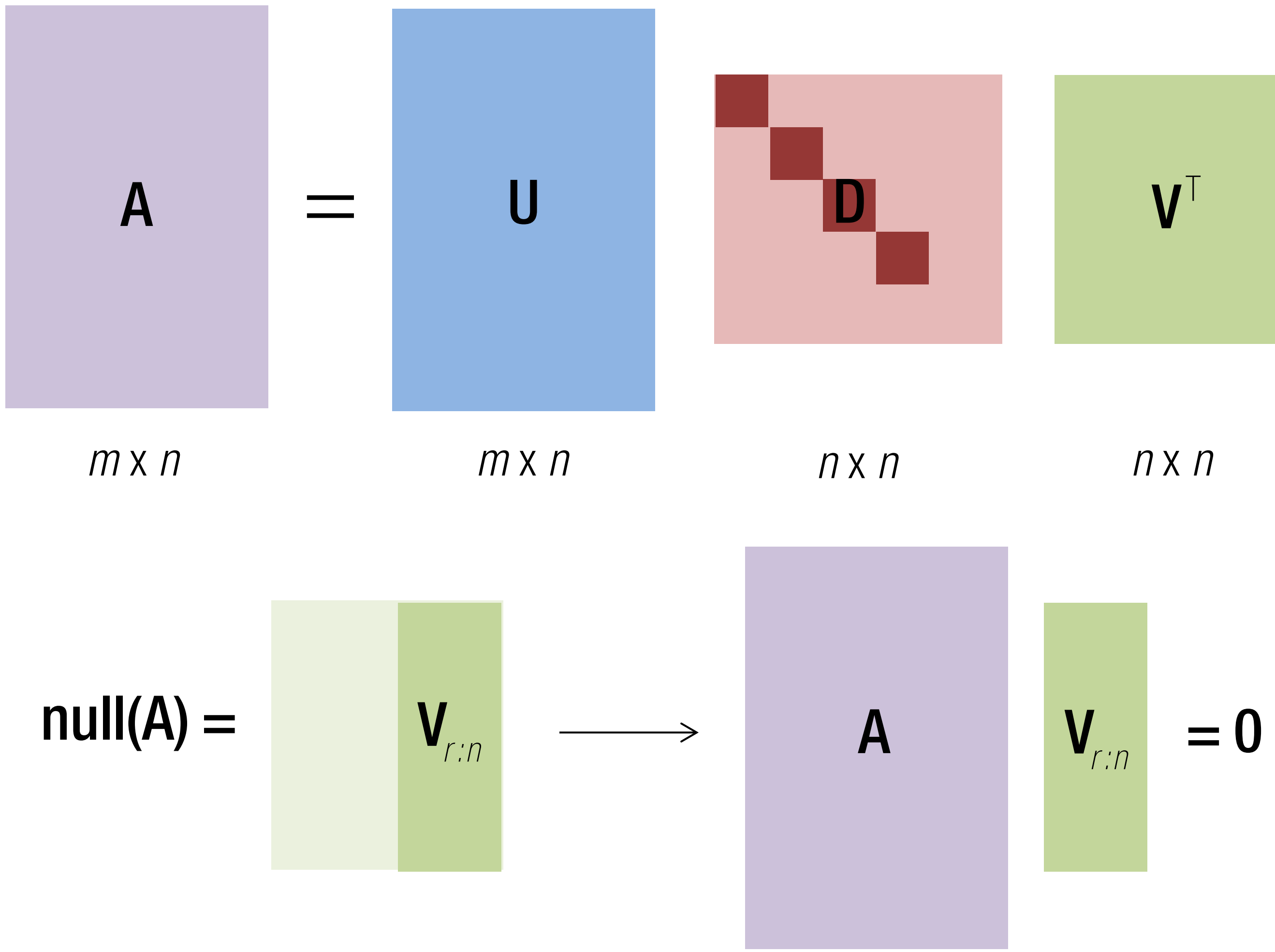
2) $\mathbf{rank(A)} = n-1$: one exact solution

$$\mathbf{x} = \mathbf{V}_n$$

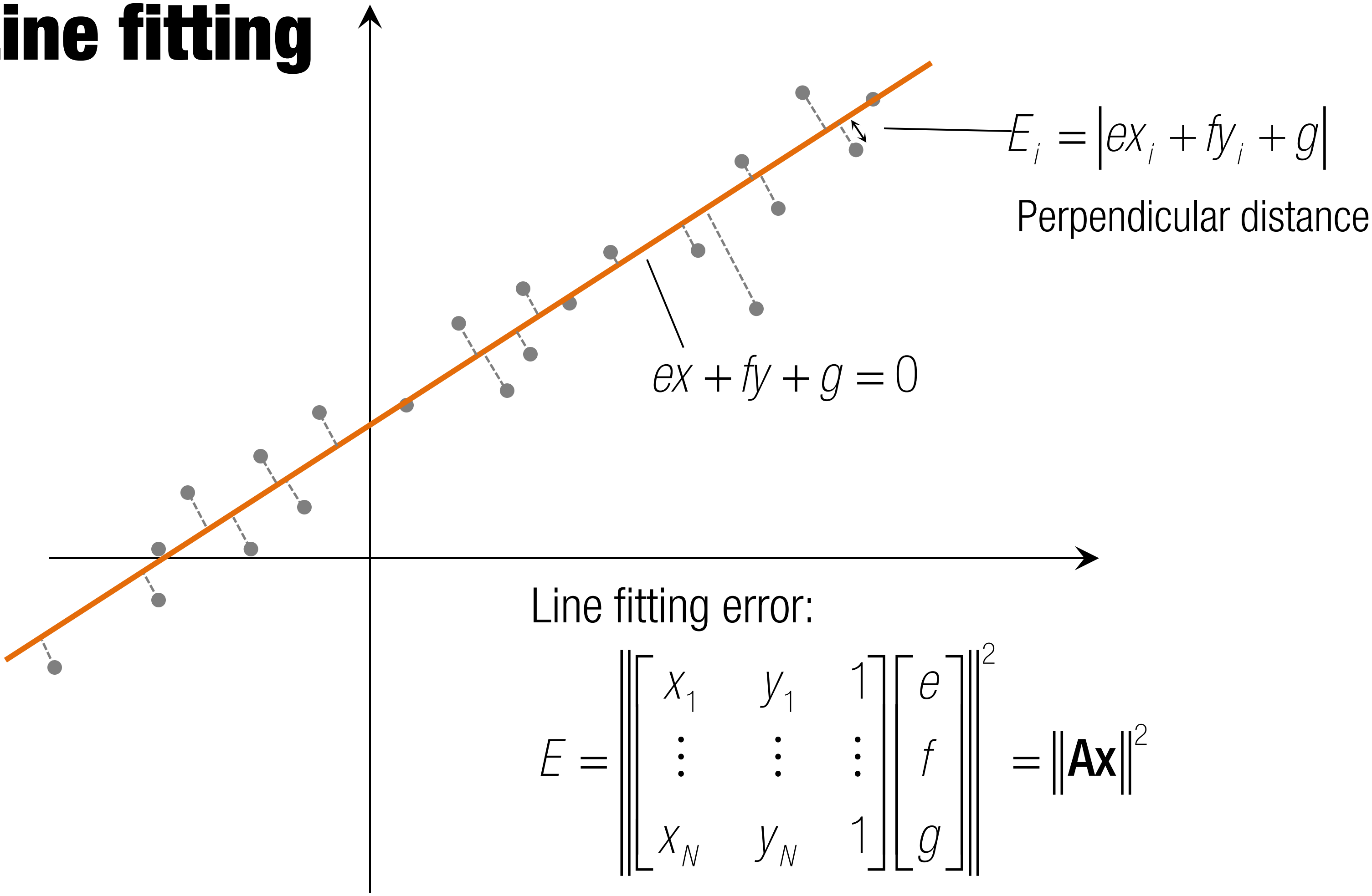
3) $n < m$: no exact solution in general (needs least squares)

$$\min_{\mathbf{x}} \|\mathbf{Ax}\|^2 \text{ subject to } \|\mathbf{x}\| = 1 \rightarrow \mathbf{x} = \mathbf{V}_n$$

Nullspace



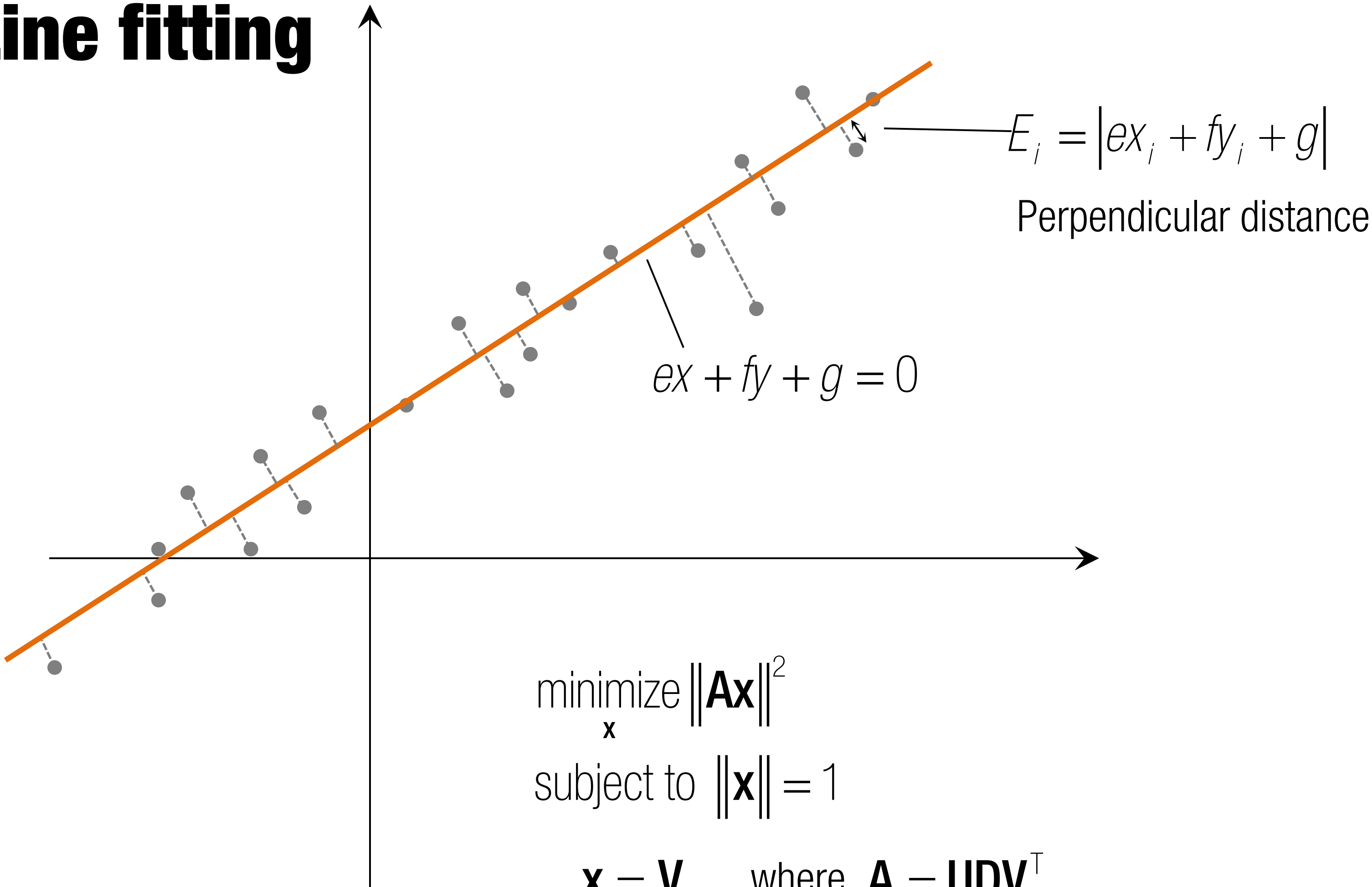
Line fitting



Line fitting error:

$$E = \left\| \begin{bmatrix} x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots \\ x_N & y_N & 1 \end{bmatrix} \begin{bmatrix} e \\ f \\ g \end{bmatrix} \right\|^2 = \|\mathbf{Ax}\|^2$$

Line fitting

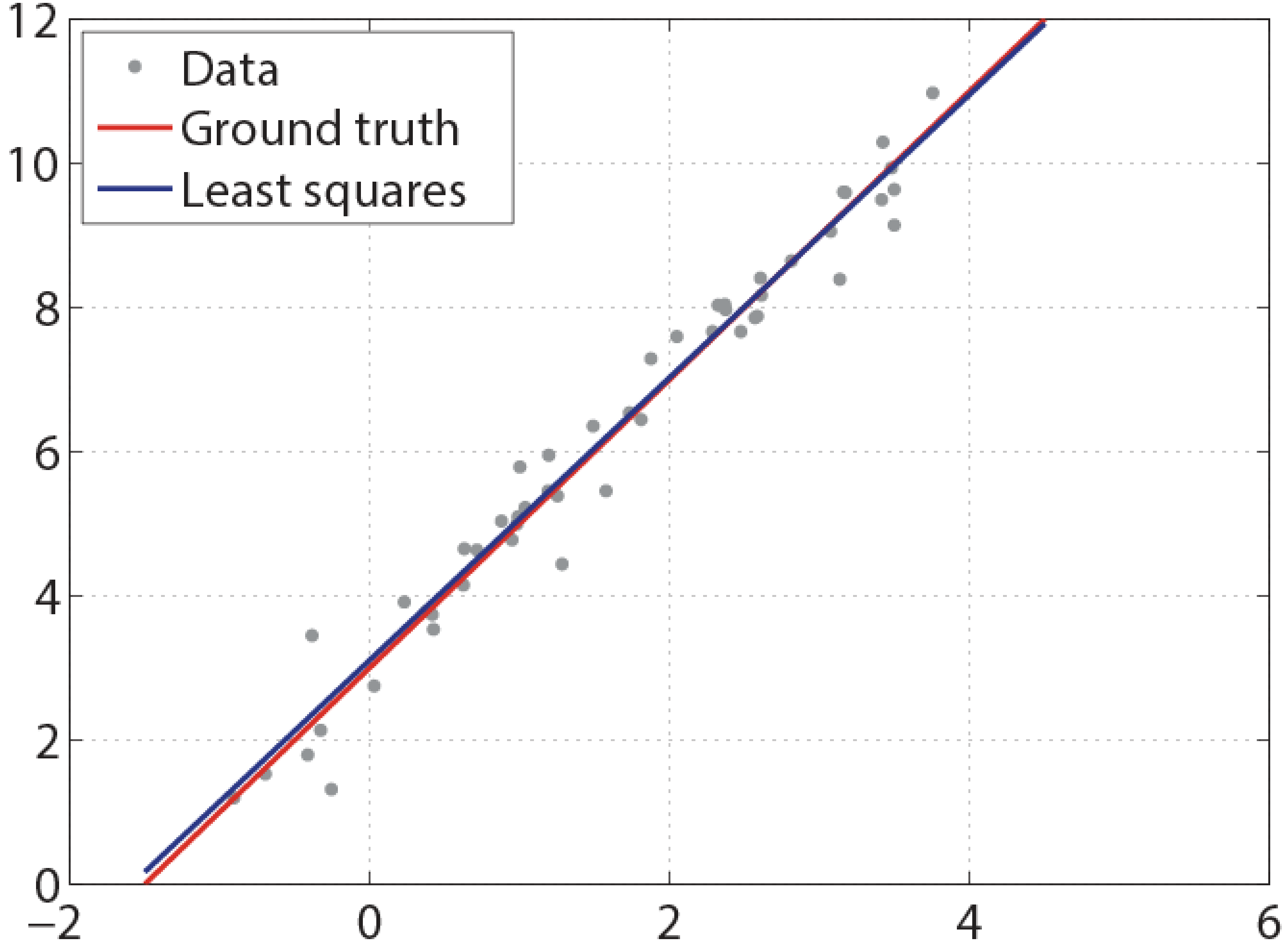


$$\underset{\mathbf{x}}{\text{minimize}} \ \|\mathbf{Ax}\|^2$$

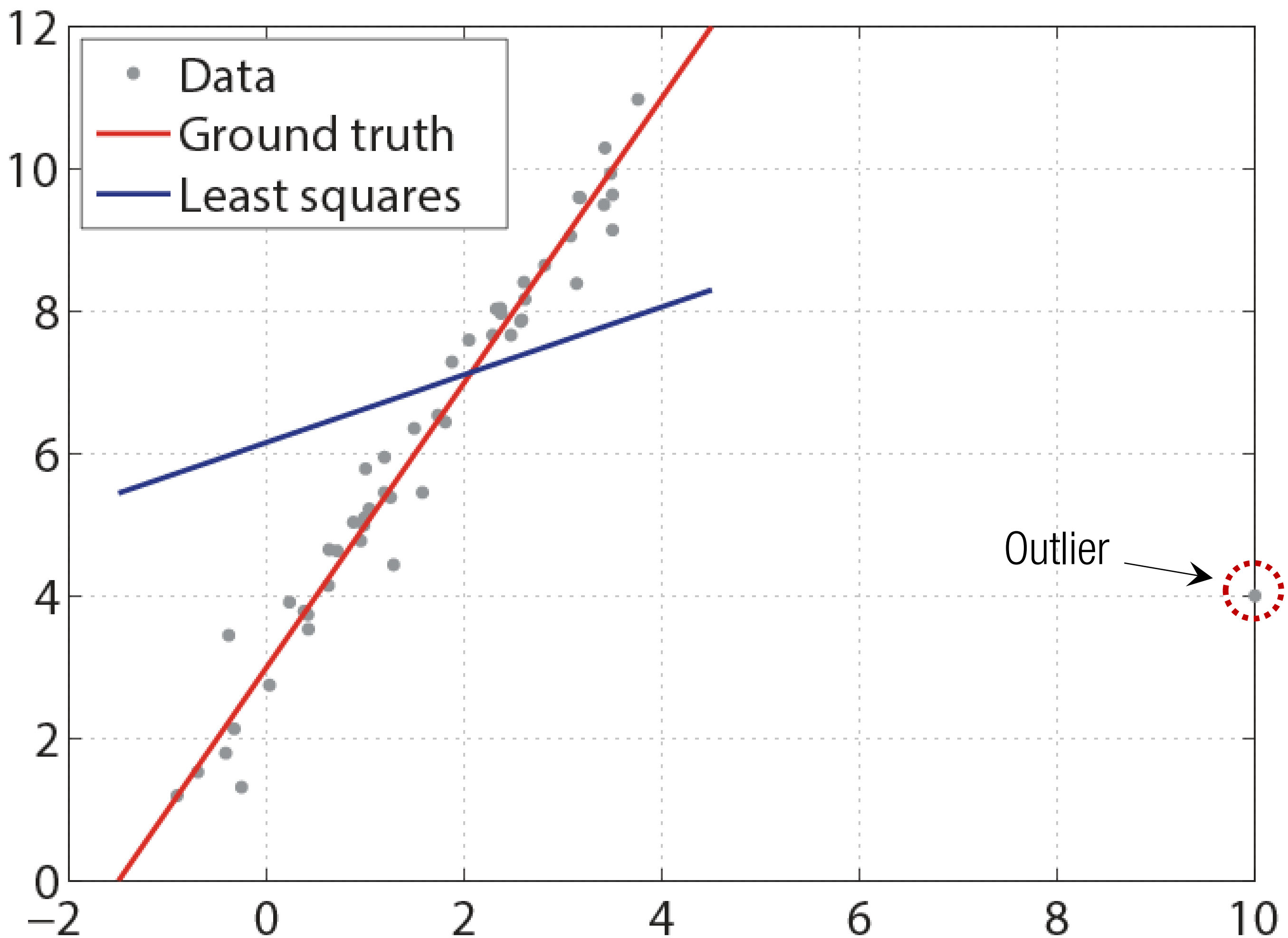
$$\text{subject to } \|\mathbf{x}\| = 1$$

$$\mathbf{x} = \mathbf{V}_3 \quad \text{where } \mathbf{A} = \mathbf{UDV}^T$$

$$\mathbf{V} = [\mathbf{V}_1 \quad \mathbf{V}_2 \quad \mathbf{V}_3]$$

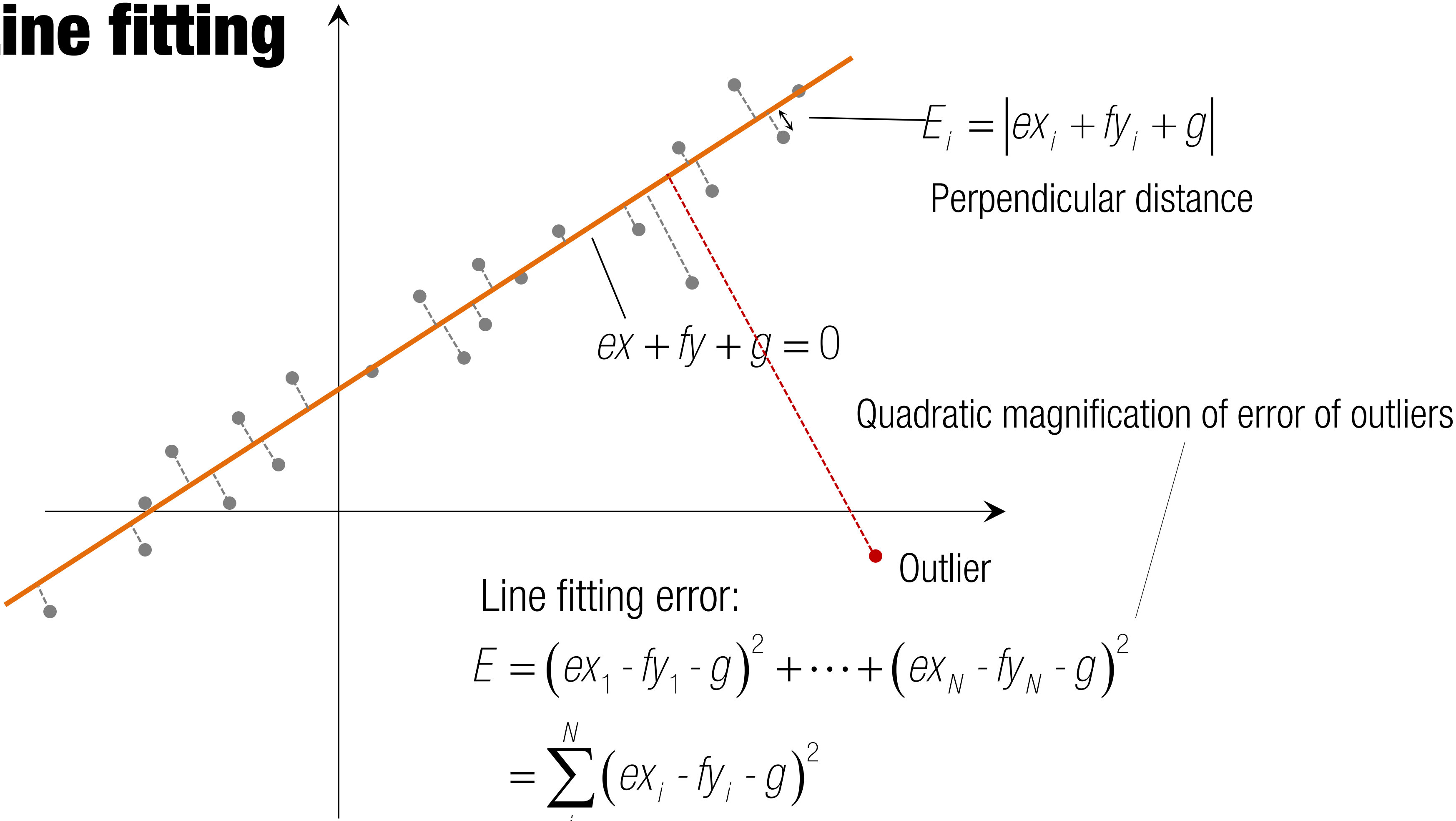


Data
Ground truth
Least squares
12
10
8
6
4
2
0
−2
0
2
4
6



Data
Ground truth
Least squares
Outlier
12
10
8
6
4
2
0
−2
0
2
4
6
8
10

Line fitting

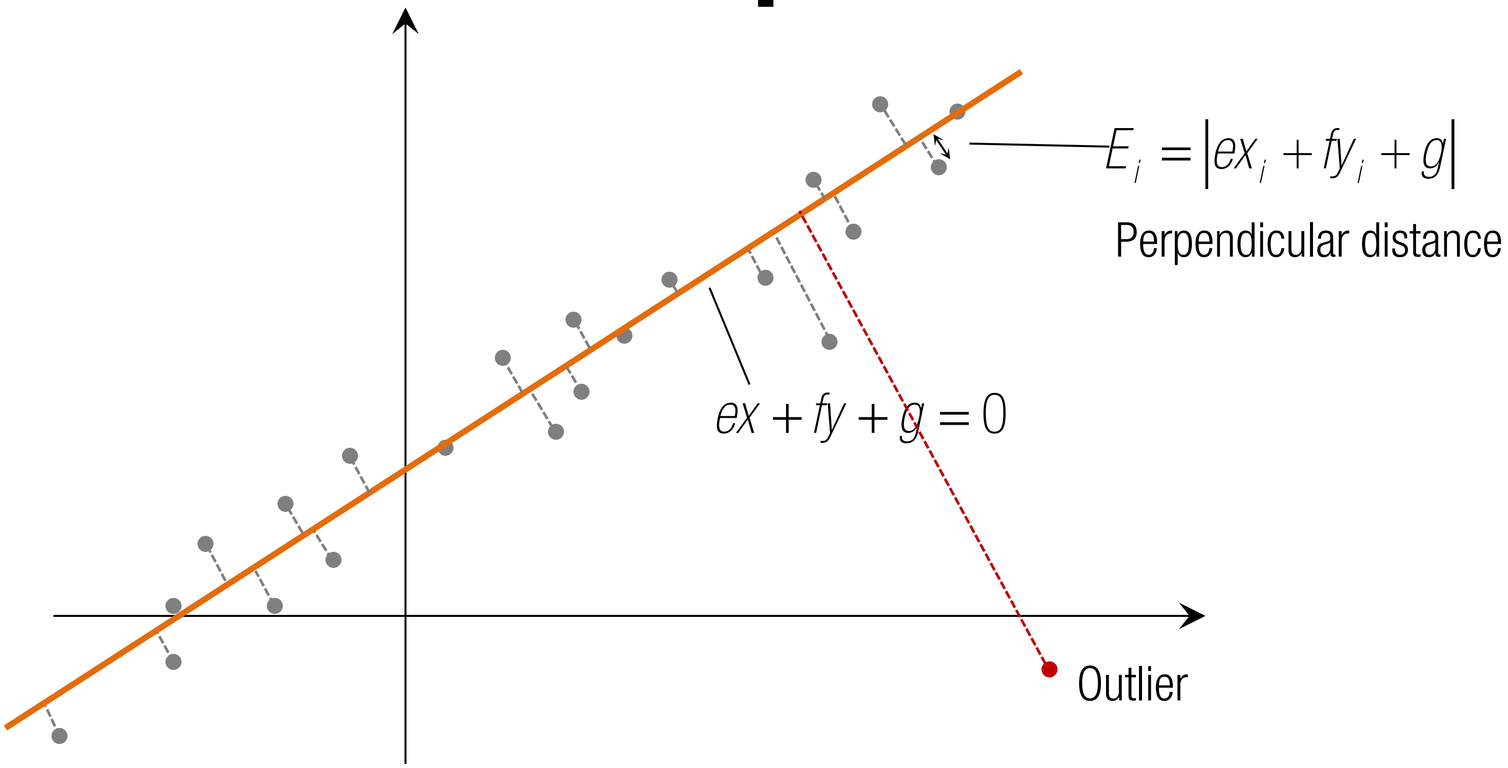


Line fitting error:

$$E = (ex_1 - fy_1 - g)^2 + \cdots + (ex_N - fy_N - g)^2$$

$$= \sum_{i}^{N} (ex_i - fy_i - g)^2$$

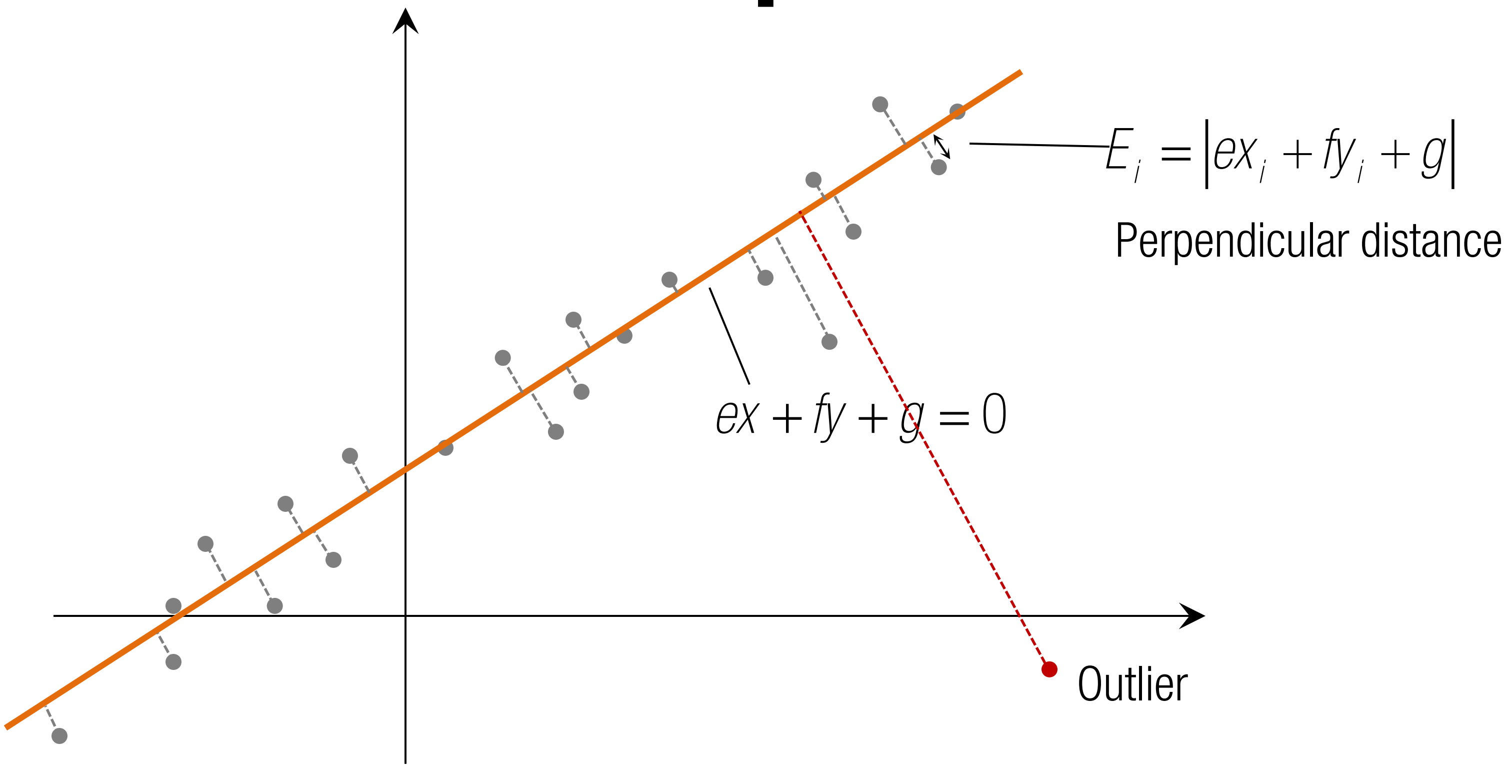
RANSAC: Random Sample Consensus



Strategy:

To find a model that accords with the maximum number of samples

RANSAC: Random Sample Consensus



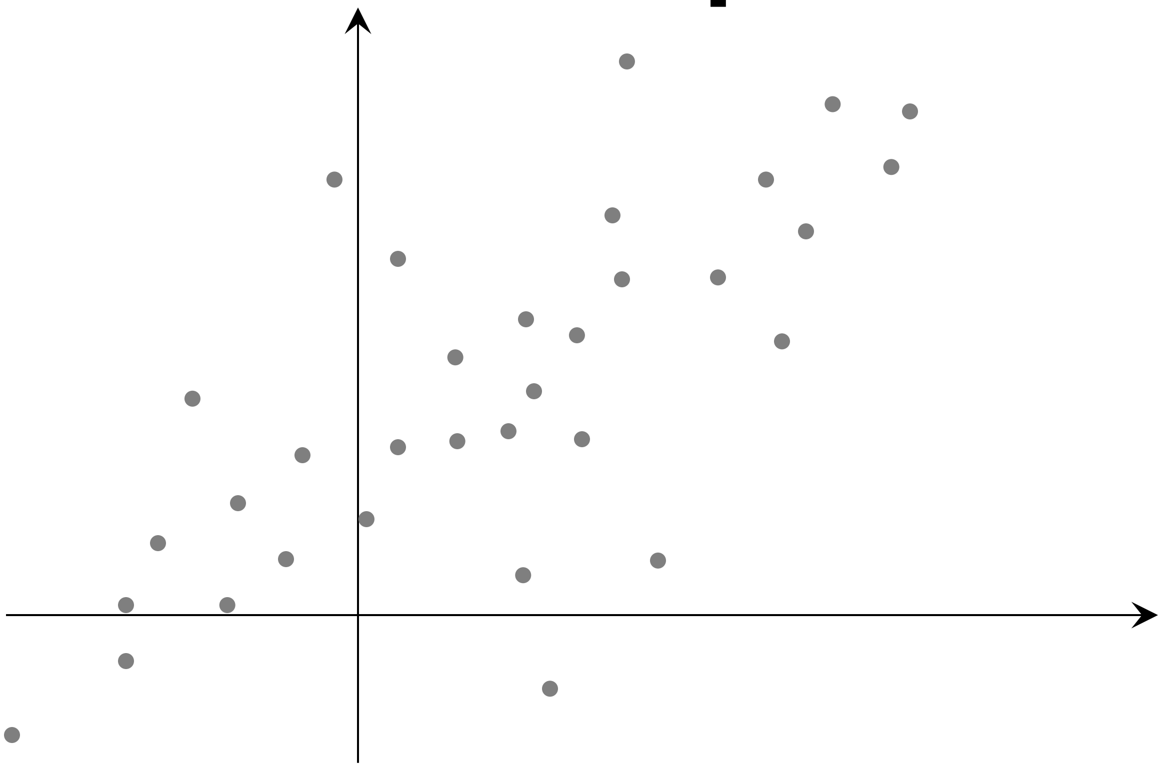
Strategy:

To find a model that accords with the maximum number of samples

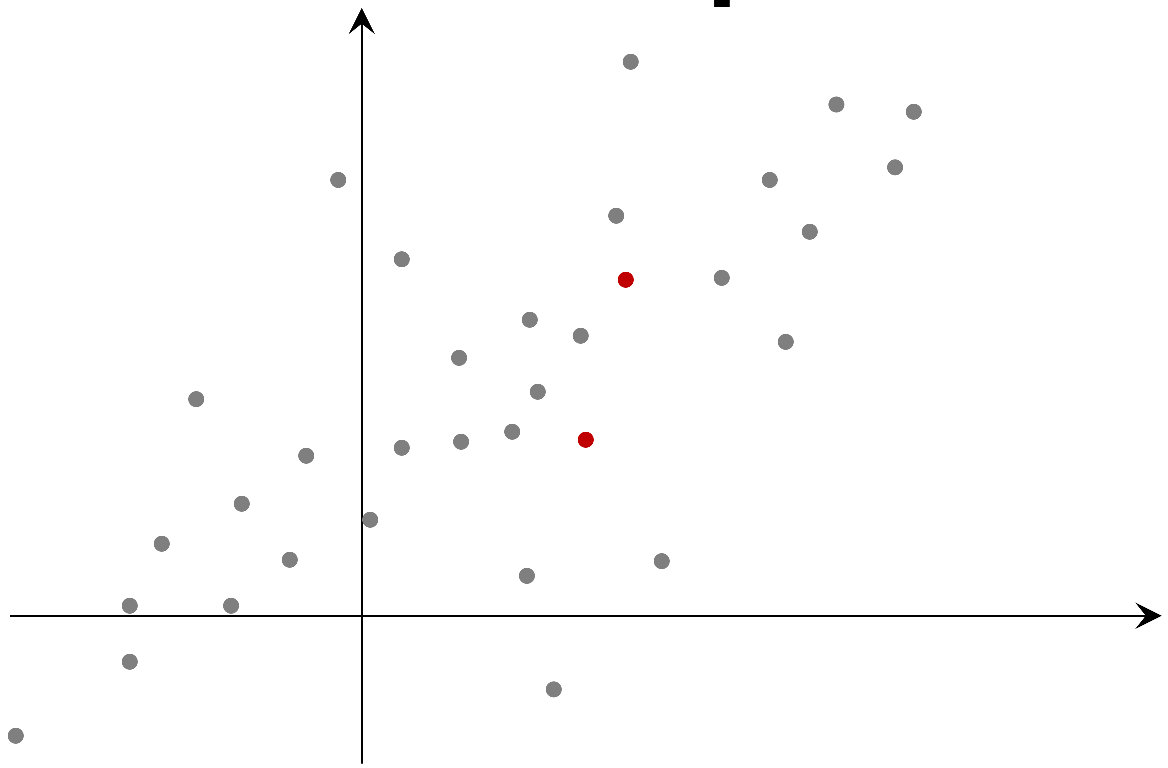
Assumptions:

1. Majority of good samples agree with the underlying model (good apples are same and simple.).
2. Bad samples does not consistently agree with a single model (all bad apples are different and complicated.).

RANSAC: Random Sample Consensus

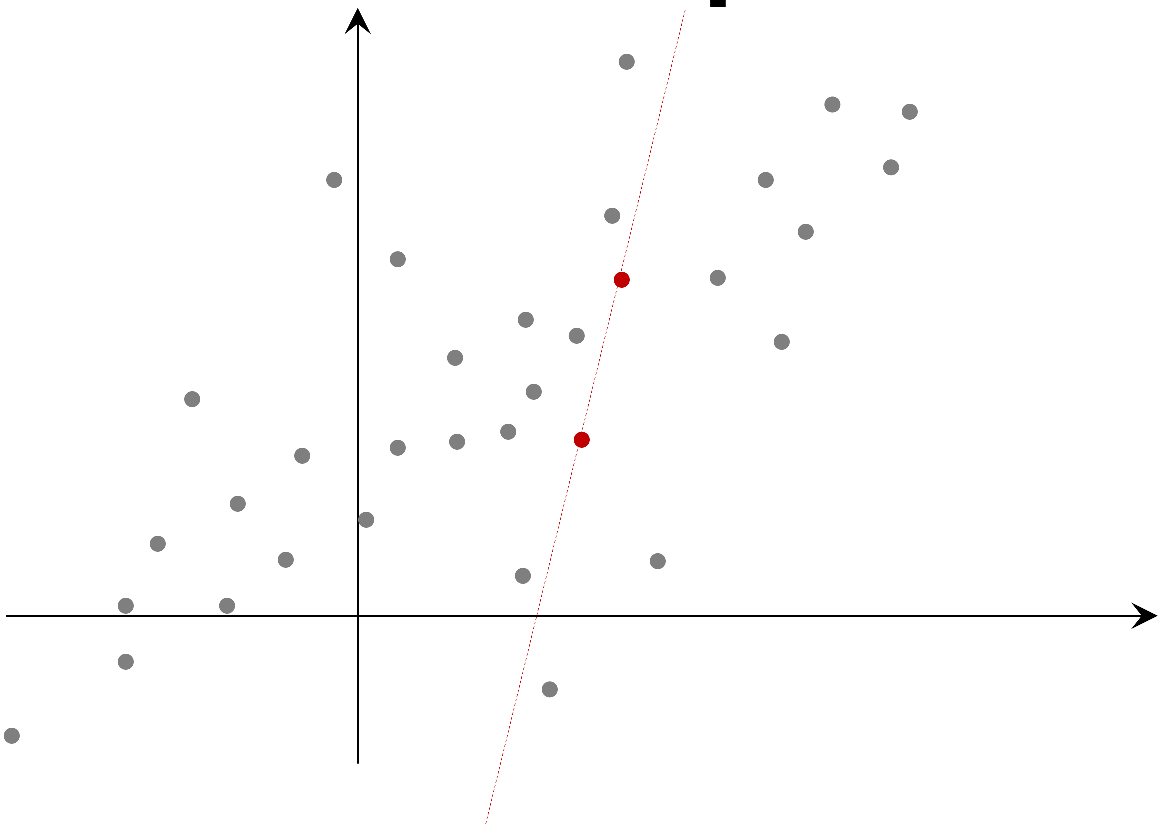


RANSAC: Random Sample Consensus



Random sampling

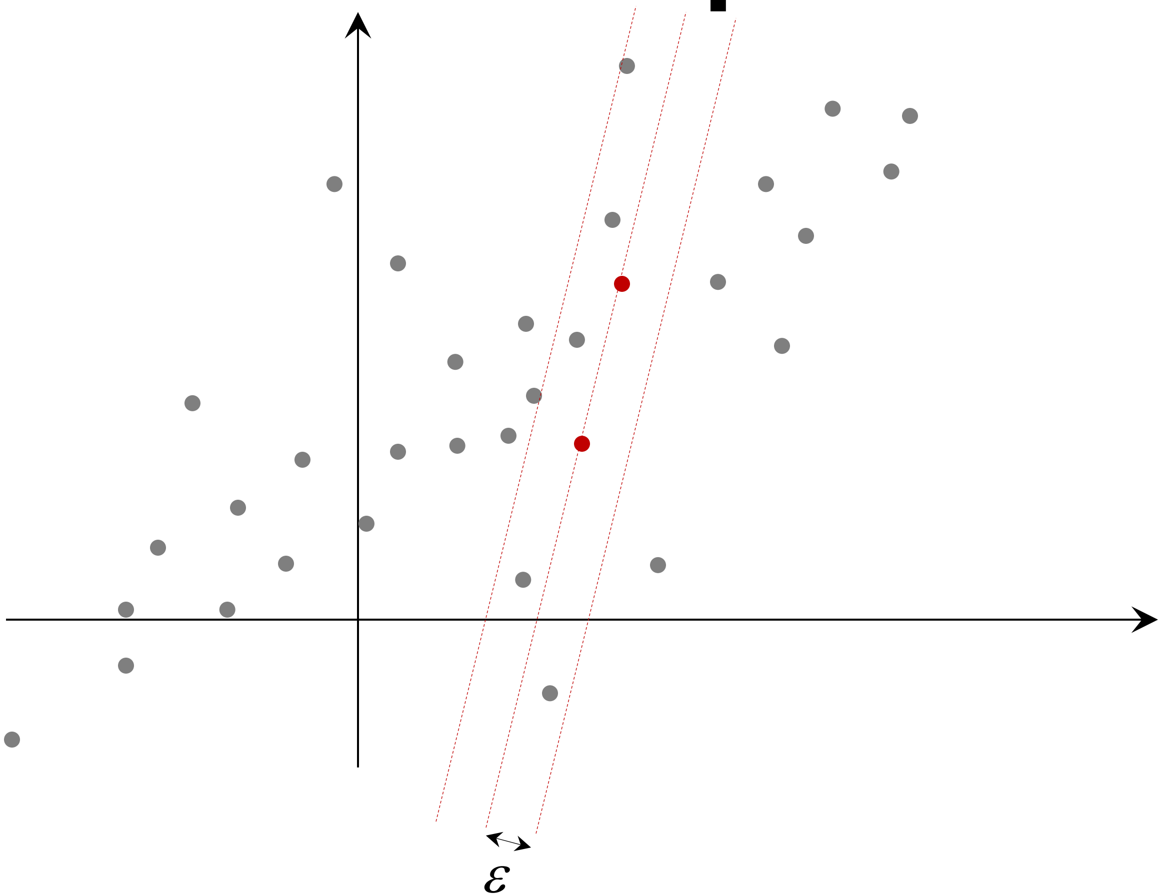
RANSAC: Random Sample Consensus



Random sampling

Model building

RANSAC: Random Sample Consensus

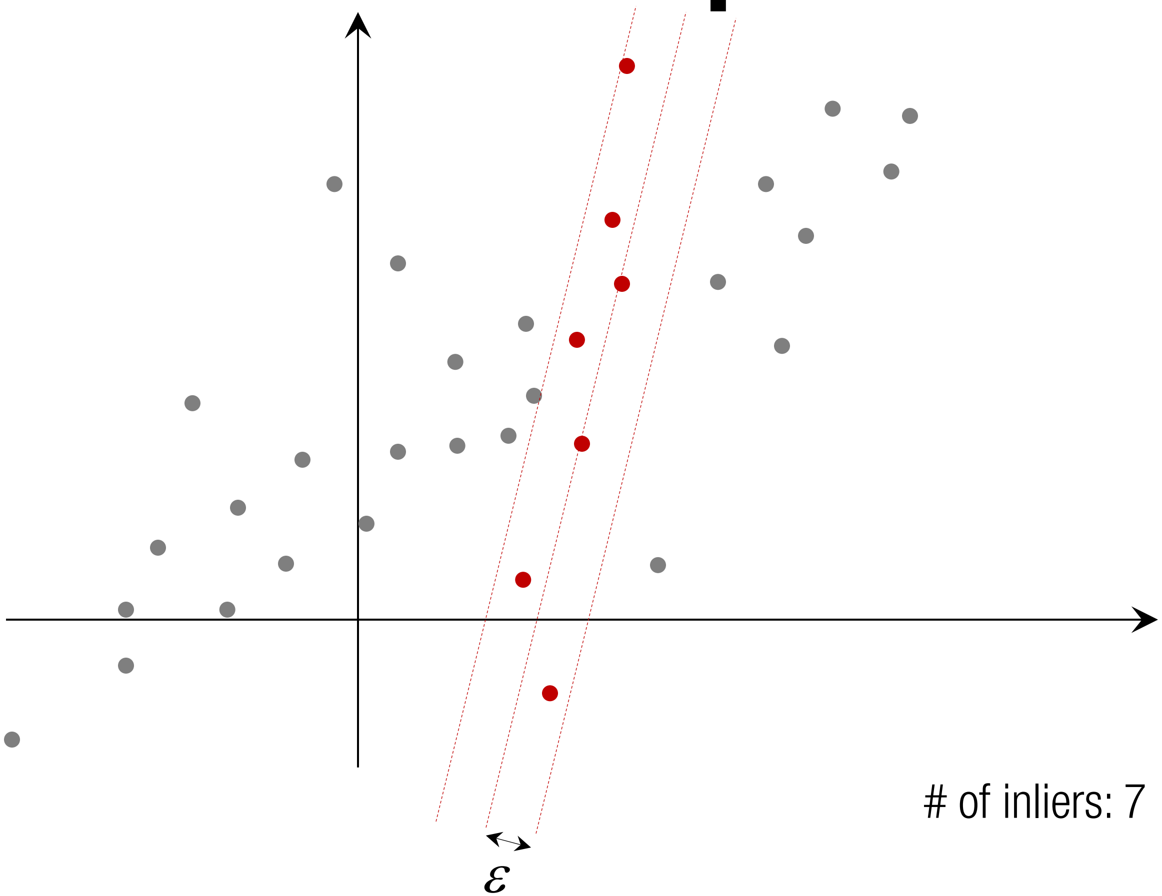


Random sampling

Model building

Thresholding

RANSAC: Random Sample Consensus



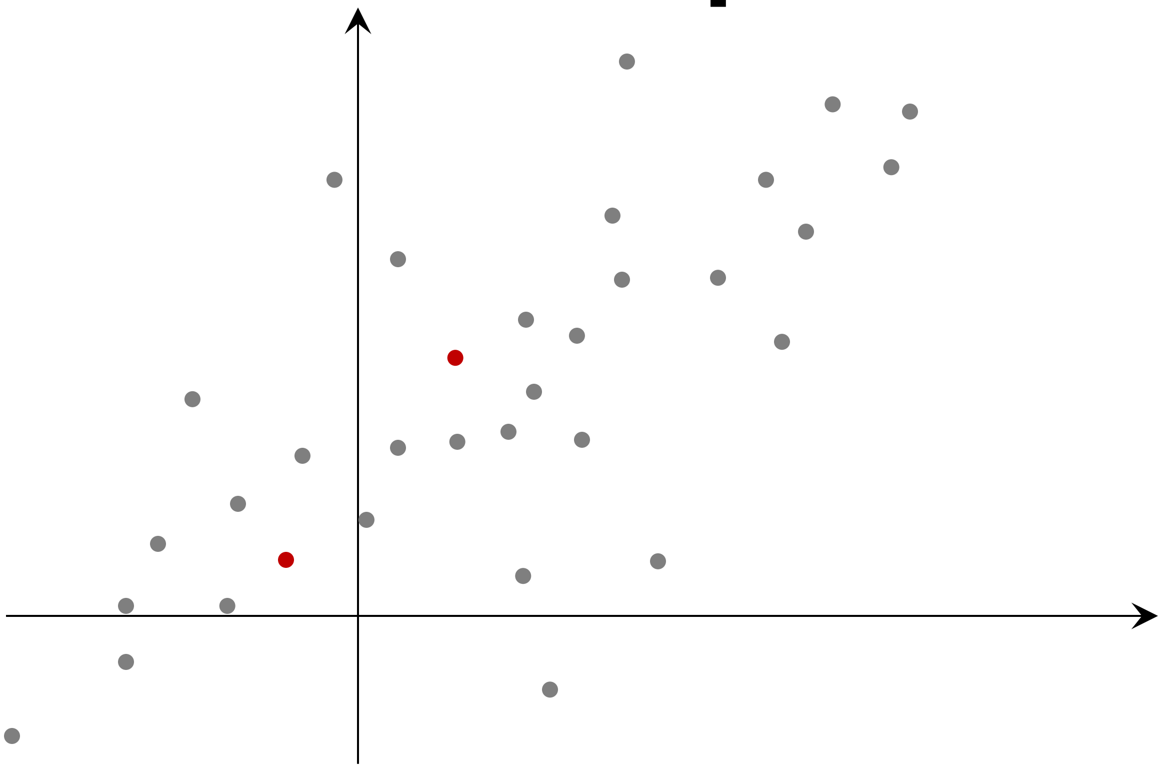
Random sampling

Model building

Thresholding

Inlier counting

RANSAC: Random Sample Consensus



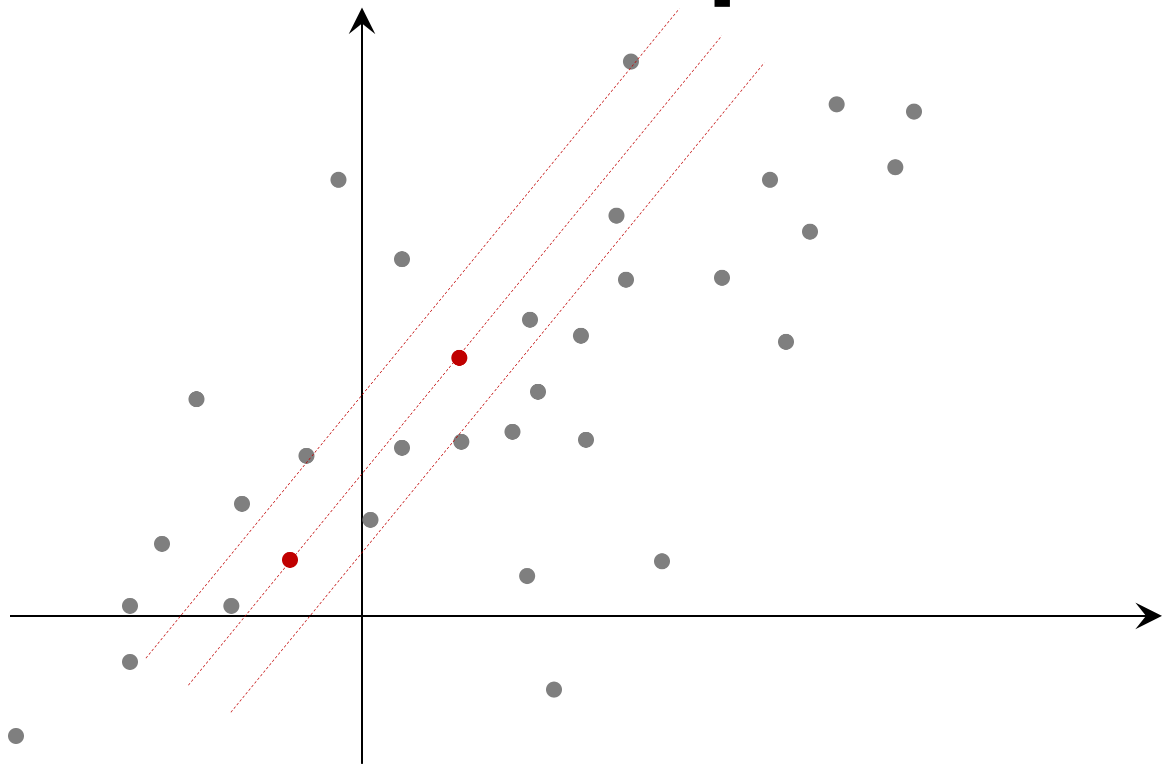
Random sampling

Model building

Thresholding

Inlier counting

RANSAC: Random Sample Consensus



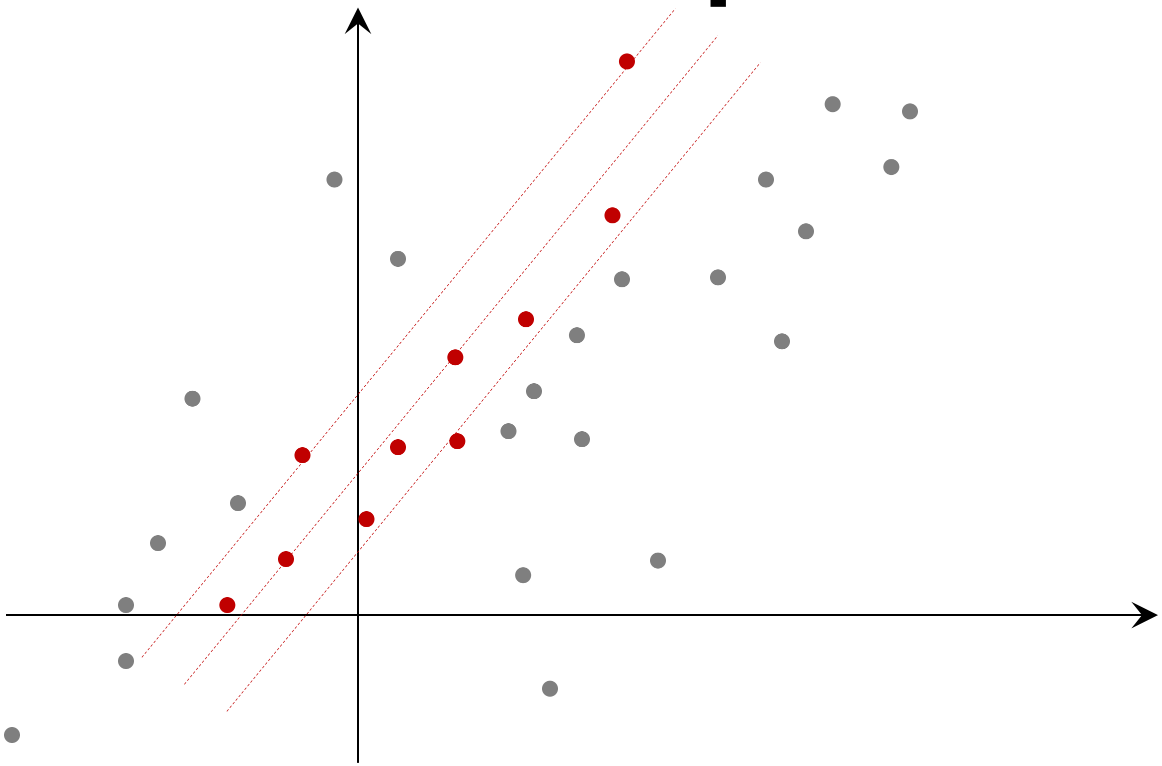
Random sampling

Model building

Thresholding

Inlier counting

RANSAC: Random Sample Consensus



Random sampling

Model building

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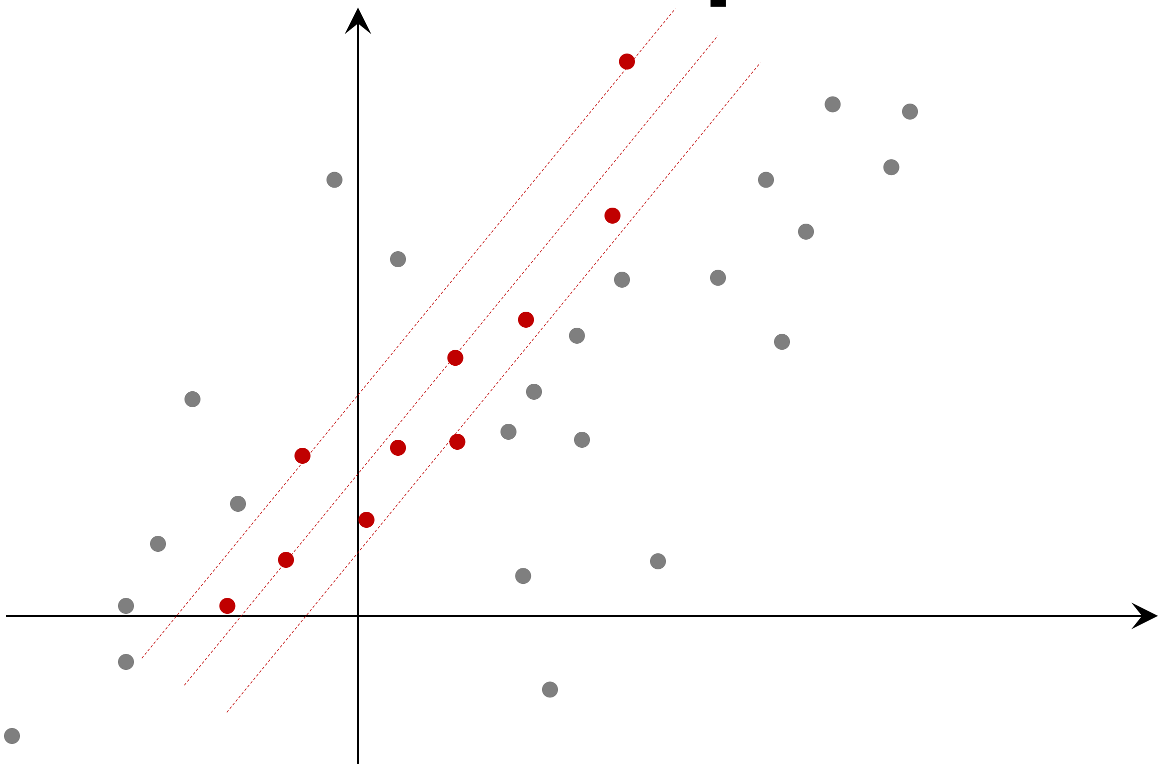
Inlier counting

RANSAC: Random Sample Consensus



Random sampling

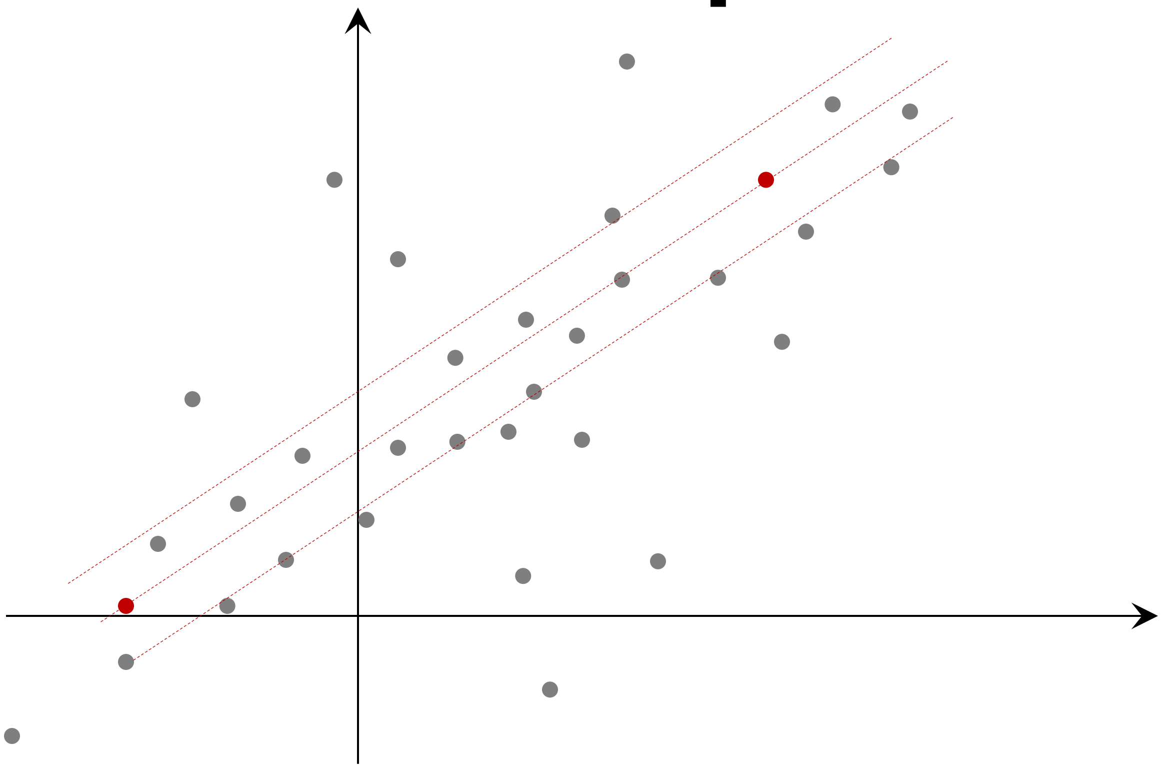
Model building

Thresholding

Inlier counting

# of inliers: 10

RANSAC: Random Sample Consensus



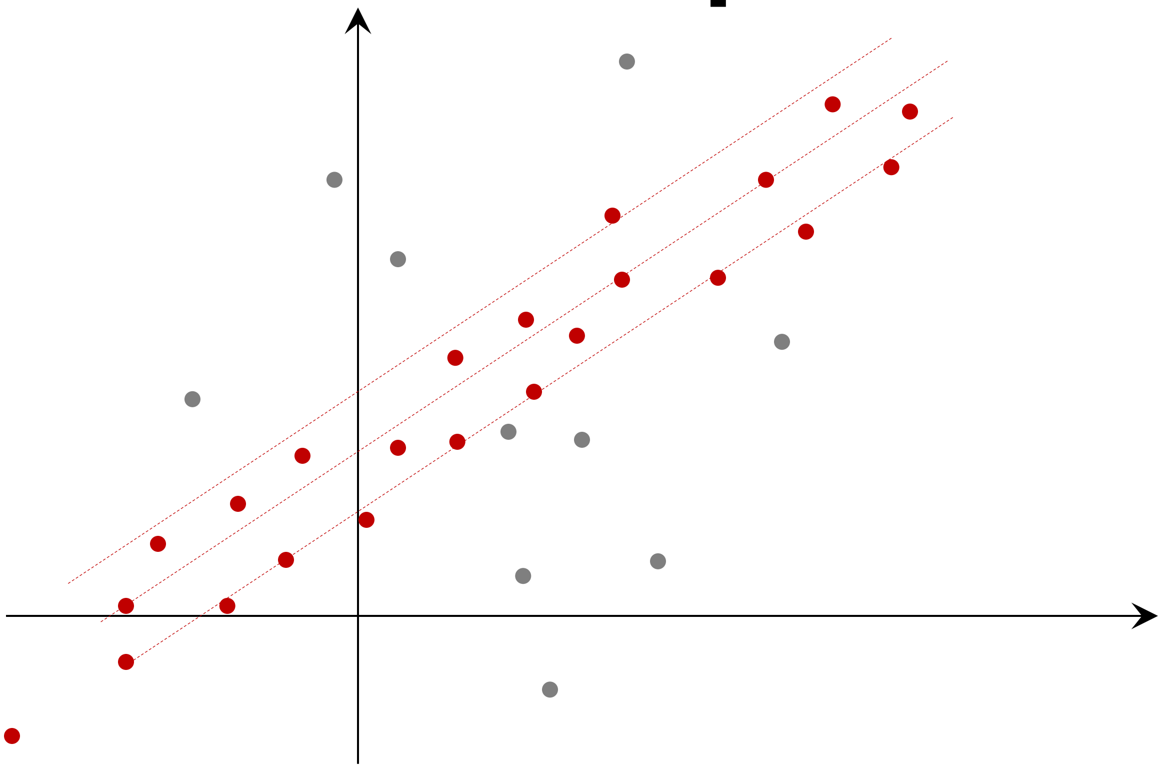
Random sampling

Model building

Thresholding

Inlier counting

RANSAC: Random Sample Consensus



Random sampling

Model building

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Inlier counting

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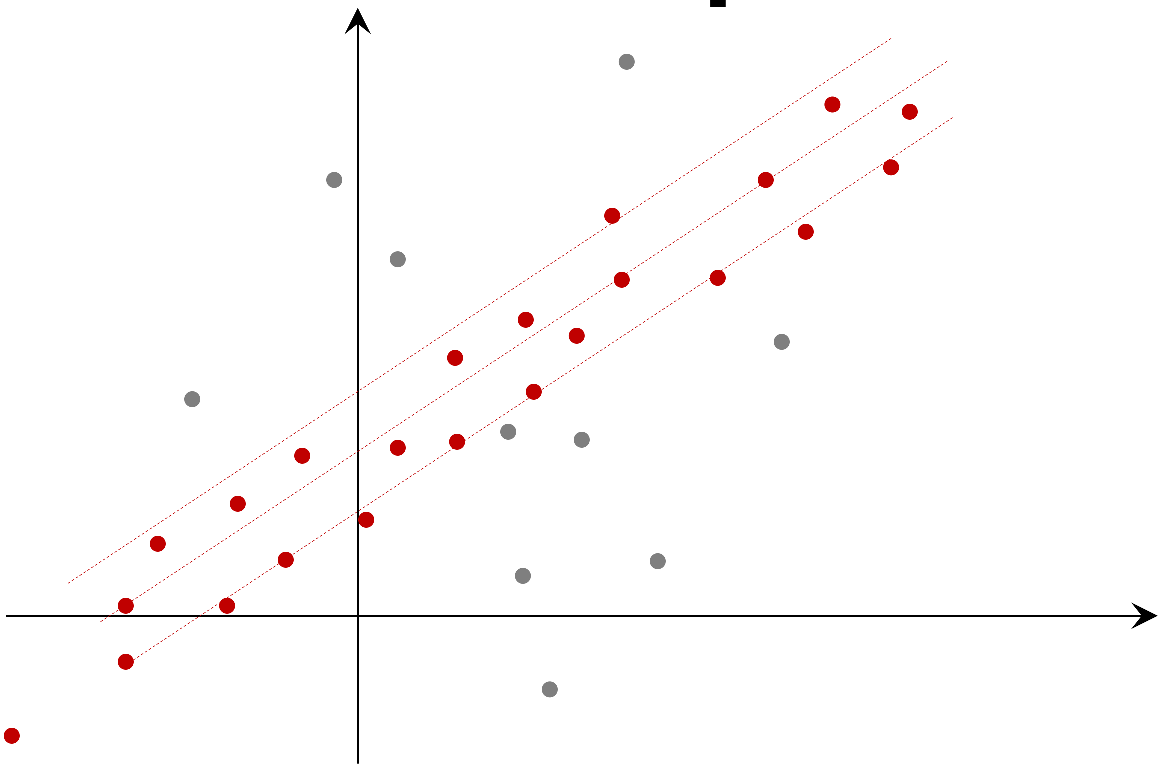

Random sampling

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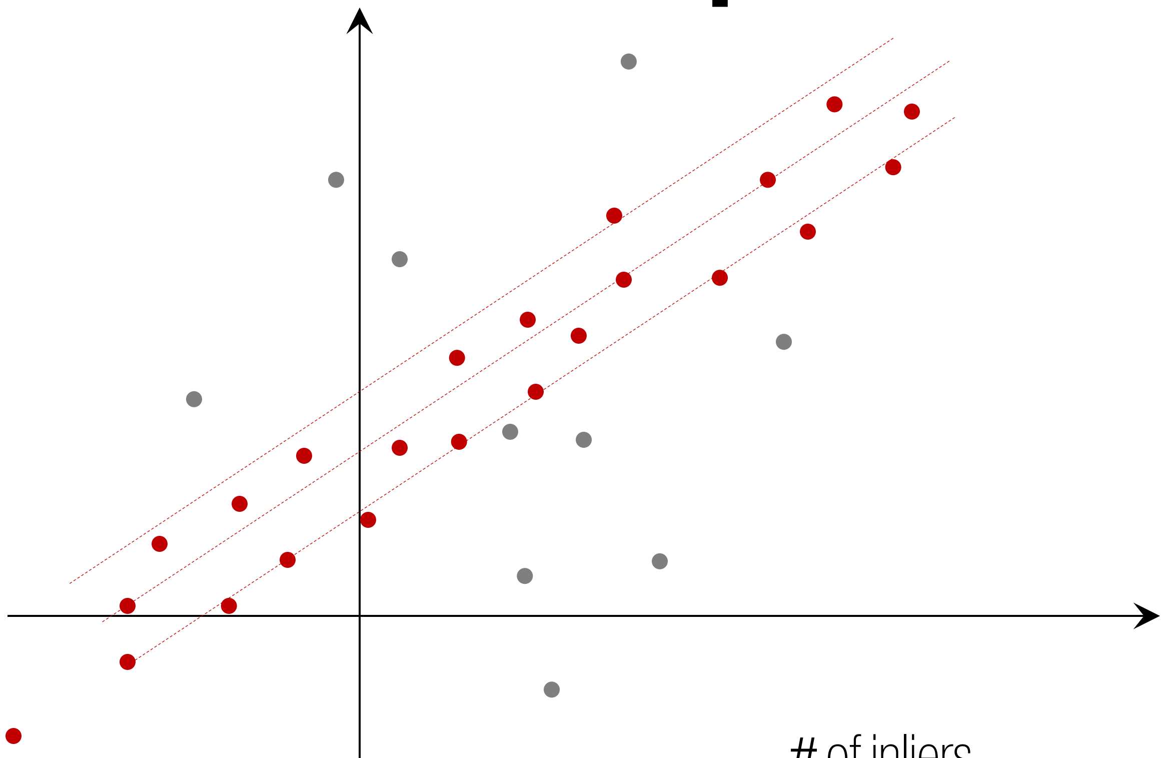
Inlier counting

# of inliers: 23

Maximum number of inliers

RANSAC: Random Sample Consensus



Random sampling

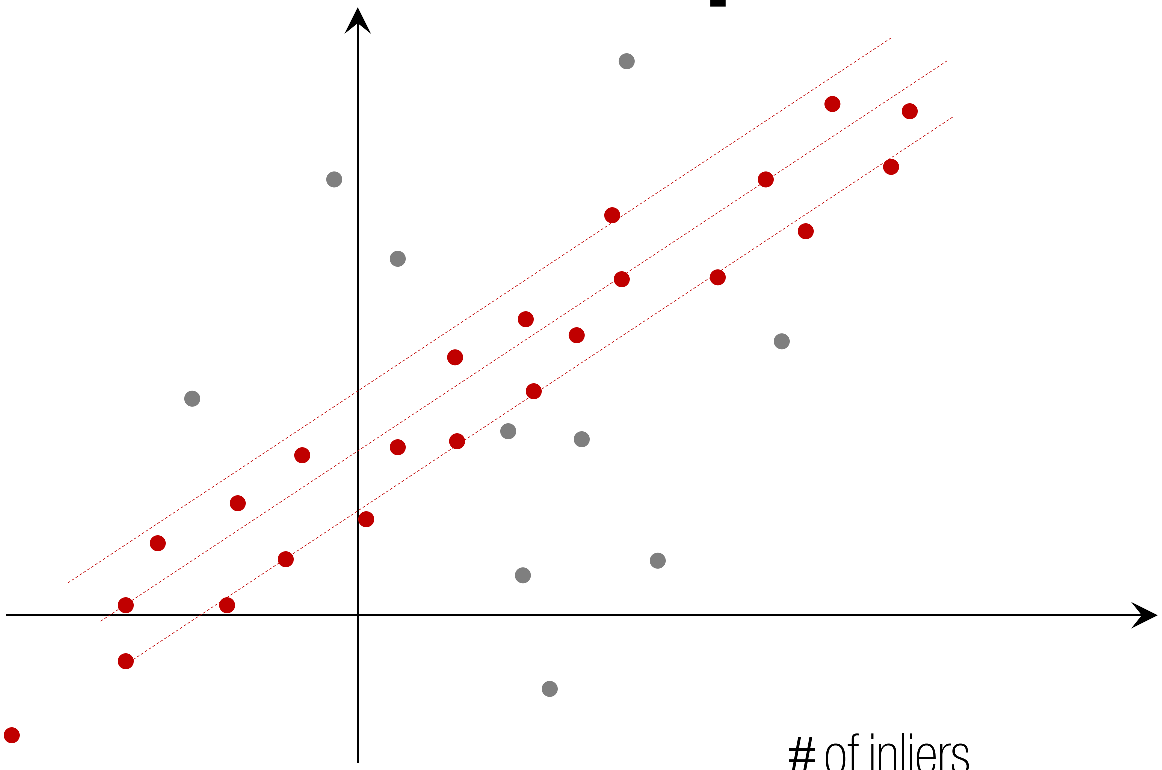
Model building

Thresholding

Inlier counting

Probability of choosing an inlier: $w = \frac{\#\text{ of inliers}}{\#\text{ of samples}}$

RANSAC: Random Sample Consensus



Random sampling

Model building

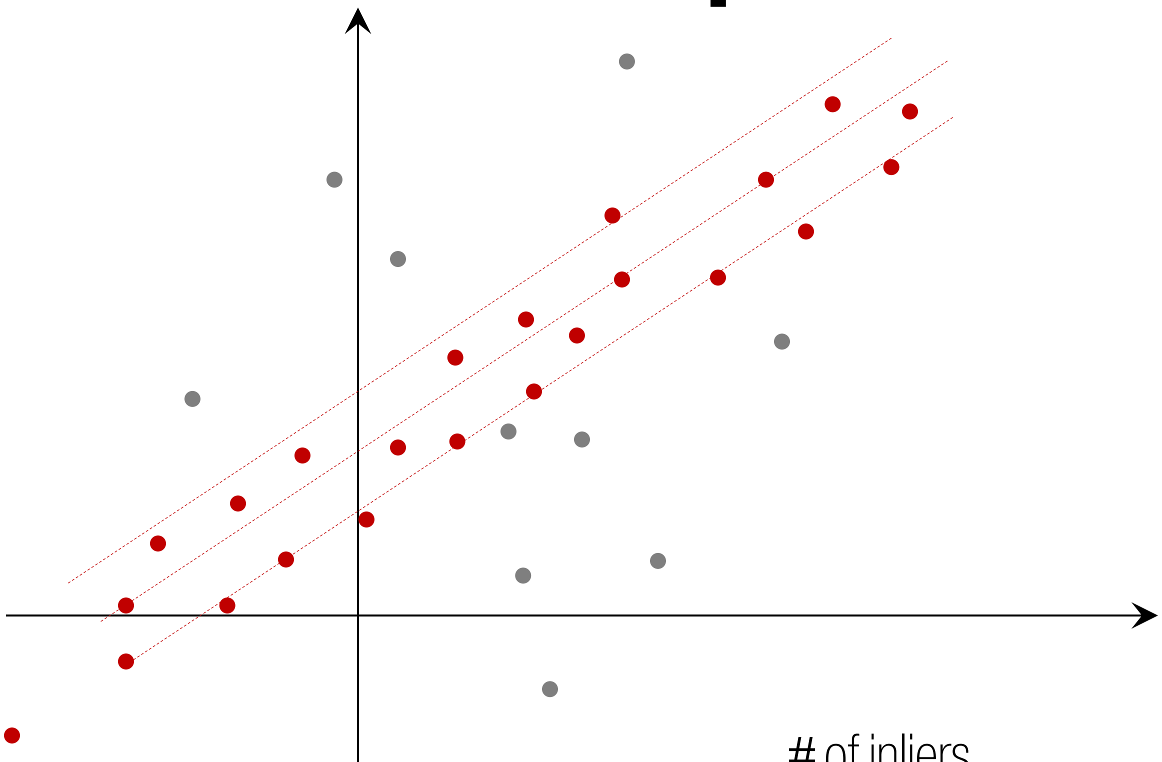
Thresholding

Inlier counting

Probability of choosing an inlier: $w = \frac{\#\text{ of inliers}}{\#\text{ of samples}}$

Probability of building a correct model: w^n where n is the number of samples to build a model.

RANSAC: Random Sample Consensus



Random sampling

Model building

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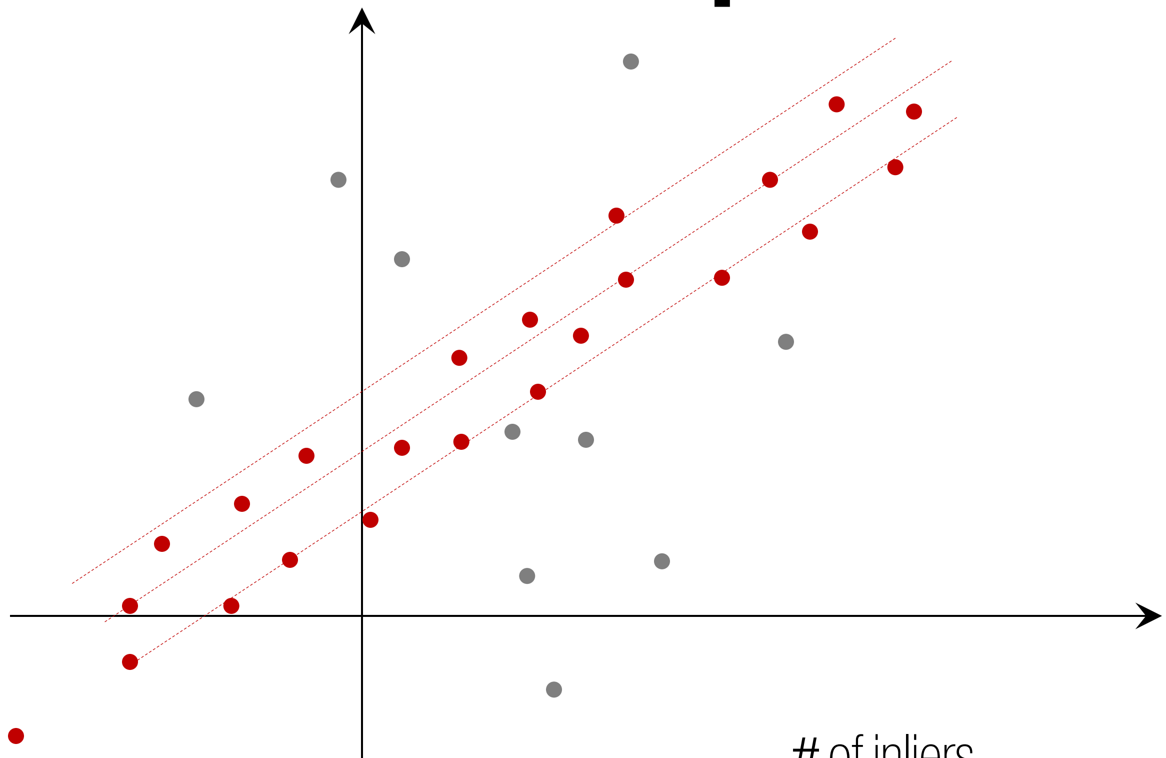
Probability of choosing an inlier: $w = \frac{\#\text{ of inliers}}{\#\text{ of samples}}$

Probability of building a correct model: w^n where n is the number of samples to build a model.

Probability of not building a correct model during *k* iterations: $\left(1 - w^n\right)^k$

RANSAC: Random Sample Consensus



Random sampling
Model building
Thresholding
Inlier counting

Probability of choosing an inlier: $w = \frac{\#\text{ of inliers}}{\#\text{ of samples}}$

Probability of building a correct model: w^n where n is the number of samples to build a model.

Probability of not building a correct model during k iterations: $\left(1-w^n\right)^k$

$\left(1-w^n\right)^k = 1-p$ where p is desired RANSAC success rate.

$$k = \frac{\log\left(1-p\right)}{\log\left(1-w^n\right)}$$