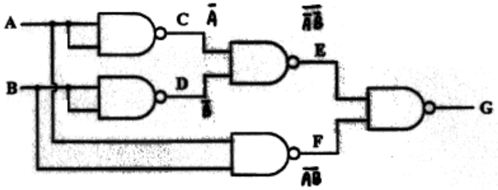
SOLUTIONS

Handout #24

Problem 1: Logic Gates and Timing Diagrams [25 points]

Consider the following digital logic circuit:



$G = \overline{\overline{\bar{A}\bar{B}}\;\overline{AB}}$

$= \bar{A}\bar{B} + AB$

by DeMorgan's Theorem

a) Fill out the truth table for the logic function G. **[8 pts]**

A	B	G
0	0	1
0	1	0
1	0	0
1	1	1

b) Write a simple logical expression for the function G. **[5 pts]**

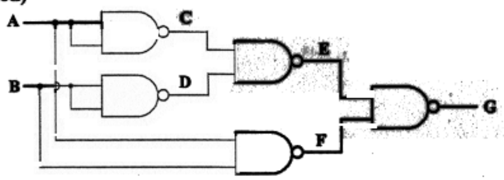
G = $\bar{A}\bar{B} + AB$

c) How many unit gate delays are there between the inputs (A and B) and the output (G)? **[2 pts]**
(In other words, how many unit gate delays must you wait, after changing A and/or B, before you can trust the value of G to be valid?)

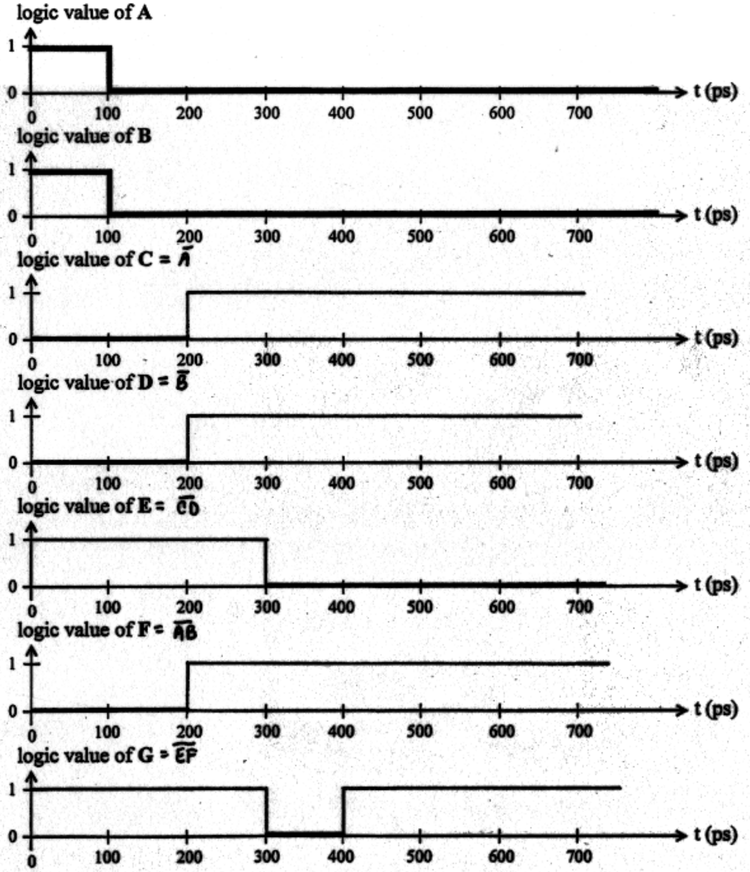
The longest path between the input variables and the output variable is 3 logic gates. Therefore, we need to wait for a period of 3 unit gate delays after an input variable is changed, before we can trust the value of G to be valid.

3 unit gate delays

Problem 1 (continued)



d) Assume each logic gate has a unit gate delay τ = 100 ps.
Draw the timing diagrams for t=0 to t=700 ps, for the given logic input values A and B. **[10 pts]**


logic value of A

logic value of B

logic value of C = Ā

logic value of D = B̄

logic value of E = $\overline{CD}$

logic value of F = $\overline{AB}$

logic value of G = $\overline{EF}$

Truth tables:

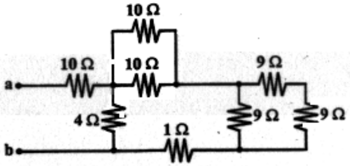
C	D	E
0	0	1
0	1	1
1	0	1
1	1	0

A	B	F
0	0	1
0	1	1
1	0	1
1	1	0

E	F	G
0	0	1
0	1	1
1	0	1
		0

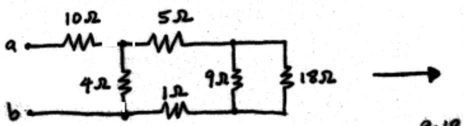
Problem 2: Resistive Circuits [30 points]

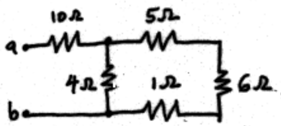
a) Find the equivalent resistance R_{ab} for the following circuit. [6 pts]



R_{ab} = 13 Ω

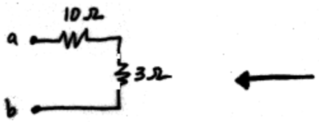
9Ω resistors in series → 18Ω resistor
10Ω resistors in parallel → 5Ω resistor

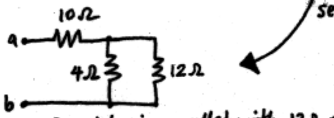




9Ω resistor in parallel with 18Ω resistor → $\frac{9\cdot18}{9+18}$ = 6Ω resistor

5Ω, 6Ω, 1Ω resistors in series → 12Ω resistor



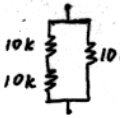


$\left(\frac{4\cdot12}{4+12}=3\right)$

4Ω resistor in parallel with 12Ω resistor → 3Ω resistor

b) Suppose you need a **6 kΩ** resistor for your Tutebot project, but your TA gives you only a supply of 10 kΩ resistors. Being a clever Cal student, how would you connect several 10 kΩ resistors together, to achieve a 6 kΩ resistance? [7 pts]

- To achieve an equivalent resistance lower than the individual resistors, we should connect resistors in parallel.
- But the parallel combination of 2 10kΩ resistors is 5kΩ — too low!
  ⇒ need to increase the resistance of one of the parallel branches
- Try parallel combination of a 10kΩ resistor and two 10kΩ resistors in series:

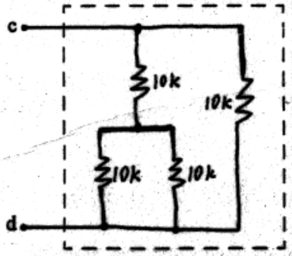


$\frac{20\cdot10}{20+10}$ = 6.7 kΩ too high

- Try increasing the resistance of one parallel branch by only 5kΩ (10kΩ // 10kΩ) instead of 10kΩ

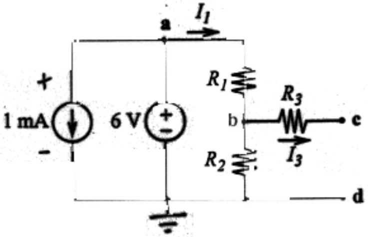
$\frac{15\cdot10}{15+10}$ = 6kΩ !

Circuit diagram of 10-kΩ resistors connected to give R_{cd} = 6 kΩ:



Problem 2 (continued)

c) Consider the following circuit:



$R_1 = 1\ \text{k}\Omega$
$R_2 = 2\ \text{k}\Omega$
$R_3 = 2\ \text{k}\Omega$

i) Find V_{cd}. [3 pts]

V_{cd} = 4 V

$I_3 = 0$ since terminal c is not connected

Thus the current flowing through R_1 equals the current flowing through R_2, i.e. we have a voltage divider.

Since there is no voltage drop across R_3 (because $I_3=0$), $V_c = V_b$

$\Rightarrow V_{cd} = V_{bd} = 2V$

$$\Rightarrow V_{bd} = \frac{R_2}{R_1+R_2}(6) = \frac{2}{1+2}(6) = 4V$$

ii) Find the power developed/absorbed by the current source, P_I. [3 pts]

P_I = 6 mW
[developed, [✓ absorbed]]
(circle correct choice)

The voltage across the current source is established by the voltage source and is equal to 6V.

$$P_I = IV = (1mA)(6V) = 6\ mW$$

Since positive current is entering the positive terminal of the current source, it is absorbing power.

iii) Indicate in the table below (by checking the appropriate boxes) how various circuit parameters would change if the terminals **c** and **d** were to be shorted together. Justify your answers. [6 pts]

Parameter	Value will:			Brief Explanation/Justification
	increase	decrease	not change	
V_{bd}		✓		The resistance between b and d decreases; by the voltage-divider formula, V_{bd} decreases
I_1	✓			Total resistance between a and d decreases; V_{ad} remains 6V; $I_1 = \frac{V_{ad}}{R_{ad}}$
Power developed by voltage source	✓			Since I_1 increases, the current supplied by the voltage source increases.

iv) What is the value of I_3 when the terminals **c** and **d** are shorted together? [5 pts]

I_3 = 1.5 mA

Equivalent resistance between terminals a and d is

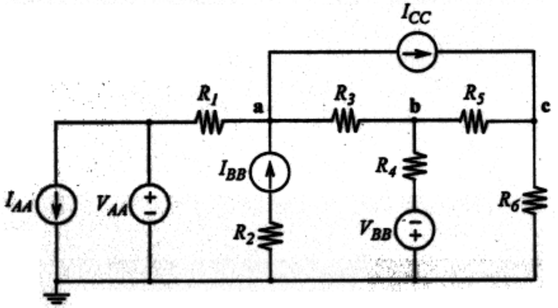
$R_1 + R_2//R_3 = 1 + \frac{2\cdot2}{2+2} = 2\ k\Omega$

$\Rightarrow I_1 = \frac{6V}{2k\Omega} = 3mA$

Using current-divider formula, $I_3 = \frac{2}{2+2}(3mA) = 1.5\ mA$

Problem 3: Nodal Analysis [20 points]

a) In the circuit below, the independent source values and resistances are known. Use the nodal analysis technique to write 3 equations sufficient to solve for V_a, V_b, and V_c. To receive credit, you must write your answer in the box below. **[10 pts]**
DO NOT SOLVE THE EQUATIONS!



Apply Kirchhoff's Current Law to nodes a, b, c:
(Sum of currents entering a node = 0)

$$\text{(a)}\quad \frac{V_{AA}-V_a}{R_1} + I_{BB} - I_{cc} + \frac{V_b-V_a}{R_3} = 0$$

$$\text{(b)}\quad \frac{V_a-V_b}{R_3} + \frac{-V_{BB}-V_b}{R_4} + \frac{V_c-V_b}{R_5} = 0$$

$$\text{(c)}\quad I_{cc} + \frac{V_b-V_c}{R_5} + \frac{-V_c}{R_6} = 0$$

3 independent equations for 3 unknowns (V_a, V_b, V_c) ⇒ can solve to find unknowns

Write the nodal equations here:

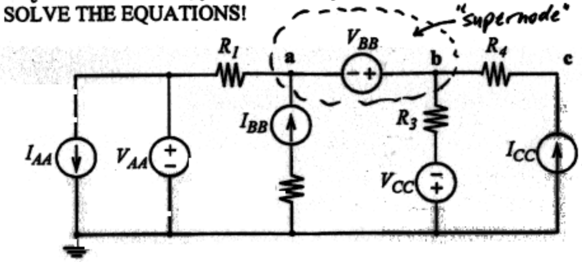
$$\frac{V_{AA}-V_a}{R_1} + I_{BB} - I_{cc} + \frac{V_b-V_a}{R_3} = 0$$

$$\frac{V_a-V_b}{R_3} - \frac{V_{BB}+V_b}{R_4} + \frac{V_c-V_b}{R_5} = 0$$

$$I_{cc} + \frac{V_b-V_c}{R_5} - \frac{V_c}{R_6} = 0$$

Problem 3 (continued)

b) Similarly to part (a), use the nodal analysis technique to write 3 equations sufficient to solve for V_a, V_b, and V_c. To receive credit, you must write your answer in the box below. **[10 pts]**
DO NOT SOLVE THE EQUATIONS!



Current flowing through the voltage source V_{BB} cannot be expressed as a function of the node voltages V_a and V_b

⇒ use the "supernode" approach.

Applying Kirchhoff's Current Law to the supernode and node c:

supernode: $$\frac{V_{AA}-V_a}{R_1} + I_{BB} + \frac{-V_{cc}-V_b}{R_3} + I_{cc} = 0$$

node c: $$\frac{V_b-V_c}{R_4} + I_{cc} = 0$$

Need one more equation in order to be able to solve for the 3 unknowns:

$$V_b - V_a = V_{BB}$$

Write the nodal equations here:

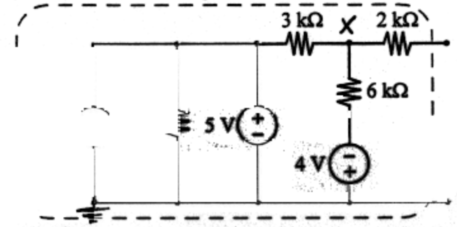
$$\frac{V_{AA}-V_a}{R_1} + I_{BB} - \frac{V_{cc}+V_b}{R_3} + I_{cc} = 0$$

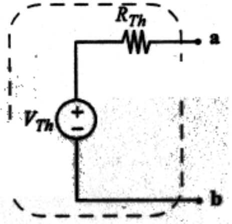
$$\frac{V_b-V_c}{R_4} + I_{cc} = 0$$

$$V_b - V_a = V_{BB}$$

Problem 4: Thevenin and Norton Equivalent Circuits [25 points]

a) Find the Thevenin Equivalent Circuit for the following circuit. [10 pts]





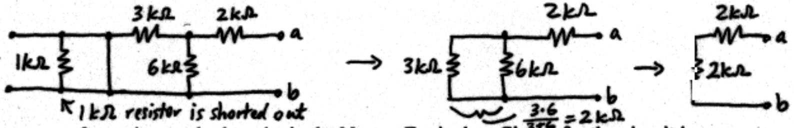
The open-circuit voltage, V_{oc}, is equal to V_{ab}, which is equal to V_{xb} since no current is flowing through the 2kΩ resistor. Applying KCL to node x (defining node b as the reference node)

$$\rightarrow \frac{5-V_x}{3} + \frac{-4-V_x}{6} = 0 \Rightarrow 6 = 3V_x \Rightarrow V_x = 2V \qquad \therefore V_{oc} = V_{Th} = 2V$$

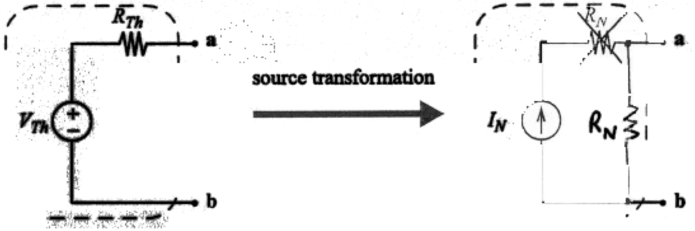
V_{Th} = 2 V

R_{Th} = 4 kΩ

To find R_{Th}, set all the independent sources to zero:



b) Use the source transformation method to obtain the Norton Equivalent Circuit for the circuit in part (a). [5 pts]



$$R_N = R_{Th} = 4k\Omega$$

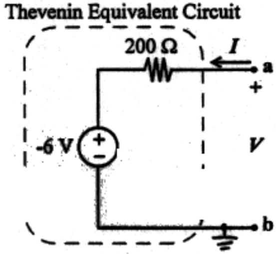
$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{2V}{4k\Omega} = 0.5mA$$

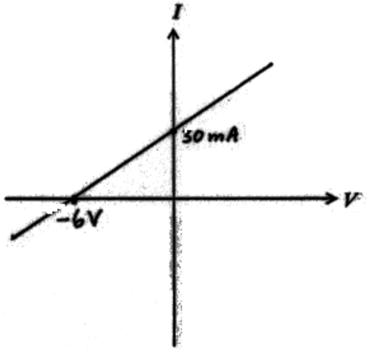
I_N = 0.5 mA

R_N = 4 kΩ

Problem 4 (continued)

c) The Thevenin Equivalent Circuit for a certain linear circuit is given below. Plot the current (I) versus the output voltage (V) for the circuit, **labelling the y-intercept and x-intercept.** [5 pts]



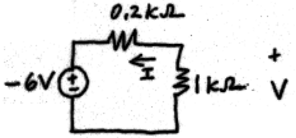


When $I=0$, $V=-6V$

When $V=0$ (i.e. terminals a and b shorted together), $I = \frac{0-(-6V)}{200}$

$I = 30\ mA$

d) The circuit in part (c) is connected to a 1 kΩ load resistor (placed between the terminals a and b). Find the power absorbed in the load resistor, P_{1k}. [5 pts]



P_{1k} = 25 mW

Using voltage-divider formula, $V = \frac{1000}{1000+200}(-6) = -5V$

$$P = IV = \left(\frac{V}{R}\right)V = \frac{V^2}{R} = \frac{(-5)^2}{1000} = 25\ mW$$