Announcements

- Homework #1 due Monday at 6pm
  - White drop box in Student Lounge on the second floor of Cory
- Books on reserve in Bechtel
  - Hambley, 2^{nd} and 3^{rd} Edition
- Next Friday's class is cancelled
  - It will be made up the following Thursday (July 7^{th}) 12-2

Review from Last Class

- KCL, KVL
  - Node and Loops
- Resistors in Series and parallel
  - Equivalent Resistance
- Voltage and Current Division
- I-V Characteristics
  - Sources
  - Resistors

Lecture #3

OUTLINE

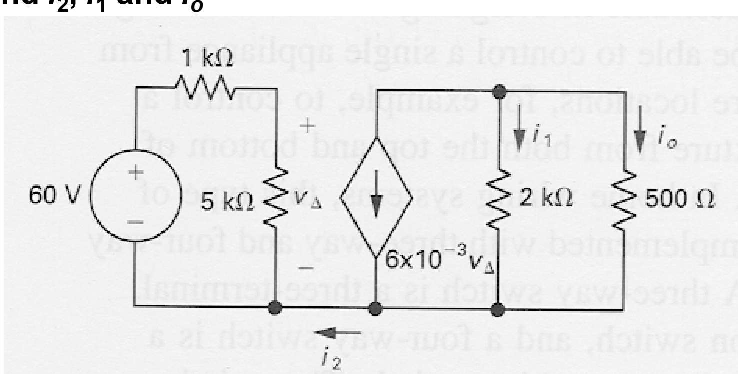
- KCL, KVL Examples
- Thevenin/Norton Equivalent circuit
- Measurement Devices

Reading

Finish Chapter 1,2

Circuit w/ Dependent Source Example

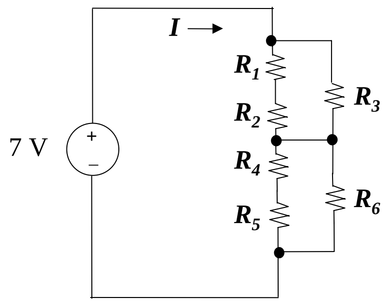
Find i_2, i_1 and i_o



Using Equivalent Resistances

Simplify a circuit before applying KCL and/or KVL:

Example: Find I



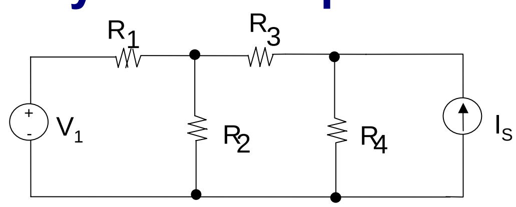
$R_1 = R_2 = 3\ \text{k}\Omega$
$R_3 = 6\ \text{k}\Omega$

$R_4 = R_5 = 5\ \text{k}\Omega$
$R_6 = 10\ \text{k}\Omega$

Node-Voltage Circuit Analysis Method

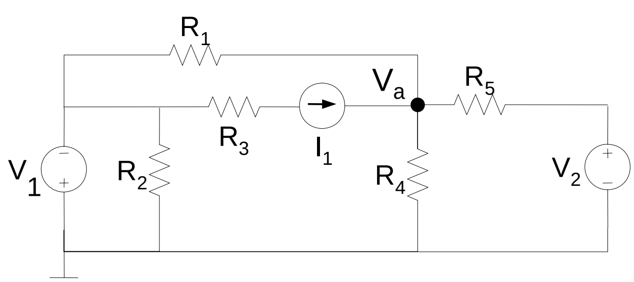
1. **Choose a reference node** ("ground")
   Look for the one with the most connections!
2. **Define unknown node voltages**
   those which are not fixed by voltage sources
3. **Write KCL at each unknown node**, expressing current in terms of the node voltages (using the *I*-*V* relationships of branch elements)
   Special cases: floating voltage sources
4. **Solve the set of independent equations**
   N equations for *N* unknown node voltages

Nodal Analysis: Example #1



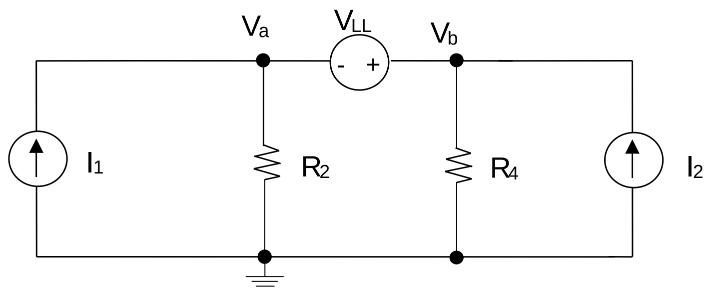
1. Choose a reference node.
2. Define the node voltages (except reference node and the one set by the voltage source).
3. Apply KCL at the nodes with unknown voltage.
4. Solve for unknown node voltages.

Nodal Analysis: Example #2



Nodal Analysis w/ "Floating Voltage Source"

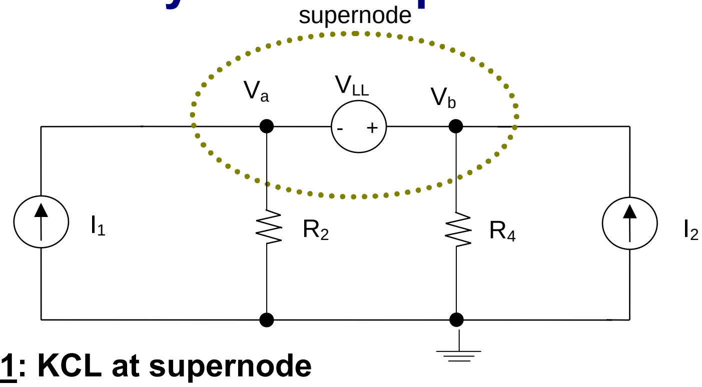
A "floating" voltage source is one for which neither side is connected to the reference node, e.g. V_{LL} in the circuit below:



Nodal Analysis w/ "Floating Voltage Source"

- Problem: We cannot write KCL at nodes a or b because there is no way to express the current through the voltage source in terms of Va-Vb.
- Solution: Define a "supernode" – that chunk of the circuit containing nodes a and b. Express KCL for this supernode. Incorporate voltage source constraint into KCL equation.

Nodal Analysis: Example #3



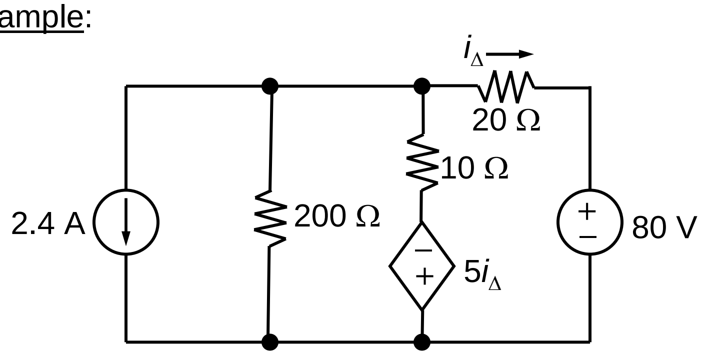
Eq'n 1: KCL at supernode

Substitute property of voltage source:

Node-Voltage Method and Dependent Sources

- If a circuit contains dependent sources, what to do?

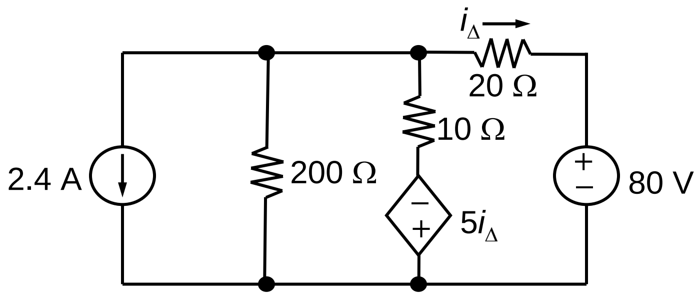
Example:



Node-Voltage Method and Dependent Sources

- Dependent current source: treat as independent current source in organizing and writing node eqns, but include (substitute) constraining dependency in terms of defined node voltages.
- Dependent voltage source: treat as independent voltage source in organizing and writing node eqns, but include (substitute) constraining dependency in terms of defined node voltages.

Example:



Mesh Circuit Analysis Method

1) **Select M independent mesh currents** such that at least one mesh current passes through each branch*

   M = #branches - #nodes + 1

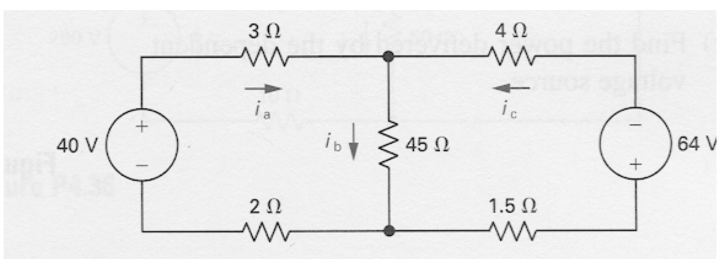
2) **Apply KVL to each mesh**, expressing voltages in terms of mesh currents

   => M equations for
   M unknown mesh currents

3) **Solve for mesh currents**

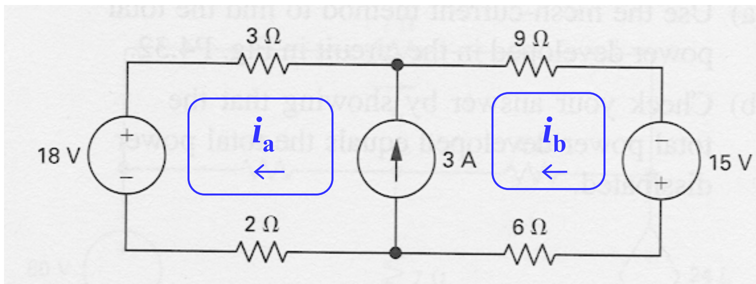
   => determine node voltages

Mesh Analysis: Example #1



1. Select M mesh currents.
2. Apply KVL to each mesh.
3. Solve for mesh currents.

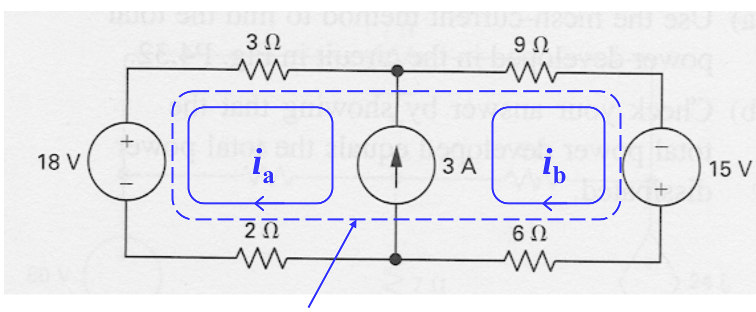
Mesh Analysis with a Current Source



Problem: We cannot write KVL for meshes a and b because there is no way to express the voltage drop across the current source in terms of the mesh currents.

Solution: Define a "supermesh" – a mesh which avoids the branch containing the current source. Apply KVL for this supermesh.

Mesh Analysis: Example #2



Eq'n 1: KVL for supermesh

Eq'n 2: Constraint due to current source:

Mesh Analysis with Dependent Sources

- Exactly analogous to Node Analysis
- Dependent Voltage Source: (1) Formulate and write KVL mesh eqns. (2) Include and express dependency constraint in terms of mesh currents
- Dependent Current Source: (1) Use supermesh. (2) Include and express dependency constraint in terms of mesh currents

Formal Circuit Analysis Methods

NODAL ANALYSIS

("Node-Voltage Method")

1) Choose a reference node
2) Define unknown node voltages
3) Apply KCL to each unknown node, expressing current in terms of the node voltages
   => N equations for N unknown node voltages
4) Solve for node voltages
   => determine branch currents

MESH ANALYSIS

("Mesh-Current Method")

1) Select M independent mesh currents such that at least one mesh current passes through each branch*
   M = #branches - #nodes + 1
2) Apply KVL to each mesh, expressing voltages in terms of mesh currents
   => M equations for M unknown mesh currents
3) Solve for mesh currents
   => determine node voltages

Superposition

A ***linear circuit*** is one constructed only of linear elements (linear resistors, **and linear capacitors and inductors,** linear dependent sources) and independent sources. Linear

means I-V charcteristic of elements/sources are straight lines when plotted

Principle of Superposition:

- In any linear circuit containing multiple independent sources, the current or voltage at any point in the network may be calculated as the algebraic sum of the individual contributions of each source acting alone.

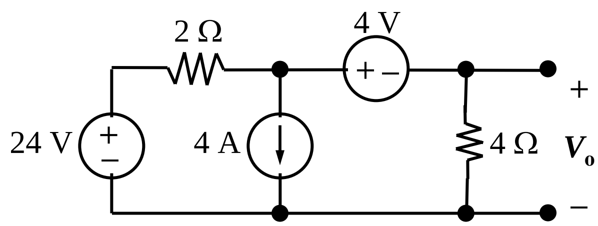
Superposition

Procedure:

1. Determine contribution due to one independent source
   - Set all other sources to 0: Replace independent voltage source by short circuit, independent current source by open circuit
2. Repeat for each independent source
3. Sum individual contributions to obtain desired voltage or current

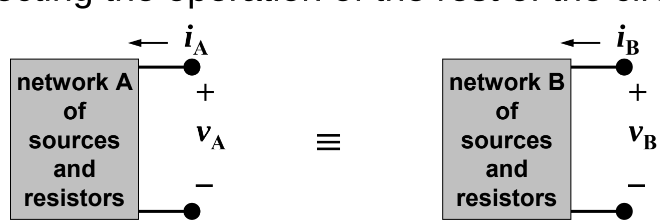
Superposition Example

- Find V_o



Equivalent Circuit Concept

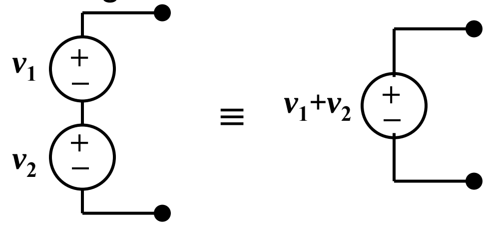
- A network of voltage sources, current sources, and resistors can be replaced by an ***equivalent circuit*** which has identical terminal properties (*I*-*V* characteristics) without affecting the operation of the rest of the circuit.



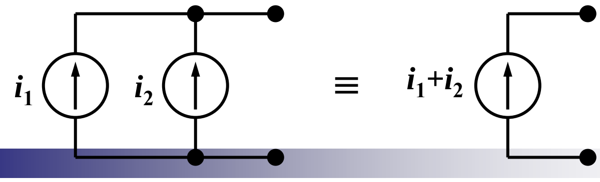
$$i_A(v_A) = i_B(v_B)$$

Source Combinations

- Voltage sources in series can be replaced by an equivalent voltage source:

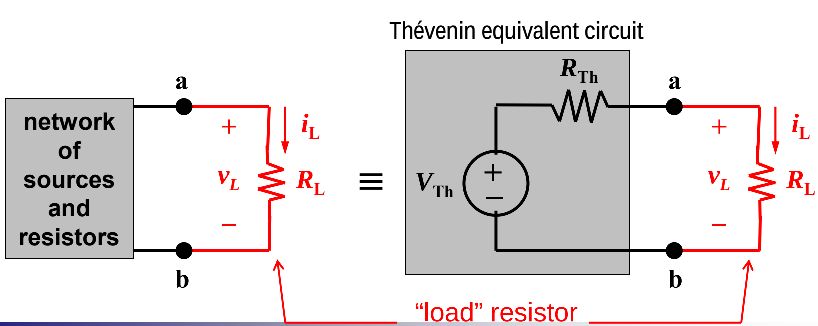


- Current sources in parallel can be replaced by an equivalent current source:



Thévenin Equivalent Circuit

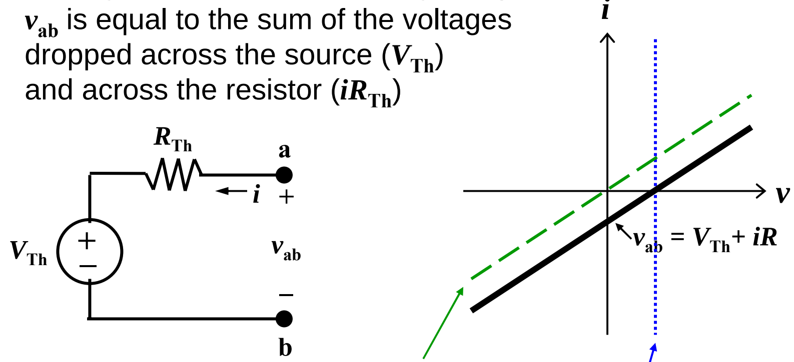
- Any* *linear* 2-terminal (1-port) network of indep. voltage sources, indep. current sources, and linear resistors can be replaced by an equivalent circuit consisting of an independent voltage source in series with a resistor without affecting the operation of the rest of the circuit.



I-V Characteristic of Thévenin Equivalent

- **The *I-V* characteristic for the series combination of elements is obtained by adding their voltage drops:**

For a given current i, the voltage drop v_{ab} is equal to the sum of the voltages dropped across the source (V_{Th}) and across the resistor (iR_{Th})

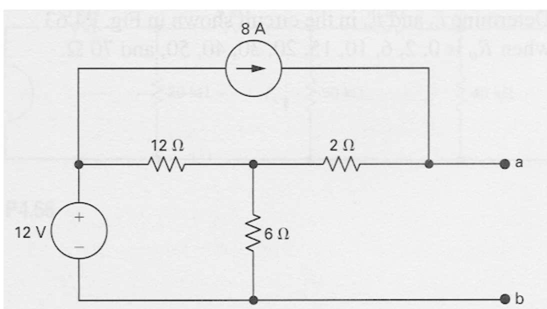


I-V characteristic of resistor: $v = iR$

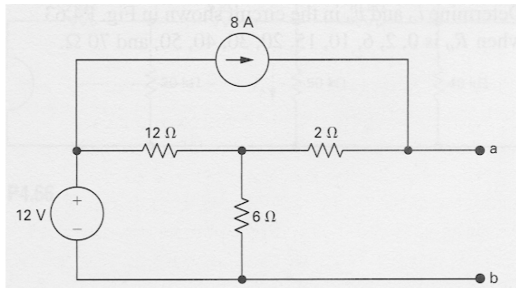
I-V characteristic of voltage source: $v = V_{Th}$

Thévenin Equivalent Voltage Example

Find the Thevenin equivalent voltage with respect to the terminals a,b:



R_{Th} Calculation Example #1



Set all independent sources to 0:

Comments on Dependent Sources

A dependent source establishes a voltage or current whose value depends on the value of a voltage or current at a specified location in the circuit.

(device model, used to model behavior of transistors & amplifiers)

To specify a dependent source, we must identify:

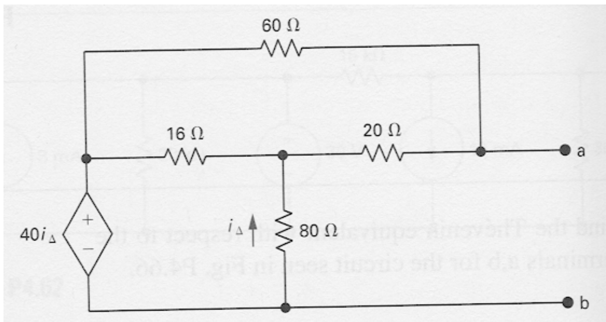
1. the controlling voltage or current (must be calculated, in general)
2. the relationship between the controlling voltage or current and the supplied voltage or current
3. the reference direction for the supplied voltage or current

The relationship between the dependent source and its reference cannot be broken!

- Dependent sources cannot be turned off for various purposes (*e.g.* to find the Thévenin resistance, or in analysis using Superposition).

Thevenin Equivalent Example #2

Find the Thevenin equivalent with respect to the terminals a,b:



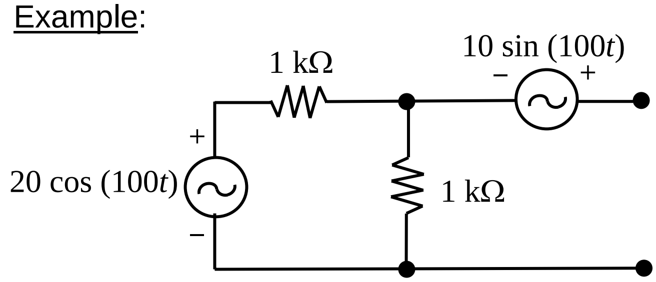
Thevenin Equivalent Calculation

- $V_{Th} = V_{OL}$
- $R_{Th} = V_{Th}/I_{SC}$

Networks Containing Time-Varying Sources

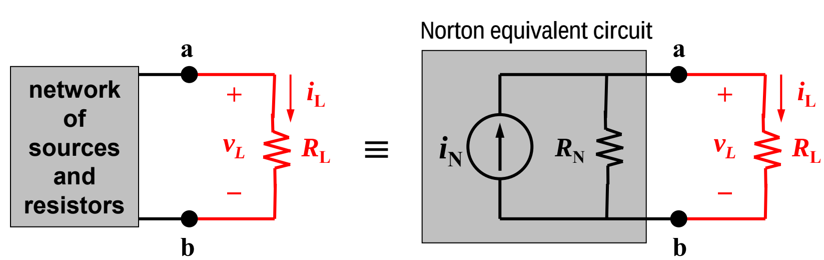
Care must be taken in summing time-varying sources!

Example:



Norton Equivalent Circuit

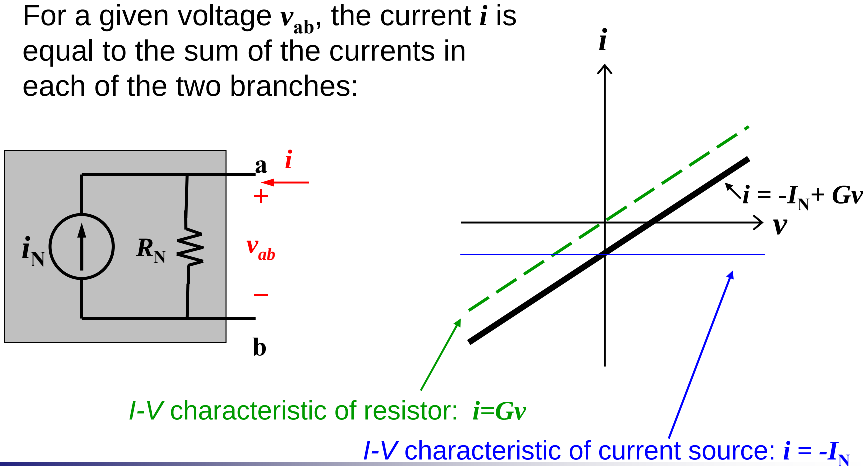
- Any* *linear* 2-terminal (1-port) network of indep. voltage sources, indep. current sources, and linear resistors can be replaced by an equivalent circuit consisting of an independent current source in parallel with a resistor without affecting the operation of the rest of the circuit.



I-V Characteristic of Norton Equivalent

- **The *I-V* characteristic for the parallel combination of elements is obtained by adding their currents:**

For a given voltage v_{ab}, the current i is equal to the sum of the currents in each of the two branches:



Finding I_N and $R_N = R_{Th}$

Analogous to calculation of Thevenin Eq. Ckt:

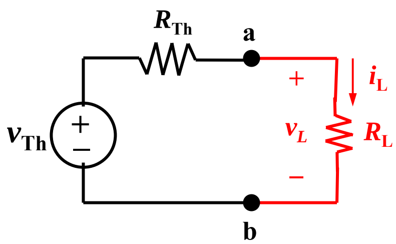
1) Find o.c voltage and s.c. current

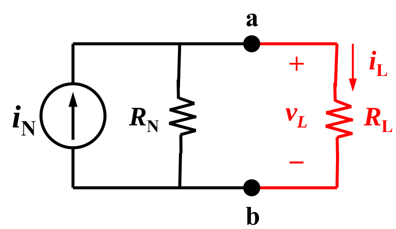
$$I_N \equiv i_{sc} = V_{Th}/R_{Th}$$

2) Or, find s.c. current and Norton (Thev) resistance

Finding I_N and R_N

- We can derive the Norton equivalent circuit from a Thévenin equivalent circuit simply by making a source transformation:

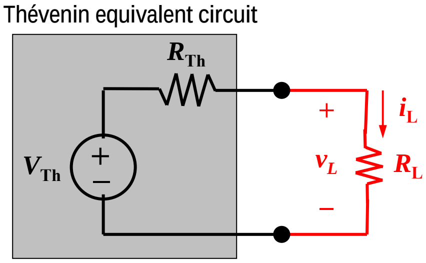




$$\boxed{R_N = R_{Th} = \frac{v_{oc}}{i_{sc}};\quad i_N = \frac{v_{Th}}{R_{Th}} = i_{sc}}$$

Maximum Power Transfer Theorem

Thévenin equivalent circuit



Power absorbed by load resistor:

$$p = i_L^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

To find the value of R_L for which p is maximum, set $\dfrac{dp}{dR_L}$ to 0:

$$\frac{dp}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th} + R_L)^2 - R_L \times 2(R_{Th} + R_L)}{(R_{Th} + R_L)^4}\right] = 0$$

$$\Rightarrow (R_{Th} + R_L)^2 - R_L \times 2(R_{Th} + R_L) = 0$$

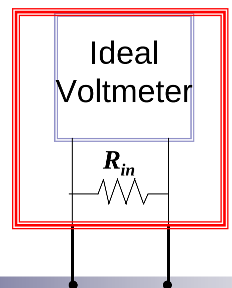
$$\Rightarrow \boxed{R_{Th} = R_L}$$

A resistive load receives maximum power from a circuit if the load resistance equals the Thévenin resistance of the circuit.

Measuring Voltage

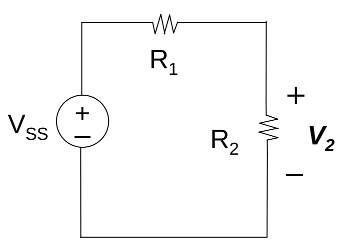
To measure the voltage drop across an element in a real circuit, insert a voltmeter (digital multimeter in voltage mode) **in parallel** with the element.

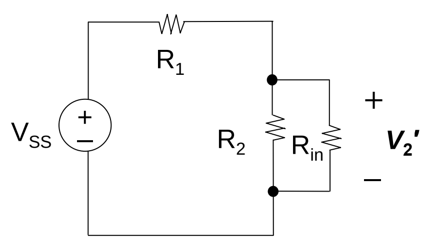
Voltmeters are characterized by their "voltmeter input resistance" (R_{in}). Ideally, this should be very high (typical value 10 MΩ)



Effect of Voltmeter

undisturbed circuit | **circuit with voltmeter inserted**





$$V_2 = V_{SS}\left[\frac{R_2}{R_1 + R_2}\right]$$

$$V_2' = V_{SS}\left[\frac{R_2 \parallel R_{in}}{R_2 \parallel R_{in} + R_1}\right]$$

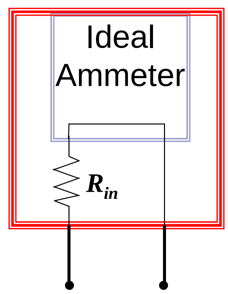
Example: $V_{SS} = 10\,V, R_2 = 100\,K, R_1 = 900\,K \Rightarrow V_2 = 1V$

$R_{in} = 10M,\ V_2' = ?$

Measuring Current

To measure the current flowing through an element in a real circuit, insert an ammeter (digital multimeter in current mode) **in series** with the element.

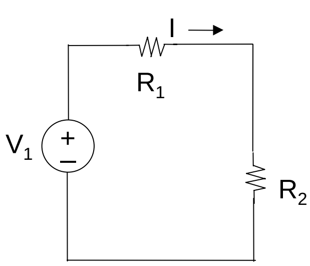
Ammeters are characterized by their “ammeter input resistance” (R_{in}). Ideally, this should be very low (typical value 1Ω).



Effect of Ammeter

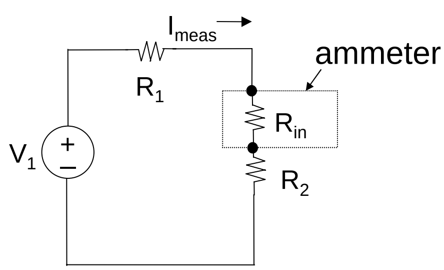
Measurement error due to non-zero input resistance:

undisturbed circuit



$$I = \frac{V_1}{R_1 + R_2}$$

circuit with ammeter inserted



$$I_{meas} = \frac{V_1}{R_1 + R_2 + R_{in}}$$

Example: V_1 = 1 V, R_1= R_2 = 500 Ω, R_{in} = 1Ω

$$I = \frac{1V}{500\Omega + 500\Omega} = 1mA, \quad I_{meas} = ?$$