

## Announcements

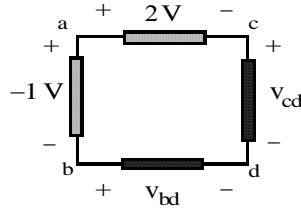
- Sections begin this week
  - Cancelled Sections: Th 12-2.
- Labs begin this week. Attend your only second lab slot this week.
  - Cancelled labs: ThF 8-11, 2-5. Please check your Lab section.
- Homework #1 online
  - Due next Monday at 4pm

## Announcements

- Office Hours
  - Tu,Th 11-12 in Cory 382
  - Or just e-mail me at ***florescu@eecs***
- TA Office Hours
  - TBD

## Review

Find  $v_{ab}$ ,  $v_{ca}$ ,  $v_{cb}$



Note that the labeling convention has nothing to do with whether or not  $v$  is positive or negative.

## Lecture #2

### OUTLINE

- Circuit Elements
- Circuit element  $I$ - $V$  characteristics
- Construction of a circuit model
- Kirchhoff's laws – a closer look

### Reading

(Chapter 1, begin Ch. 2)

## Circuit Elements

- 5 ideal basic circuit elements:
    - voltage source
    - current source
    - resistor
    - inductor
    - capacitor
- } **active elements**, capable of generating electric energy
- } **passive elements**, incapable of generating electric energy
- Many practical systems can be modeled with just sources and resistors
  - The basic analytical techniques for solving circuits with inductors and capacitors are similar to those for resistive circuits

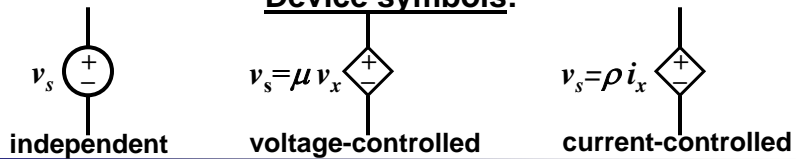
## Electrical Sources

- An **electrical source** is a device that is capable of converting non-electric energy to electric energy and *vice versa*.
    - Examples:
      - battery: chemical  $\longleftrightarrow$  electric
      - dynamo (generator/motor): mechanical  $\longleftrightarrow$  electric  
(Ex. gasoline-powered generator, Bonneville dam)
- Electrical sources can either deliver or absorb power

## Ideal Voltage Source

- Circuit element that maintains a prescribed voltage across its terminals, **regardless of the current flowing in those terminals**.
  - Voltage is known, but current is determined by the circuit to which the source is connected.
- The voltage can be either **independent** or **dependent** on a voltage or current elsewhere in the circuit, and can be constant or time-varying.

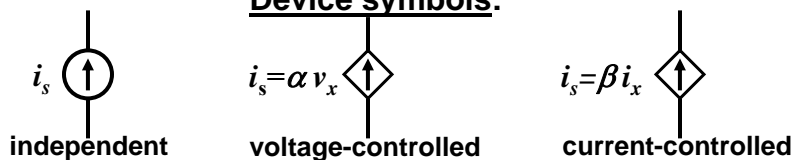
### Device symbols:



## Ideal Current Source

- Circuit element that maintains a prescribed current through its terminals, **regardless of the voltage across those terminals**.
  - Current is known, but voltage is determined by the circuit to which the source is connected.
- The current can be either **independent** or **dependent** on a voltage or current elsewhere in the circuit, and can be constant or time-varying.

### Device symbols:



## Electrical Resistance

- **Resistance:** the ratio of voltage drop and current. The circuit element used to model this behavior is the **resistor**.

Circuit symbol:



Units: Volts per Ampere  $\equiv$  ohms ( $\Omega$ )

- The current flowing in the resistor is proportional to the voltage across the resistor:

$$v = i R \quad (\text{Ohm's Law})$$

where  $v$  = voltage (V),  $i$  = current (A), and  $R$  = resistance ( $\Omega$ )



Georg Simon Ohm  
1789-1854

## Electrical Conductance

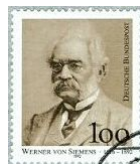
- **Conductance** is the reciprocal of resistance.

Symbol:  $G$

Units: siemens (S) or mhos ( $\mathcal{O}$ )

Example:

Consider an  $8 \Omega$  resistor. *What is its conductance?*



Werner von Siemens  
1816-1892

## Short Circuit and Open Circuit

### ■ Short circuit

- $R = 0 \rightarrow$  no voltage difference exists
- all points on the wire are at the same potential.
- Current can flow, as determined by the circuit

### ■ Open circuit

- $R = \infty \rightarrow$  no current flows
- Voltage difference can exist, as determined by the circuit

## Circuit Nodes and Loops

- A **node** is a point where two or more circuit elements are connected.
- A **loop** is formed by tracing a closed path in a circuit through selected basic circuit elements without passing through any intermediate node more than once

## Kirchhoff's Laws

### ■ Kirchhoff's Current Law (KCL):

- The algebraic sum of all the **currents** entering any **node** in a circuit equals zero.

### ■ Kirchhoff's Voltage Law (KVL):

- The algebraic sum of all the **voltages** around any **loop** in a circuit equals zero.



Gustav Robert Kirchhoff  
1824-1887

## Example: Power Absorbed by a Resistor

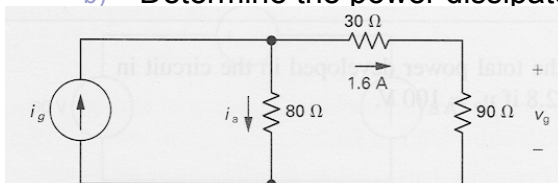
$$p = vi = (iR)i = i^2R$$

$$p = vi = v(v/R) = v^2/R$$

**Note that  $p > 0$  always, for a resistor → a resistor  
dissipates electric energy**

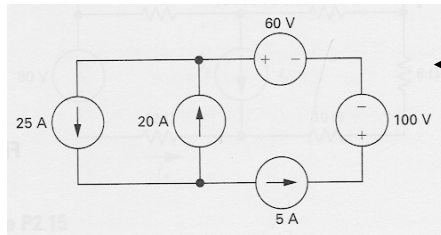
Example:

- Calculate the voltage  $v_g$  and current  $i_a$ .
- Determine the power dissipated in the  $80\Omega$  resistor.



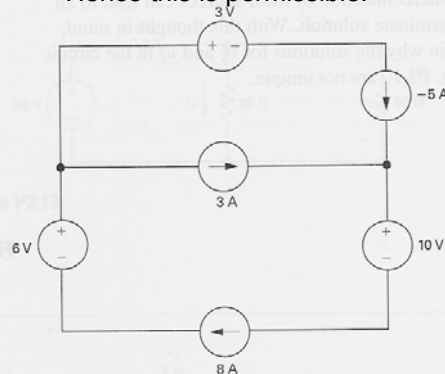
## More Examples

- Are these interconnections permissible?



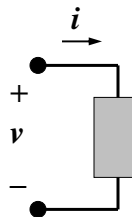
This circuit connection is NOT permissible. It violates the KCL.

This circuit connection is permissible. This is because the current sources can sustain any voltage across; Hence this is permissible.



## Current vs. Voltage ( $I$ - $V$ ) Characteristic

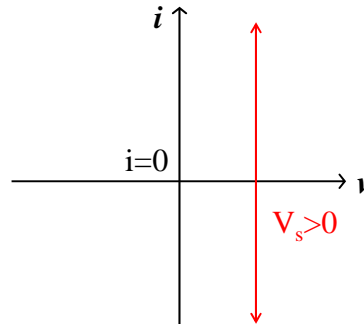
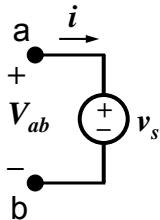
- Voltage sources, current sources, and resistors can be described by plotting the current ( $i$ ) as a function of the voltage ( $v$ )



Passive? Active?



## I-V Characteristic of an Ideal Voltage Source

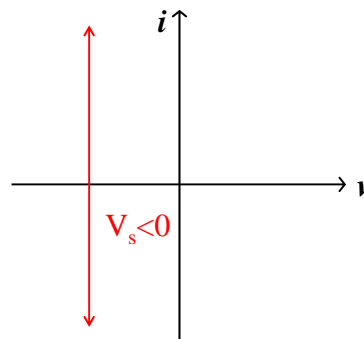
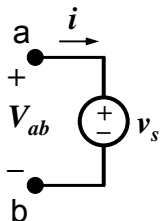


1. Plot the  $I$ - $V$  characteristic for  $v_s > 0$ . For what values of  $i$  does the source absorb power? For what values of  $i$  does the source release power?

$V_s > 0 \rightarrow i < 0$  release power;  $i > 0$  absorb power

What is the  $I$ - $V$  characteristic for an ideal wire?

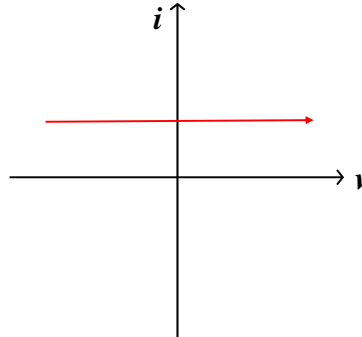
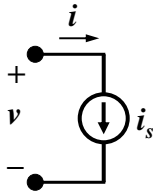
## I-V Characteristic of an Ideal Voltage Source



2. Plot the  $I$ - $V$  characteristic for  $v_s < 0$ . For what values of  $i$  does the source absorb power? For what values of  $i$  does the source release power?

$V_s < 0 \rightarrow i > 0$  release power;  $i < 0$  absorb power

## I-V Characteristic of an Ideal Current Source



1. Plot the  $I$ - $V$  characteristic for  $i_s > 0$ . For what values of  $v$  does the source absorb power? For what values of  $v$  does the source release power?

$V > 0$  absorb power;  $V < 0$  release power

## Short Circuit and Open Circuit

### Wire (“short circuit”):

- $R = 0 \rightarrow$  no voltage difference exists  
(all points on the wire are at the same potential)
- Current can flow, as determined by the circuit

### Air (“open circuit”):

- $R = \infty \rightarrow$  no current flows
- Voltage difference can exist,  
as determined by the circuit

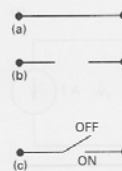
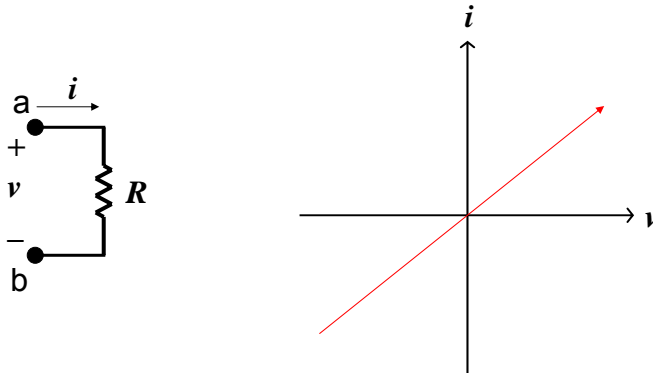


Figure 2.10 Circuit symbols. (a) Short circuit. (b) Open circuit. (c) Switch.

## I-V Characteristic of Ideal Resistor



1. Plot the  $I$ - $V$  characteristic for  $R = 1 \text{ k}\Omega$ . What is the slope?

## “Lumped Element” Circuit Modeling

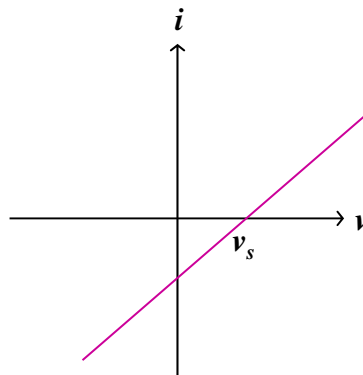
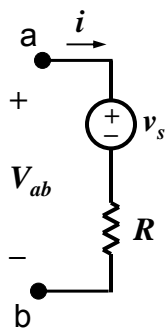
(Model = representation of a real system which simplifies analysis)

- In circuit analysis, important characteristics are grouped together in “lumps” (separate circuit elements) connected by perfect conductors (“wires”)
- An electrical system can be modeled by an **electric circuit** (combination of paths, each containing 1 or more **circuit elements**).

## Construction of a Circuit Model

- The electrical behavior of each physical component is of primary interest.
- We need to account for undesired as well as desired electrical effects.
- Simplifying assumptions should be made wherever reasonable.

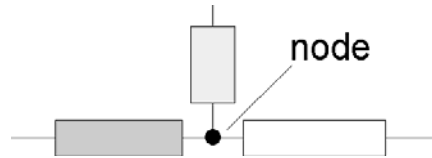
## $I$ - $V$ Characteristic of a real Voltage Source



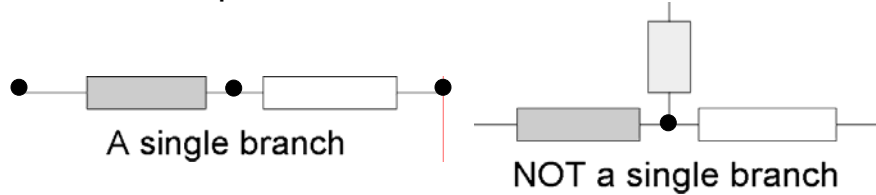
1. What is the  $I$ - $V$  characteristic for an real current source?
2. What is the  $I$ - $V$  characteristic for an ideal wire?

## Terminology: Nodes and Branches

**Node:** A point where two or more circuit elements are connected



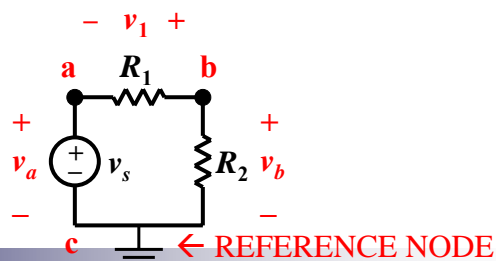
**Branch:** A path that connects two nodes



## Notation: Node and Branch Voltages

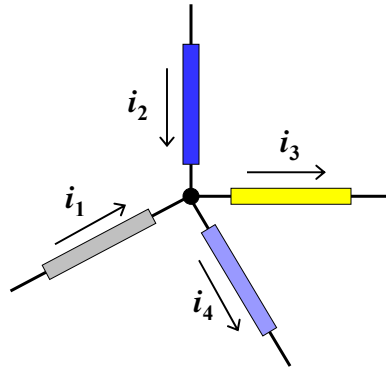
- Use one node as the reference (the “common” or “ground” node) – label it with a symbol
- The voltage drop from node  $x$  to the reference node is called the **node voltage**  $v_x$ .
- The voltage across a circuit element is defined as the difference between the node voltages at its terminals

Example:



## Using Kirchhoff's Current Law (KCL)

Consider a node connecting several branches:



- Use **reference directions** to determine whether currents are “entering” or “leaving” the node – **with no concern about actual current directions**

## Formulations of Kirchhoff's Current Law

(Charge stored in node is zero.)

### Formulation 1:

Sum of currents entering node  
= sum of currents leaving node

### Formulation 2:

Algebraic sum of currents entering node = 0

- Currents leaving are included with a minus sign.

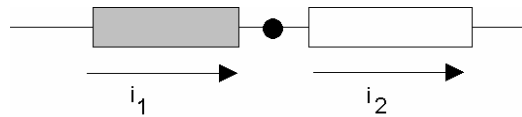
### Formulation 3:

Algebraic sum of currents leaving node = 0

- Currents entering are included with a minus sign.

## A Major Implication of KCL

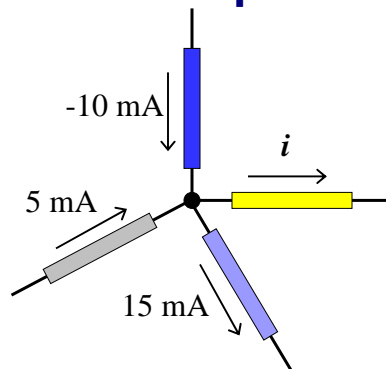
- KCL tells us that **all of the elements in a single branch carry the same current.**
- We say these elements are connected ***in series***.



Current entering node = Current leaving node

$$i_1 = i_2$$

## KCL Example



**Currents entering the node:**

**Currents leaving the node:**

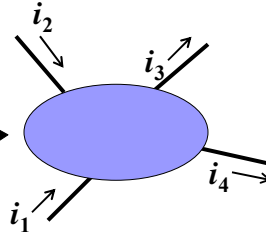
### 3 formulations of KCL:

- 1.
- 2.
- 3.

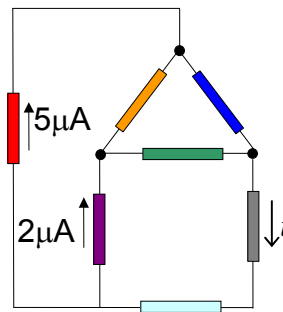
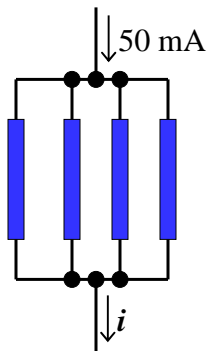
## Generalization of KCL

- The sum of currents entering/leaving a **closed surface** is zero. Circuit branches can be inside this surface, *i.e.* the surface can enclose more than one node!

This could be a big chunk of a circuit, *e.g.* a “black box”



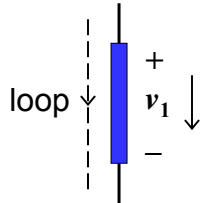
## Generalized KCL Examples



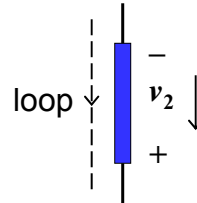


## Using Kirchhoff's Voltage Law (KVL)

Consider a branch which forms part of a loop:



Moving from + to -  
We add  $V_1$



Moving from - to +  
We subtract  $V_1$

- Use **reference polarities** to determine whether a voltage is dropped
- **No concern about actual voltage polarities**

## Formulations of Kirchhoff's Voltage Law

(Conservation of energy)

### Formulation 1:

Sum of voltage drops around loop  
= sum of voltage rises around loop

### Formulation 2:

Algebraic sum of voltage drops around loop = 0

- Voltage rises are included with a minus sign.

(Handy trick: Look at the first sign you encounter on each element when tracing the loop.)

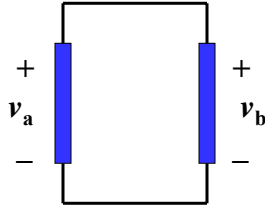
### Formulation 3:

Algebraic sum of voltage rises around loop = 0

- Voltage drops are included with a minus sign.

## A Major Implication of KVL

- KVL tells us that any set of elements which are connected at both ends carry the same voltage.
- We say these elements are connected **in parallel**.

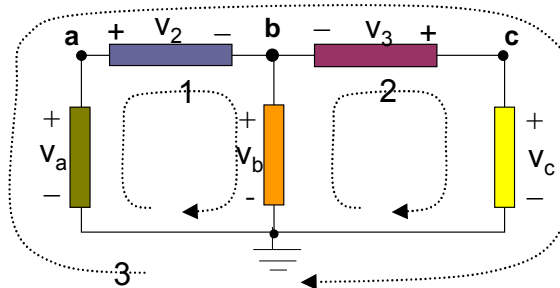


Applying KVL in the clockwise direction, starting at the top:

$$v_b - v_a = 0 \rightarrow v_b = v_a$$

## KVL Example

Three closed paths:



Path 1:

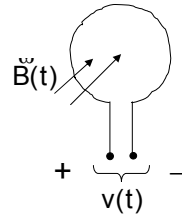
Path 2:

Path 3:

## An Underlying Assumption of KVL

- No time-varying magnetic flux through the loop  
Otherwise, there would be an induced voltage (Faraday's Law)
- Note: Antennas are designed to “pick up” electromagnetic waves; “regular circuits” often do so undesirably.

Avoid these loops!



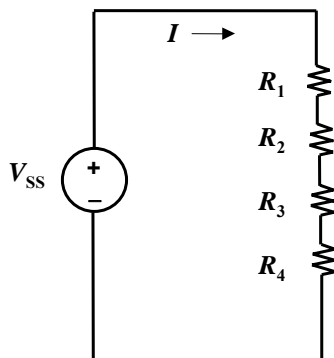
How do we deal with antennas (EECS 117A)?

Include a voltage source as the circuit representation of the induced voltage or “noise”.

(Use a **lumped model** rather than a distributed (wave) model.)

## Resistors in Series

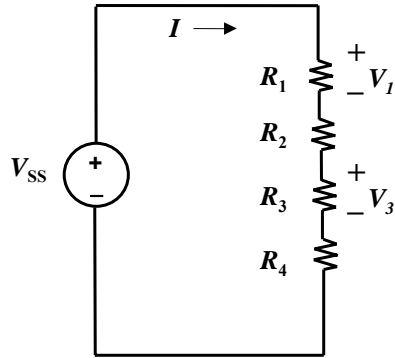
Consider a circuit with multiple resistors connected in series. Find their “equivalent resistance”.



- KCL tells us that the same current ( $I$ ) flows through every resistor
- KVL tells us

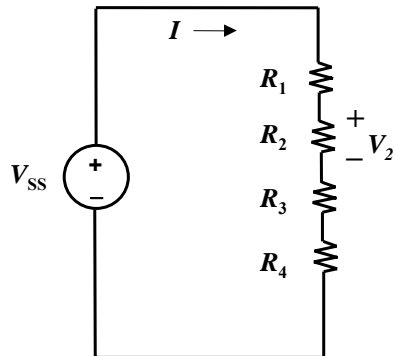
**Equivalent resistance of resistors in series is the sum**

## Voltage Divider



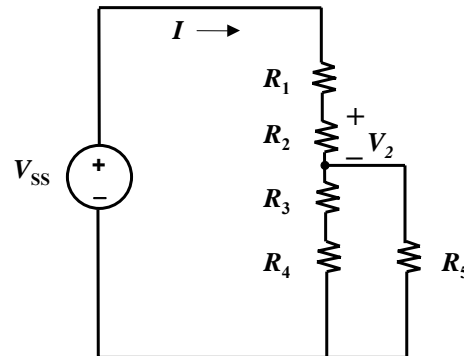
$$I = V_{SS} / (R_1 + R_2 + R_3 + R_4)$$

## When can the Voltage Divider Formula be Used?



$$V_2 = \frac{R_2}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS}$$

Correct, if nothing else  
is connected to nodes

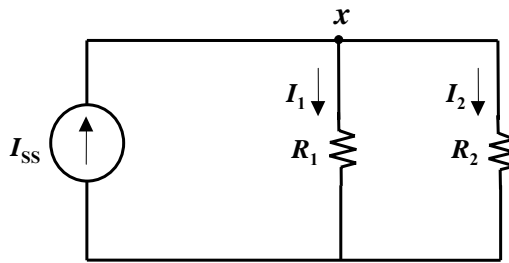


$$V_2 \neq \frac{R_2}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS}$$

Why? What is  $V_2$ ?

## Resistors in Parallel

Consider a circuit with two resistors connected in parallel. Find their “equivalent resistance”.



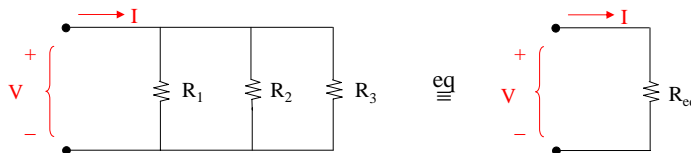
- KVL tells us that the same voltage is dropped across each resistor

$$V_x = I_1 R_1 = I_2 R_2$$

- KCL tells us

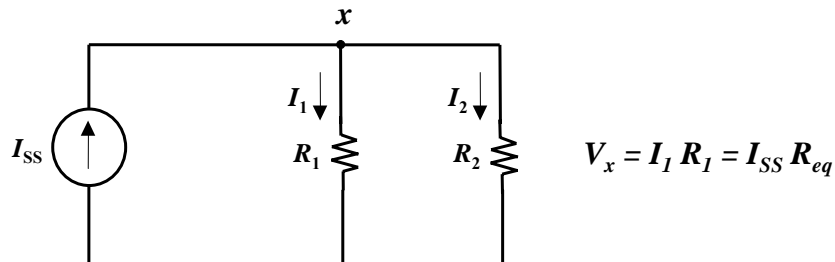
## General Formula for Parallel Resistors

What single resistance  $R_{eq}$  is equivalent to three resistors in parallel?



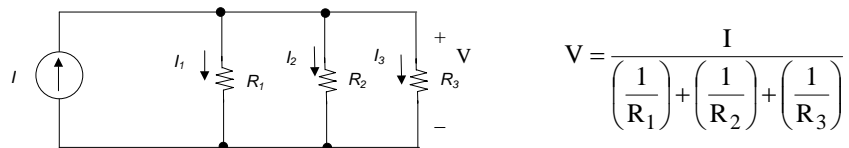
**Equivalent conductance of resistors in parallel is the sum**

## Current Divider



## Generalized Current Divider Formula

Consider a current divider circuit with >2 resistors in parallel:



$$I_3 = \frac{V}{R_3} = I \left[ \frac{1/R_3}{1/R_1 + 1/R_2 + 1/R_3} \right]$$

## Summary

- An **ideal voltage source** maintains a prescribed voltage regardless of the current in the device.
- An **ideal current source** maintains a prescribed current regardless of the voltage across the device.
- A **resistor** constrains its voltage and current to be proportional to each other:

$$v = iR \quad (\text{Ohm's law})$$

## Summary (cont'd)

- **Kirchhoff's current law (KCL)** states that the algebraic sum of all currents at any node in a circuit equals zero.
- **Kirchhoff's voltage law (KVL)** states that the algebraic sum of all voltages around any closed path in a circuit equals zero.
- **Resistors in Series** – Voltage Divider
- **Conductances in Parallel** – Current Divider