

**Midterm Exam # 2**  
**April 15, 2004**  
**Time Allowed: 80 minutes**

Name: Solution, \_\_\_\_\_  
Last First

Student ID #: \_\_\_\_\_, Signature: \_\_\_\_\_

Discussion Section: \_\_\_\_\_

This is a closed-book exam, except for use of two 8.5 x 11 inch sheets of your notes. Show all your work to receive full or partial credit. Write your answers clearly in the spaces provided.

Problem #:	Points:
1	/10
2	/20
3	/20
Total	/50

1.

a) (5 points)

A silicon sample is uniformly doped with Boron to a concentration of  $10^{16}$  atoms/cm<sup>3</sup>. Determine the resistivity of the sample at room temperature.

Use electron mobility =  $\mu_n = 1000$  cm<sup>2</sup>/v-s, hole mobility =  $\mu_p = 400$  cm<sup>2</sup>/v-s,

$q = 1.6 \cdot 10^{-19}$  C and  $n_i = 10^{10}$  at room temperature.

$$N_a = 10^{16} \text{ cm}^{-3} \quad p = 10^{16} \text{ cm}^{-3} \gg n_i$$

p-type :

$$\rho = \frac{1}{q p \mu_p} = \frac{1}{1.6 \times 10^{-19} \text{ C} \cdot 10^{16} \text{ cm}^{-3} \cdot 400 \text{ cm}^2/\text{v-s}} = \frac{1}{0.64} \text{ } \Omega \cdot \text{cm} \\ = 1.56 \text{ } \Omega \cdot \text{cm}$$

b) (5 points)

The same sample is then to be counter doped to a depth of  $5 \mu\text{m}$  with Arsenic atoms to create a resistor technology with resistance of  $100 \Omega/\square$ .

Determine the required Arsenic doping density.

$$R_s = \frac{\rho'}{t}$$

$$\rho' = R_s t = 100 \text{ } \Omega \cdot 5 \mu\text{m} = 0.05 \text{ } \Omega \cdot \text{cm}$$

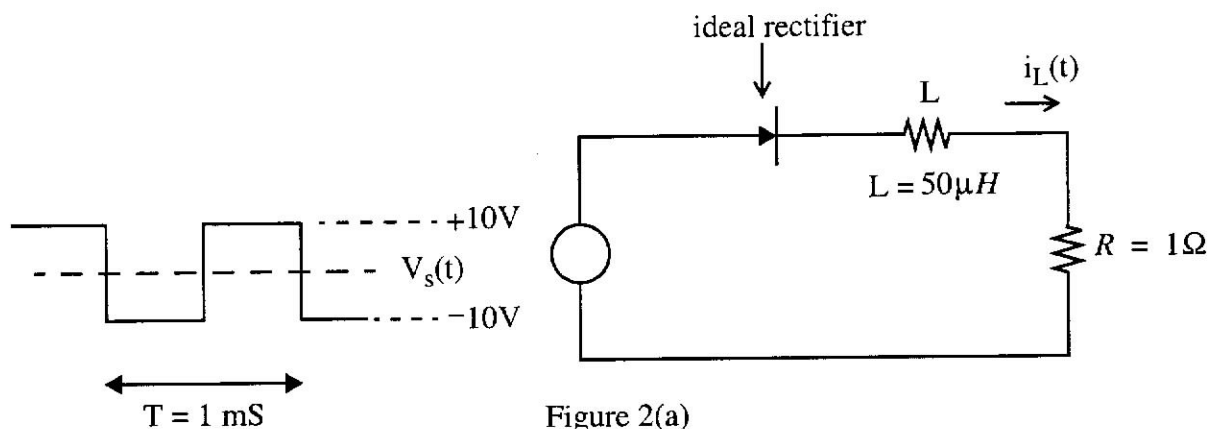
$\rho' \ll \rho \Rightarrow$  the sample becomes n-type

$$\rho' = \frac{1}{q n \mu_n + q p \mu_p} \approx \frac{1}{q n \mu_n} = \frac{1}{q (N_d - N_a) \mu_n}$$

$$\therefore N_d - N_a = n = \frac{1}{q \mu_n \rho'} = \frac{1}{1.6 \times 10^{-19} \text{ C} \cdot 1000 \text{ cm}^2/\text{v-s} \cdot 0.05 \text{ } \Omega \cdot \text{cm}} \\ = 1.25 \times 10^{17} \text{ cm}^{-3}$$

$$N_d = n + N_a = 1.25 \times 10^{17} \text{ cm}^{-3} + 10^{16} \text{ cm}^{-3} \\ = 1.35 \times 10^{17} \text{ cm}^{-3}$$

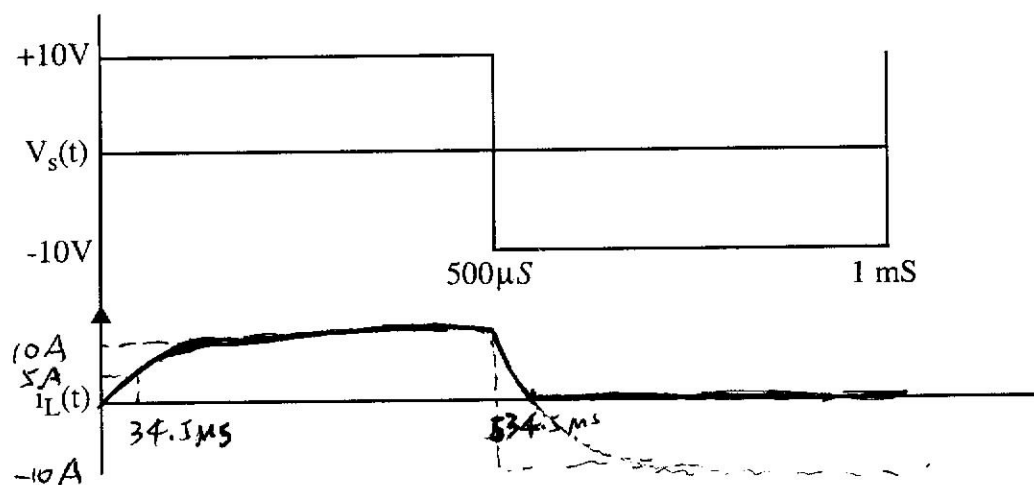
2.



a) (10 points)

The diode in Figure 2(a) is ideal. The waveform  $V_s(t)$  is a balanced square wave with amplitude of 10 V and period of 1mS. Take  $L = 50\mu H$  and  $R = 1\Omega$ .

The circuit operates in a periodic steady state. Sketch and carefully dimension one period of the  $i_L(t)$  waveform on the axes below. Make reasonable approximations.



$$\tau = \frac{L}{R} = 0.05 \text{ ms}$$

$$0.69 \tau = 0.0345 \text{ ms} = 34.5 \mu\text{s}$$

b) ( 10 points)

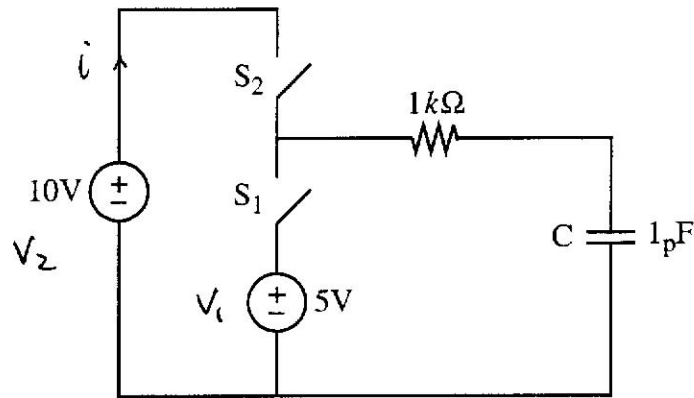


Figure 2(b)

In the circuit of Figure 2(b), switch  $S_1$  is initially closed and the circuit is in equilibrium. Switch  $S_1$  is then opened and switch  $S_2$  is closed for a sufficiently long time so that the circuit can be considered to be in equilibrium. How much energy is dissipated in the  $1\text{ k}\Omega$  resistor during the transient?

**Hint:** Think in terms of net charge and energy flow. Detailed transient analysis is **NOT** needed.

Energy changed in capacitor

$$\begin{aligned}\Delta W_c &= \frac{1}{2} C V_2^2 - \frac{1}{2} C V_1^2 \\ &= \frac{1}{2} C (100\text{ V}^2 - 25\text{ V}^2) \\ &= \frac{1}{2} \times 10^{-12}\text{ F} \times 75\text{ V}^2 \\ &= 3.75 \times 10^{-11}\text{ J}\end{aligned}$$

Total energy delivered by voltage source  $V_2$  :

$$\begin{aligned}W_{\text{source}} &= \int V_2 i dt = V_2 \int i dt = V_2 \Delta Q \\ &= V_2 (C V_2 - C V_1) \\ &= 5 \times 10^{-11}\text{ J}\end{aligned}$$

Energy dissipated in resistor

$$W_R = W_S - \Delta W_c = 1.25 \times 10^{-11}\text{ J}$$

3.

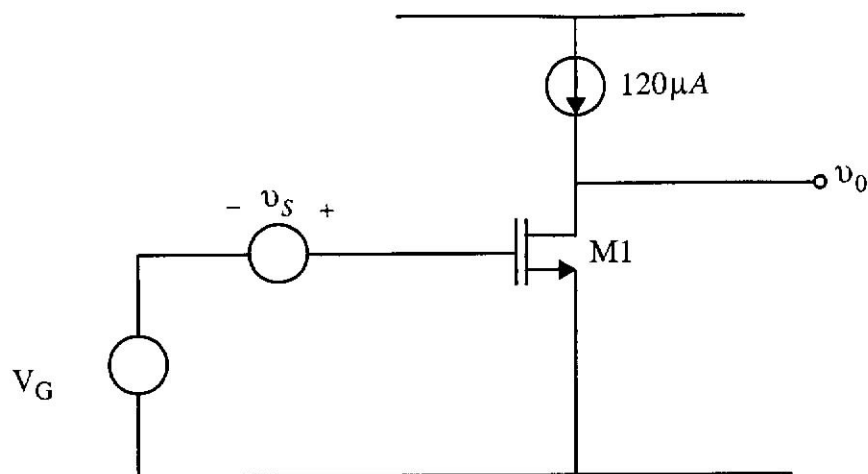


Figure 3

$$\lambda = 0.1 \text{ V}^{-1}$$

$$V_T = 0.5 \text{ V}$$

$$k' = 100 \mu\text{A}/\text{V}^2$$

$$\frac{W}{L} = 2$$

Mosfet M1 in Figure 3 is modeled by  $i_D = \frac{1}{2} k' \frac{W}{L} (v_{GS} - V_T)^2 (1 + \lambda v_{DS})$  in saturation with parameters listed in Figure 3.

a) (5 points)

Determine the required bias voltage  $V_G$  so that M1 is biased in saturation with  $V_{DS} = 2\text{V}$ . Take  $v_S = 0$  for this calculation.

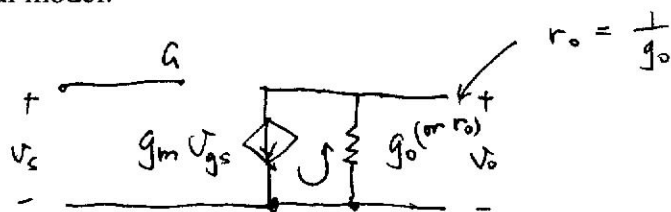
$$i_D = \frac{1}{2} k' \frac{W}{L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

$$\Rightarrow 120 \mu\text{A} = \frac{1}{2} \times 100 \mu\text{A}/\text{V}^2 \times 2 \times (V_{GS} - 0.5\text{V})^2 (1 + 0.1\text{V}^{-1} \times 2\text{V})$$

$$\Rightarrow V_{GS} = 1.5 \text{ V}$$

b) (10 points)

Draw the small signal model for this circuit. Compute the parameters of this small signal model.



$$g_m = \frac{\partial i_D}{\partial v_{gs}} = k' \frac{W}{L} (v_{gs} - V_T) (1 + \lambda v_{ds})$$

$$= 100 \mu\text{A}/\text{V}^2 \times 2 \times (1.5\text{V} - 1\text{V}) (1 + 0.1\text{V}^{-1} \times 2\text{V})$$

$$= 2.4 \times 10^{-4} \text{ S}$$

$$g_o = \frac{\partial i_D}{\partial v_{ds}} = \frac{1}{2} k' \frac{W}{L} (v_{gs} - V_T)^2 \cdot \lambda$$

$$= \frac{1}{2} \times 100 \mu\text{A}/\text{V}^2 \times 2 \times (1.5\text{V} - 1\text{V})^2 \times 0.1\text{V}^{-1}$$

$$= 10^{-5} \text{ S}$$

c) (5 points)

Determine the small signal gain  $A_v = \frac{v_o}{v_s}$ .

$$v_o = -g_m v_{gs} \cdot r_o$$

$$= -g_m v_s \cdot \frac{1}{g_o}$$

$$A_v = \frac{v_o}{v_s} = - \frac{g_m}{g_o} = - \frac{2.4 \times 10^{-4} \text{ S}}{10^{-5} \text{ S}} = -24$$