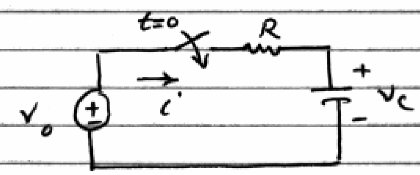
EECS 40 HW3 Solution
summer 05

(1)

a) $V_{out} = \dfrac{8\mu F}{8\mu F + 2\mu F}(10V) = \boxed{8V}$

Vout will decrease because the capacitance between terminals a and b will increase.

b)

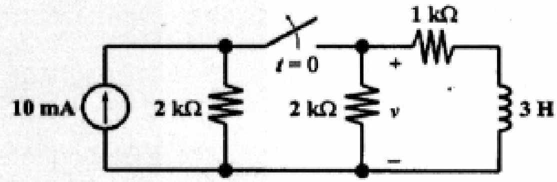


$$i = \frac{V_0 - V_C}{R} = C\frac{dV_C}{dt}$$

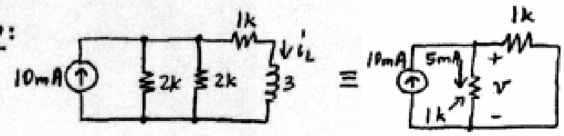
The current flowing throug R (to charge C) will decrease, hence it will take longer to deliver the charge $Q = CV_0$

problem 1 continued

(c)



$t<0$:

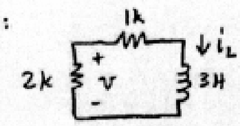


$i_L = 5\,mA$

← inductor behaves as short circuit in steady state

$v(t<0) = 5V$

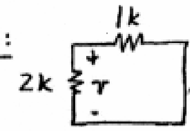
$t>0$:



$i_L(0^+) = 5\,mA$

$v(0^+) = -10V$

$t=\infty$:



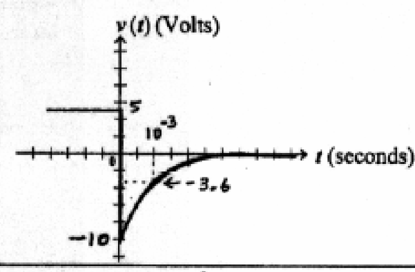
$i_L = 0$

$v(t=\infty) = 0$

$$\tau = \frac{L}{R} = \frac{3H}{2k\Omega + 1k\Omega} = 1\,ms$$

$$v = 0 + [-10 - 0]e^{-t/1ms} = -10e^{-1000t}$$

For $t<0$: $v(t) =$ 5 V. For $t>0$: $v(t) = -10e^{-1000t}$

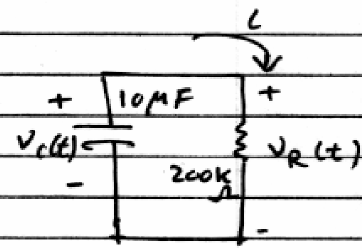


② P 4.20

$\underline{t<0}$ $\quad v_R(t) = 0$

$$v_c(t) = \frac{10k\Omega}{(20k\Omega + 10k\Omega)} \cdot 30V = 10V$$

$\underline{t>0}$ $\quad v_R(t) = Ri = v_c(t)$



$$\Rightarrow RC\frac{dv_c(t)}{dt} = -v_c(t)$$

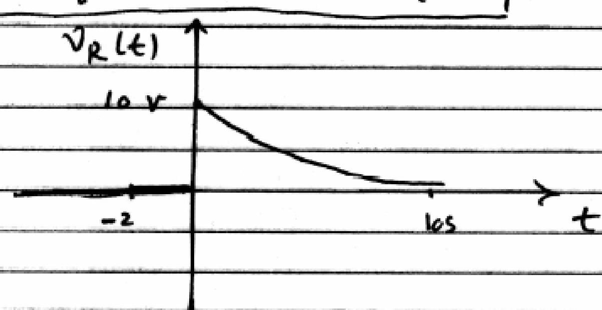
$$\Rightarrow \frac{dv_c(t)}{dt} - \frac{1}{RC} v_c(t) = 0$$

$$v_c(t) = K_1 e^{\frac{-t}{\tau}} + K_2$$

$$\begin{cases} v_c(0^-) = v_c(0^+) = 10V \\ v_c(\infty) = 0 \end{cases} \Rightarrow \begin{cases} K_1 = 10V \\ K_2 = 0 \end{cases}$$

$$v_c(t) = v_R(t) = 10\, e^{\frac{-t}{\tau}} \qquad \tau = RC = 2S$$

$$\therefore \boxed{v_R(t) = \begin{cases} 0\,V & t<0 \\ 10\, e^{\frac{-t}{2}}\,V & t>0 \end{cases}}$$



③ P. 4.26

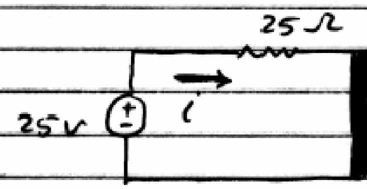
$t < 0 \qquad i(t) = \dfrac{100\,V}{25\Omega + 75\Omega} = 1A$

$t < 0$

$$i(t) = K_1 e^{\frac{-t}{\tau}} + K_2 \qquad \frac{1}{\tau} = \frac{R}{L} = \frac{25\Omega}{2H} = 12.5 s$$

① $i(0^-) = i(0^+) = 1A \Rightarrow K_1 + K_2 = 1$

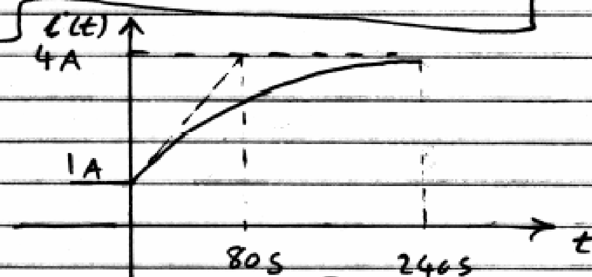
② $i(\infty) = \dfrac{100\,V}{25\Omega} = 4A$



note that the Inductor becomes a short circuit in steady state, when $t \to \infty$.

$\therefore$ ①, ② $\Rightarrow K_2 = 4$, $K_1 = -3$

$i(t) = \begin{cases} 1A & \text{for } t < 0 \\ [-3e^{-12.5t} \text{ [redacted] } + 4]\,A & \text{for } t \geq 0 \end{cases}$



④

4 P 4.33

KVL: $\frac{V_c(t) - V(t)}{R} + \frac{C\, dV_c(t)}{dt} = 0$

Rearranging this equation and substituting $V(t) = t$, we have

$$RC \frac{dV_c(t)}{dt} + V_c(t) = t \quad \text{for } t \geq 0 \qquad (1)$$

$$V_{cp}(t) = A + Bt \qquad (2)$$

Substituting equation 2 into equation 1:

$$RCB + A + Bt = t$$

$$\Rightarrow \quad B = 1$$
$$A = -RC$$

$$\Rightarrow \quad V_{cp}(t) = -RC - t$$

The complementary solution to the homogeneous equation is of the form:

$$V_{cc}(t) = K_1 e^{\frac{-t}{RC}}$$

Thus the complete solution is

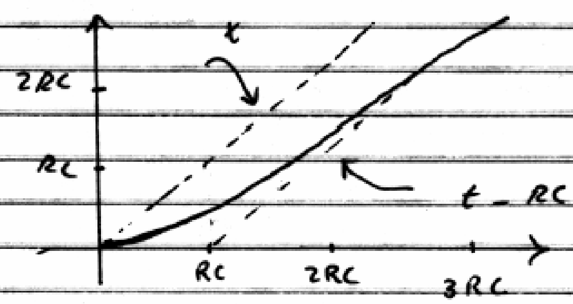
$$V_c(t) = V_{cp}(t) + V_{cc}(t) = -RC + t + K_1 e^{\frac{-t}{RC}}$$

problem 4 continued

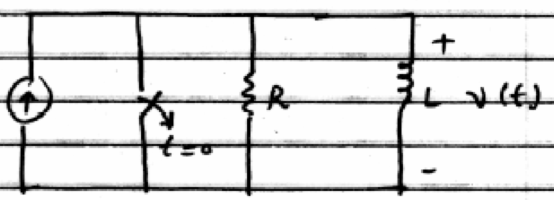
The solution must meet the given initial condition

$$V_C(0) = 0 = -RC + K_1 \Rightarrow K_1 = RC$$

$$\boxed{V_C(t) = V_{C_p}(t) + V_{C_c}(t) = -RC + t + RC\,e^{-\frac{t}{RC}}}$$



(5) P 4.35



$R = 10\,\Omega$

$L = 1\,H$

KCL: $$\frac{v(t)}{R} + \frac{1}{L}\int v(t)\,dt = 5\cos(10t)$$

$$\frac{1}{R}\frac{dv(t)}{dt} + \frac{1}{L}v(t) = -50\sin(10t)$$

$$v(t) + \frac{L}{R}\frac{dv(t)}{dt} = -50L\sin(10t) \quad (1)$$

We know $v(t) = v_p(t) + v_c(t)$
where $v_p(t)$ is the particular solution
and $v_c(t)$ is the complementary
solution to the differential equation.

$$v_p(t) = A\cos(10t) + B\sin(10t) \quad (2)$$

we substitute (2) into equation (1):

$$A\cos(10t) + B\sin(10t) + \frac{L}{R}\left(-10A\sin(10t) + 10B\cos(10t)\right)$$
$$= -50L\sin(10t)$$

$$\Rightarrow \left[A + (10B)\left(\frac{L}{R}\right)\right]\cos(10t) + \left[B - (10A)\left(\frac{L}{R}\right)\right] = -50L\sin$$

$$\Rightarrow \begin{cases} A + (10B)\left(\frac{L}{R}\right) = 0 \\ B - (10A)\left(\frac{L}{R}\right) = -50L \end{cases} \xrightarrow{\frac{L}{R} = \frac{1}{10}} \begin{cases} A + B = 0 \\ B - A = -50 \end{cases}$$

⑤ continued

$$\boxed{\begin{aligned} A &= 25 \\ B &= -25 \end{aligned}}$$

$$\Rightarrow \quad v_p(t) = 25\cos(10t) - 25\sin(10t)$$

$$v_c(t) = Ke^{\frac{-t}{\tau}} \qquad \tau = \frac{L}{R} = 0.1s$$

$$\therefore \quad v(t) = v_p(t) + v_c(t)$$

$$v(t) = Ke^{\frac{-t}{\tau}} + 25\cos(10t) - 25\sin(10t) \quad (3)$$

Because the current in the inductor is zero at $t = 0^+$, the 5A supplied by the source must flow through through the 10Ω resistor and we have $v(0^+) = 50$. substituting this into the general solution, equation (3), we get $\boxed{K_1 = 25}$

Thus

$$\boxed{\begin{aligned} &v(t) = 25e^{\frac{-t}{\tau}} + 25\cos(10t) - 25\sin(10t) \quad t \geq 0 \\ &\tau = 0.1s \end{aligned}}$$

⑥ P4.48

(a) $\alpha = \frac{1}{2RC} = 20 \times 10^{6}$

$\omega_0 = \frac{1}{\sqrt{LC}} = 10 \times 10^{6} \ \frac{1}{s}$

$\boxed{\zeta = \frac{\alpha}{\omega_0} = 2}$

Thus we have an overdamped circuit

(b) writing a current equation at $t = 0^+$, we have

$\frac{v(0+)}{R} + i_L(0+) + Cv'(0^+) = 1$

substituting $v(0+) = 0$ and $i_L(0+) = 0$, yields

$\boxed{v'(0+) = \frac{1}{C} = 10^{9} \ \frac{V}{s}}$

(c)

under steady-state conditions, the inductance acts as a short circuit Therefore, the particular solution for $v(t)$ is:

$\boxed{v_p(t) = 0}$

note: $v(t) = v_c(t) + v_p(t)$

we know that $v_c(t)$ is a multiple of e^{-st}, so $v_c(t) = 0$ for $t \to \infty$

We know $v(t) = 0$ when $t \to \infty$. Thus

$V_p(t) = 0$

(d) $S_1 = -\alpha + \sqrt{\alpha^2 - w_o^2} = -2.679 \times 10^6$

$S_2 = -\alpha - \sqrt{\alpha^2 - w_o^2} = -37.32 \times 10^6$

The complementary solution is

$V_c(t) = K_1 e^{S_1 t} + K_2 e^{S_2 t}$

Since the particular solution is zero
the complete solution is

$V(t) = K_1 e^{S_1 t} + K_2 e^{S_2 t}$

$v(0^+) = 0$ and $V'(0^+) = 10^9\ V$

Thus we have:

$V(0^+) = 0 = K_1 + K_2$

$V'(0^+) = 10^9 = S_1 K_1 + S_2 K_2$

$\Rightarrow K_1 = 28.87$
$K_2 = -28.87$

$$\boxed{V(t) = 28.87 e^{S_1 t} - 28.87 e^{S_2 t}}$$

(7) P5.41

$$\boxed{I_S = 10\angle 0^\circ \text{ mA}}$$

$$V = I_S \frac{1}{1/R + 1/j\omega L + j\omega C} = 10^{-2} \frac{1}{1/1000 - j0.005 + j0.005}$$

$$\boxed{V = 10\angle 0^\circ \text{ V}}$$

$$I_R = \frac{V}{R} = \boxed{10\angle 0^\circ \text{ mA}}$$

$$I_L = \frac{V}{j\omega L} = \boxed{50\angle -90^\circ \text{ mA}}$$

$$I_C = V(j\omega C) = \boxed{50\angle 90^\circ \text{ mA}}$$

The peak value of $i_L(t)$ is five times larger than the source current! This is possible because current in the capacitance balances the current in the inductance.

(8) P 5.42

$$\boxed{V_1 = 100\angle -90^\circ = -j100}$$

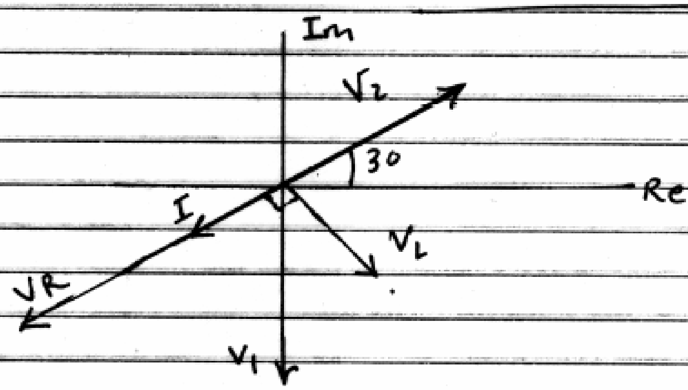
$$\boxed{V_2 = 100\angle 30^\circ}$$

$$I = \frac{V_1 - V_2}{R + j\omega L} = \frac{-j100 - (86.60 + j50)}{100 + j50}$$

$$I = \frac{-86.60 - j150}{100 + j50} = \boxed{1.549\angle -146.6^\circ \text{ A}}$$

$$V_R = 100 I = \boxed{154.9\angle -146.6^\circ \text{ V}}$$

$$V_L = j50 I = \boxed{77.45\angle -56.6^\circ}$$



I lags V_1 by 56.6°

I lags V_L by 90.0°

(9) P.5.43

$$Z_{total} = j\omega L + \frac{1}{\frac{1}{R} + j\omega C}$$

$$= j100 + \frac{1}{0.01 + j0.01}$$

$$\boxed{= j100 + 50 - j50}$$

$$= 50 + j50 = \boxed{70.71\angle 45^\circ}$$

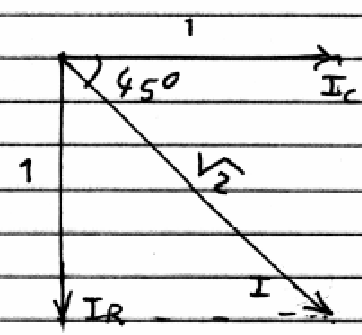
$$I = \frac{100\angle 0^\circ}{Z_{total}} = \boxed{1.414\angle -45^\circ}$$

$$I_R = I\frac{Z_C}{R + Z_C} = (1.414\angle -45^\circ) \times \frac{-j100}{100 - j100}$$

$$\boxed{I_R = 1\angle -90^\circ C}$$

$$I_C = I\frac{R}{R + Z_C} = (1.414\angle -45^\circ) \times \frac{100}{100 - j100}$$

$$\boxed{I_C = 1\angle 0^\circ}$$



(10) P. 5. 44

writing KCL equations at nodes 1 and 2 we obtain

$$\frac{V_1}{10} + \frac{V_1 - V_2}{5+j15} = 1\angle 0^\circ$$

$$\frac{V_2}{-j10} + \frac{V_2 - V_1}{5+j15} = 1\angle 30^\circ$$

Solving these equations, we obtain

$$\boxed{V_1 = 6.735\angle -38.54^\circ}$$

$$\boxed{V_2 = 16.25\angle -55.52^\circ}$$