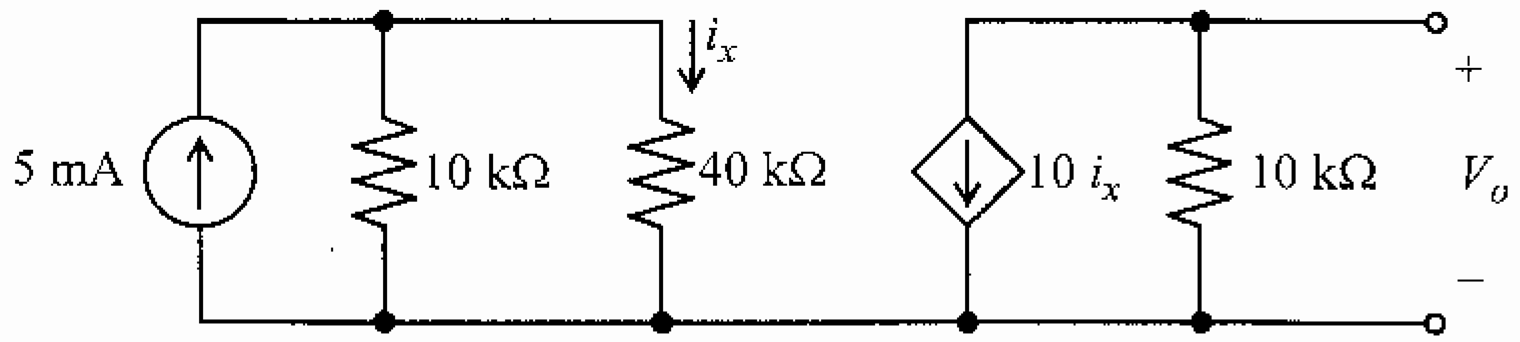
Problem 1 **Circuits with Dependent Sources [20 points]**

a) Find V_o. **[4 pts]**



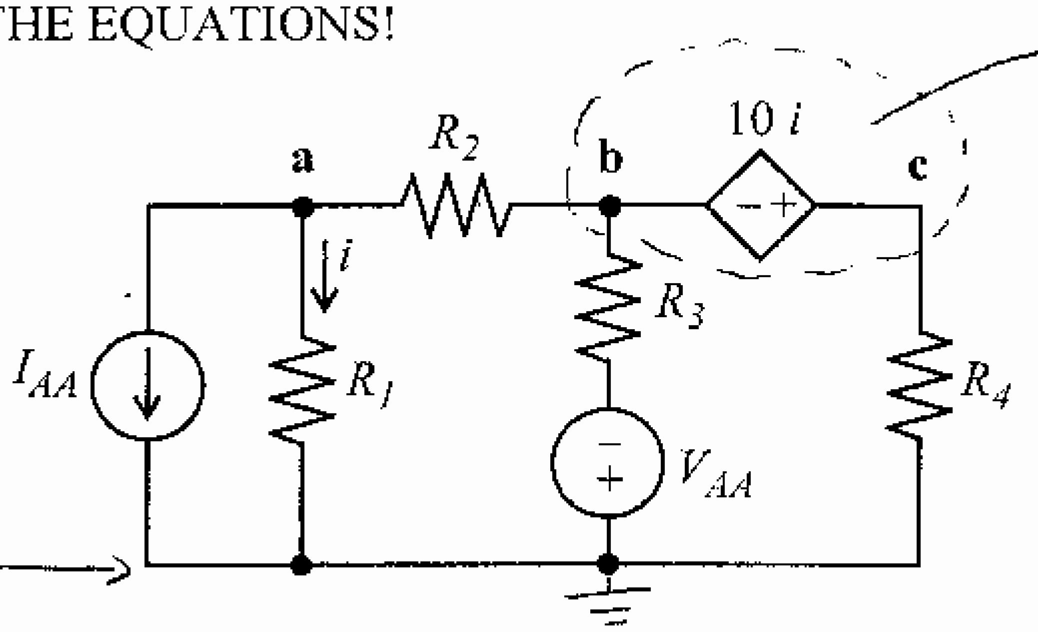
Current divider formula: $i_x = \dfrac{10\,k\Omega}{10\,k\Omega + 40\,k\Omega}(5mA)$
$= 1\,mA$

$V_o =$ $\underline{-100}$ V

$V_0 = (-10\,i_x)(10\,k\Omega) = (-10 \times 1\,mA)(10\,k\Omega) = -100\,V$

b) In the circuit below, the independent source values and resistances are known.
Use the **nodal analysis technique** to write **3 equations sufficient to solve for V_a, V_b, and V_c**.
To receive credit, you must write your answer in the box below. **[6 pts]**
DO NOT SOLVE THE EQUATIONS!

"supernode"



reference node

$i = \dfrac{V_a}{R_1}$ $\Rightarrow$ Value of dependent voltage source $= 10\,i$
$= 10\,(V_a/R_1)$

Write the nodal equations here: Note that the only unknowns in these equations are V_a, V_b, V_c.

KCL applied

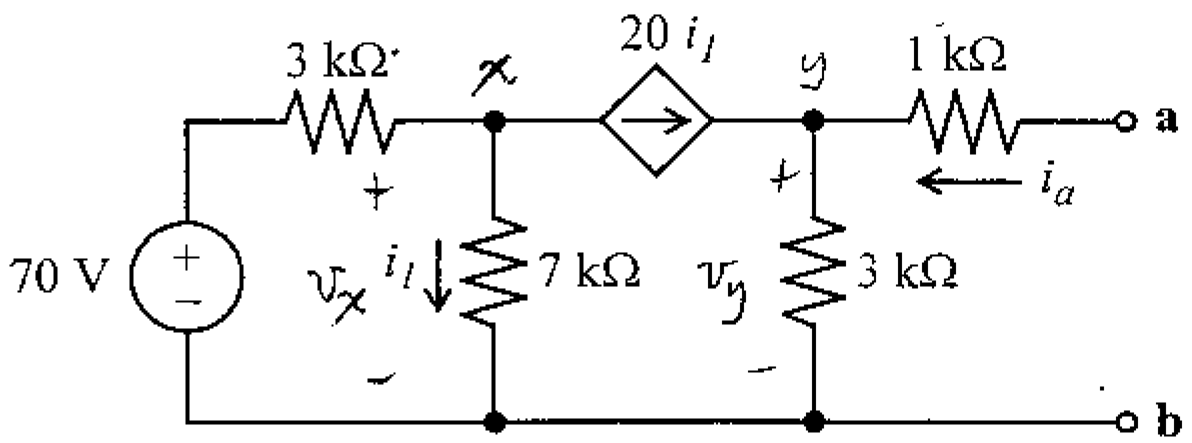
node a: $I_{AA} + \dfrac{V_a}{R_1} + \dfrac{V_a - V_b}{R_2} = 0$

supernode: $\dfrac{V_b - V_a}{R_2} + \dfrac{V_b + V_{AA}}{R_3} + \dfrac{V_c}{R_4} = 0$

relationship due to dependent source: $V_c - V_b = 10\,\dfrac{V_a}{R_1}$

Problem 1 (continued)

c) Consider the following circuit:



i) Find the voltage V_{ab}. **[5 pts]**

V_{ab} = 60 V

$$i_1 = \frac{v_x}{7k\Omega}$$

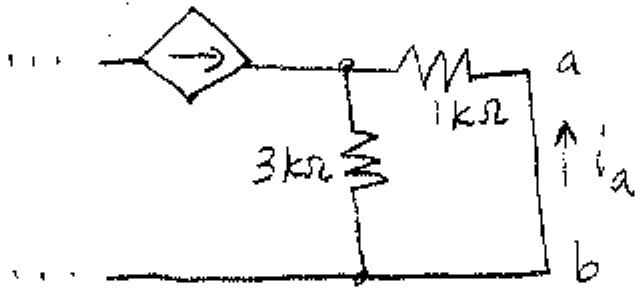
Applying KCL to node x: $\frac{70 - v_x}{3k\Omega} = i_1 + 20 i_1 = 21 i_1 = 21 \left(\frac{v_x}{7k\Omega} \right)$

$$70 - v_x = 9 v_x \Rightarrow v_x = 7V$$

$i_1 = \frac{7V}{7k\Omega} = 1mA$; $i_a = 0 \Rightarrow V_{ab} = v_y = 20 i_1 (3k\Omega) = 60V$

ii) What is the current i_a when the terminals **a** and **b** are shorted together? **[3 pts]**

$20 i_1 = 20 \, mA$



i_a = −15 mA

Current divider formula:

$$i_a = \frac{3k\Omega}{3k\Omega + 1k\Omega} (-20mA) = -15mA$$

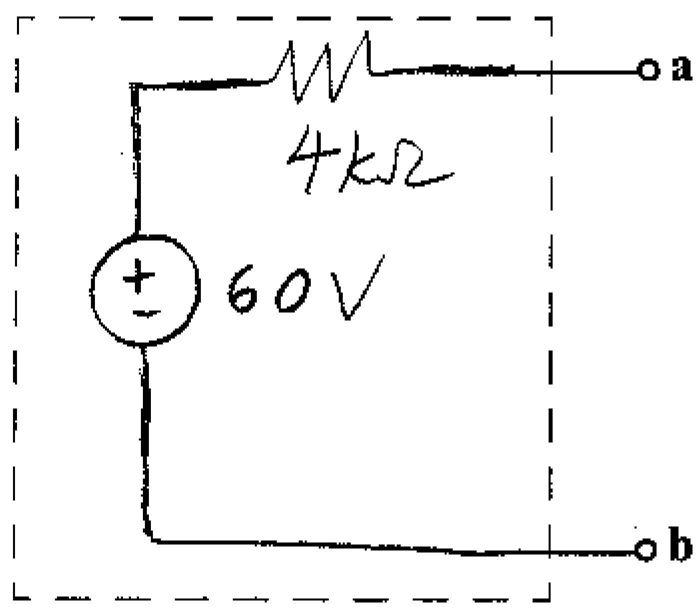
iii) Draw the Thevenin Equivalent Circuit. **[2 pts]**

$V_{Th} = V_{oc} = V_{ab}$ from part (i)

$$R_{Th} = -\frac{V_{oc}}{I_{sc}} = \frac{V_{ab} \text{ from part (i)}}{i_a \text{ from part (ii)}}$$

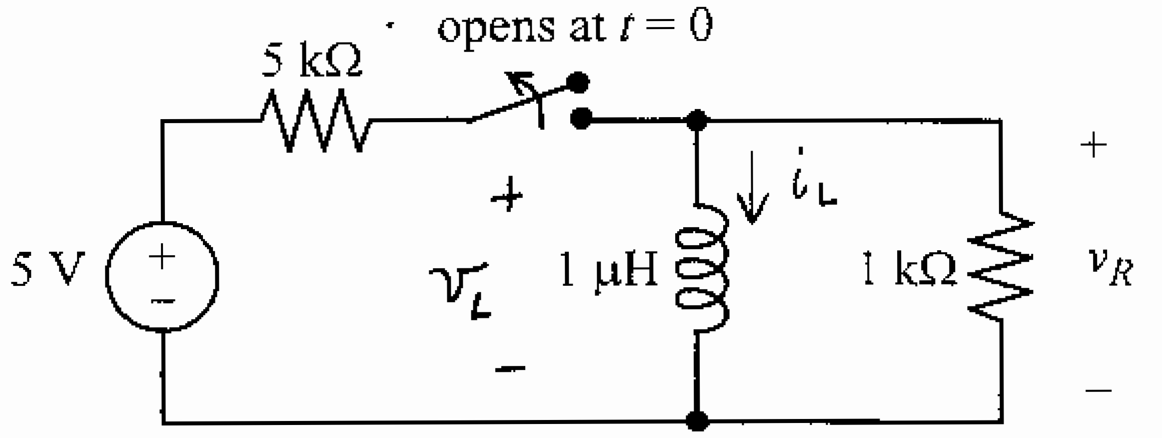
$$= -\frac{60V}{-15mA} = 4k\Omega$$

Thevenin Equivalent Circuit:



Problem 2: Transient Response [30 points]

a) In the circuit below, the switch has been in the closed position for a long time.



$v_R(0+) = (-i_L(0+))(1k\Omega)$
$= (-1mA)(1k\Omega) = -1V$

i) Find the value of v_R just after the switch opens ($t = 0^+$). **[3 pts]**

Before the switch is opened, the voltage across the inductor is zero ($v_L = L\frac{di_L}{dt} = 0$), i.e. the 1 kΩ resistor is "shorted out" by the inductor.

$v_R(0^+) =$ −1 V

⇒ The current flowing through the inductor $i_L(0-) = 5V/5k\Omega = \underline{1mA}$

$i_L(0+) = i_L(0-) = 1mA$, since inductor current cannot change instantaneously.

ii) How much energy is dissipated in the 1 kΩ resistor after the switch is opened? **[2 pts]**

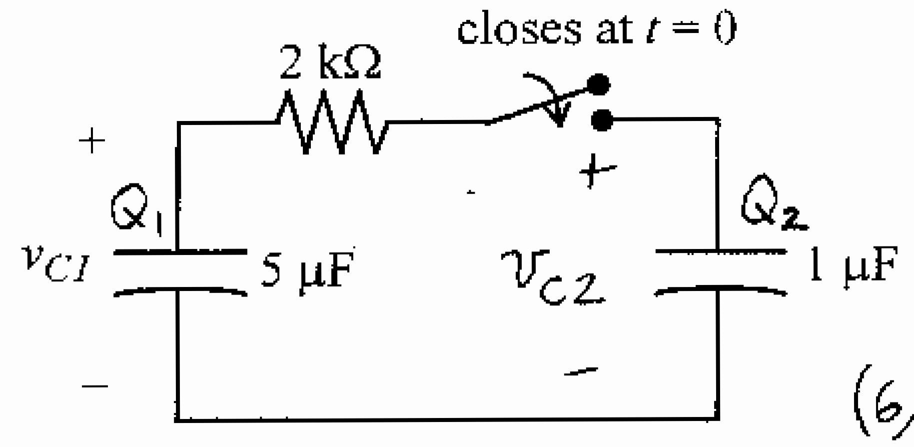
All of the energy which was stored in the inductor at $t=0$ is dissipated in the 1kΩ resistor after the switch is closed.

Energy dissipated = 0.5 pJ

$$\frac{1}{2}L[i(0)]^2 = \frac{1}{2}(10^{-6}\,H)(10^{-3}\,A)^2 = \frac{1}{2}\times 10^{-12}\ J$$

b) In the circuit below, the 5 μF capacitor is initially charged to 5 V ($v_{C1}(0^-) = 5$ V). (The 1 μF capacitor is initially uncharged.) The switch is then closed at time $t = 0$. What is the final value of v_{C1}? **[5 pts]**

Charge stored on a capacitor is $Q = CV$



$(6\mu F)v_{C1final} = (5\mu F)(5V)$

$v_{C1\,final} = \frac{5}{6}(5V) = 4.17\,V$

Conservation of charge

$\Rightarrow Q_{1\,final} + Q_{2\,final} = Q_{1\,initial}$

$(5\mu F)v_{C_1\,final} + (1\mu F)v_{C2final} = (5\mu F)v_{C1\,initial}$

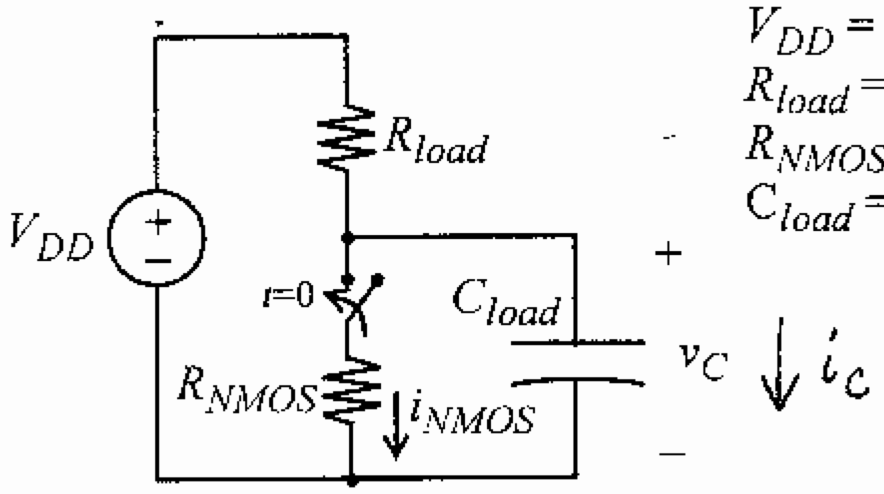
Final value of v_{C1} = 4.2 V

In the final state ($t = \infty$), the voltages across the capacitors are equal, i.e. $\underline{v_{C1final} = v_{C2\,final}}$

Problem 2 (continued)

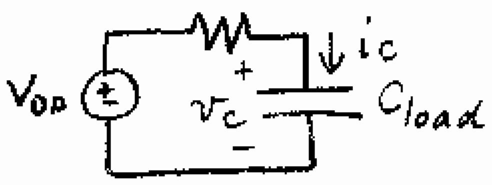
c) The following is a circuit model for an NMOS inverter, in which the transistor is turned on at time $t = 0$:

NMOS is off for $t < 0$
$\Rightarrow i_{NMOS} = 0$ for $t \le 0^-$



$V_{DD} = 5$ V
$R_{load} = 90$ kΩ
$R_{NMOS} = 10$ kΩ
$C_{load} = 10$ nF

i) What is the value of v_C at $t = 0^-$? **[3 pts]**



In steady state (before the switch closes), $i_c = 0$
$\Rightarrow v_c = V_{DD}$

$v_C(0^-) =$ 5 V

ii) What is the value of i_{NMOS} at $t = 0^+$? **[3 pts]**

$v_c(0+) = v_c(0^-)$, since capacitor voltage cannot change instantaneously

$i_{NMOS}(0+) = v_c(0+)/R_{NMOS} = 5V/10\,k\Omega$

$i_{NMOS}(0^-) =$ 0.5 mA

iii) What is the final value of v_C? **[3 pts]**

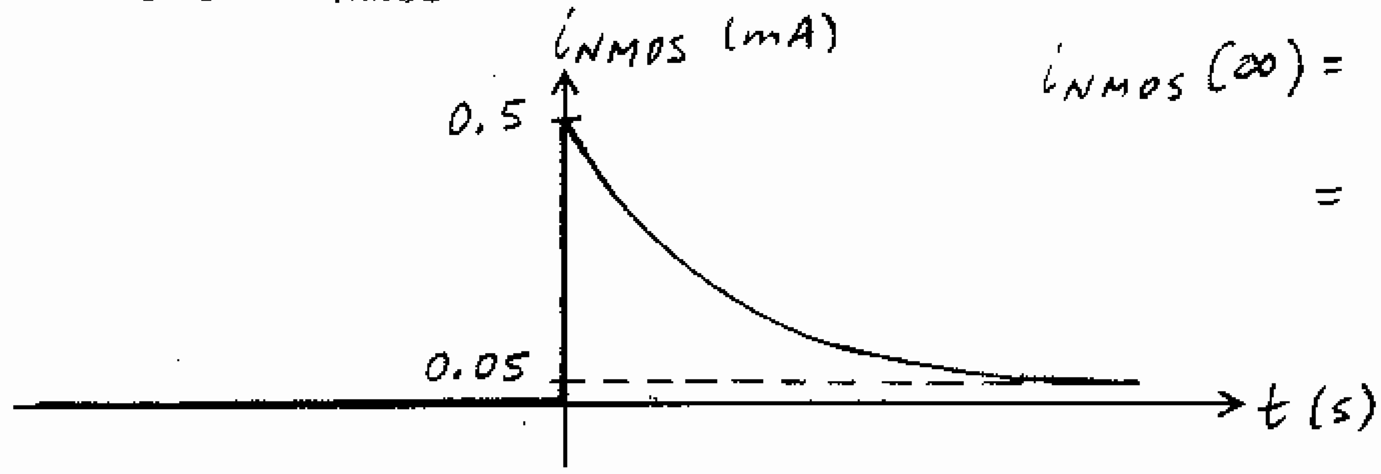
In steady state, $i_c = 0$

Therefore, we have a simple voltage divider:
$v_c(\infty) = [R_{NMOS}/(R_{NMOS} + R_{load})]V_{DD}$

$v_c(\infty) = \frac{10}{10+90}(5V) = 0.5V$

final value of $v_C =$ 0.5 V

iv) Neatly sketch the graph of i_{NMOS} **for all *t***, labelling the axes. **[5 pts]**



$$i_{NMOS}(\infty) = \frac{v_c(\infty)}{R_{NMOS}} = \frac{0.5V}{10\,k\Omega} = 0.05\,mA$$

v) Write an equation for i_{NMOS} as a function of time, for $t > 0$. **[6 pts]**

$$i_{NMOS} = i_{NMOS\,final} + [i_{NMOS}(0+) - i_{NMOS\,final}]\,e^{-t/R_{eq}C}$$

R_{eq} is equivalent resistance seen by the capacitor : $R_{eq} = R_{load} \| R_{NMOS}$

Equation for i_{NMOS}: $0.05 + 0.45e^{-t/9\times10^{-5}}$ mA

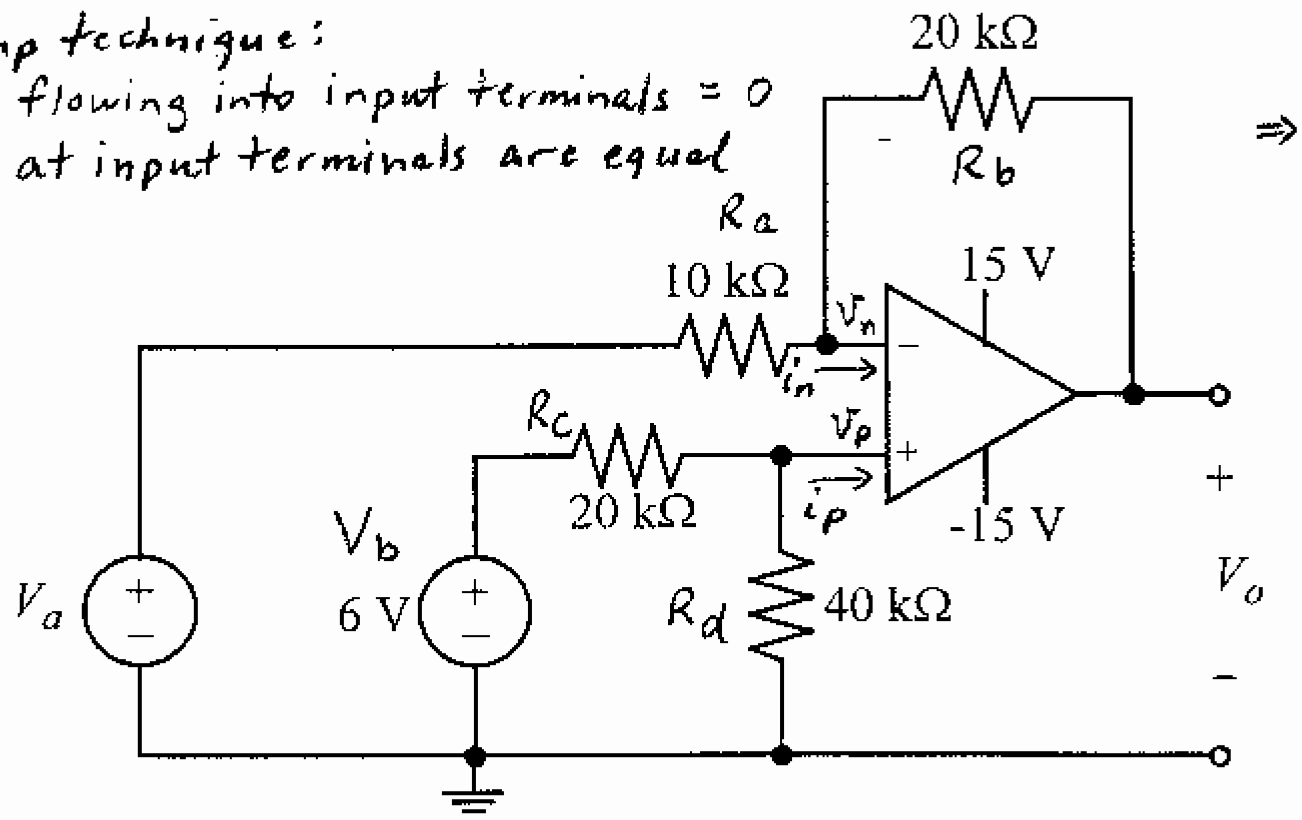
$$R_{eq} = \frac{(90)(10)}{90+10} = 9\,k\Omega \;;\; R_{eq}C = (9\,k\Omega)(10\,nF) = 9\times10^{-5}s$$

Problem 3: Op-Amp Circuits [25 points]

Assume the op-amps in this problem are ideal.

a) Consider the following circuit:

ideal op-amp technique:
- *currents flowing into input terminals = 0*
- *voltages at input terminals are equal*



Apply KCL at (-) node:

$$\frac{V_a - v_n}{R_a} + \frac{V_o - v_n}{R_b} = 0$$

$$\Rightarrow V_o = -\frac{R_b}{R_a}(V_a - v_n) + v_n$$

Since $i_p = 0$, we can use the voltage-divider formula:

$$v_p = \frac{R_d}{R_c + R_d} V_b = v_n$$

$$\Rightarrow V_o = -\frac{R_b}{R_a}\left[V_a - \frac{R_d}{R_c + R_d} V_b\right] + \frac{R_d}{R_c + R_d} V_b$$

i) Find an expression for V_o as a function of V_a. **[6 pts]**

This is a difference-amplifier circuit (which you've studied in the lab)

with $\frac{R_a}{R_b} = \frac{R_c}{R_d} = \frac{1}{2}$

$$V_o = \frac{R_b}{R_a}(6 - V_a) = 2(6 - V_a)$$

Expression for V_o: $12 - 2V_a$

ii) Find V_o for $V_a = 2$ V. **[3 pts]**

$$V_o = 12 - 2(2) = 8 \text{ V}$$

$V_o =$ 8 V

iii) For what values of V_a will the op-amp be saturated? **[6 pts]**

$V_o = 12 - 2V_a \Rightarrow V_a = \frac{12 - V_o}{2}$

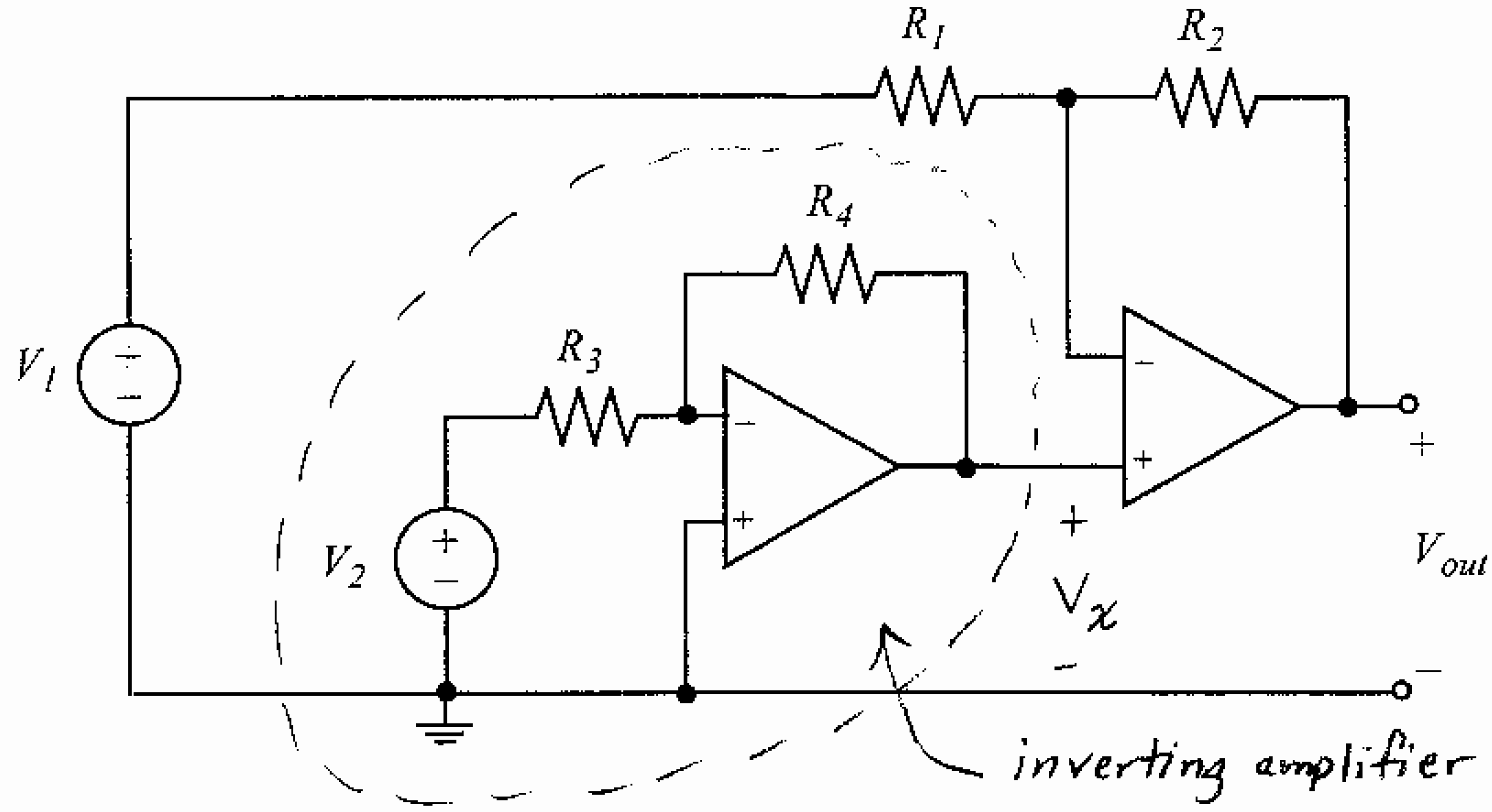
V_o saturated at 15V : $V_a \le \frac{12-15}{2} = -\frac{3}{2}$ V

V_o saturated at -15V : $V_a \ge \frac{12-(-15)}{2} = \frac{27}{2}$ V

Values of V_a for which the op-amp will be saturated: $V_a \le -1.5$ V ; $V_a \ge 13.5$ V

Problem 3 (continued)

b) In the following circuit, the op-amps are operating linearly.



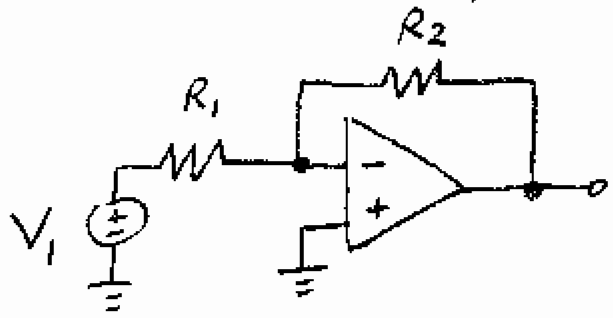
inverting amplifier: $V_x = -\frac{R_4}{R_3} V_2$

Find V_{out} in terms of V_1, V_2, R_1, R_2, R_3, R_4. **[10 pts]**

(Hint: The superposition method might be helpful here.)

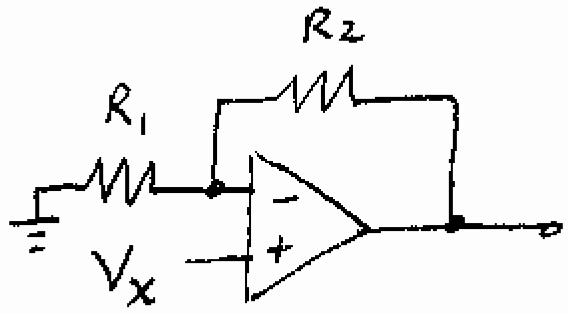
Find the individual contributions of each voltage source:

i) Set V_2 to 0 V: $V_x = 0$, so the circuit simplifies to a simple inverting amplifier



$$V_{out}' = -\frac{R_2}{R_1} V_1$$

ii) Set V_1 to 0 V: Circuit simplifies to simple non-inverting amplifier



$$V_{out}'' = \left(1 + \frac{R_2}{R_1}\right) V_x = \left(1 + \frac{R_2}{R_1}\right)\left(-\frac{R_4}{R_3} V_2\right)$$

Add the contributions of each source together:

$$V_{out} = V_{out}' + V_{out}'' = -\frac{R_2}{R_1} V_1 - \frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right) V_2$$

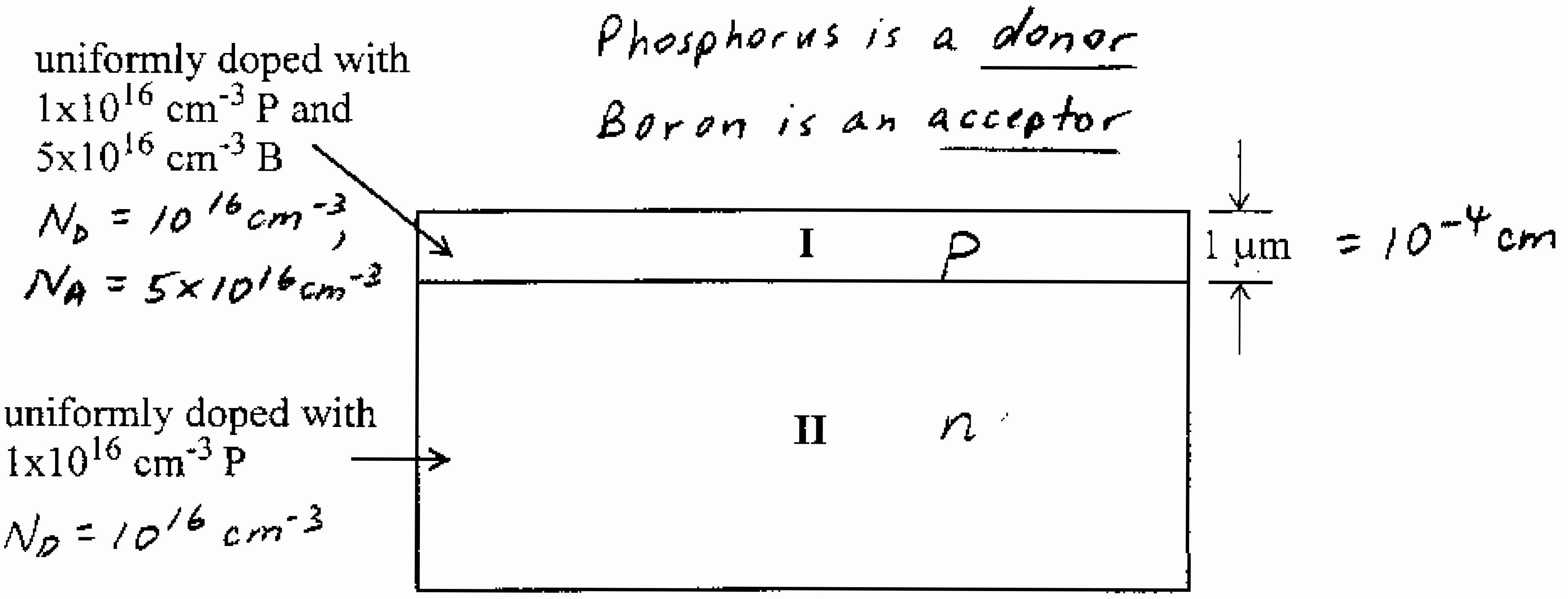
$$\boxed{V_{out} = -\frac{R_2}{R_1} V_1 - \frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right) V_2}$$

Problem 4: Semiconductor properties; p-n diodes [25 points]

a) Consider a silicon sample maintained at 300K under equilibrium conditions, uniformly doped with $1\text{x}10^{16}$ cm^{-3} phosphorus atoms. The surface region of the sample is **additionally** doped uniformly with $5\text{x}10^{16}$ cm^{-3} boron atoms, to a depth of 1 μm, as shown in the figure below.

Phosphorus is a donor

Boron is an acceptor



Schematic cross-sectional view of silicon sample

i) In the figure above, indicate the type of the regions (I and II) by labelling them as "n" or "p" type. **[2 pts]**

ii) What are the electron and hole concentrations in Region I? **[5 pts]**

$N_A > N_D$, and $N_A >> n_i$ so

$$p = N_A - N_D = 5\times10^{16} - 1\times10^{16} = 4\times10^{16}$$

$$pn = n_i^2 \Rightarrow n = \frac{n_i^2}{p} = \frac{(1.45\times10^{10})^2}{4\times10^{16}} = 5256$$

$n =$ 5256 cm^{-3}

$p =$ 4×10^{16} cm^{-3}

iii) What is the sheet resistance of Region I? **[5 pts]**

$$\rho = \frac{1}{q\mu_n n + q\mu_p p} \simeq \frac{1}{q\mu_p p}$$

From plot on Page 2, $\mu_p \cong 350$ $cm^2/V\cdot s$ for $N_A + N_D = 6\times10^{16} cm^{-3}$

$$R_s = \frac{\rho}{t} = \frac{1}{q\mu_p p t} = \left[(1.602\times10^{-19})(350)(4\times10^{16})(10^{-4})\right]^{-1} = 4458\,\Omega/\square$$

$R_s =$ 4458 Ω/square

iv) Suppose any voltage between 0 V and 5 V can be applied to Region I. What fixed voltage ("bias") would you apply to Region II, to guarantee that no current would ever flow between Region I and Region II? **Briefly explain your answer. [3 pts]**

To prevent current from flowing, we need to ensure that the p-n junction will never be forward biased. Thus, the n-type region must be biased at 5V or higher.

Region II bias voltage = 5 V

Problem 4 **(continued)**

b) If a diode is operated only within a small range of forward-bias voltages, its behavior can be accurately modelled by a resistor, whose value is dependent on the bias voltage. Derive an expression for the diode "small-signal" resistance:

$$R_{diode} = \left(\frac{\partial I}{\partial V}\right)^{-1}$$

in terms of the saturation current I_S, the bias voltage V, and the absolute temperature T. **[5 pts]**

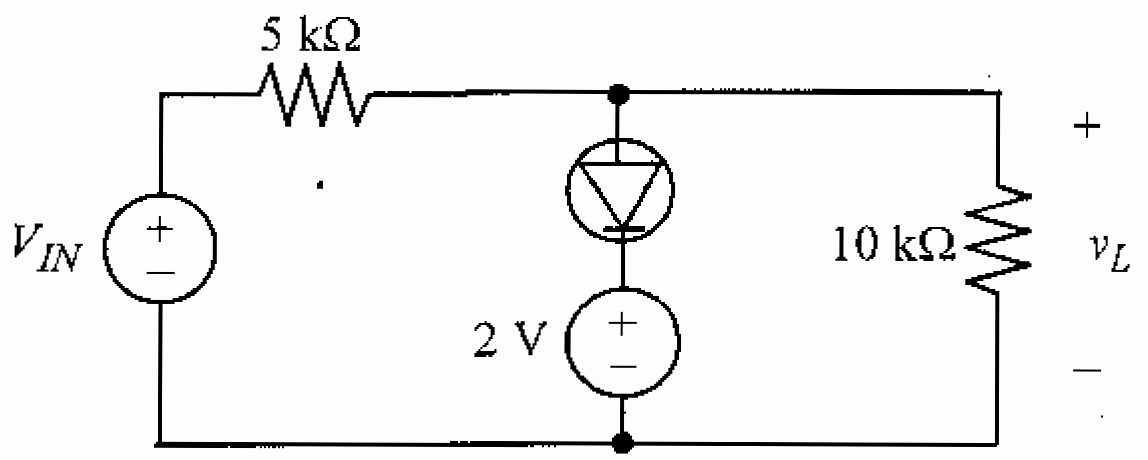
$$I = I_S\left(e^{qV/kT} - 1\right)$$

$$\frac{\partial I}{\partial V} = I_S\left(\frac{q}{kT}\right)e^{qV/kT}$$

$$R_{diode} = \frac{kT}{qI_S}\, e^{-qV/kT}$$

$$\boxed{R_{diode} = \left(\frac{kT}{qI_S}\right)e^{-qV/kT}}$$

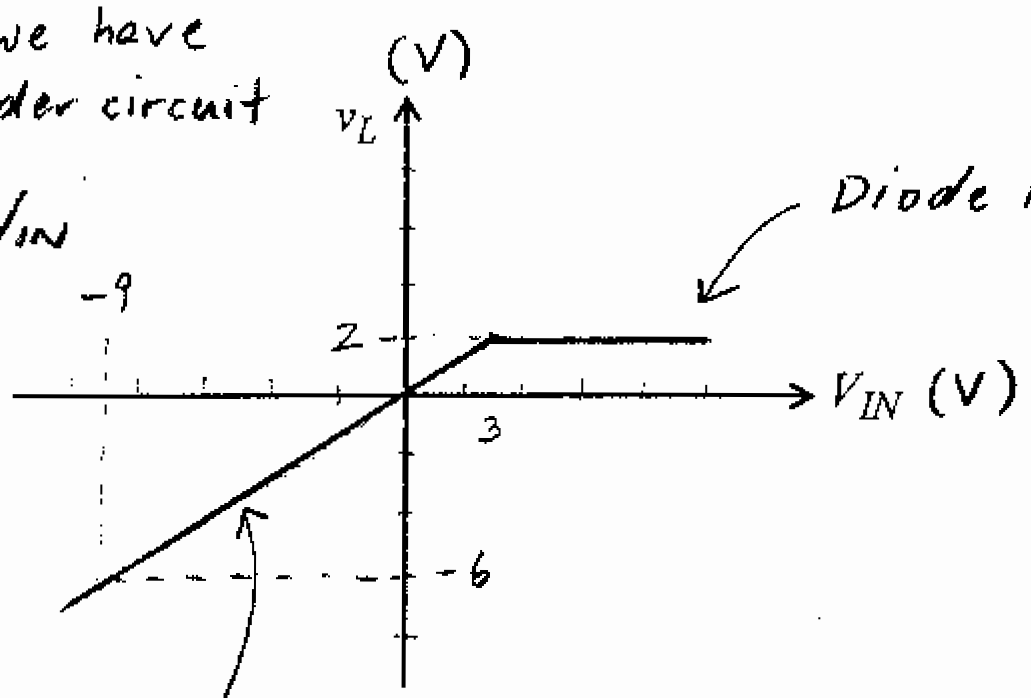
c) Plot v_L vs. V_{IN} for **-10 V < V_{IN} < 10 V** on the axes provided, for the circuit below. Note that the diode is a perfect rectifier. **Label the axes. [5 pts]**



When the diode is off, we have a simple voltage-divider circuit

$$v_L = \frac{10}{5+10}V_{IN} = \frac{2}{3}V_{IN}$$

Diode turns on when v_L reaches 2V, i.e. when $\frac{2}{3}V_{IN} = 2V$, or $V_{IN} = 3V$



Diode limits v_L to 2V

slope = $\frac{2}{3}$