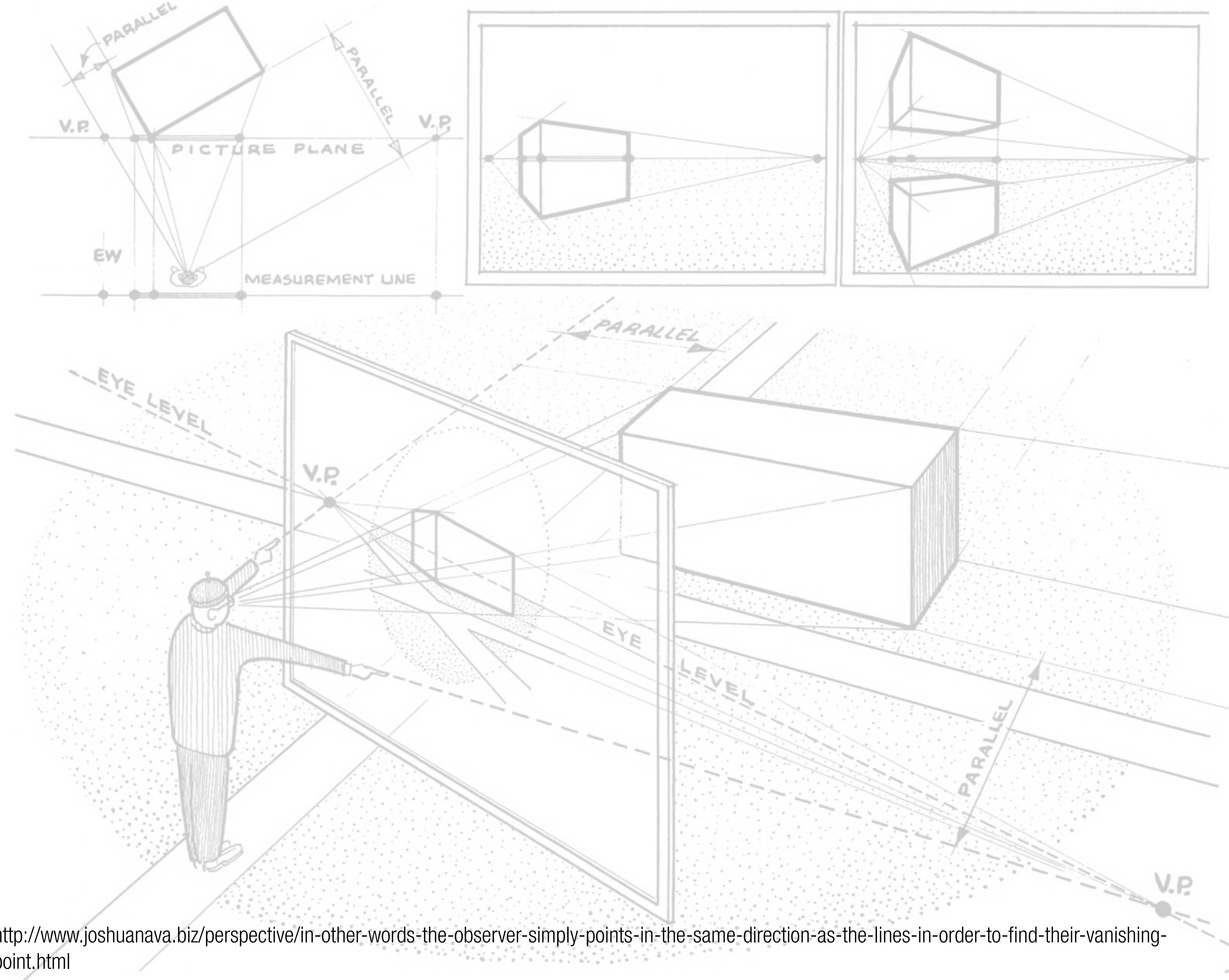
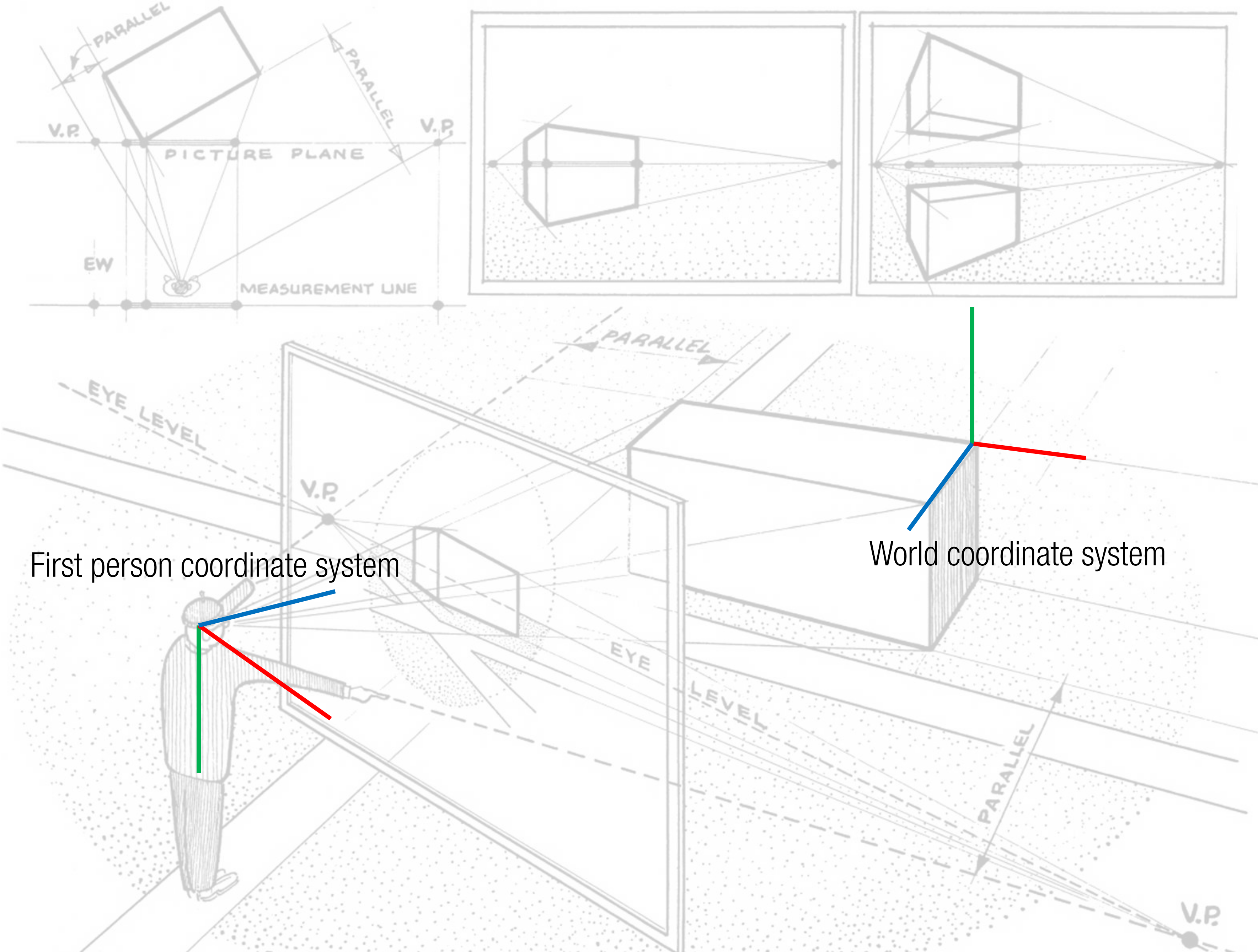
$$
z\begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & s & p_x \\ & f_x & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}
$$

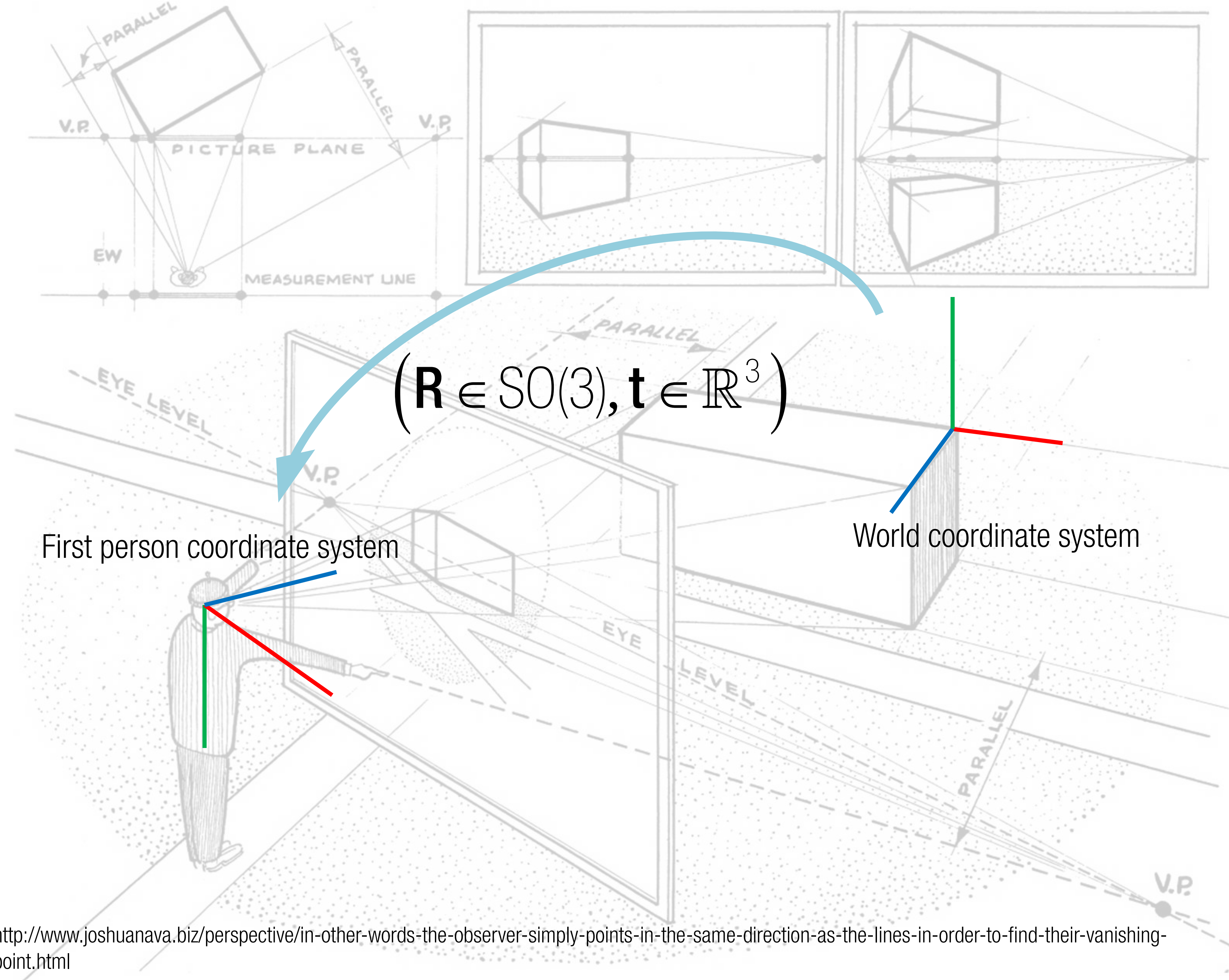
$\mathbf{x}$ $\mathbf{K}$ $\mathbf{R} \in \mathbb{R}^{3\times 3}$ $\mathbf{t}$ $\mathbf{X}$



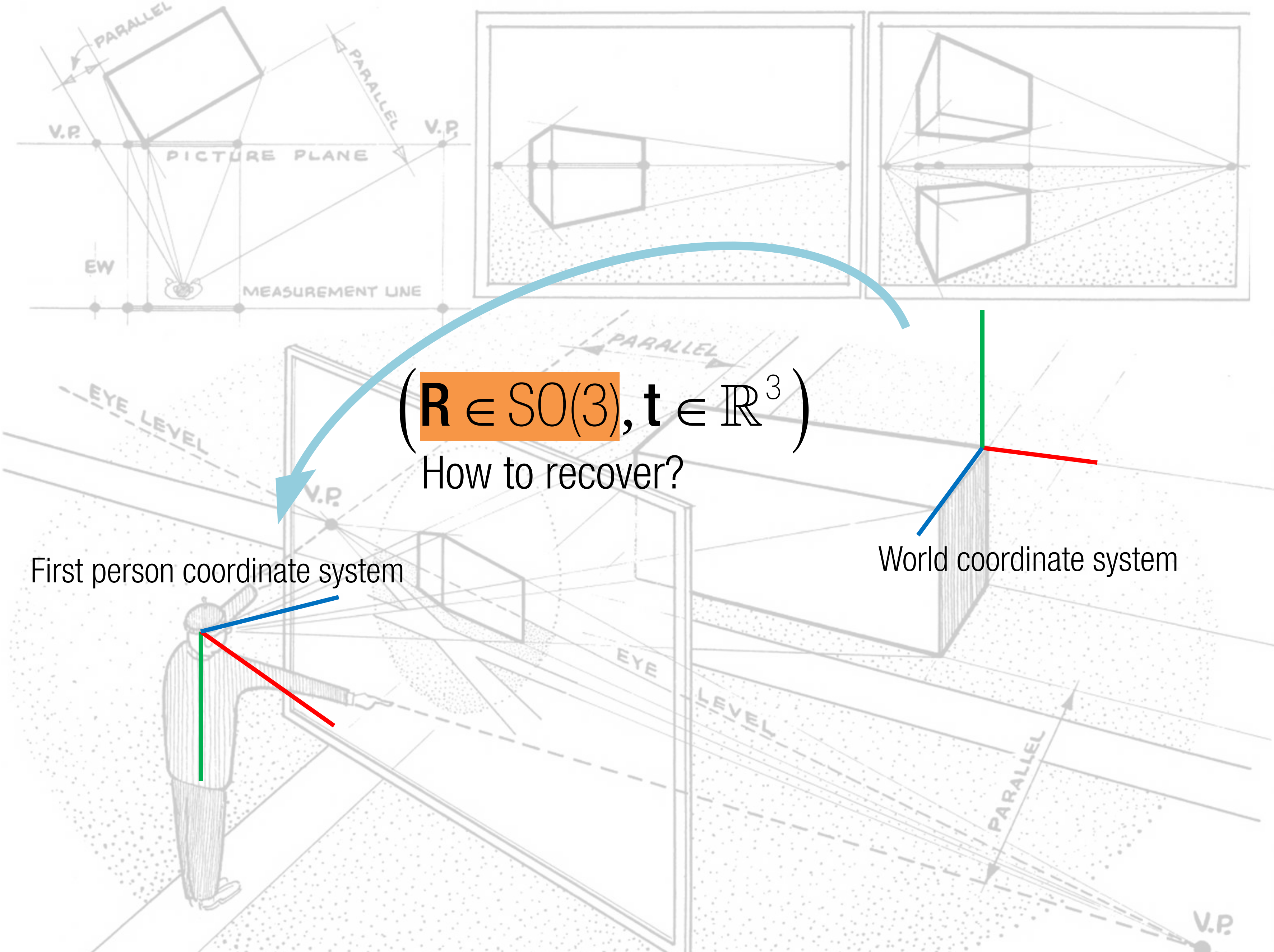
http://www.joshuanava.biz/perspective/in-other-words-the-observer-simply-points-in-the-same-direction-as-the-lines-in-order-to-find-their-vanishing-point.html



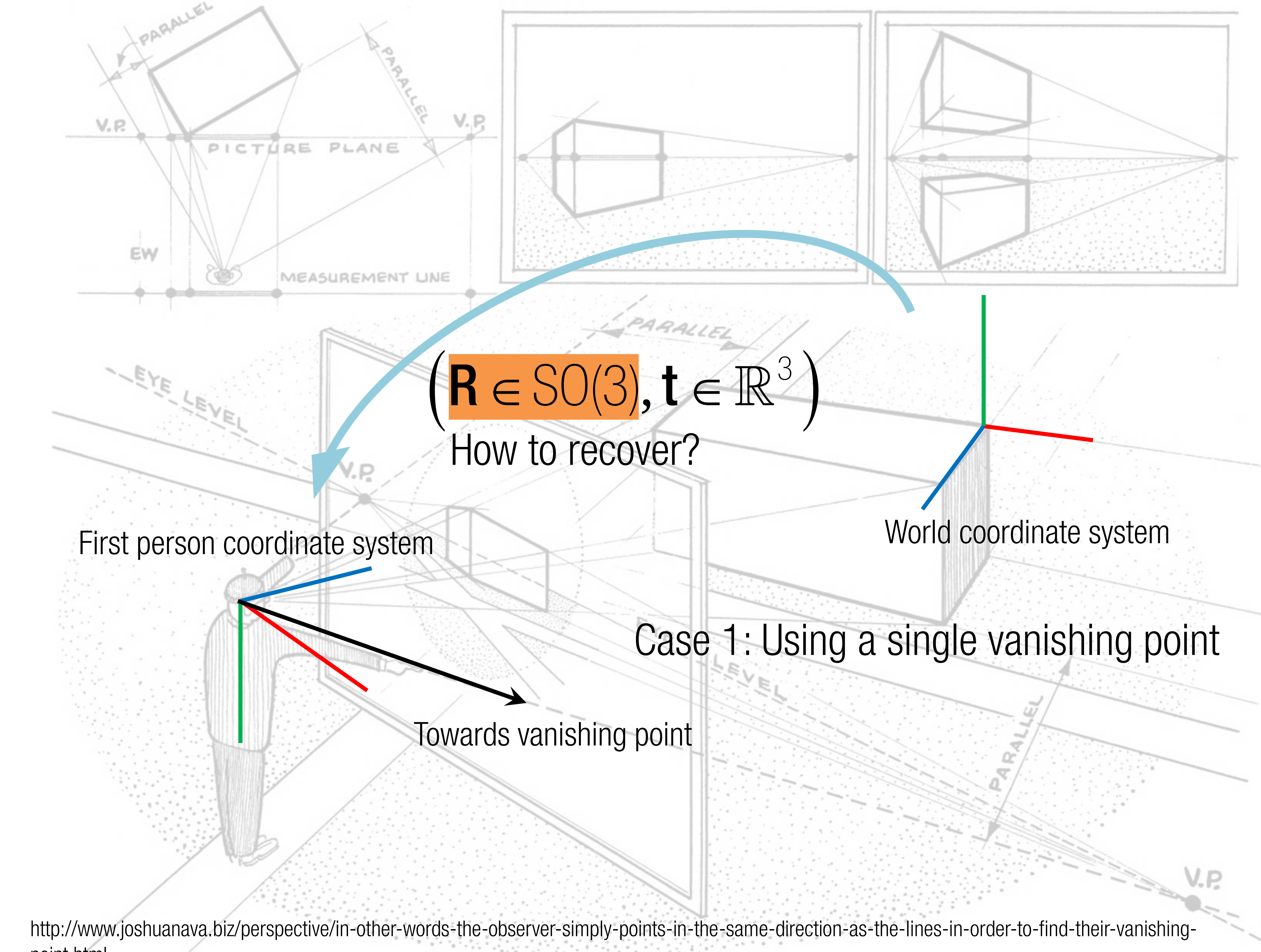
http://www.joshuanava.biz/perspective/in-other-words-the-observer-simply-points-in-the-same-direction-as-the-lines-in-order-to-find-their-vanishing-point.html



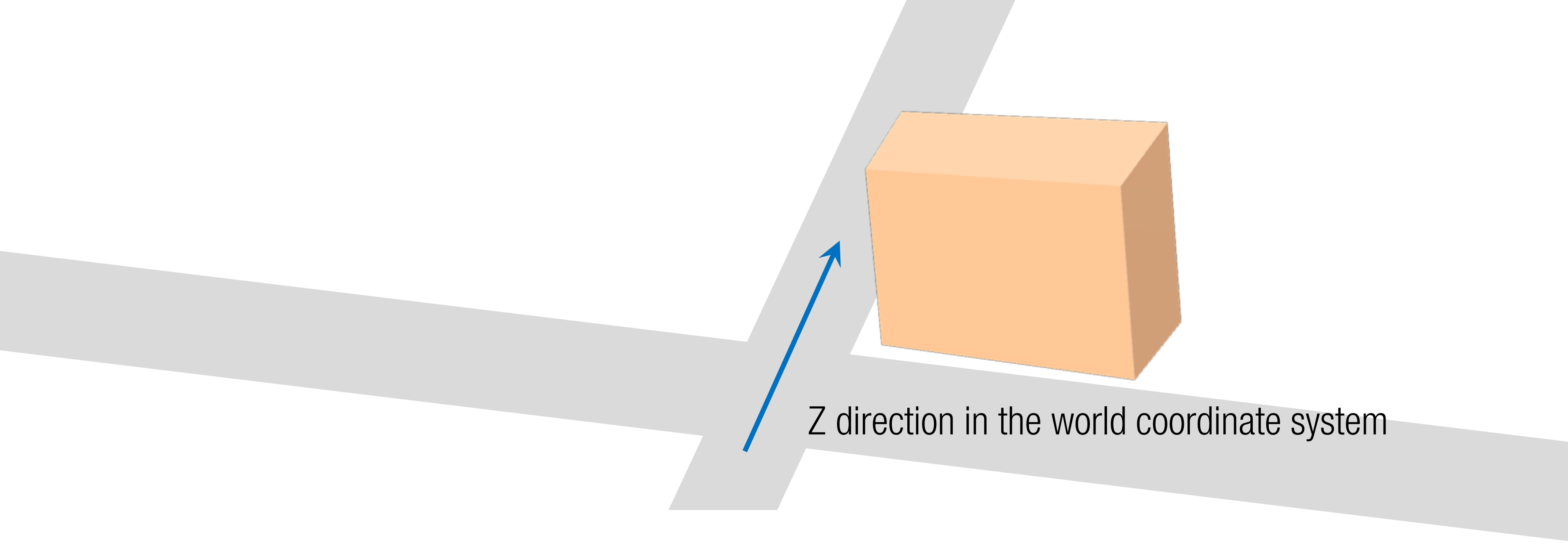
http://www.joshuanava.biz/perspective/in-other-words-the-observer-simply-points-in-the-same-direction-as-the-lines-in-order-to-find-their-vanishing-point.html

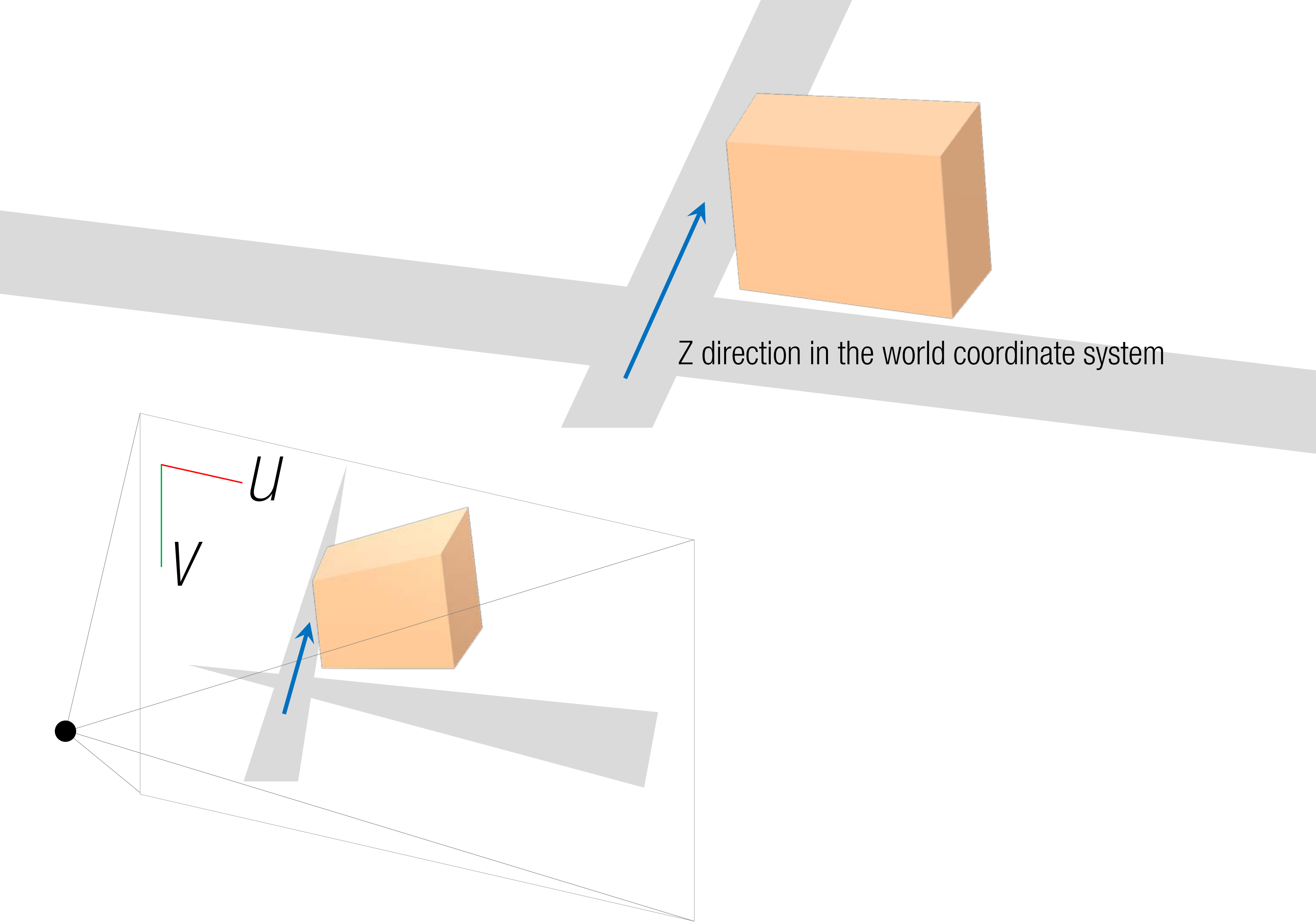


http://www.joshuanava.biz/perspective/in-other-words-the-observer-simply-points-in-the-same-direction-as-the-lines-in-order-to-find-their-vanishing-point.html

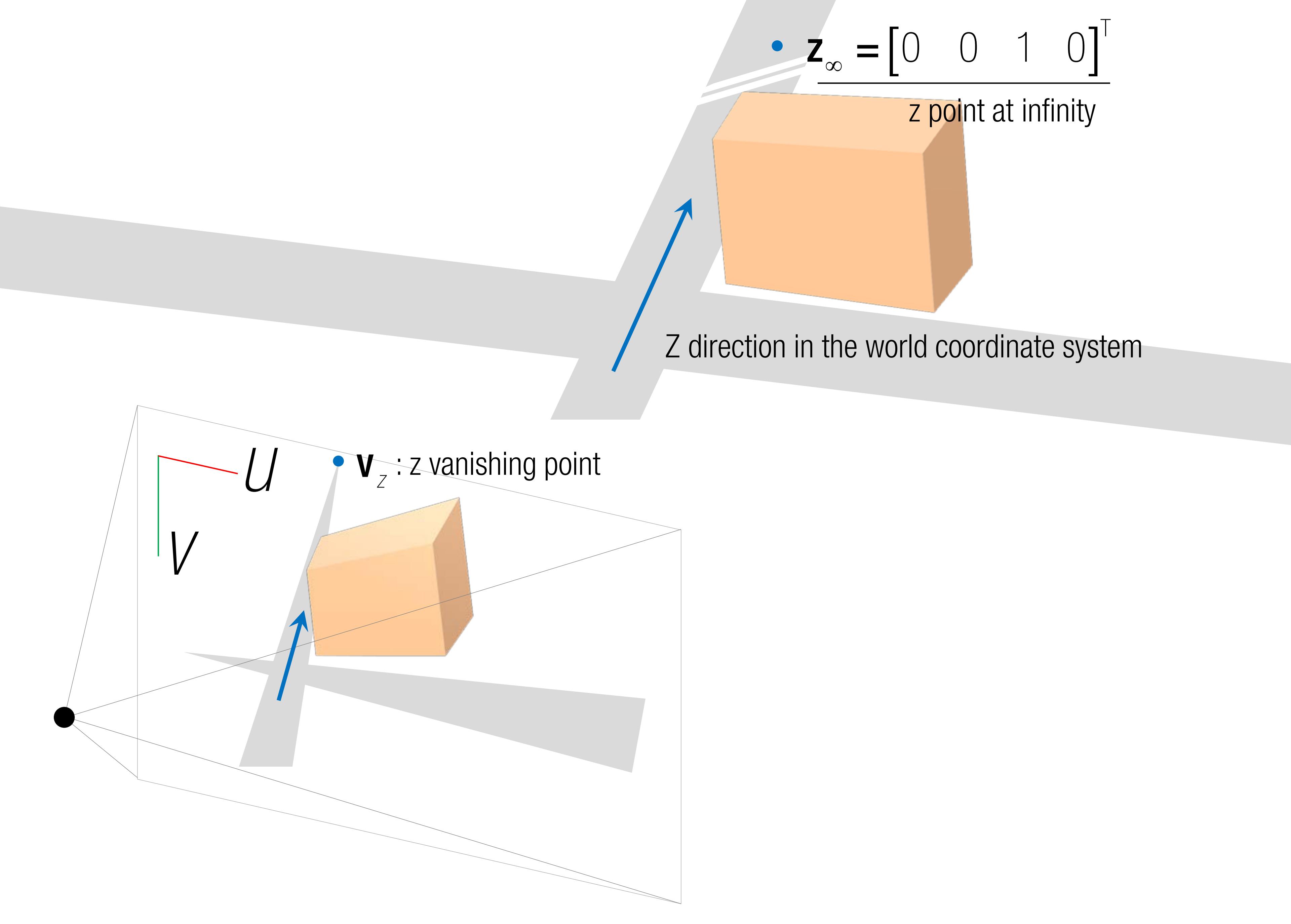


http://www.joshuanava.biz/perspective/in-other-words-the-observer-simply-points-in-the-same-direction-as-the-lines-in-order-to-find-their-vanishing-point.html

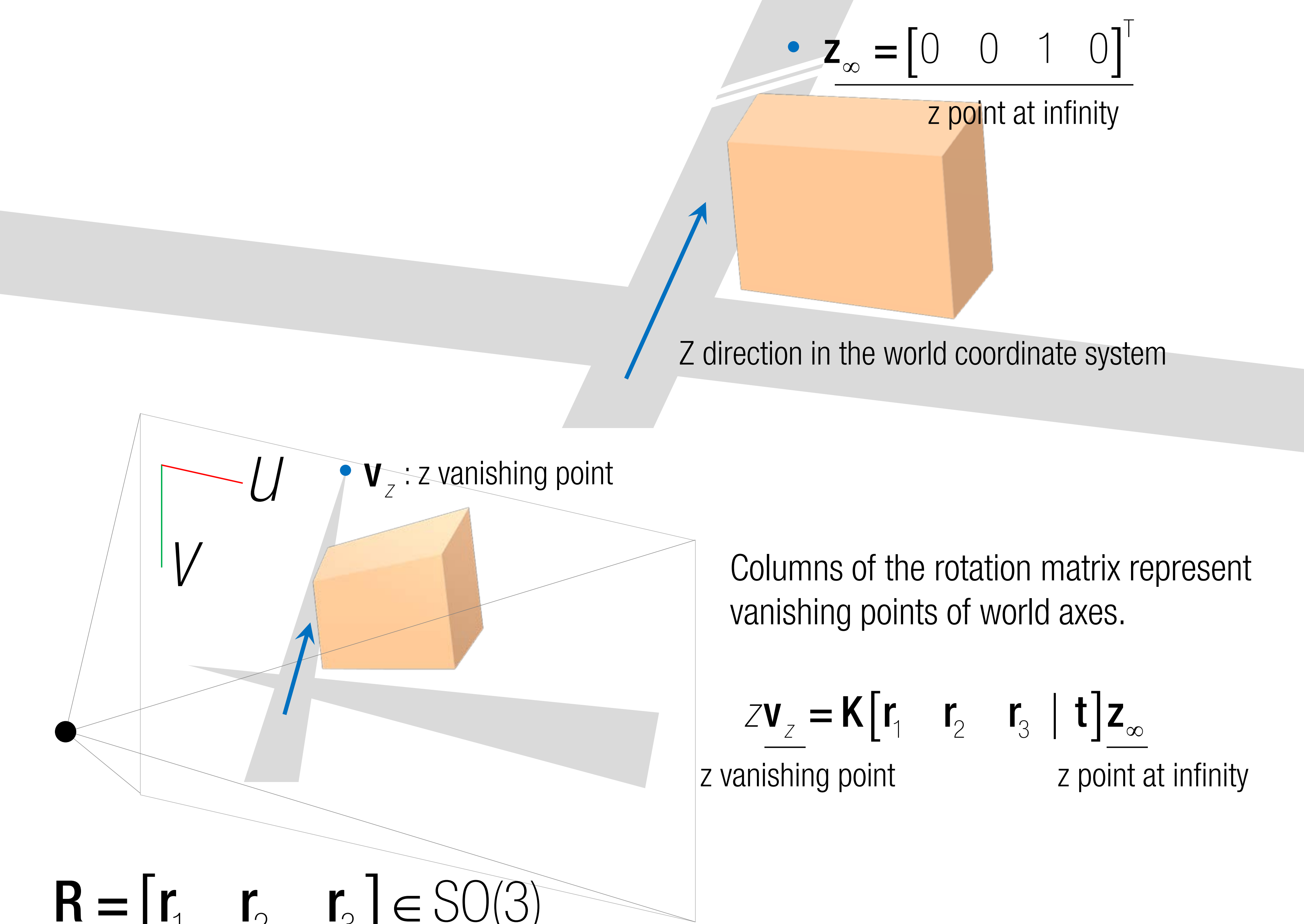

Z direction in the world coordinate system



Z direction in the world coordinate system
U
V



$\mathbf{z}_{\infty} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}^T$
z point at infinity
Z direction in the world coordinate system
U
V
$\mathbf{v}_Z$: z vanishing point

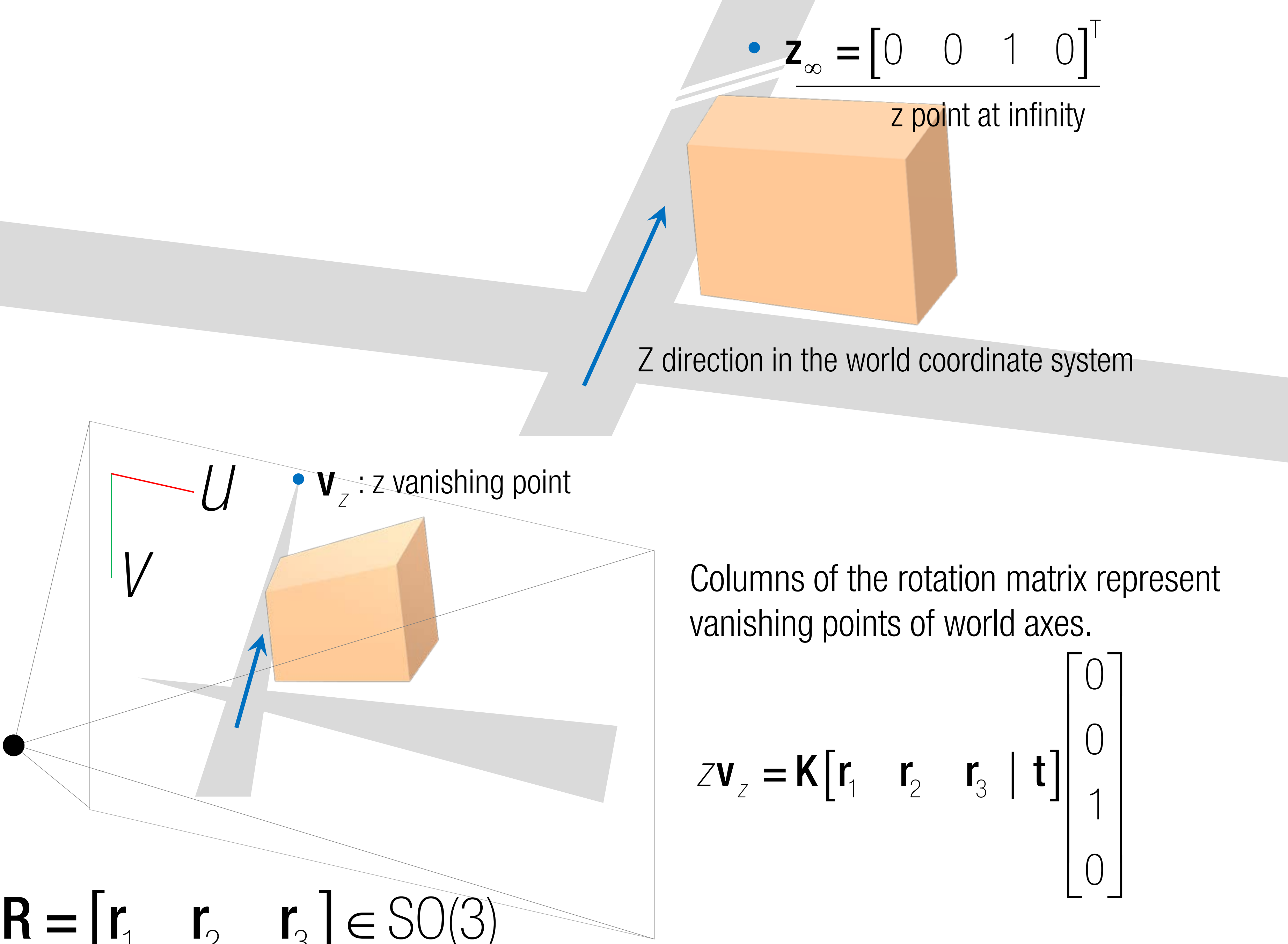


Columns of the rotation matrix represent vanishing points of world axes.

$$z\mathbf{v}_z = \mathbf{K}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 \mid \mathbf{t} \end{bmatrix}\mathbf{z}_\infty$$

z vanishing point — z point at infinity

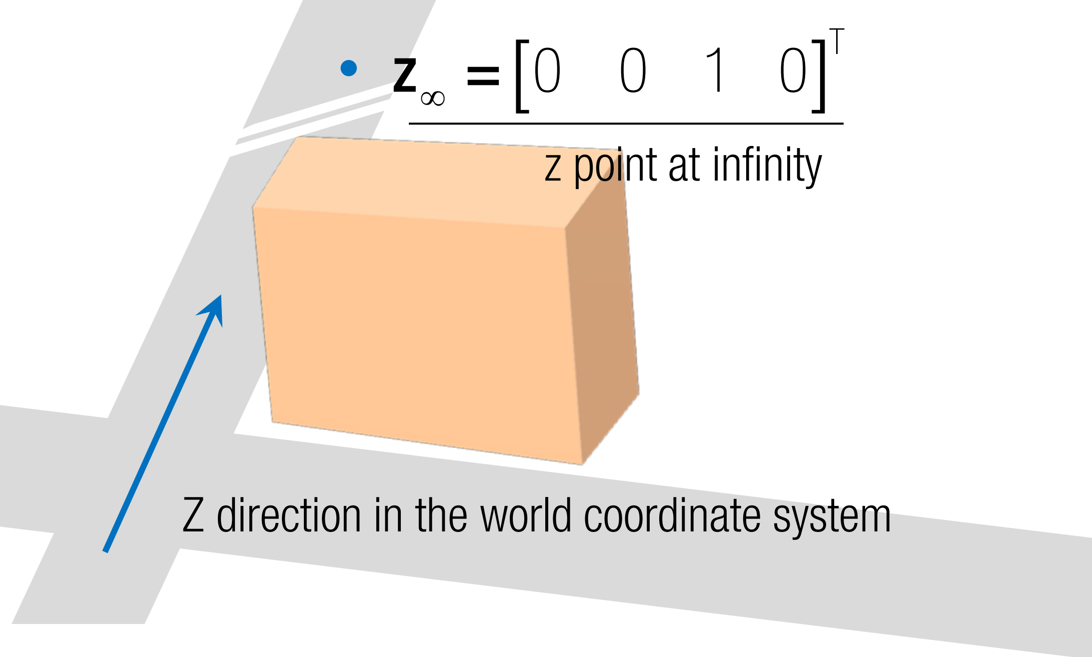
$$\mathbf{R} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 \end{bmatrix} \in SO(3)$$

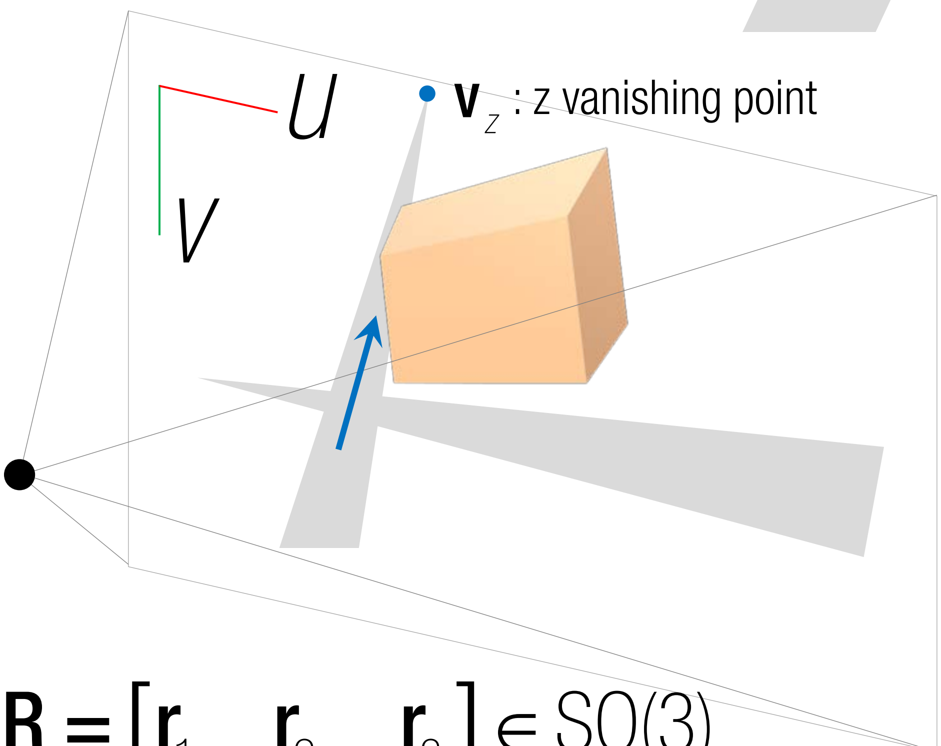


Columns of the rotation matrix represent vanishing points of world axes.

$$z\mathbf{v}_z = \mathbf{K}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & | & \mathbf{t} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 \end{bmatrix} \in SO(3)$$



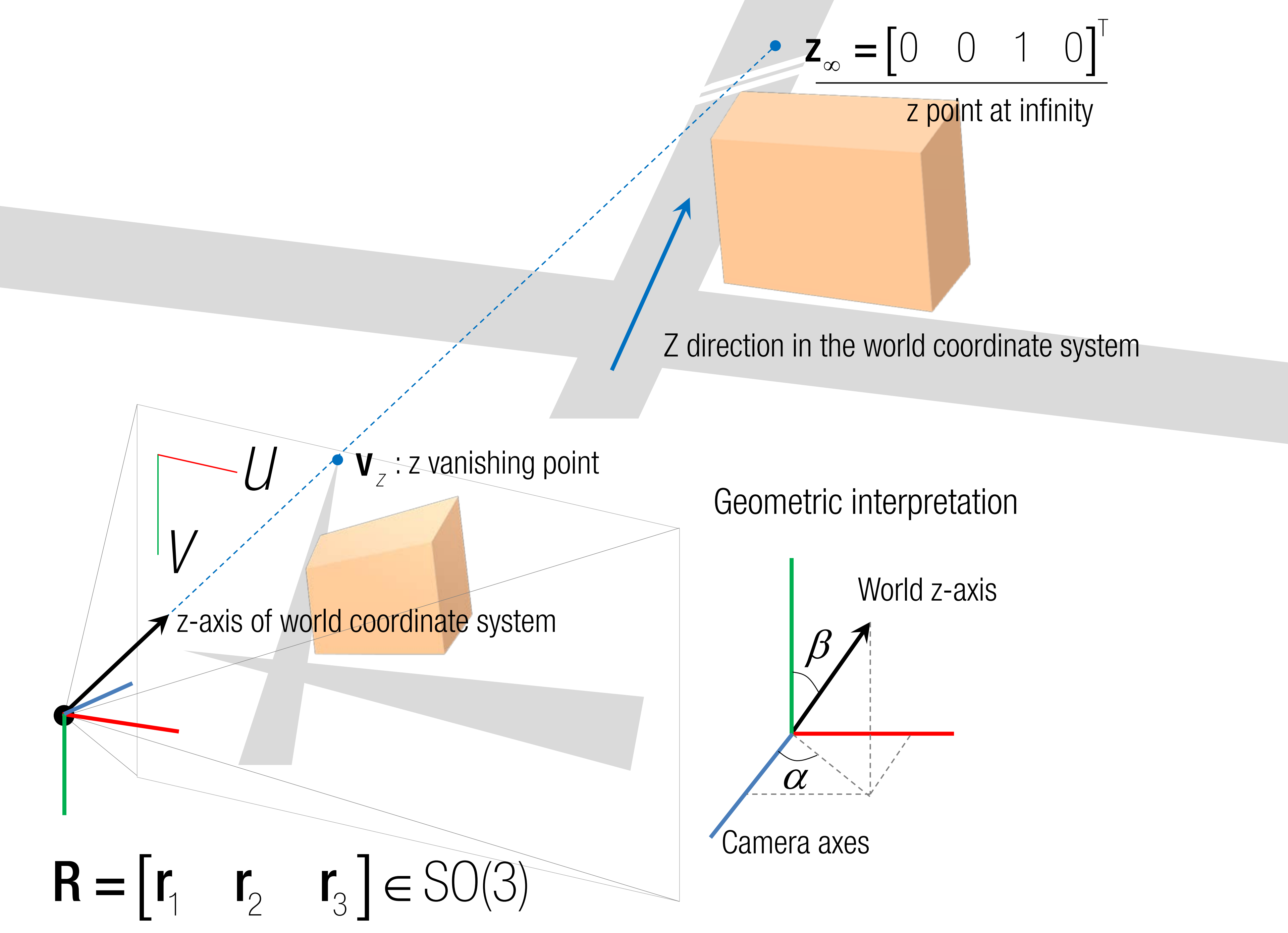


Columns of the rotation matrix represent vanishing points of world axes.

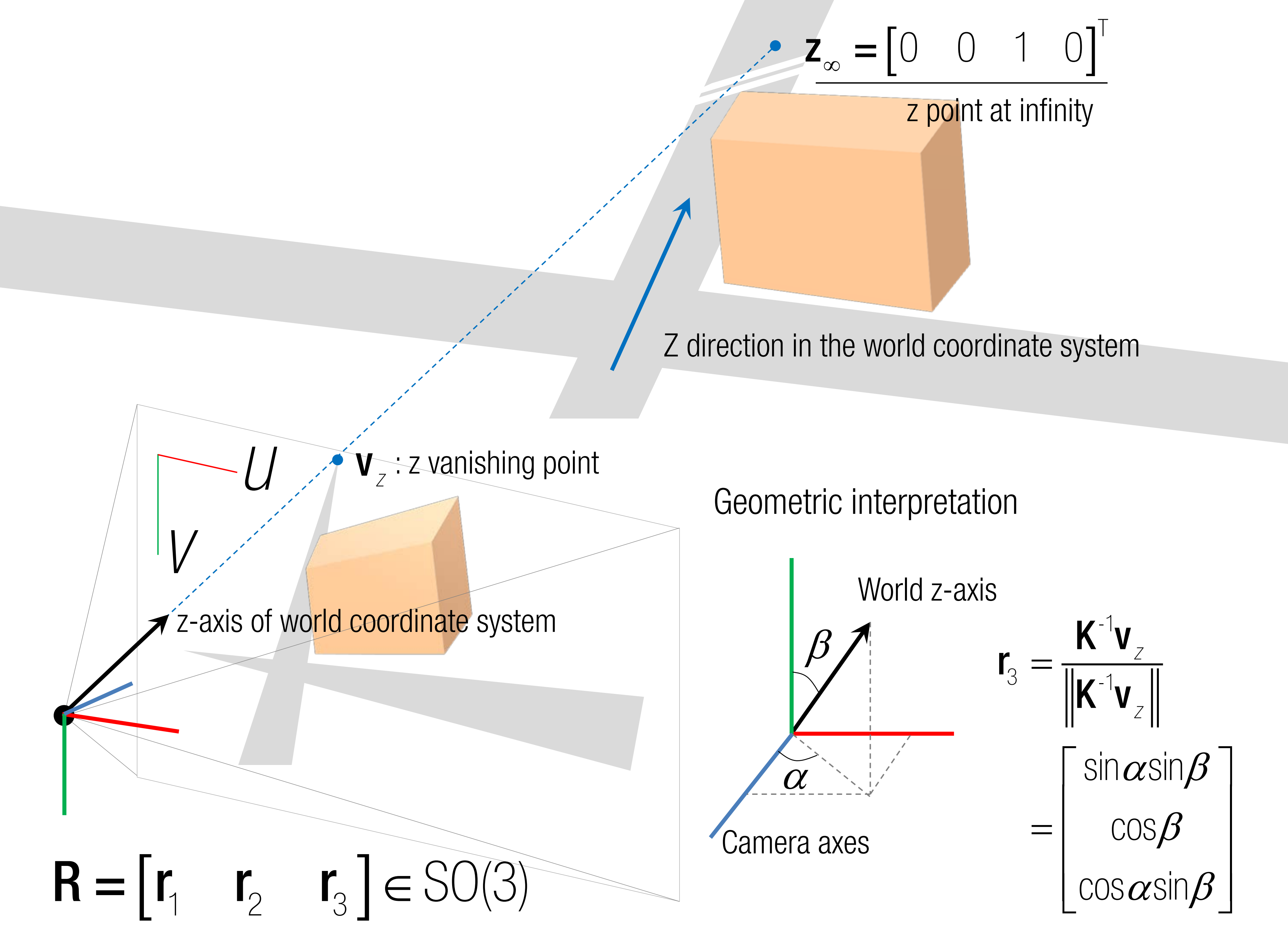
$$z\mathbf{v}_z = \mathbf{K}\mathbf{r}_3$$

$$\mathbf{r}_3 = \mathbf{K}^{-1}\mathbf{v}_z / \left\|\mathbf{K}^{-1}\mathbf{v}_z\right\|$$

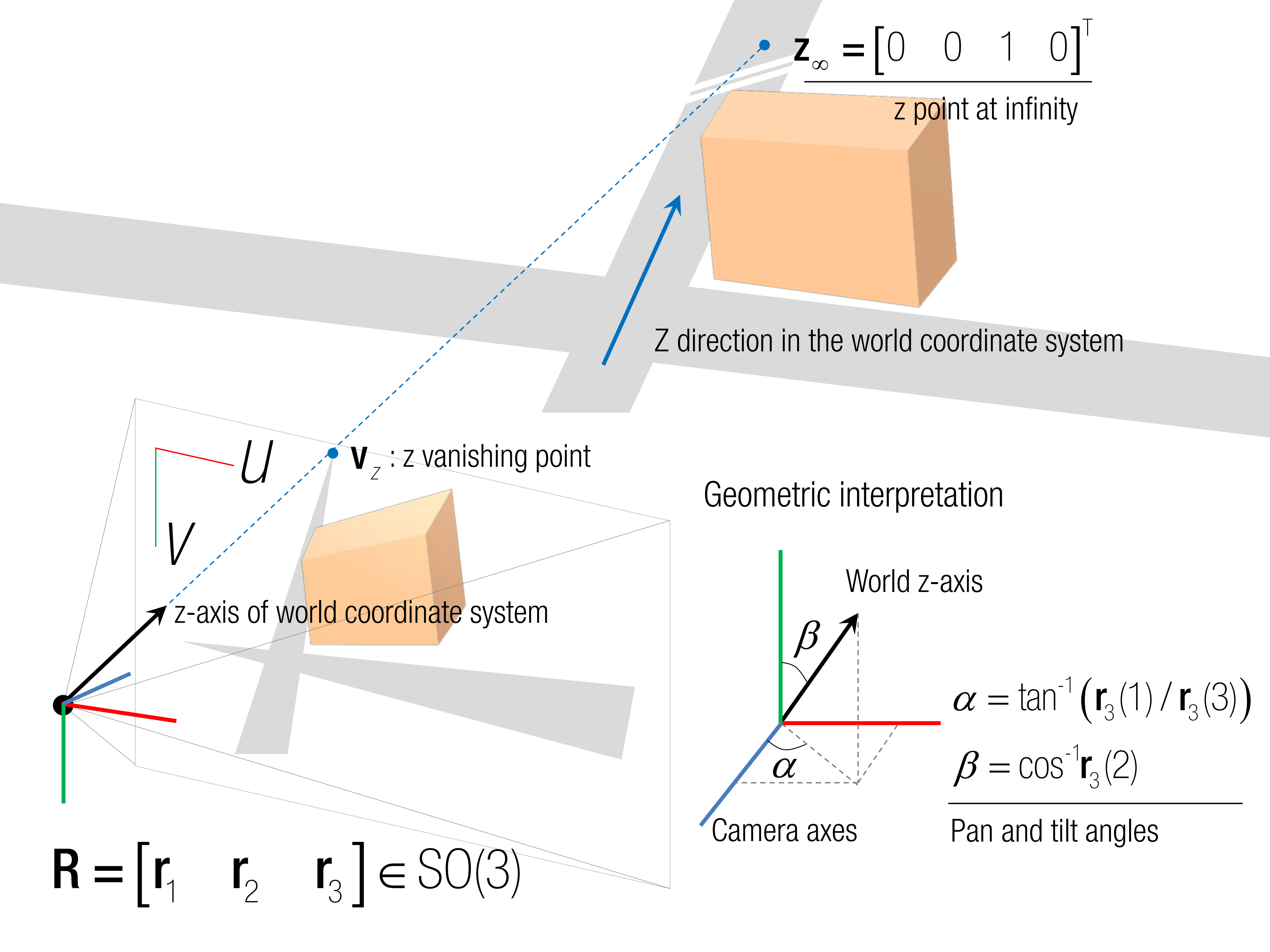
$$\mathbf{R} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 \end{bmatrix} \in SO(3)$$

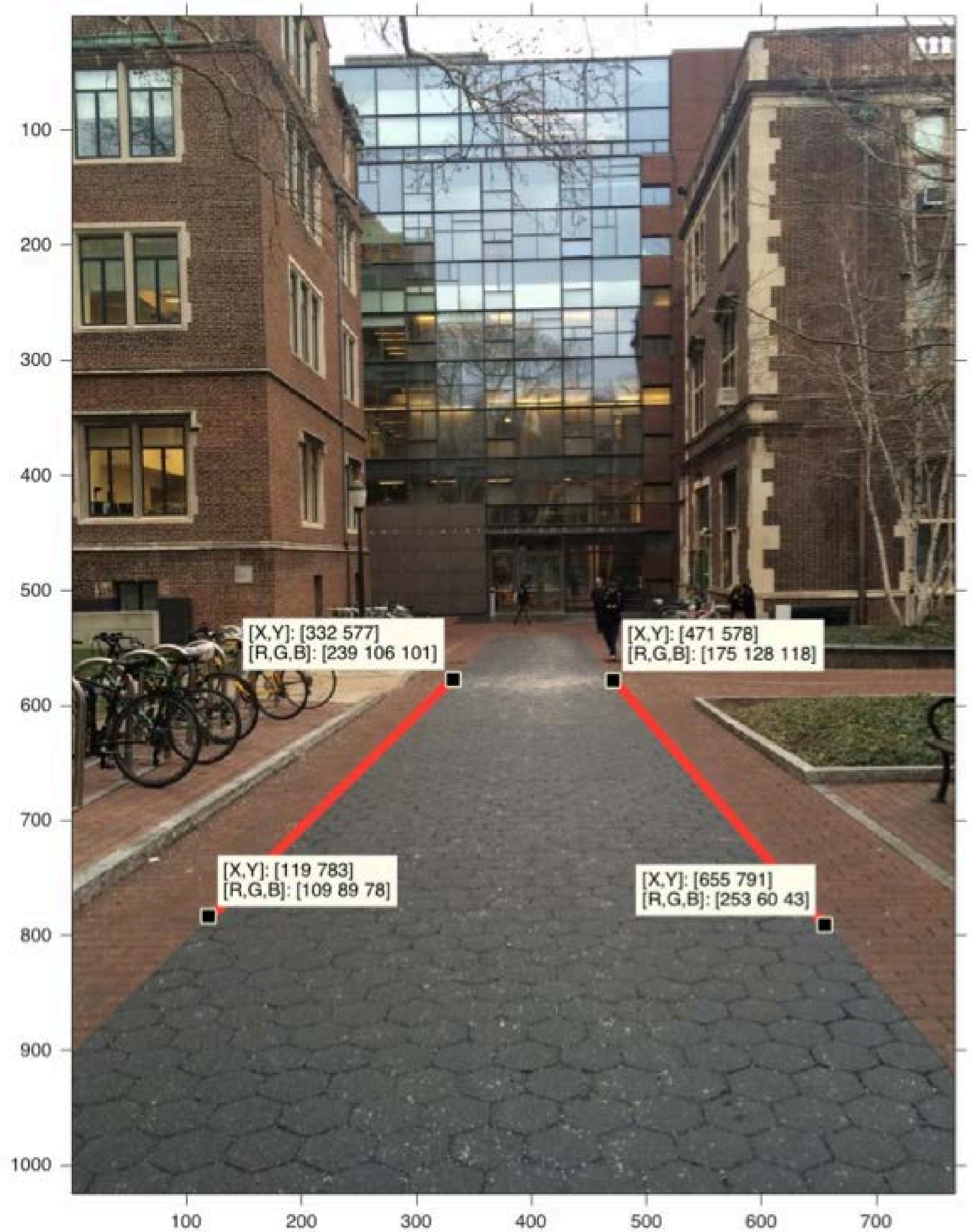


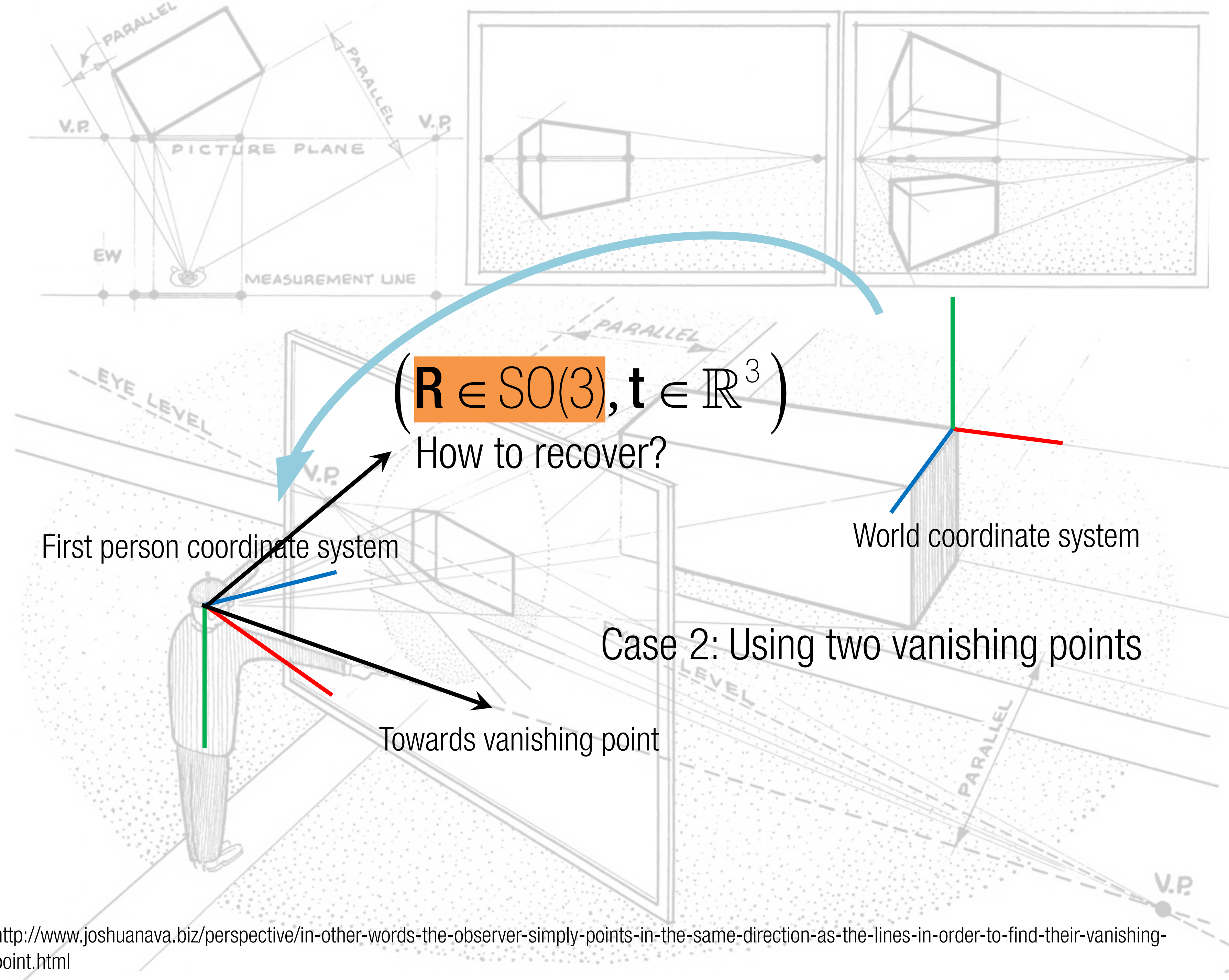
$\mathbf{z}_\infty = [0 \quad 0 \quad 1 \quad 0]^T$
z point at infinity
Z direction in the world coordinate system
U
V
$\mathbf{v}_z$: z vanishing point
z-axis of world coordinate system
Geometric interpretation
World z-axis
β
α
Camera axes
$\mathbf{R} = [\mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{r}_3] \in SO(3)$



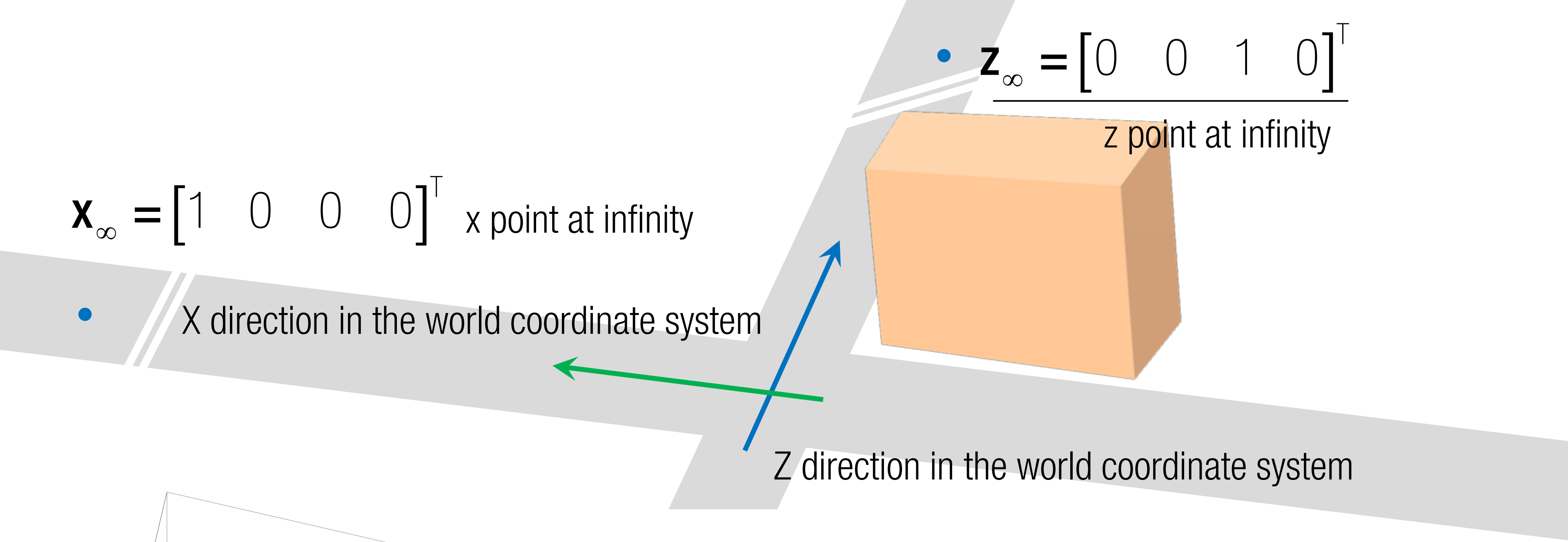
$\mathbf{Z}_\infty = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}^T$
z point at infinity
Z direction in the world coordinate system
U
V
$\mathbf{v}_Z$: z vanishing point
z-axis of world coordinate system
Geometric interpretation
World z-axis
β
α
Camera axes
$\mathbf{r}_3 = \dfrac{\mathbf{K}^{-1}\mathbf{v}_Z}{\left\|\mathbf{K}^{-1}\mathbf{v}_Z\right\|} = \begin{bmatrix} \sin\alpha\sin\beta \\ \cos\beta \\ \cos\alpha\sin\beta \end{bmatrix}$
$\mathbf{R} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 \end{bmatrix} \in SO(3)$


$\mathbf{z}_\infty = [0 \quad 0 \quad 1 \quad 0]^T$
z point at infinity
Z direction in the world coordinate system
U
V
$\mathbf{v}_z$: z vanishing point
z-axis of world coordinate system
Geometric interpretation
World z-axis
β
α
Camera axes
$\alpha = \tan^{-1}(\mathbf{r}_3(1) / \mathbf{r}_3(3))$
$\beta = \cos^{-1}\mathbf{r}_3(2)$
Pan and tilt angles
$\mathbf{R} = [\mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{r}_3] \in SO(3)$


[X,Y]: [332 577]
[R,G,B]: [239 106 101]
[X,Y]: [471 578]
[R,G,B]: [175 128 118]
[X,Y]: [119 783]
[R,G,B]: [109 89 78]
[X,Y]: [655 791]
[R,G,B]: [253 60 43]
100
200
300
400
500
600
700
800
900
1000
100
200
300
400
500
600
700

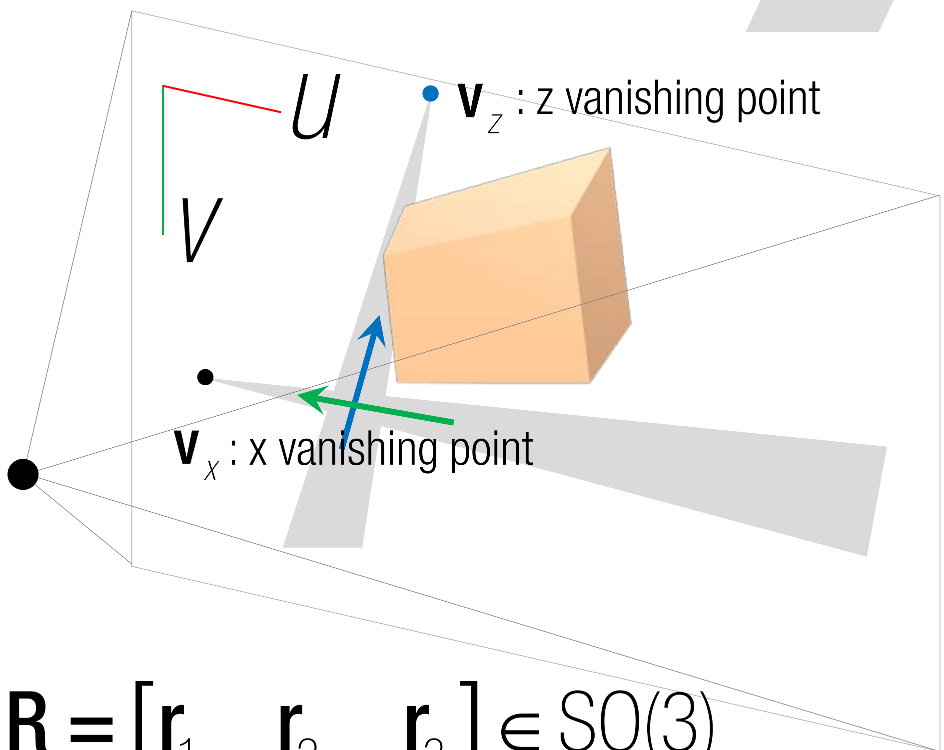


http://www.joshuanava.biz/perspective/in-other-words-the-observer-simply-points-in-the-same-direction-as-the-lines-in-order-to-find-their-vanishing-point.html



$$\mathbf{X}_\infty = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T$$

$$\mathbf{Z}_\infty = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}^T$$



Columns of the rotation matrix represent vanishing points of world axes.

$$\mathbf{r}_3 = \mathbf{K}^{-1} Z \mathbf{v}_Z$$

$$\mathbf{r}_1 = \mathbf{K}^{-1} Z \mathbf{v}_X$$

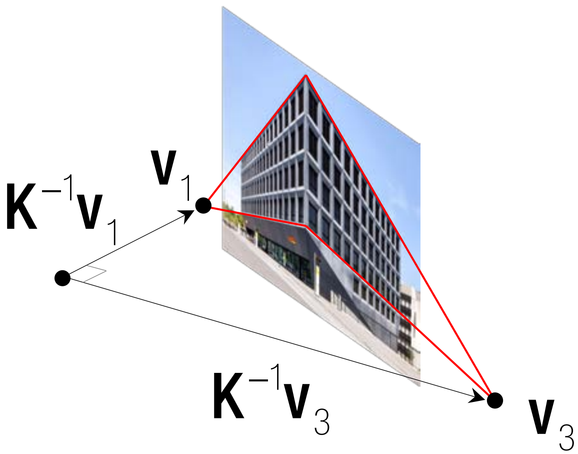
$$\mathbf{r}_2 = \mathbf{r}_3 \times \mathbf{r}_1$$

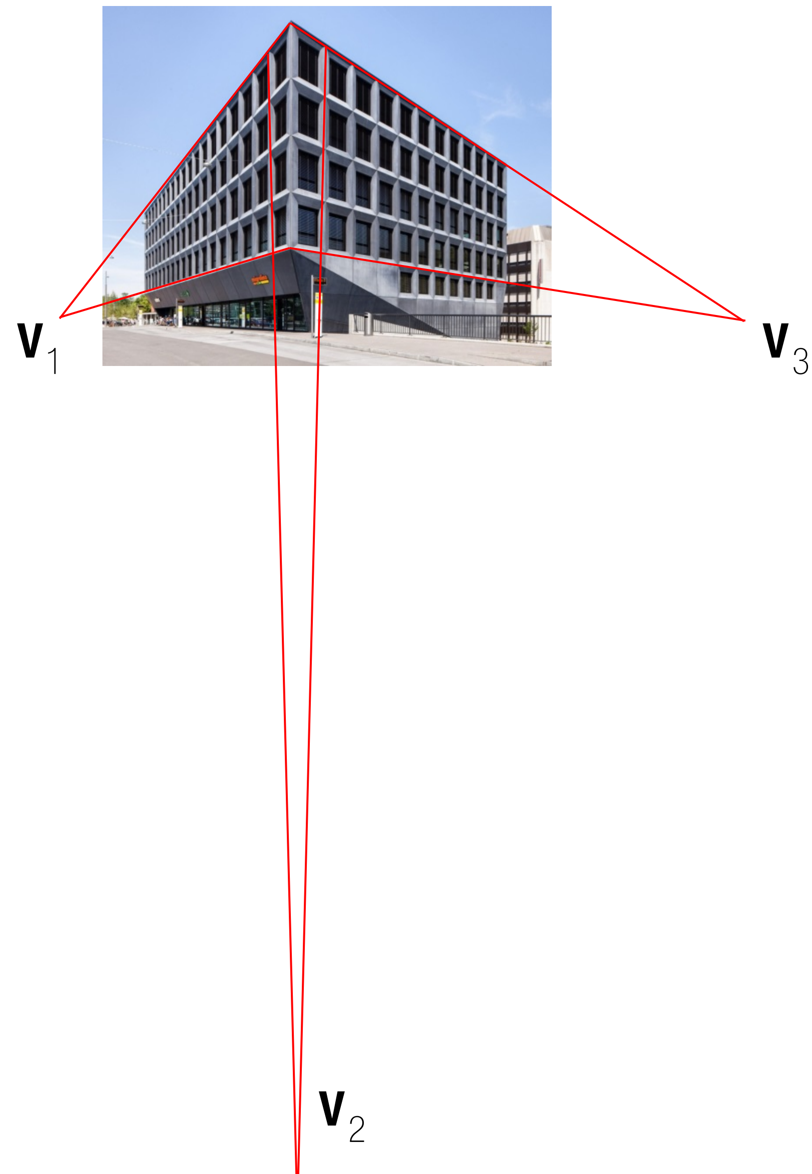
Orthogonal rotation matrix

$$\mathbf{R} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 \end{bmatrix} \in SO(3)$$


migrolino


migrolino





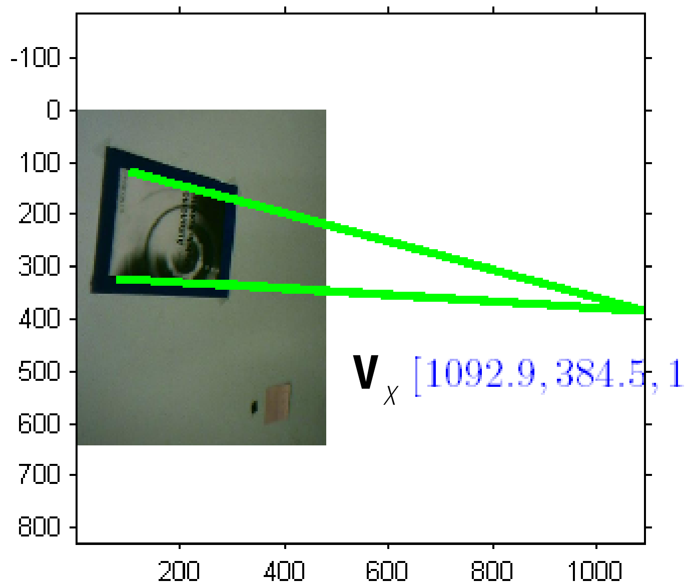
$$\mathbf{r}_3 = \mathbf{K}^{-1} Z \mathbf{v}_Z$$

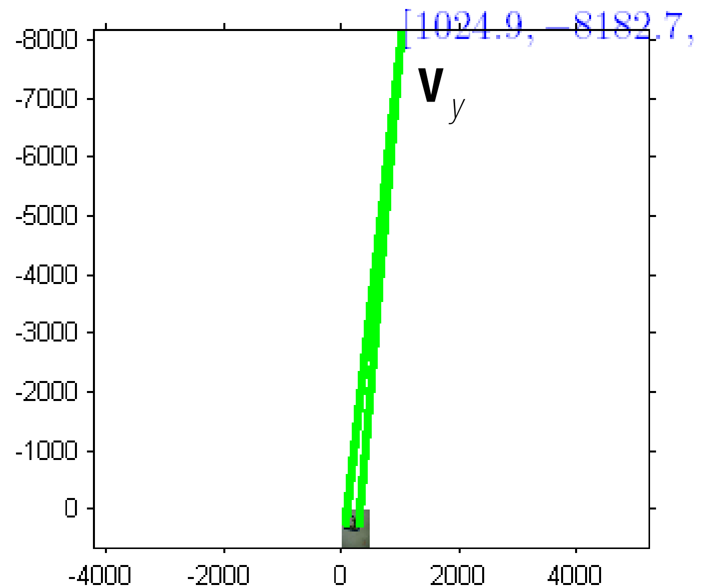
$$\mathbf{r}_1 = \mathbf{K}^{-1} Z \mathbf{v}_X$$

$$\mathbf{r}_2 = \mathbf{r}_3 \times \mathbf{r}_1$$

$$\mathbf{R} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 \end{bmatrix}$$

Exercise I



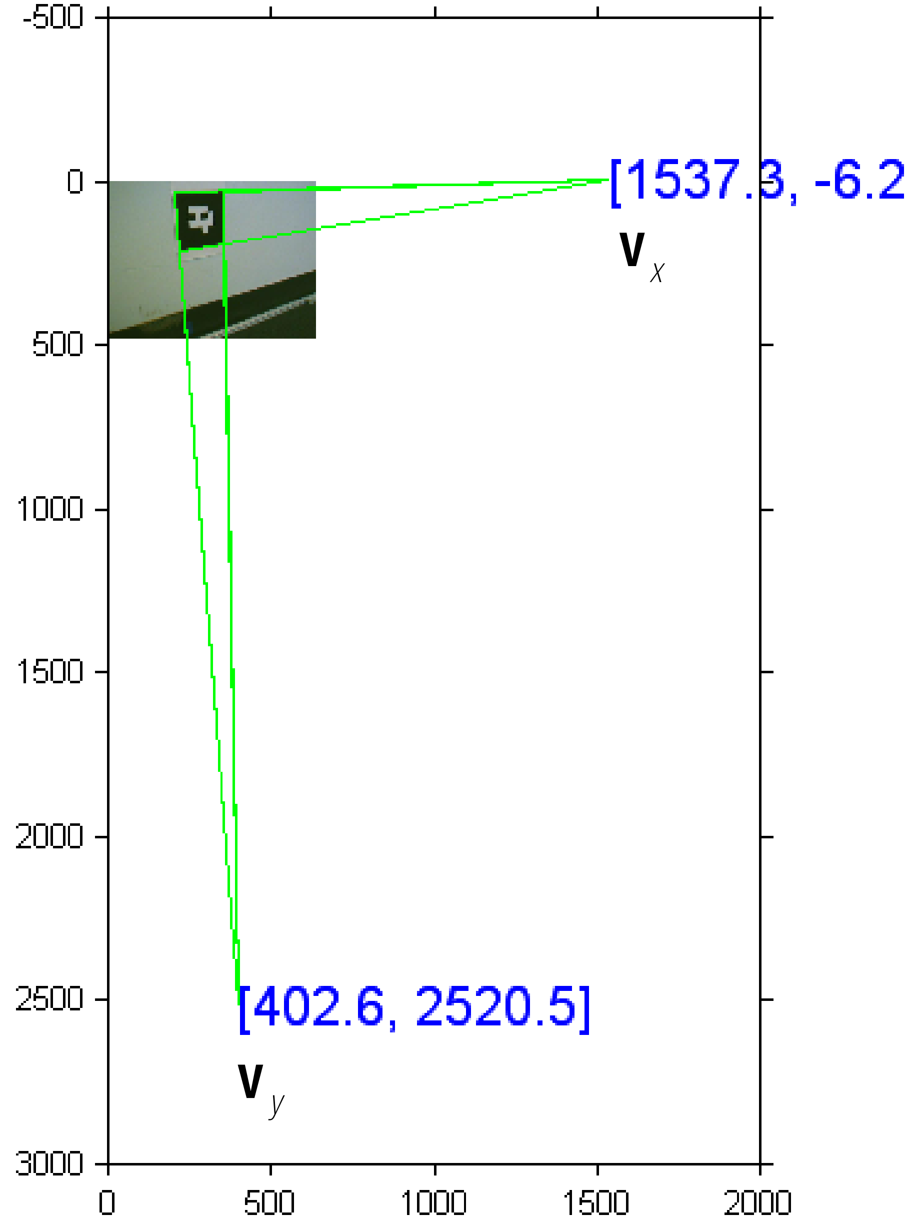


$$\mathbf{r}_1 = \mathbf{K}^{-1} Z \mathbf{v}_x$$

$$\mathbf{r}_2 = \mathbf{K}^{-1} Z \mathbf{v}_y$$

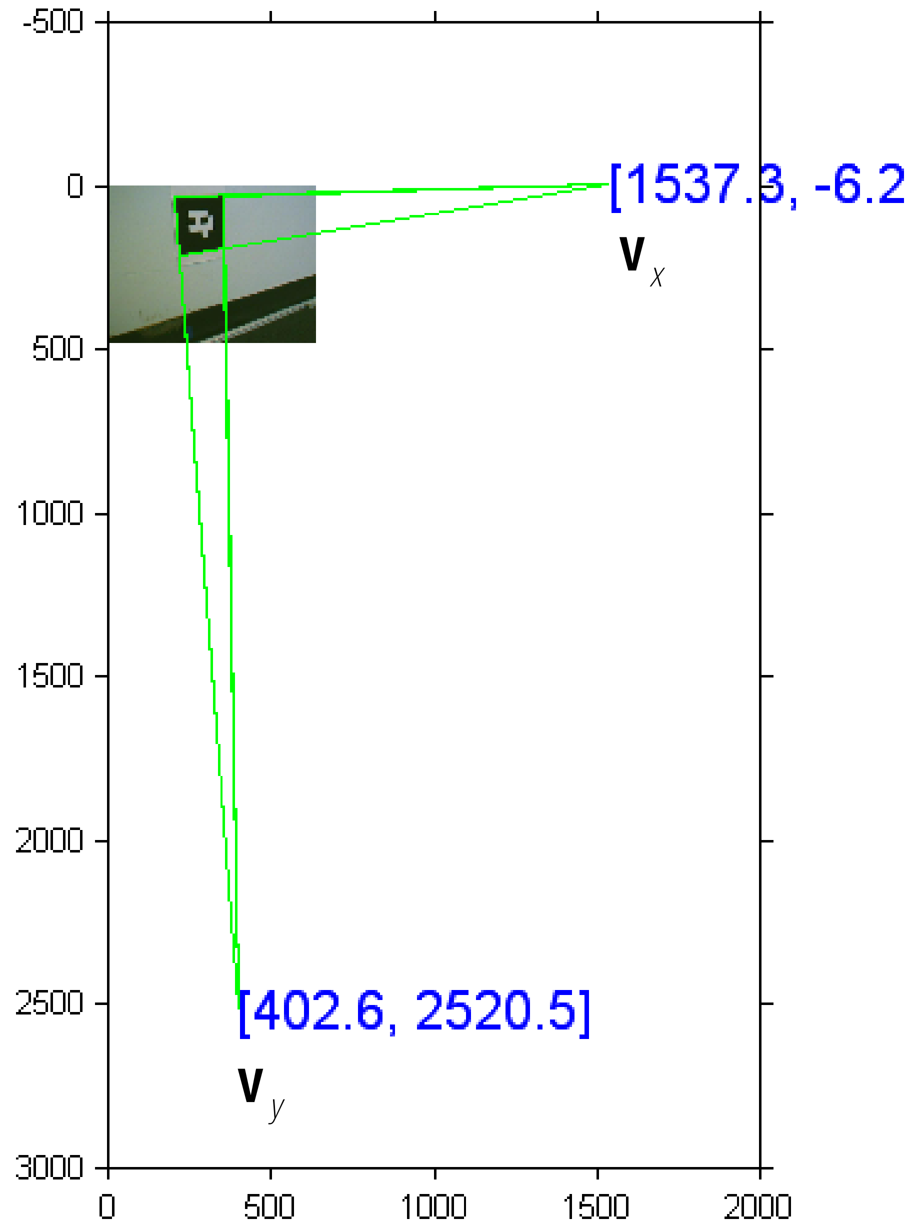
$$\mathbf{R} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_1 \times \mathbf{r}_2 \end{bmatrix}$$

Exercise II



-500
0
500
1000
1500
2000
2500
3000
0
500
1000
1500
2000
[1537.3, -6.2]
$\mathbf{v}_x$
[402.6, 2520.5]
$\mathbf{v}_y$

Exercise II



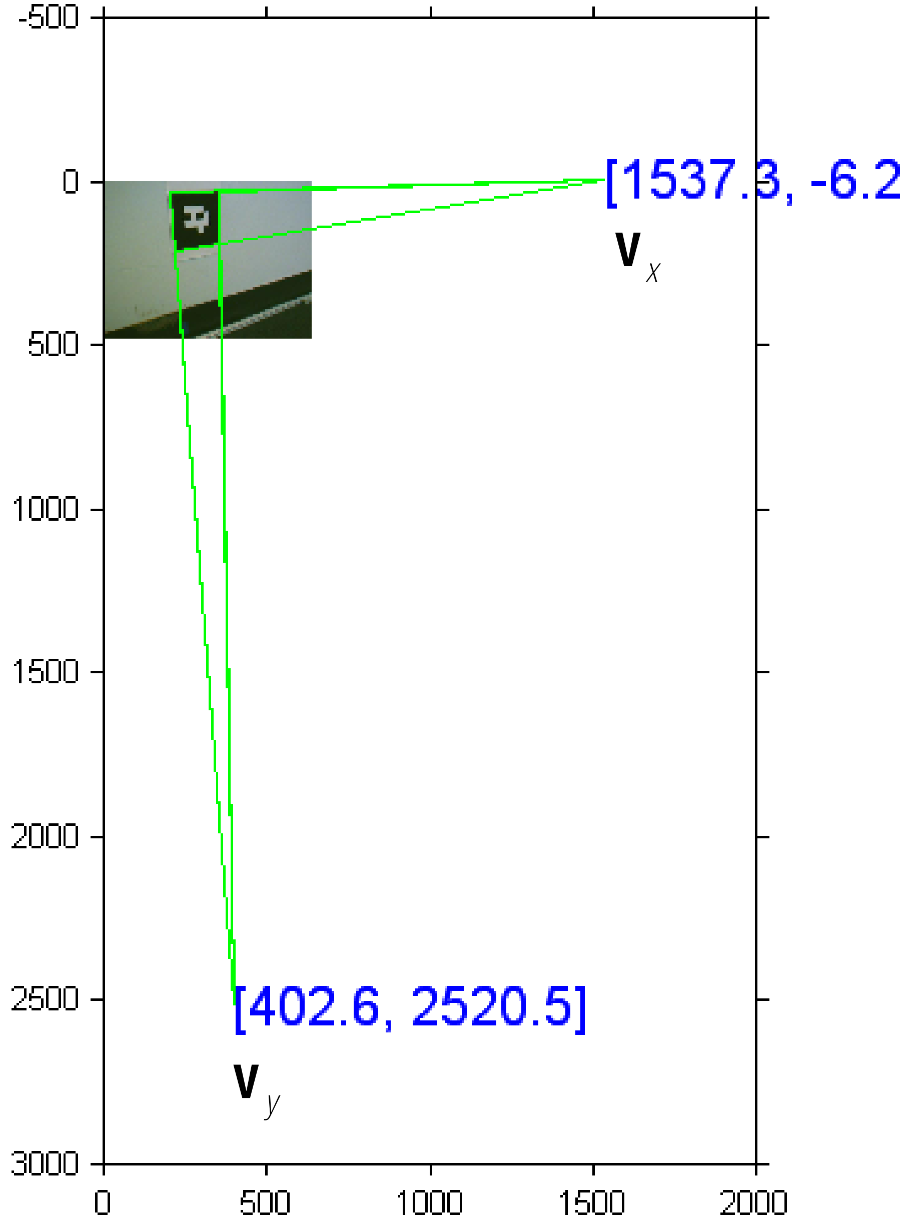
$$\mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{v}_x \,/\, \left\|\mathbf{K}^{-1}\mathbf{v}_x\right\|$$

$$\mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{v}_y \,/\, \left\|\mathbf{K}^{-1}\mathbf{v}_y\right\|$$

Scale normalization

Exercise II

$$r_1 = (0.8017, -0.2086, 0.5602)^T$$
$$r_2 = (0.0067, 0.9411, 0.3382)^T$$
$$r_3 = r_1 \times r_2 = (-0.5988, -0.2673, 0.7558)^T$$



Exercise II

Estimate pan/tilt from $\mathbf{r}_3$.

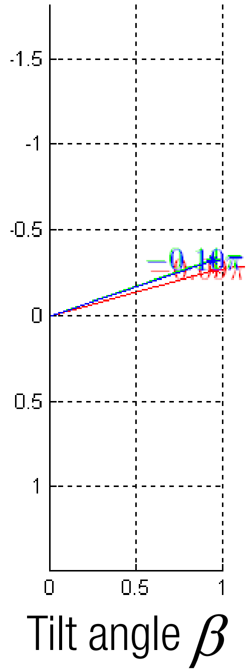
$$\alpha = \tan^{-1}\left(\mathbf{r}_3(1) / \mathbf{r}_3(3)\right)$$

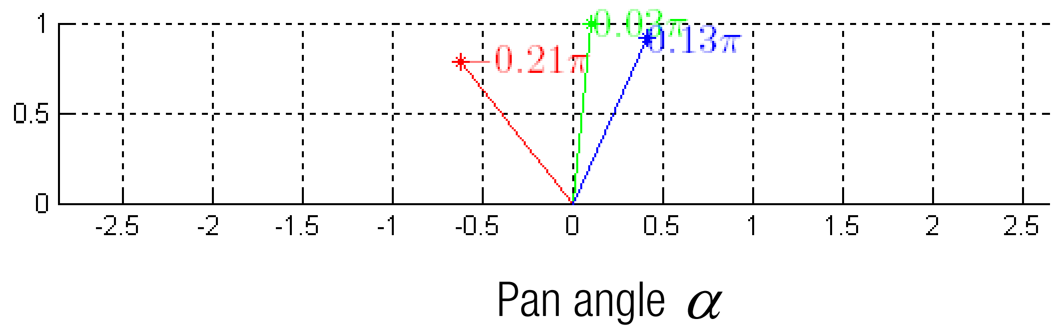
$$\beta = \sin^{-1}\mathbf{r}_3(2)$$

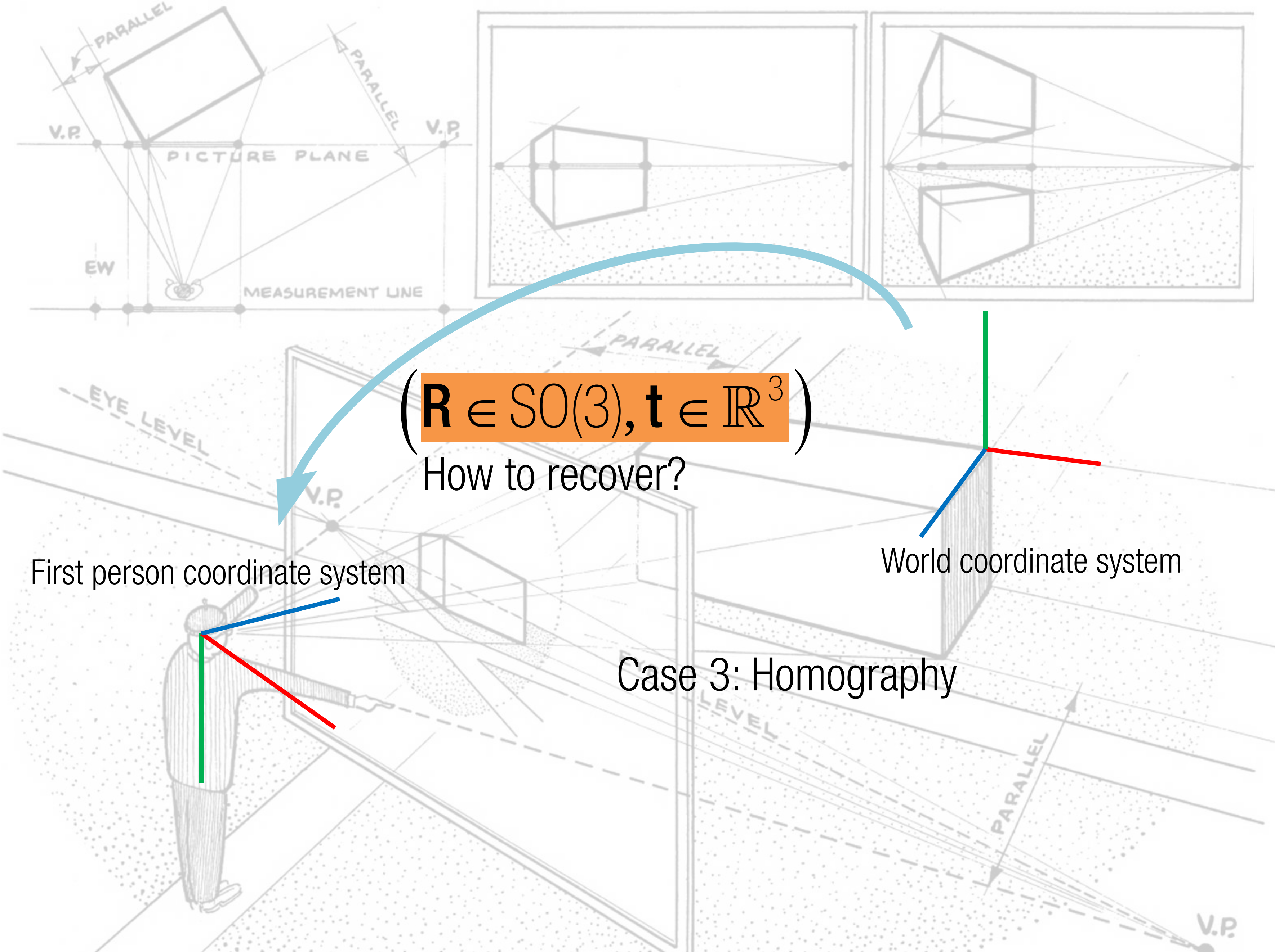
$$\begin{aligned} \alpha &= -0.6691 = -0.2130\pi \\ \beta &= -0.2706 = -0.0861\pi \end{aligned}$$

$$R = \begin{pmatrix} 0.8017 & 0.0067 & -0.5977 \\ -0.2086 & 0.9411 & -0.2673 \\ 0.5602 & 0.3382 & 0.7558 \end{pmatrix}$$

Exercise II

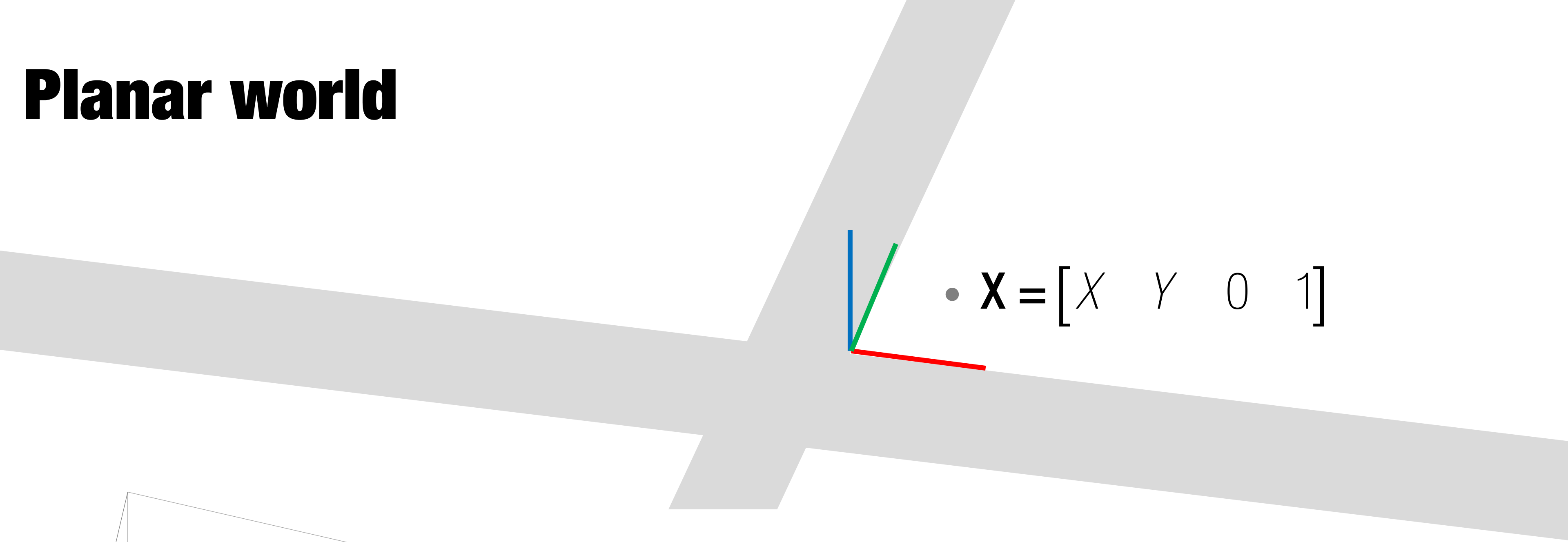

-1.5
-1
-0.5
0
0.5
1
0
0.5
1
Tilt angle β


-0.21π
0.03π
0.13π
1
0.5
0
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
Pan angle α

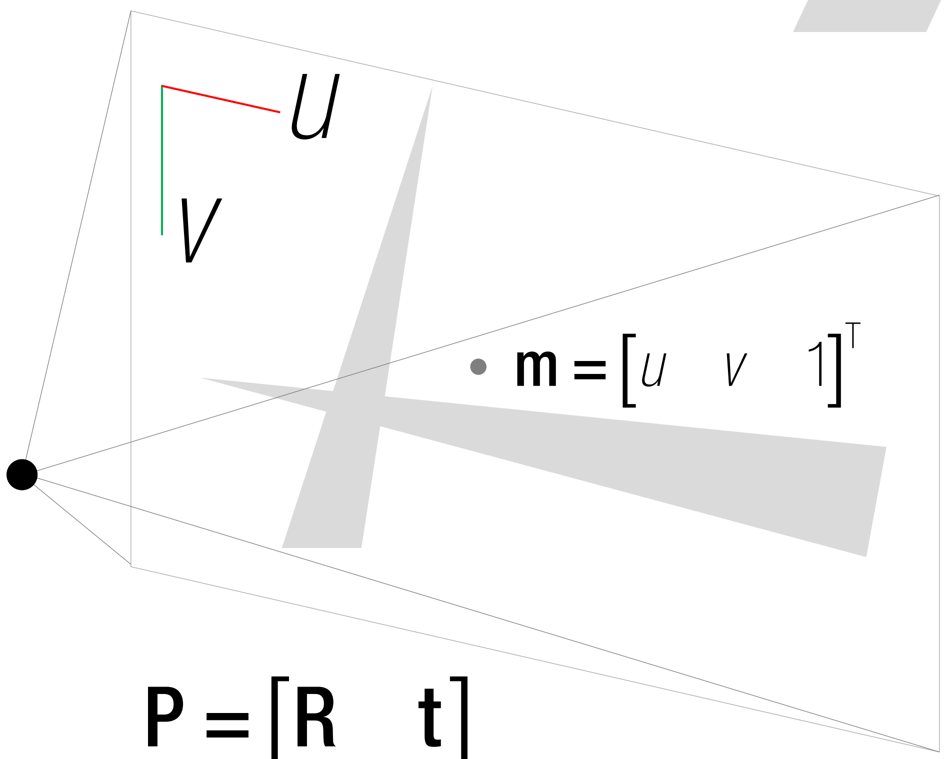


http://www.joshuanava.biz/perspective/in-other-words-the-observer-simply-points-in-the-same-direction-as-the-lines-in-order-to-find-their-vanishing-point.html

Planar world



$$\mathbf{X} = \begin{bmatrix} X & Y & 0 & 1 \end{bmatrix}$$



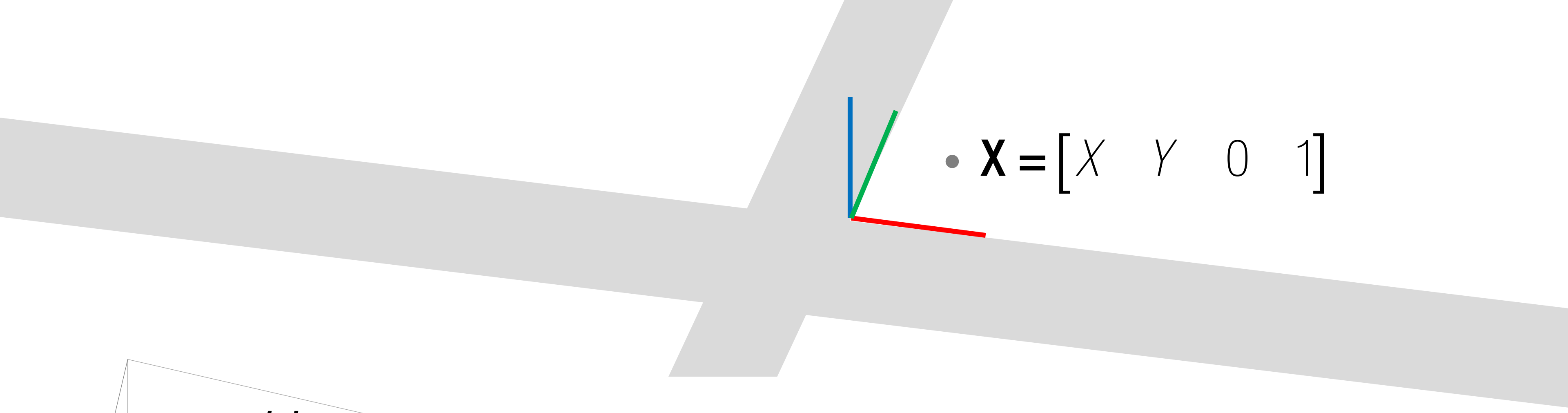
$$\mathbf{m} = \begin{bmatrix} u & v & 1 \end{bmatrix}^T$$

$$z\mathbf{m} = \mathbf{K}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & | & \mathbf{t} \end{bmatrix}\mathbf{X}$$

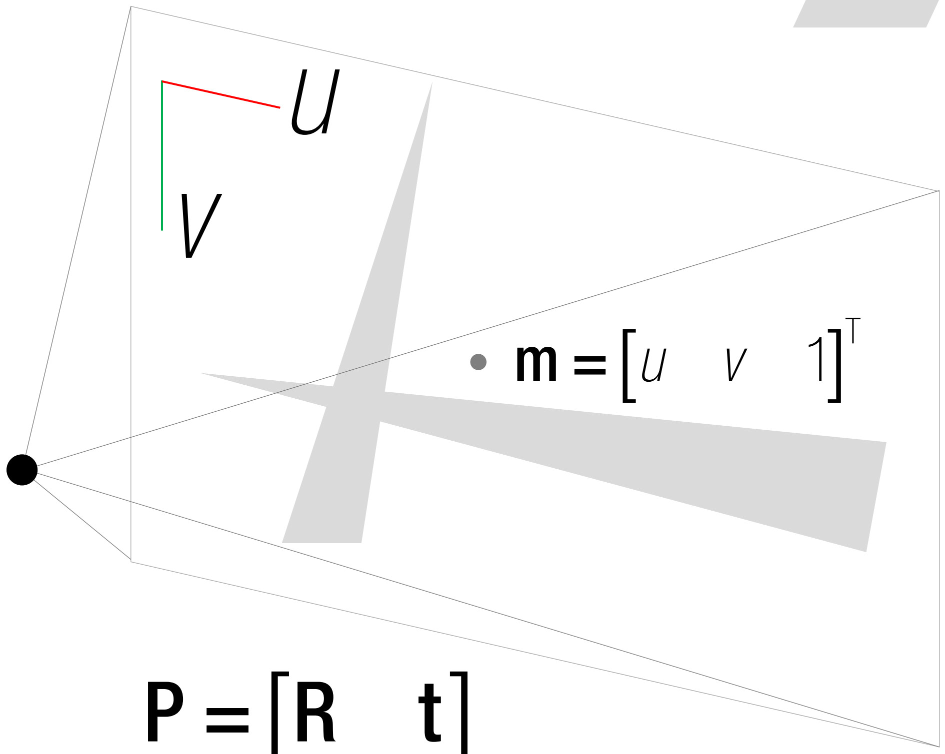
$$= \mathbf{K}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & | & \mathbf{t} \end{bmatrix}\begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix} = \underbrace{\mathbf{K}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}}_{\text{2D homography}}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}$$

Planar world



- $\mathbf{X} = \begin{bmatrix} X & Y & 0 & 1 \end{bmatrix}$

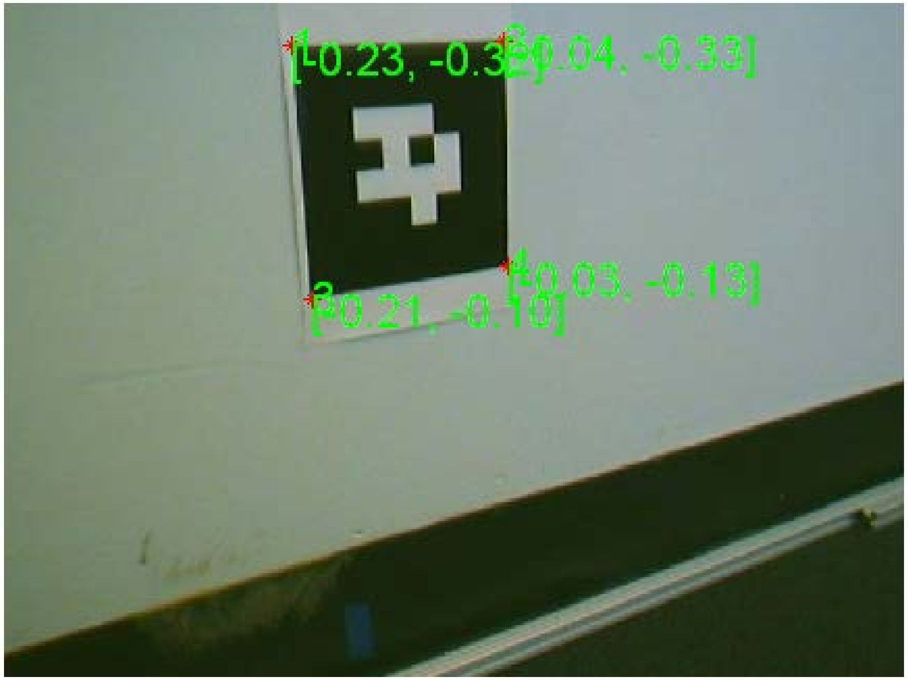


- $\mathbf{m} = \begin{bmatrix} u & v & 1 \end{bmatrix}^T$

$$z\mathbf{m} = \tilde{\mathbf{H}} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} \text{ where } \tilde{\mathbf{H}} = \mathbf{K}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}$$

Exercise

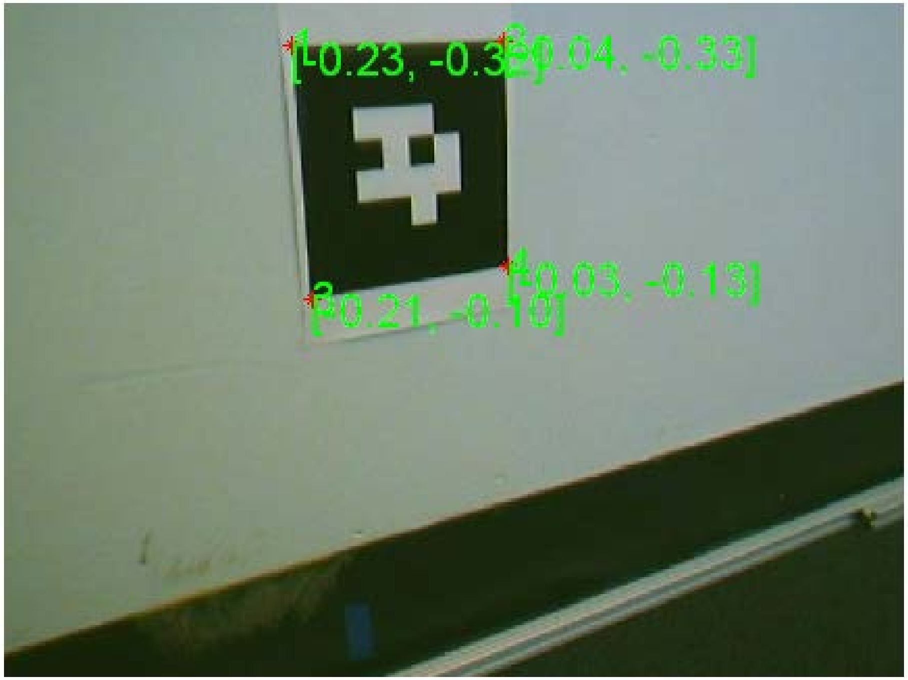


Homography from four points:

$$H = \begin{pmatrix} 0.4430 & 0.0037 & -0.1071 \\ -0.1153 & 0.5216 & 0.1506 \\ 0.3096 & 0.1875 & 0.5944 \end{pmatrix}$$

$$\mathbf{H} = \mathbf{K}^{-1}\tilde{\mathbf{H}} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}$$ Note that $\|\mathbf{r}_1\| = \|\mathbf{r}_2\| = 1$

Exercise



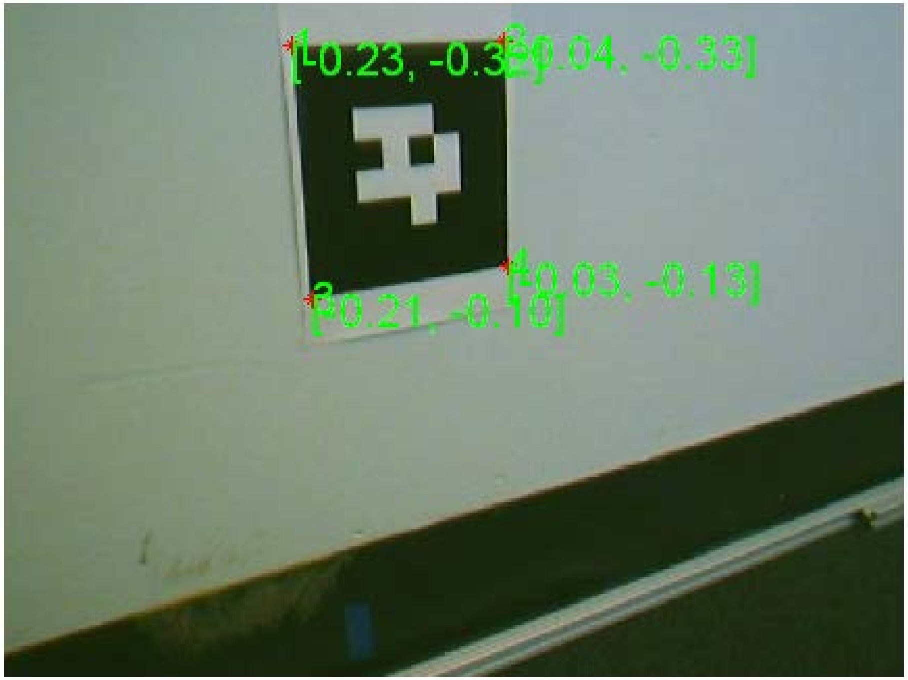
Homography from four points:

$$H = \begin{pmatrix} 0.4430 & 0.0037 & -0.1071 \\ -0.1153 & 0.5216 & 0.1506 \\ 0.3096 & 0.1875 & 0.5944 \end{pmatrix}$$

$$a = \|(H_{11}, H_{21}, H_{31})\|$$: Normalization factor

$$\begin{aligned} t &= H(:,3)/a = (-0.1937, 0.2726, 1.0756)^T \\ r_1 &= H(:,1)/a = (0.8017, -0.2086, 0.5602)^T \\ r_2 &= H(:,2)/a = (0.0067, 0.9439, 0.3392)^T \end{aligned}$$

Exercise



Homography from four points:

$$H = \begin{pmatrix} 0.4430 & 0.0037 & -0.1071 \\ -0.1153 & 0.5216 & 0.1506 \\ 0.3096 & 0.1875 & 0.5944 \end{pmatrix}$$

$$a = \|(H_{11}, H_{21}, H_{31})\|$$: Normalization factor

$$\begin{aligned} t &= H(:,3)/a = (-0.1937, 0.2726, 1.0756)^T \\ r_1 &= H(:,1)/a = (0.8017, -0.2086, 0.5602)^T \\ r_2 &= H(:,2)/a = (0.0067, 0.9439, 0.3392)^T \\ r_3 &= r_1 \times r_2 = (-0.1937, 0.2726, 1.0756)^T \end{aligned}$$

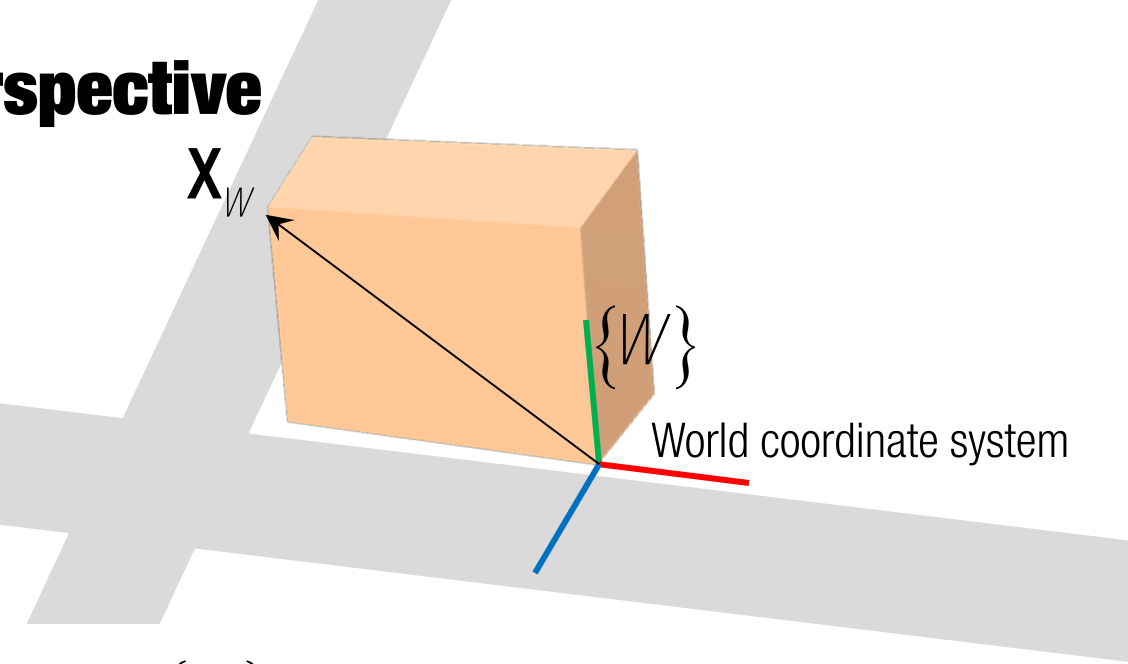
How to estimate the rotation and translation of the robot from the world point of view?

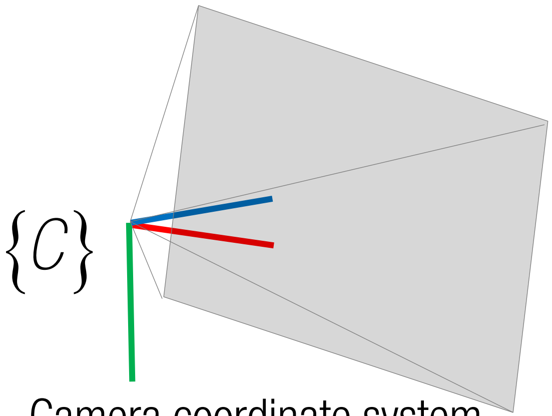
In the case of moving robot(rather than moving target), we need to know the orientation/position of the robot in the world ==>

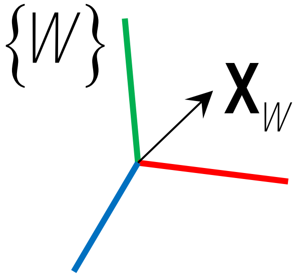
we need to how to pan/tilt the world oriented to the robot.

Note: pan/tilt of the camera is very different from the pan/tilt of the world!

Third person (world) perspective

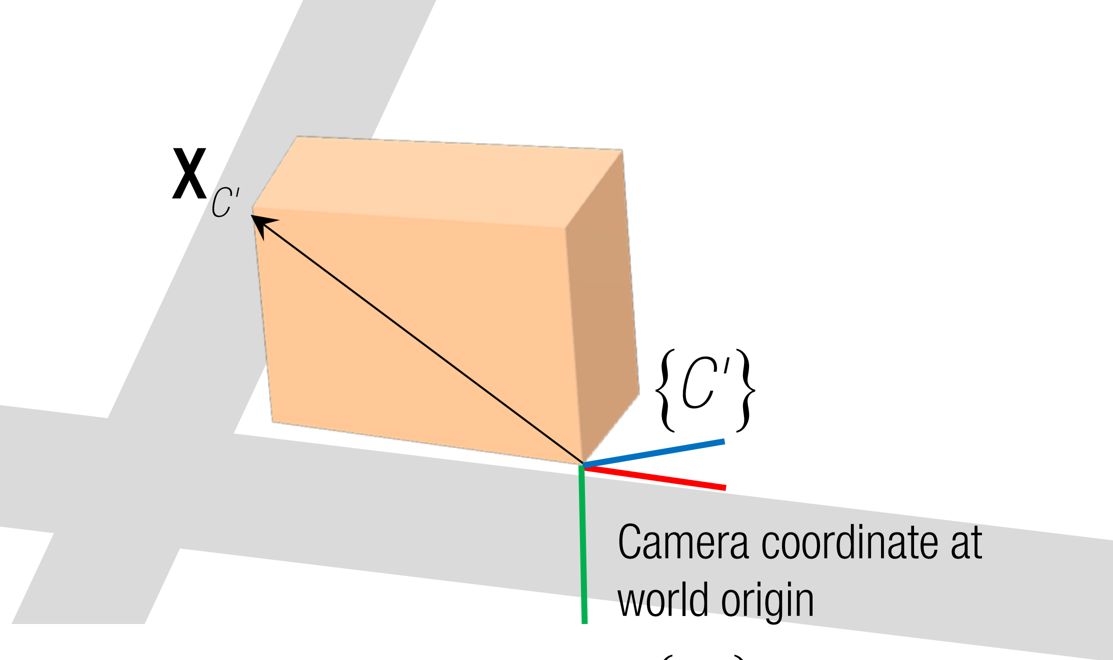


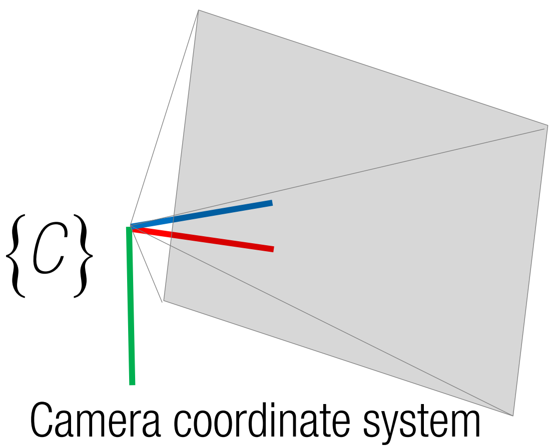


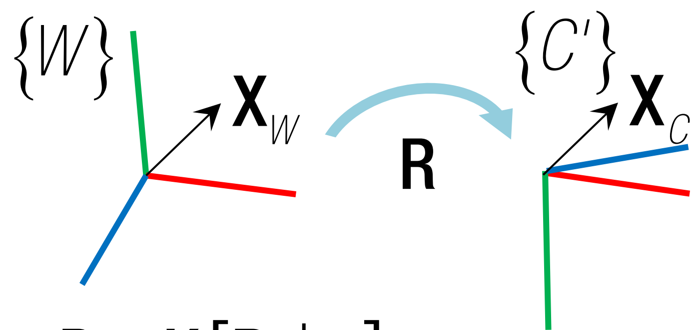


$$\mathbf{P} = \mathbf{K}\left[\mathbf{R} \mid \mathbf{t}\right]$$

First person perspective





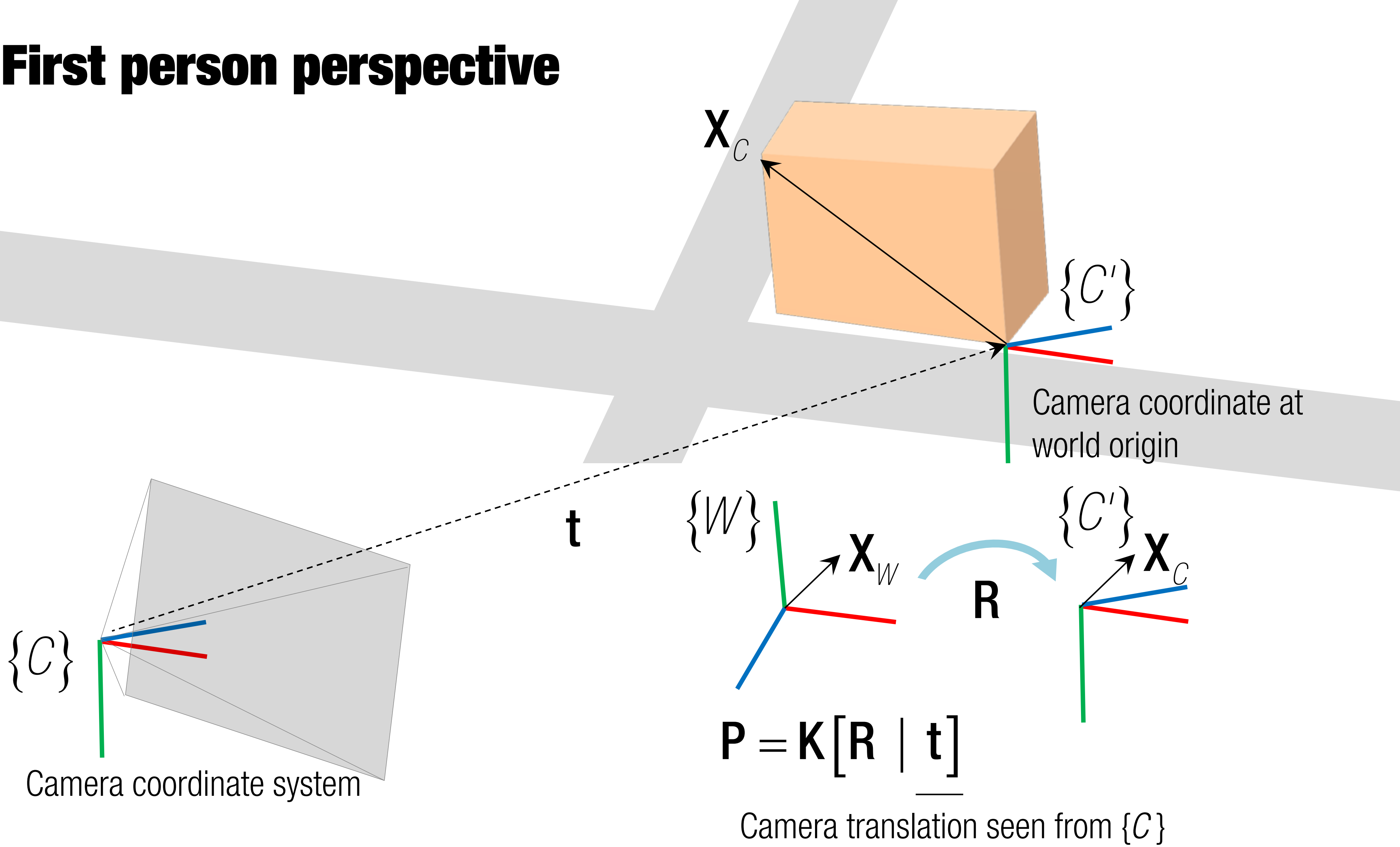


$$\mathbf{P} = \mathbf{K}\left[\underline{\mathbf{R}} \mid \mathbf{t}\right]$$

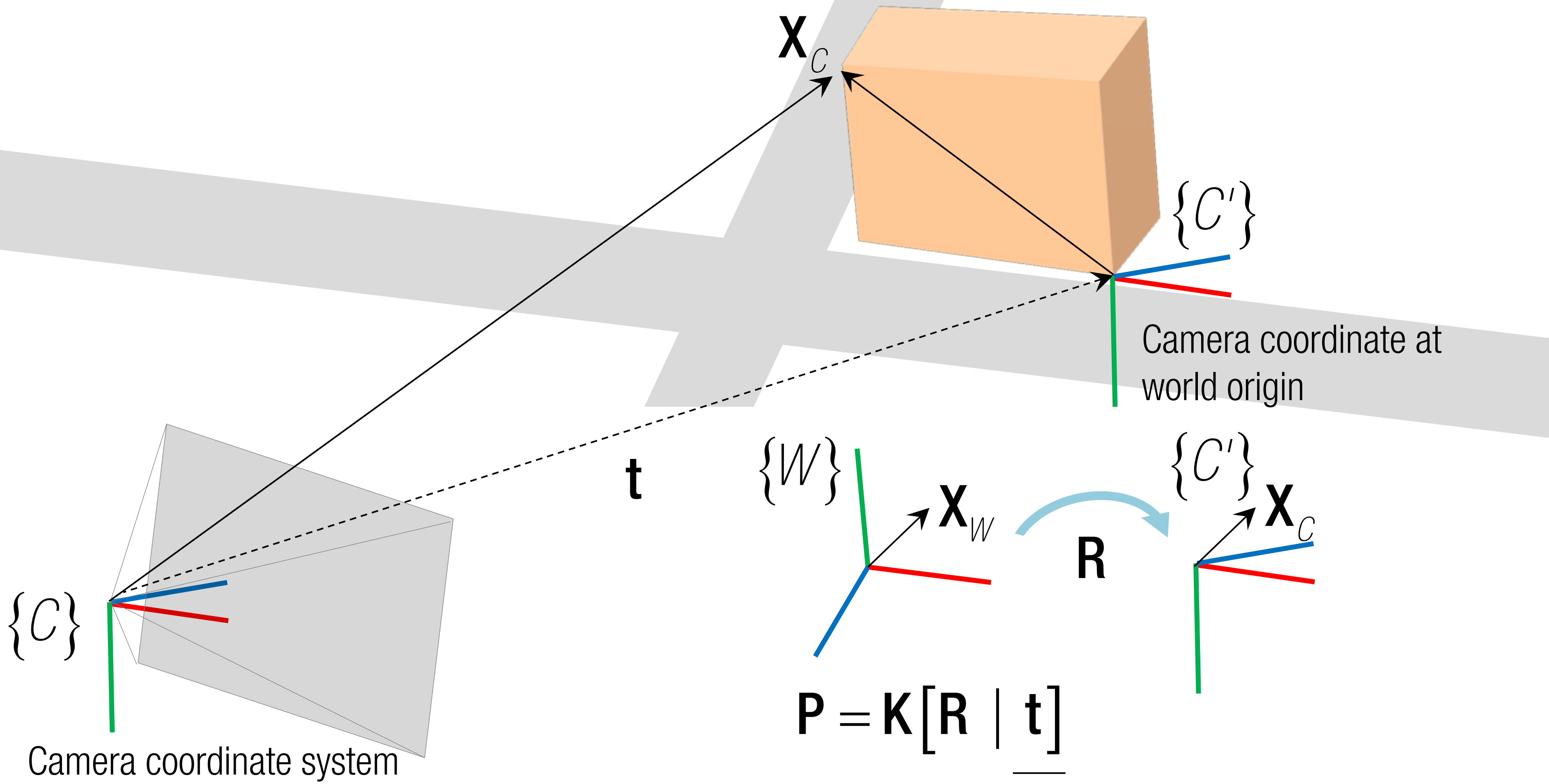
Coordinate transform from $\{W\}$ to $\{C'\}$

$$\mathbf{X}_{C'} = \mathbf{R}\mathbf{X}_W$$

First person perspective



First person perspective



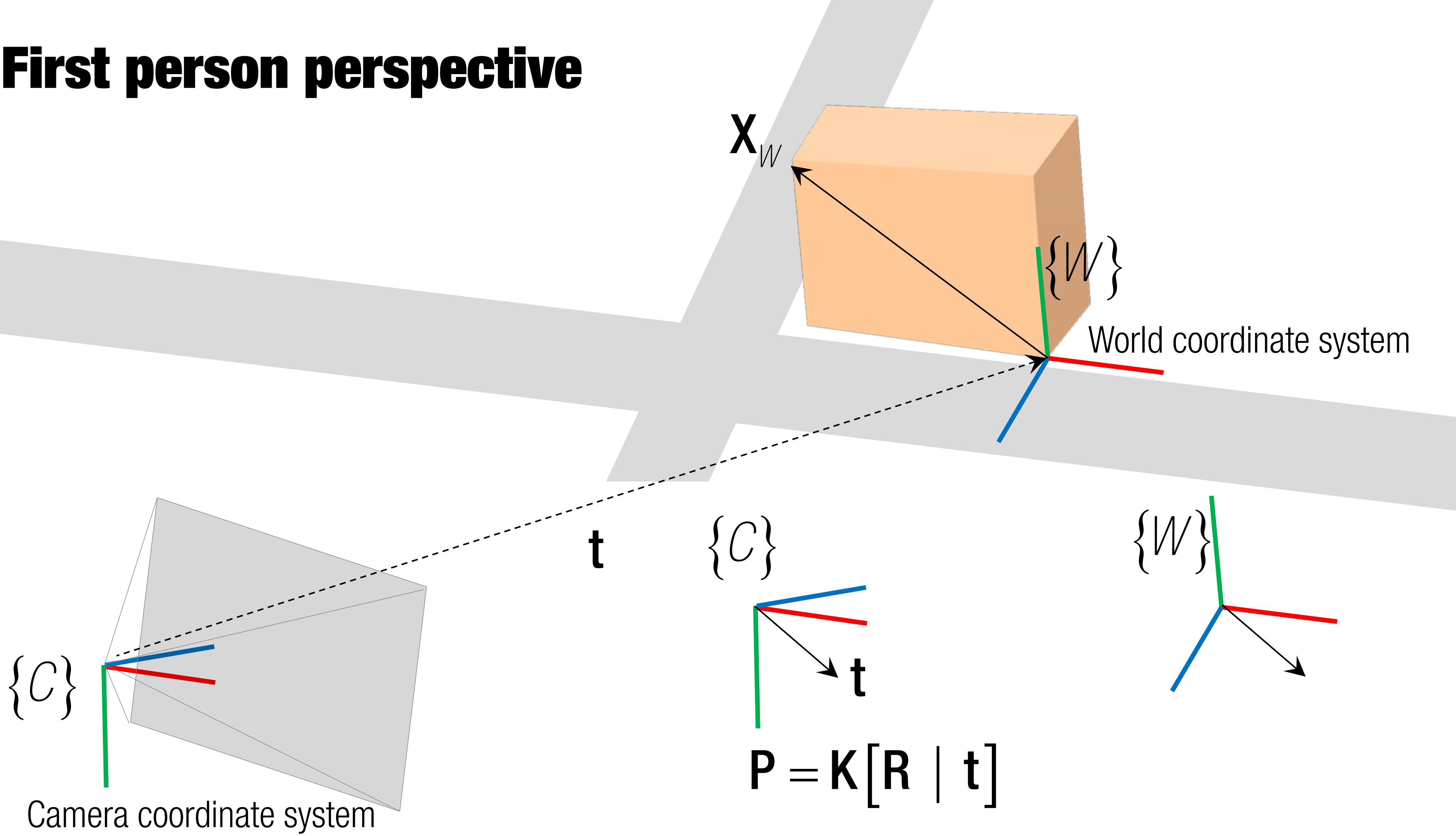
$$\mathbf{P} = \mathbf{K}[\mathbf{R} \mid \underline{\mathbf{t}}]$$

Camera translation seen from $\{C\}$

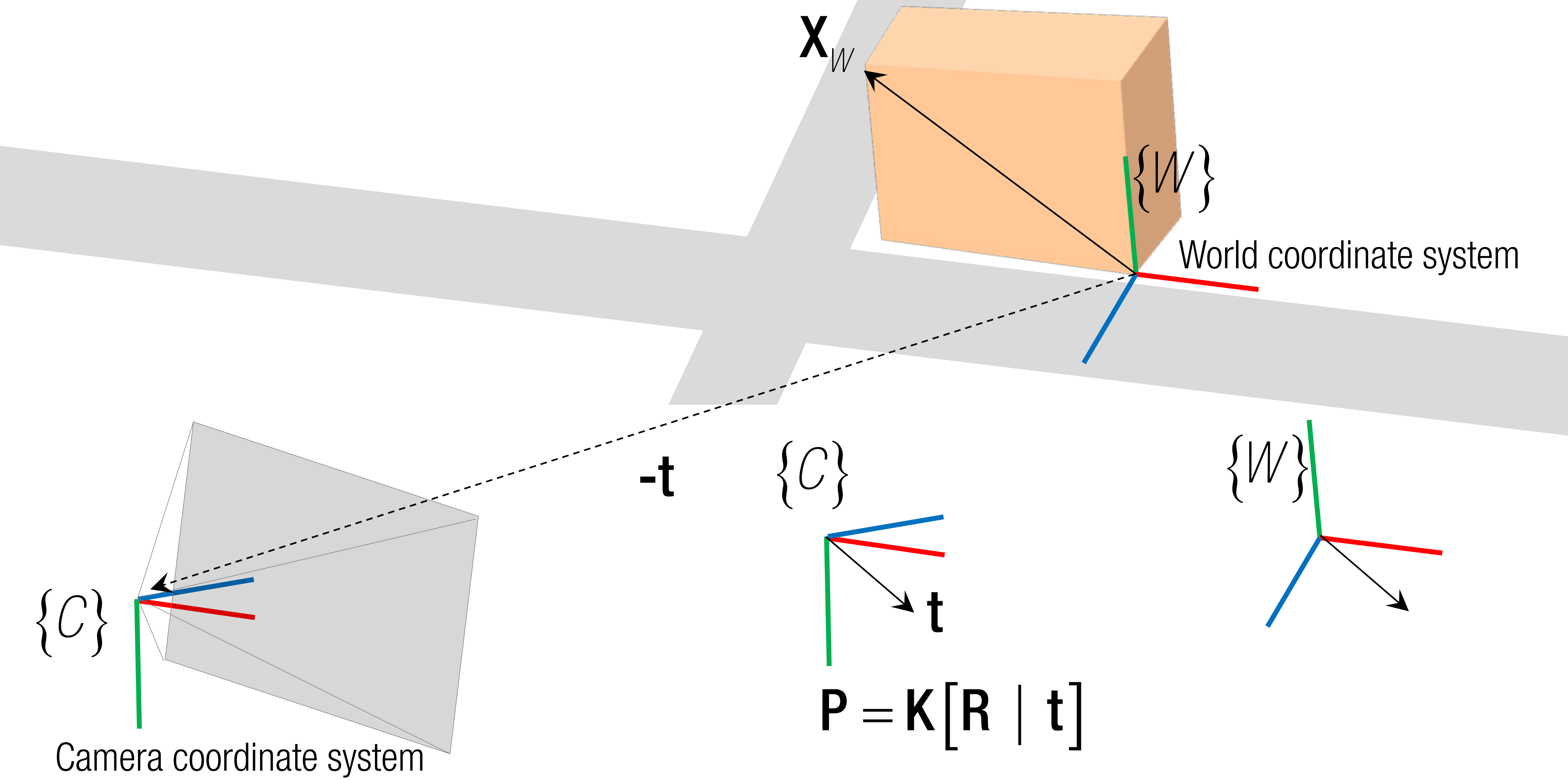
$$\mathbf{X}_C = \mathbf{R}\mathbf{X}_W + \mathbf{t}$$

Looking a point in world through the camera view point

First person perspective



Third person (world) perspective



Third person (world) perspective