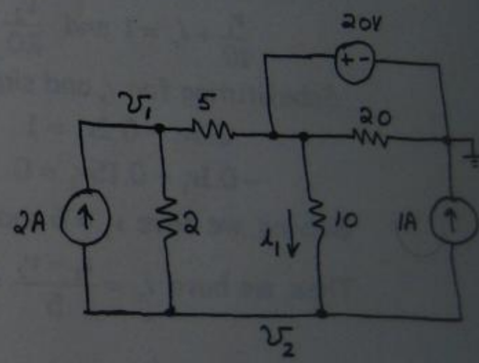
P2.43 To minimize the number of unknowns, we select the reference node at one end of the voltage source. Then we define the node voltages and write a KCL equation at each node.

$$\frac{v_1 - 20}{5} + \frac{v_1 - v_2}{2} = 2$$

$$\frac{v_2 - v_1}{2} + \frac{v_2 - 20}{10} = -3$$



Solving, we find $v_1 = 18.24$ V and $v_2 = 13.53$ V.

Then, we have $i_1 = \frac{20 - v_2}{10} = 0.647$ A.

P2.48* $v_x = v_2 - v_1$

Writing KCL at nodes 1 and 2:

$$\frac{v_1}{5} + \frac{v_1 - 2v_x}{15} + \frac{v_1 - v_2}{10} = 1$$

$$\frac{v_2}{5} + \frac{v_2 - 2v_x}{10} + \frac{v_2 - v_1}{10} = 2$$

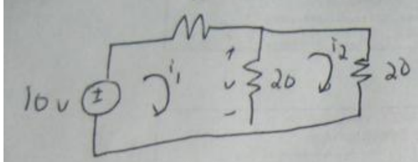
Substituting and simplifying, we have

$15v_1 - 7v_2 = 30$ and $v_1 + 2v_2 = 20$.

Solving, we find $v_1 = 5.405$ and $v_2 = 7.297$.

I have not shown here how to actually solve the system of equations. This is an intentional omission. We leave it up to you guys to figure out how you want to solve them. Personally, I use substitution for 2 variable systems, and use matrices for systems with more variables.

2.54)



i_1: $20i_1 + 20(i_1 - i_2) - 10 = 0$

i_2: $20i_2 + 20(i_2 - i_1) = 0$

$40i_1 - 20i_2 = 10$ $\qquad$ $i_1 = 2i_2$

$-20i_1 + 40i_2 = 0$ $\qquad$ $40(2i_2) - 20i_2 = 10$

$60i_2 = 10$

$i_2 = 1/6$

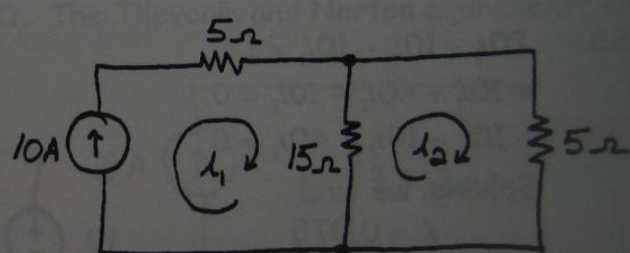
$i_1 = 1/3$

$20(i_1 - i_2) = 20(1/3 - 1/6) = 20(1/6) = \frac{20}{6} V = \frac{10}{3} V$

2.60 *Mesh equations:*

$i_1 = 10$ A

$15(i_2 - i_1) + 5i_2 = 0$

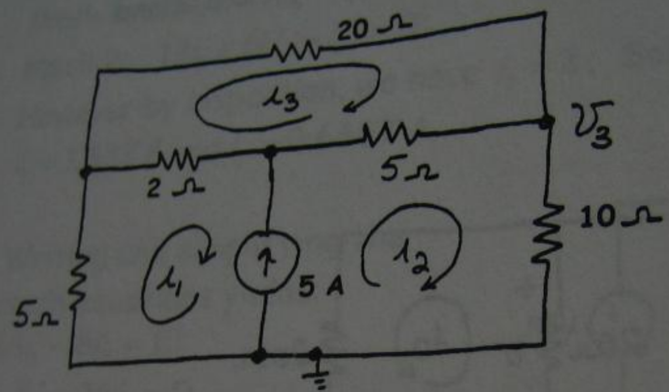


Solving, we find $i_2 = 7.5$ A.

However i_3 shown in Figure P2.37 is the same as i_2, so the answer is i_3 = 7.5 A.

The current through the resistor going downwards is (i1 – i2) = 10 A – 7.5 A= 2.5 A
The current is the difference of the two mesh currents that go through the resistor.

P2.62



Current source in terms of mesh currents: $-i_1 + i_2 = 5$

KVL for mesh 3: $-2i_1 - 5i_2 + 27i_3 = 0$

KVL around outside of network: $5i_1 + 10i_2 + 20i_3 = 0$

Solving, we find

$i_1 = -3.3945$, $i_2 = 1.6055$ and $i_3 = 0.0459$

Then, we have

$v_3 = 10i_2 = 16.06$ V

P3.28* $C = \dfrac{\varepsilon_r \varepsilon_0 A}{d} = \dfrac{15 \times 8.85 \times 10^{-12} \times 10 \times 10^{-2} \times 30 \times 10^{-2}}{0.01 \times 10^{-3}} = 0.398\ \mu\text{F}$

P3.34

$$i_c(t) = C\frac{dv_c(t)}{dt} = 10^{-7}\frac{d}{dt}[10\cos(100t)] = -10^{-4}\sin(100t)$$
$$v_r(t) = Ri_c(t) = -10^{-3}\sin(100t)$$
$$v(t) = v_c(t) + v_r(t)$$
$$v(t) = 10\cos(100t) - 10^{-3}\sin(100t)$$

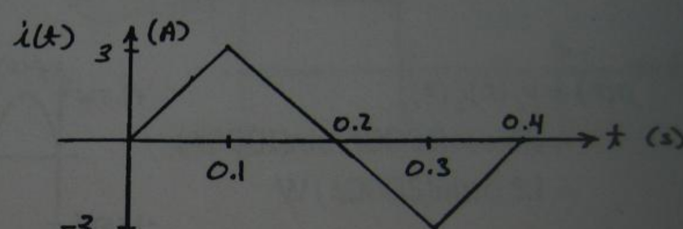
Thus, $v(t) = v_c(t)$ to within 1% accuracy, and the resistance can be neglected.

Repeating for $v_c(t) = 0.1\cos(10^7 t)$, we find

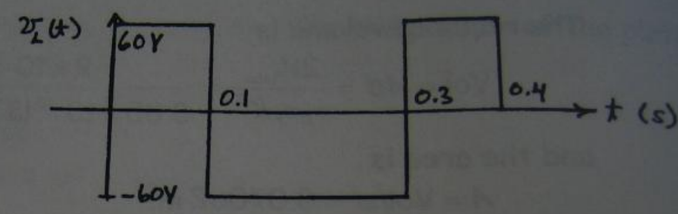
$$i_c(t) = -0.1\sin(10^7 t)$$
$$v_r(t) = -\sin(10^7)$$
$$v(t) = v_c(t) + v_r(t) = 0.1\cos(10^7 t) - \sin(10^7 t)$$

Thus, in this case, the voltage across the parasitic resistance is larger in magnitude than the voltage across the capacitance.

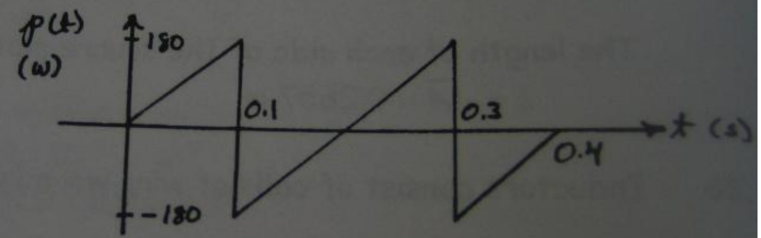
P3.40* $L = 2\text{H}$



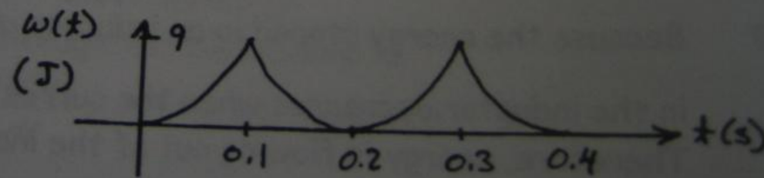
$$v_L(t) = L\frac{di_L(t)}{dt}$$



$$p(t) = v_L(t)i_L(t)$$



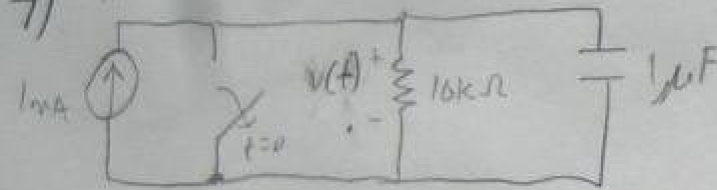
$$w(t) = \frac{1}{2}L[i_L(t)]^2$$



P3.46* $v_L = L\frac{di}{dt} = 0.5\frac{4}{0.2} = 10\text{ V}$

For problem 4.7, I have included three solutions. The first is an explicit step-by-step use of the technique we discussed in class (which involves finding a separate complementary and particular solution). The second solution is a shorter version of the first, and it is the way I used to do 1^{st} order problems as an undergrad (it's really fast). With practice you should be able to do this second version. The third solution uses the method given in the book. In my opinion, this method is not as good as the one discussed in class, but you are free to use whatever method you are comfortable with. Just like with solving systems of equations, there are many ways to solve differential equations.

4.7) Alternate Solution #1:



KCL: $\frac{V(t)}{10K} + 1\mu F \frac{dV(t)}{dt} - 1mA = 0$

Complimentary solution:

$$\frac{V_c(t)}{10K} + 1\mu F \cdot \frac{dV_c(t)}{dt} = 0$$

$$V_c(t) + 1\mu F \cdot 10K \cdot \frac{dV_c(t)}{dt} = 0$$

$$V_c(t) + 0.01 \cdot \frac{dV_c(t)}{dt} = 0$$

$$V_c(t) = Ke^{-t/0.01} = Ke^{-100t}$$

Initial conditions.

$v(0^-) = 0\,V$ (since there is no current

$v(\infty) = 10\,V$ (Steady state)

Particular Solution!

$$\frac{v(t)}{10K} + 1\,\mu F \cdot \frac{dV(t)}{dt} = 1\,mA$$

$$V_c(t) + 0{,}01\frac{dV_c(t)}{dt} = 10$$

$$f(t) = 10$$

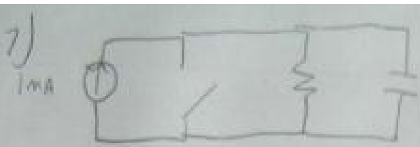
so, $V_p(t) = 10a + b \cdot \frac{d(10)}{dt} = 10a$

$$v(t) = Ke^{-100t} + 10a$$

$v(\infty) = 10a = 10$, so $a = 1$ $\qquad v(t) = 10 - 10e^{-100t}$

$v(0) = K + 10 = 0$, so $K = -10$, so

4.7)

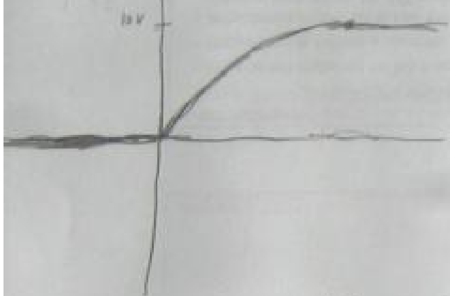


Alternate Solution #2:

$v(0^-) = 0V$

$v(\infty) = 10V$

we know from experience that v(t) will look something like:



and that the solution will be of the form:

$v(t) = K_1 - K_2 e^{-t/\alpha}$

It is easy to see that since v(t) must eventually be 10V, that $K_1 = 10$, and since $v(0) = 0V$, K_2 must therefore also be 10.

Finally we know that the exponential will decrease at a rate $\alpha = RC = 0.01$,

so:

$v(t) = 10 - 10e^{-100t}$

P4.7 Prior to $t = 0$, we have $v(t) = 0$ because the switch is closed. After $t = 0$, we can write the following KCL equation at the top node of the circuit:

$$\frac{v(t)}{R} + C\frac{dv(t)}{dt} = 1\,\text{mA}$$

Multiplying both sides by R and substituting values, we have

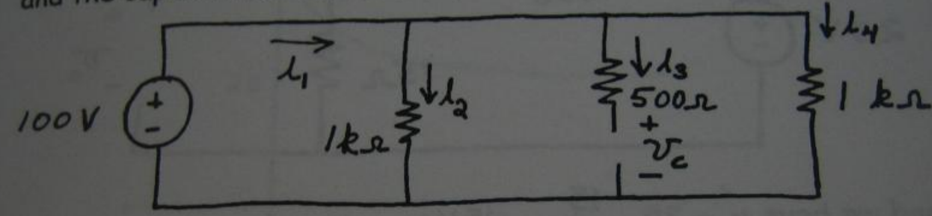
$$0.01\frac{dv(t)}{dt} + v(t) = 10 \qquad (1)$$

The solution is of the form

$$v(t) = K_1 + K_2 \exp(-t/RC) = K_1 + K_2 \exp(-100t) \qquad (2)$$

Substituting Equation (2) into Equation (1), we eventually obtain

P4.14 In steady state with a dc source, the inductance acts as a short circuit and the capacitance acts as an open circuit. The equivalent circuit is:



$$i_4 = (100\ \text{V})/(1\ \text{k}\Omega) = 100\ \text{mA}$$
$$i_3 = 0$$
$$i_2 = (100\ \text{V})/(1\ \text{k}\Omega) = 100\ \text{mA}$$
$$i_1 = i_2 + i_3 + i_4 = 200\ \text{mA}$$
$$v_C = 100\ \text{V}$$

P4.23 The solution is of the form

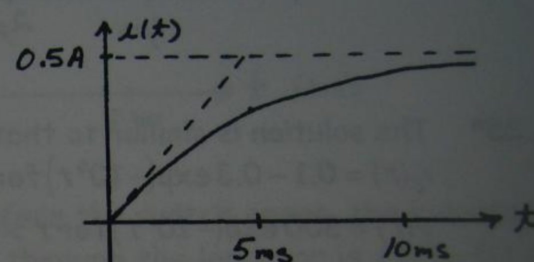
$$i(t) = K_1 + K_2 \exp(-Rt/L)$$

At $t = 0$, we have

$$i(0+) = 0 = K_1 + K_2$$

and at $t = \infty$, we have

$$i(\infty) = (100\ \text{V})/(200\ \Omega)$$
$$= 0.5 = K_1$$

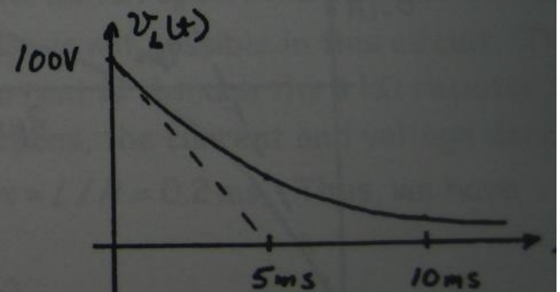


The time constant is $\tau = L/R = 5\ \text{ms}$. Thus, the solution is

$$i(t) = 0.5 - 0.5\exp(-200t)$$

The voltage across the inductor is

$$v_L(t) = L\frac{di(t)}{dt} = 100\exp(-200t)$$



Here the book uses the second method I used for problem 4.7.

P4.30* In steady state with the switch closed, we have $i(t) = 0$ for $t < 0$ because the closed switch shorts the source.

In steady state with the switch open, the inductance acts as a short circuit and the current becomes $i(\infty) = 1\,\text{A}$. The current is of the form

$$i(t) = K_1 + K_2 \exp(-Rt/L) \text{ for } t \geq 0$$

in which $R = 20\,\Omega$, because that is the Thévenin resistance seen looking back from the terminals of the inductance with the switch open. Also, we have

$$i(0+) = i(0-) = 0 = K_1 + K_2$$

$$i(\infty) = 1 = K_1$$

Thus, $K_2 = -1$ and the current (in amperes) is given by

$$\begin{aligned} i(t) &= 0 && \text{for } t < 0 \\ &= 1 - \exp(-20t) && \text{for } t \geq 0 \end{aligned}$$