EECS 40, Spring 2005
Prof. Chang-Hasnain
Midterm #1

March 3, 2005
Total Time Allotted: 80 minutes

1. This is a closed book exam. However, you are allowed to bring one page (8.5" x 11"), double-sided notes
2. No electronic devices, i.e. calculators, cell phones, computers, etc.
3. Numerical answers within a factor of 1.5 will not get points deducted, provided the steps are all correct and the errors are due to the lack of a calculator. (e.g. if the correct answer is 1, the acceptable range will be 0.67~1.5).
4. SHOW all the steps on the exam. Answers without steps will be given only a small percentage of credits. Partial credits will be given if you have proper steps but no final answers.
5. Write your answers in the spaces (lines, boxes or plots) provided.
6. Remember to put down units. Points will be taken off for answers without units.
7. **NOTE: nH=10^{-9} H; pF=10^{-12} F; GHz=10^{9} Hz; MHz=10^{6} Hz**

Last (Family) Name:____

First Name: ____

Student ID: ____

Signature: ____

Score:	
Problem 1 (20 pts)	
Problem 2 (20 pts):	
Problem 3 (30 pts):	
Problem 4 (30 pts):	
Total 100 pts	

1: A voltage source with voltage V is connected to N identical resistors with resistance R.

(a) All of the resistors are connected in parallel, as in Figure 1a. What is the equivalent resistance R_{eq}? What is the total power consumed in the resistors?

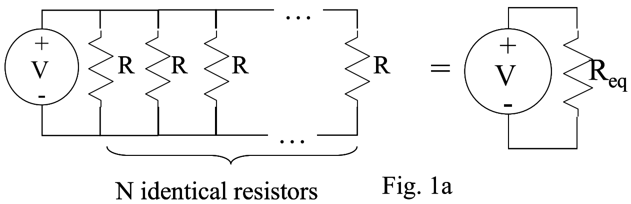


Fig. 1a

Solution:
The resistors are in parallel so:

$$\frac{1}{R_{eq}} = \sum_{n=1}^{N} \frac{1}{R} = \frac{N}{R}$$
$$\Rightarrow R_{eq} = \frac{R}{N}$$

Thus, the total power dissipated is:

$$P = \frac{V^2}{R_{eq}} = N\frac{V^2}{R}$$

Answer: R_{eq}= R/N
Power = NV^2/R

(b) All the resistors are all connected in series, as in Figure 1b. What is the equivalent resistance R_{eq}? What is the total power consumed in the resistors?

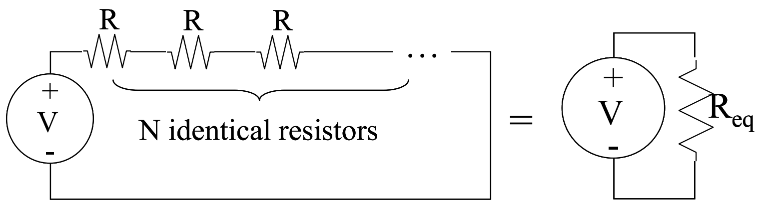


Fig. 1b

Solution:
The resistors are in series so:

$$R_{eq} = NR$$

Thus, the total power dissipated is:

$$P = \frac{V^2}{R_{eq}} = \frac{1}{N}\frac{V^2}{R}$$

Answer: R_{eq}= NR
Power = V^2/NR

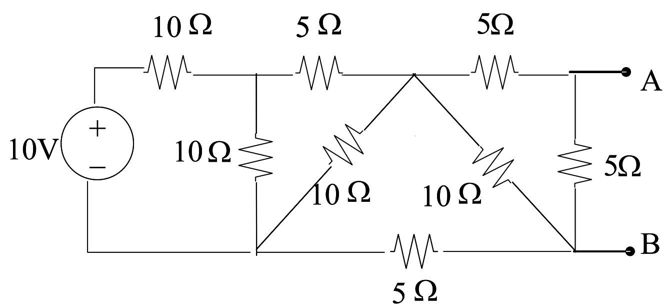
2. Equivalent circuit

(a) What is equivalent resistance R_{eq} for points A-B?

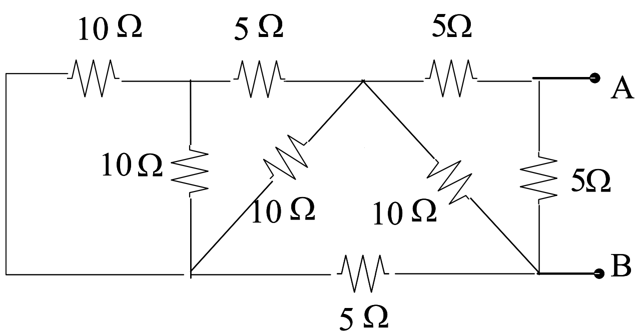
(b) What is the open circuit voltage across points A-B?

(c) What is the short circuit current through points A-B?

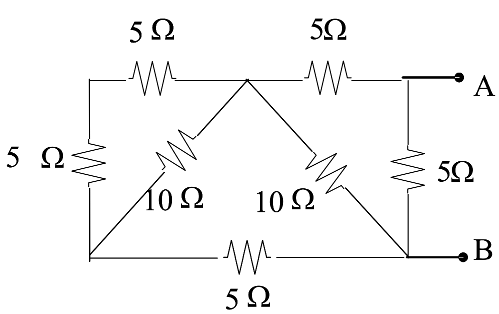
(d) Draw both Thevenin equivalent circuits.



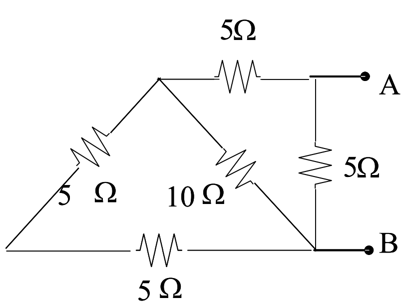
(a) To get the equivalent resistance, we short voltage source:



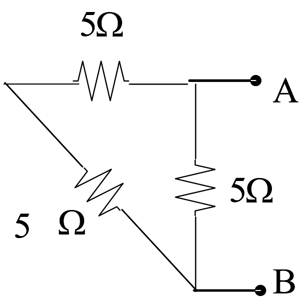
Two 10 Ω in parallel is equivalent to one resistor of 5 Ω. So the equivalent circuit is:



Two 5 Ω in series and then in parallel with 10 Ω. R=(5 + 5) || 10 = 5 Ω. So the equivalent circuit is:



Again two 5 Ω in series then in parallel with one 10 Ω resistor. R=(5 + 5) || 10 = 5Ω. So the equivalent circuit is:



Finally, two 5Ω in series then in parallel with another 5Ω.

$$R_{eq} = (5+5) \,||\, 5 = 10/3\Omega$$

(b) Open circuit

We set nodes C, D, E as shown below besides nodes A and B.

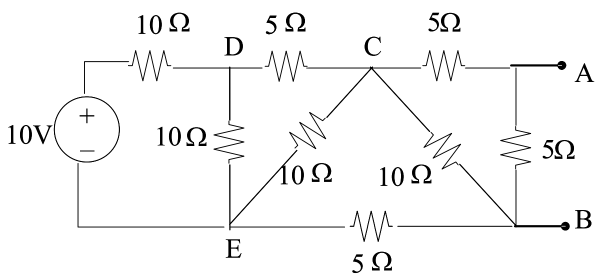


Figure 1

Now we try to find the equivalent resistance to the right of D and E.

Two 5Ω in series then in parallel with one 10Ω, R=(5 + 5) || 10 = 5Ω. We get:

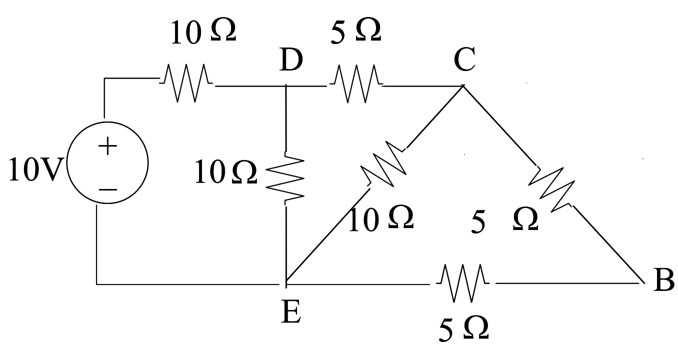


Figure 2

To further simplify that, two 5Ω in series then in parallel with 10Ω, R=(5 + 5) || 10 = 5Ω. We get:

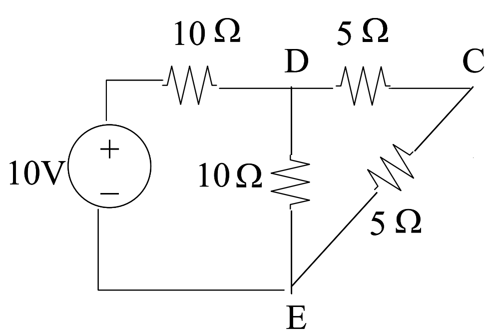
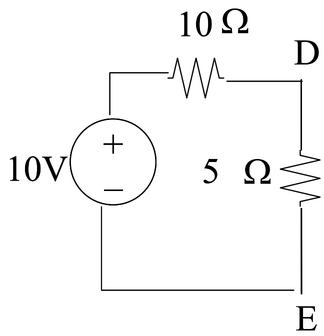


Figure 3

To further simplify, 5Ω in series then in parallel, R=(5 + 5) || 10 = 5Ω, we have:



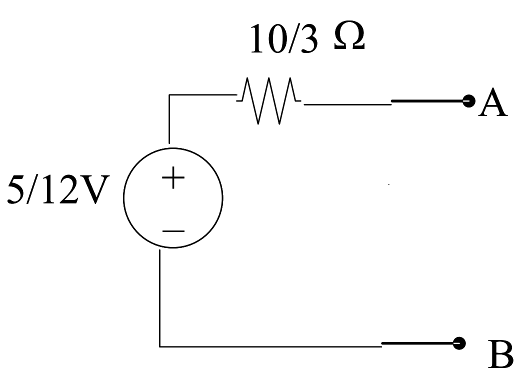
Then $V_{DE} = \frac{5}{10+5} \times 10 = \frac{10}{3}\mathrm{V}$

From Figure 3, we know $V_{CE} = \frac{5}{5+5} \times V_{DE} = \frac{5}{3}\mathrm{V}$

From Figure 2, we know $V_{CB} = \frac{5}{5+5} \times V_{CE} = \frac{5}{6}\mathrm{V}$

Finally from Figure 1, we know $V_{oc} = V_{AB} = \frac{5}{5+5} \times V_{CB} = \frac{5}{12}\mathrm{V}$

(c) $I_{sc} = \frac{V_{oc}}{R_{eq}} = \frac{5}{12} \times \frac{3}{10} = \frac{1}{8}\mathrm{A}$

<table>
<tr><td>R_{eq}=10/3Ω</td><td>V_{oc}=5/12V</td><td>I_{sc}=1/8A</td></tr>
<tr><td colspan="3">Thevenin equivalent:

</td></tr>
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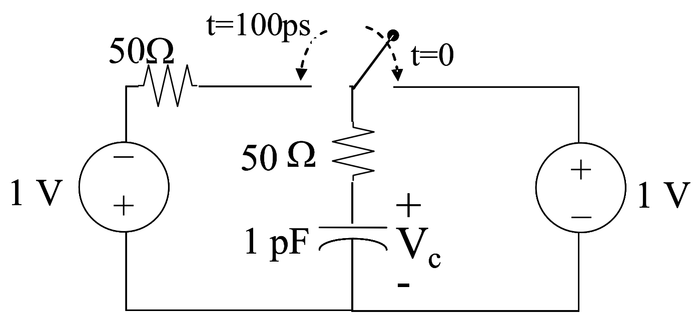
3. Transient Analysis: 1st order circuit

(a) At t<0, the switch is open and V_c= 0. At t=0, the switch is closed towards the right.
What is V_c(t)? (*Hint: You can leave terms of the form* e^{-x} *as they are*)

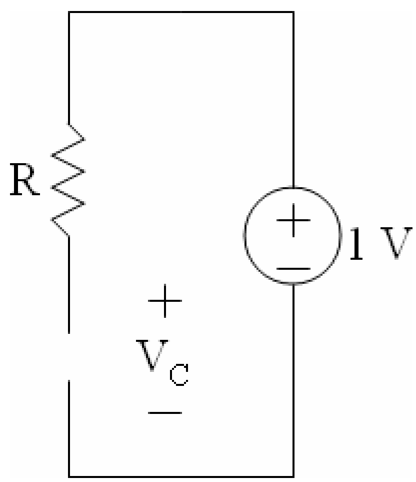
(b) At t=100 ps, the switch is closed towards the left and shall stay closed towards the left.
What is V_c(t) when t>100 ps?

(c) Qualitatively draw V_c(t) when t>0.



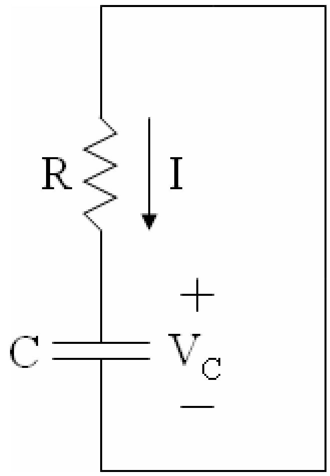
Solution:

(a) *Steady-state response*: At DC steady-state, the capacitor behaves like an open circuit so we can redraw the circuit like so:



No current flows in this circuit, so there is no voltage drop across the resistor and KVL says that the steady state voltage is 1 V.

Transient response: To see the natural response of the circuit, we turn off all power sources:



KVL:

$$-V_C(t) - RI(t) = 0$$

where

$$I = C\frac{dV_C(t)}{dt}$$
$$\Rightarrow \frac{dV_C(t)}{dt} + \frac{1}{RC}V_C(t) = 0$$

This first-order linear differential equation gives solutions of the form

$$V_C(t) = Ke^{-t/\tau}$$

Substituting into the homogeneous equation, we have:

$$-\frac{K}{\tau}e^{-t/\tau} + \frac{K}{RC}e^{-t/\tau} = \left(-\frac{1}{\tau} + \frac{1}{RC}\right)Ke^{-t/\tau} = 0$$
$$\Rightarrow \tau = RC = 50\text{ps}$$

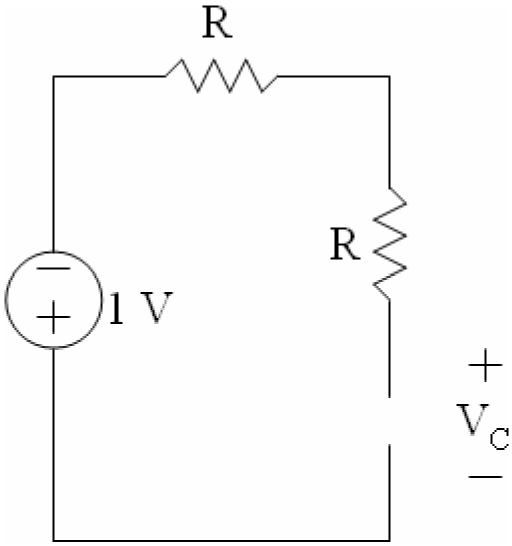
Total response: From the above, we have that the total response is in the form:

$$V_C(t) = 1 + Ke^{-t/50\text{ps}}$$

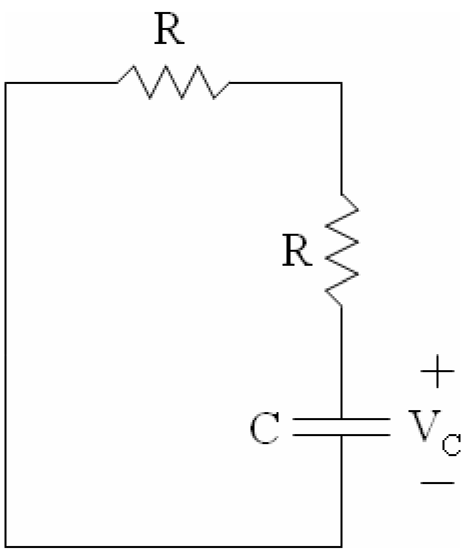
And since the voltage across a capacitor must be continuous, we have the initial condition:

$$V_C(0^+) = 1 + K = 0$$
$$\Rightarrow K = -1$$
$$\Rightarrow V_C(t) = 1 - e^{-t/50\text{ps}}\ \text{V}$$

(b) The analysis here is much the same as part (a). *Steady-state response*: V_C = -1 V.



Transient response:



$$V_C(t) = Ke^{-t/\tau}$$
$$\tau = (50\Omega + 50\Omega)(1\text{pF}) = 100\text{ps}$$

Total response:

$$V_C(t) = -1 + Le^{-t/100\text{ps}}$$

Note that this is equivalent to writing:

$$V_C(t) = -1 + Me^{-(t-100\text{ps})/100\text{ps}}$$

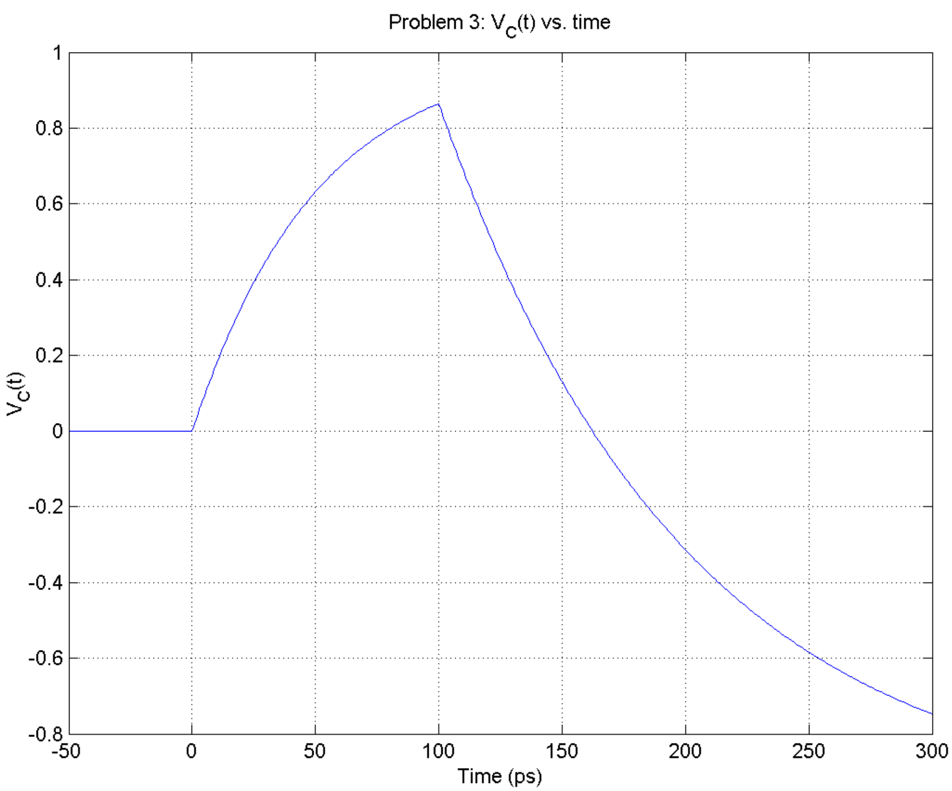
So now when we use part (a) to find initial conditions:

$$V_C(100^- \text{ps}) = 1 - e^{-100\text{ps}/50\text{ps}} = 1 - e^{-2}$$
$$V_C(100^+ \text{ps}) = -1 + Me^{-(100\text{ps}-100\text{ps})/100\text{ps}} = 1 - e^{-2}$$
$$\Rightarrow M = 2 - e^{-2}$$
$$\Rightarrow V_C(t) = -1 + (2 - e^{-2})e^{-(t-100\text{ps})/100\text{ps}} \text{ V}$$

(c)

0<t<100 ps	t>100 ps
V_c(t)= $V_C(t) = 1 - e^{-t/50\text{ps}}$ V	V_c(t)= $V_C(t) = -1 + (2 - e^{-2})e^{-(t-100\text{ps})/100\text{ps}}$ V

Draw V_c(t) vs t:



4. Transient Analysis: 2nd order circuit

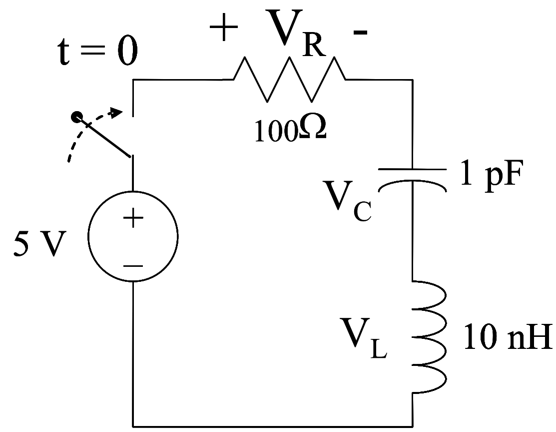
At t=0, the switch is closed.

(a) What is the KVL equation of the circuit in terms of $V_c(t)$?

(b) Calculate ω_0 , α, and ζ . Is the circuit underdamped, critically damped or overdamped?

(c) What is the solution of $V_c(t)$?

(*Hint:* $\omega_n = \sqrt{\omega_0^{\,2} - \alpha^2} = \frac{\sqrt{3}}{2} \times 10^{10}$ rad/s)



(a) $V_L + V_R + V_c = 5$

$$\because i = C\frac{dV_c}{dt}, \qquad V_L = L\frac{di}{dt} = LC\frac{d^2V_c}{dt^2}, \qquad V_R = R \times i = RC\frac{dV_c}{dt}$$

$$\therefore LC\frac{d^2V_c}{dt^2} + RC\frac{dV_c}{dt} + V_c = 5$$

or $$\frac{d^2V_c}{dt^2} + \frac{R}{L}\frac{dV_c}{dt} + \frac{V_c}{LC} = \frac{5}{LC}$$

So $$\frac{d^2V_C}{dt^2} + 10^{10}\frac{dV_c}{dt} + 10^{20}V_c = 5 \times 10^{20}$$

(b) $$\omega_0 = \frac{1}{\sqrt{LC}} = 10^{10} \text{ rad/s}$$

$$\alpha = \frac{R}{2L} = 5 \times 10^9$$

$$\zeta = \frac{\alpha}{\omega_0} = \frac{1}{2}$$ underdamped case

(c) Since it's underdamped, the solution has the following form:

$$V_c = K_1 e^{-\alpha t}\cos\omega_n t + K_2 e^{-\alpha t}\sin\omega_n t + 5$$

Initial condition: there is no charge on the capacitor $V_c(0) = 0$

So $V_c = K_1 + 5 = 0 \Rightarrow K_1 = -5$

Initial condition: there is no current in the circuit $i(0) = C\left.\frac{dV_c}{dt}\right|_{t=0} = 0$

So $i(0) = -\alpha K_1 + \omega_n K_2 = 0 \Rightarrow K_2 = \frac{\alpha K_1}{\omega_n} = -\frac{5}{\sqrt{3}} = -\frac{5\sqrt{3}}{3}$

$$V_c = -5e^{-5\times10^9 t}\cos\frac{\sqrt{3}}{2}\times10^{10}t - \frac{5\sqrt{3}}{3}e^{-5\times10^9 t}\sin\frac{\sqrt{3}}{2}\times10^{10}t + 5$$

<table>
<tr><td colspan="4">KVL in terms of V_c(t): $\frac{d^2V_c}{dt^2} + 10^{10}\frac{dV_c}{dt} + 10^{20}V_c = 5\times10^{20}$</td></tr>
<tr><td>$\omega_0 =$ 10^{10}rad/s</td><td>$\alpha =$ 5*10^9</td><td>$\zeta =$ 1/2</td><td>Damping case:
underdamped</td></tr>
<tr><td colspan="4">V_c(t): $V_c = -5e^{-5\times10^9 t}\cos\frac{\sqrt{3}}{2}\times10^{10}t - \frac{5\sqrt{3}}{3}e^{-5\times10^9 t}\sin\frac{\sqrt{3}}{2}\times10^{10}t + 5$</td></tr>
</table>