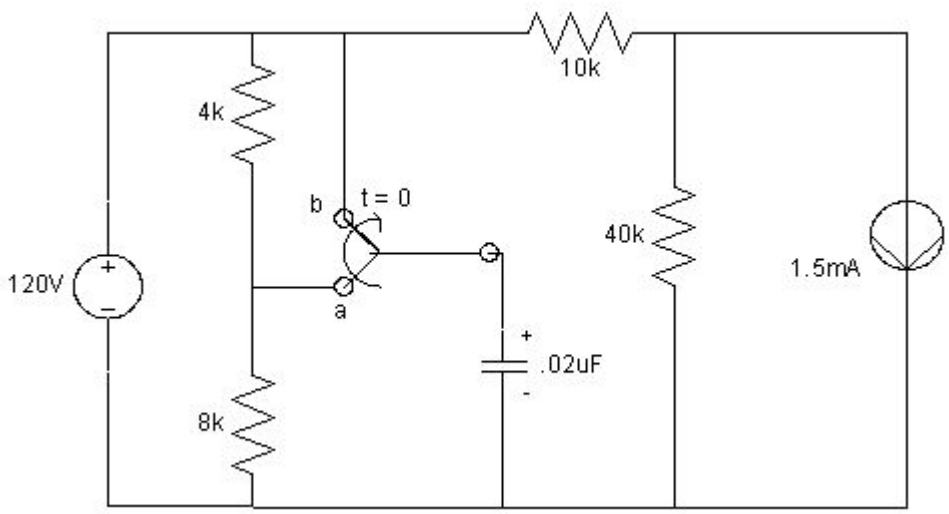
EECS 40/42/100, Spring 2007
Prof. Chang-Hasnain

Homework #5

Due at 6 pm in 240 Cory on Wednesday, 2/28/07
Total Points: 100

- Put (1) your name and (2) **discussion section number** on your homework.
- You need to put down all the derivation steps to obtain full credits of the problems. Numerical answers alone will at best receive low percentage partial credits.
- No late submission will be accepted expect those with prior approval from Prof. Chang-Hasnain.

1. Consider the following circuit: [11 points]



The switch in the circuit shown has been in position a for a long time, At t = 0 the switch is moved to position b. Calculate:

(a) The initial voltage on the capacitor

Just prior to the switch flipping at time t=0, the circuit is in steady-state, so the capacitor acts like an open circuit. Thus the voltage across the capacitor is given by a voltage divider:

$$v_c(0-) = \frac{8k}{4k+8k}(120V) = 80V$$

Because capacitor voltage is continuous:

$$v_c(0+) = v_c(0-) = 80V$$ [2 points]

(b) The final voltage on the capacitor

Again, the circuit has reached steady state, so the capacitor acts like an open circuit. Thus all the current goes through the 40k resistor.

By KVL:
$v_c(\infty) = v_{40k} - v_{10k} = -1.5mA(40k) - 0A(10k) = -60V$ [2 points]

(c) The time constant for t > 0

To find the Thevenin equivalent resistance, we zero the current source and get
$R_{eq} = 10k + 40k = 50k$
So the time constant is
$\tau = R_{eq}C = (50k)(0.02\mu F) = 1ms$ [3 points]

(d) The length of time required for capacitor voltage to reach zero after the switch is moved to position b.

Solving the differential equation we get
$v_c(t) = K_1 + K_2 \exp\{-t/1ms\}$
$v_c(\infty) = K_1 = -60V$
$v_c(0+) = -60V + K_2 = 80V \Rightarrow K_2 = 140V$ [2 points]
$v_c(t) = -60V + 140V \exp\{-t/1ms\}$
So to find the time it takes to reach zero, we set this equal to zero and solve for t.
$0 = -60V + 140V \exp\{-t/1ms\} \Rightarrow t = 1ms \times \ln(7/3) = 0.8473ms$ [2 points]

2. Hambley, P4.33 [10 points]

The differential equation obtained using KCL:
$$\frac{v_c(t) - v(t)}{R} + C\frac{dv_c(t)}{dt} = 0$$
Rearranging and substituting in $v_c=t$:
$$RC\frac{dv_c(t)}{dt} + v_c(t) = t \text{ for } t > 0.$$ [2 point]
As per the hint, we try a particular solution of the form:
$v_{cp}(t) = A + Bt$
Which gives us
$$\frac{dv_c(t)}{dt} = B$$
Plugging these into the differential equation, we get:
$RCB + A + Bt = t$
Matching coefficients, we get B=1 and A=-RC
So the particular solution is:
$v_{cp}(t) = -RC + t$ [5 points]
The complementary solution is of the form:
$v_{cc}(t) = K \exp\{-t/RC\}$

Which gives us a complete solution of the form:
$v_c(t) = -RC + t + K\exp\{-t/RC\}$
Using our initial condition,
$v_c(0) = -RC + 0 + K = 0 \Rightarrow K = RC$
So our complete solution is:
$v_c(t) = -RC + t + RC\exp\{-t/RC\}$ [3 points]

3. Hambley, P4.35 [11 points]

KCL at the top node gives us:

$$5\cos(10t) = \frac{v(t)}{R} + \frac{1}{L}\int_0^t v(t')dt' + i_L(0)$$

To get a differential equation, we get the time derivative:

$$-50\sin(10t) = \frac{1}{R}\frac{dv(t)}{dt} + \frac{v(t)}{L}.$$ [2 points]

Our particular solution is of the form:

$$v(t) = A\cos(10t) + B\sin(10t)$$

$$\frac{dv(t)}{dt} = -10A\sin(10t) + 10B\cos(10t)$$

Substituting this into the differential equation, we get:

$$-50\sin(10t) = \frac{1}{R}\left[-10A\sin(10t) + 10B\cos(10t)\right] + \frac{1}{L}\left[A\cos(10t) + B\sin(10t)\right]$$

Equating the coefficients of the sine and cosine terms, we get:

$$-50 = \frac{-10A}{R} + \frac{B}{L}$$

$$0 = \frac{10B}{R} + \frac{A}{L}$$

Solving for A and B, we get A=25 and B=-25. [6 points]
The time constant is L/R=0.1s and the general form of the solution is:
$v(t) = 25\cos(10t) - 25\sin(10t) + K\exp\{-t/0.1s\}$
Because $i_L(0+)$=0A, all the current from the source goes through the resitor making v(0+)=50V.
Substituting this in and solving gives us K=25, and our final solution is:
$v(t) = 25\cos(10t) - 25\sin(10t) + 25\exp\{-t/0.1s\}$ [3 points]

4. Hambley, P4.39 [11 points]

Using KVL, we obtain the differential equation

$$L\frac{di(t)}{dt} + Ri(t) = v(t)$$

[2 points]

$$\frac{di(t)}{dt} + 300i(t) = 10\sin(300t)$$

We try a particular soltuion of the form:

$$i_p(t) = A\cos(300t) + B\sin(300t)$$

$$\frac{di_p(t)}{dt} = -300A\cos(300t) + 300B\cos(300t)$$

Plugging this in:

$$-300A\sin(300t) + 300B\cos(300t) + 300A\cos(300t) + 300B\sin(300t) = 10\sin(300t)$$

By matching coefficients we get:

$$300B + 300A = 0$$

$$-300A + 300B = 10$$

Solving gives us B = 1/60, A = -1/60. [6 points]

The complementary solution is of the typical form with time constant L/R=1/300. So the complete solution is of the form:

$$i(t) = -\frac{1}{60}\cos(300t) + \frac{1}{60}\sin(300t) + K\exp\{-300t\}$$

Using the initial condition i(0)=0, we can find that K=1/60.

So the final solution is:

$$i(t) = -\frac{1}{60}\cos(300t) + \frac{1}{60}\sin(300t) + \frac{1}{60}\exp\{-300t\}$$ [3 points]

5. Hambley, P4.41 [7 points]

We first combine all the capacitors and inductors that we can combine in series and parallel. After this we can count the total number of energy storage elements. This gives us the order of the circuit.

6. Hambley, P4.48 [14 points]

(a) KCL at the top node gives us:

$$\frac{v(t)}{R} + \frac{1}{L}\int_0^t v(t')dt' + i_L(0) + C\frac{dv(t)}{dt} = 1$$

Differentiating with respect to time:

$$\frac{1}{R}\frac{dv(t)}{dt} + \frac{1}{L}v(t) + C\frac{d^2v(t)}{dt^2} = 0$$

Dividing by C:

$$\frac{d^2v(t)}{dt^2} + \frac{1}{RC}\frac{dv(t)}{dt} + \frac{1}{LC}v(t) = 0$$ [2 points]

This gives us a characteristic equation:

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

We can then easily read out:

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10\times10^6, \alpha = \frac{1}{2RC} = 20\times10^6, \varsigma = \frac{\alpha}{\omega_0} = 2$$ [3 points]

(b) At t=0+, KCL gives us:

$$\frac{v(0+)}{R} + i_L(0+) + Cv'(0+) = 1A$$

Plugging in the given initial conditions and solving gives v'(0+)=1A/C=10^9V/s.

[3 points]

(c) We find the particular solution by looking at the circuit as time goes to infinity. In steady-state, the inductor acts as a short, so $v_p(t)=0$ [1 point]

(d) Because the particular solution is zero, the solution is just the complementary solution. Because the damping ratio is greater than 1, the circuit is overdamped. Thus the soltuion is of the form:

$$v(t) = K_1 \exp(s_1 t) + K_2 \exp(s_2 t)$$ [1 point]

Where s_1 and s_2 are the roots of the characteristic equation:

$$s_1 = -2.679 \times 10^6, s_2 = -37.32 \times 10^6$$ [2 points]

The initial conditions are v(0+)=0 and v'(0+)=10^9. Plugging these in:

$$v(0+) = 0 = K_1 + K_2$$

$$v'(0+) = 10^9 = s_1 K_1 + s_2 K_2$$

Solving, we find K_1=28.87 and K_2=-28.87. This gives us a final solution:

$$v(t) = 28.87 \exp(-2.679 \times 10^6 \times t) - 28.87 \exp(-37.32 \times 10^6 \times t)$$ [2 points]

7. Hambley, P5.19 [8 points]

The magnitudes of the phasors for the two known voltages are $8\sqrt{2}$ and $3\sqrt{2}$, but the phases are unknown. To get the smallest magnitude of the sum, we would want the two phasors to be perfectly out of phase. Then the magnutide of the sum is $8\sqrt{2} - 3\sqrt{2} = 5\sqrt{2}$, which corresponds to an rms voltage of 5V. [4 points]

To get the max, we want them to be perfectly in phase, then the rms voltage will be the sum: 11 V. [4 points]

8. Hambley, P5.22 [9 points]

$I_m = \sqrt{2} \times I_{rms} = 7.071$ [3 points]

$\theta = 30° + 20° = 50°$ [3 points]

$i_1(t) = I_m \cos(\omega \cdot t + \theta) = 7.071 \cos(\omega \cdot t + 50°)$ [3 points]

9. Hambley, P5.24 [10 points]

We are given:

$$5\sin(\omega \cdot t) + 5\cos(\omega \cdot t + 30°) + 5\cos(\omega \cdot t + 150°)$$

Converting to phasors: [3 points per conversion]

$$5\angle -90° + 5\angle 30° + 5\angle 150° = -j5 + \left(\frac{5\sqrt{3}}{2} + j2.5\right) + \left(-\frac{5\sqrt{3}}{2} + j2.5\right) = 0$$

So $5\sin(\omega \cdot t) + 5\cos(\omega \cdot t + 30°) + 5\cos(\omega \cdot t + 150°) = 0$ [1 point]

10. Hambley, P5.30 [9 points]

(a) We are given:

$V = 100\angle -60°, I = 1\angle 30°$

$Z = \frac{V}{I} = 100\angle -90° = -j100$ [1 point]

Because the Z is imaginary and negative, the element is a capacitance. [1 point]

$Z = \frac{1}{j\omega C} \Rightarrow C = \frac{1}{j\omega Z} = \frac{1}{j200 \times -j100} = 50\mu F$ (since $j \times -j = 1$) [1 point]

(b) We are given:

$V = 500\angle 50°, I = 2\angle 50°$

$Z = \frac{V}{I} = 250\angle 0° = 250$ [1 point]

Because the Z is real, the element is a resistance of 250 ohms. [2 points]

(c) We are given:

$V = 100\angle 30°, I = 1\angle -60°$

$Z = \frac{V}{I} = 100\angle 90° = j100$ [1 point]

Because the Z is imaginary and positive, the element is an inductance. [1 point]

$Z = j\omega L \Rightarrow L = \frac{Z}{j\omega} = \frac{j100}{j400} = 0.25H$ [1 point]