Announcements

- Midterm 2 this Thursday 11:30 to 1pm
  - 145 Dwinelle
  - Lectures 8-13, HW #4,5 (no small signal model)
  - Review Session this Wednesday 5-8pm in Cory 277

Lecture #14

OUTLINE

- Load Line and Small signal analyses of:
  - Common source amplifier
  - Source follower
  - Common gate amplifier

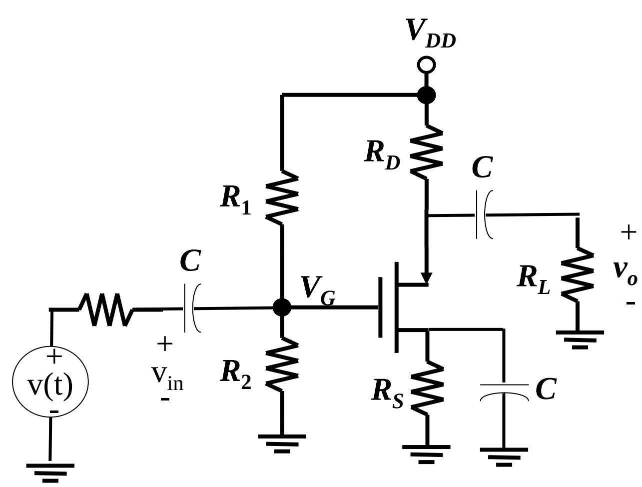
Reference Reading

- Hambley: Chapter 12.1-12.5

MOSFET Circuit

- First look at DC case to find Q point
  - Use load line technique
  - All capacitors are open circuit
  - From Q-point, get g_m and r_d for small signal AC model
- AC Small signal analysis
  - DC source is AC ground (because there is no AC signal variation).
  - All capacitors are short circuit (unless otherwise specified).

Common Source Amplifier

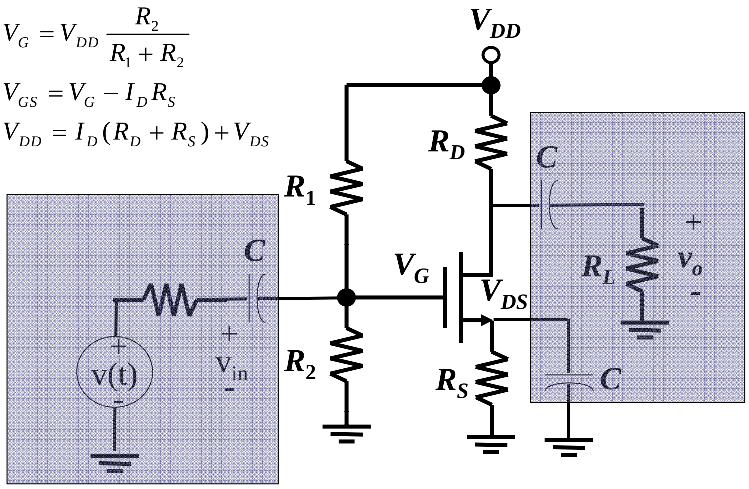


Step 1: find Q point

$$V_G = V_{DD}\frac{R_2}{R_1 + R_2}$$

$$V_{GS} = V_G - I_D R_S$$

$$V_{DD} = I_D (R_D + R_S) + V_{DS}$$



Load line:

From load lines, we get I_D → and hence g_m and r_d

Small Signal Model:

$$v_g = v_{in}, v_s = 0 \rightarrow v_{gs} = v_{in}$$

$$v_o = \frac{R_L R_D}{R_L + R_D}(-g_m v_{gs})$$

$$A_v = \frac{v_o}{v_{in}} = -g_m \frac{R_L R_D}{R_L + R_D}$$

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{R_1 R_2}{R_1 + R_2}$$

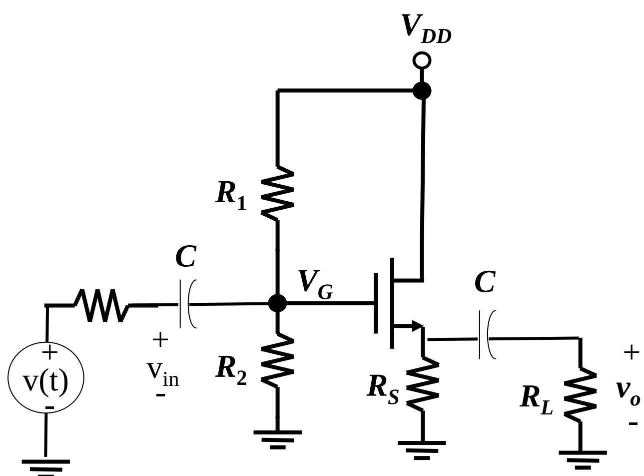
For output impedance R_{out}:

1. Turn off all independent sources.
2. Take away load impedance R_L

$$v_{in} = 0, v_{gs} = 0, g_m v_{gs} = 0$$

$$R_{out} = \frac{r_d R_D}{r_d + R_D}$$

Source Follower

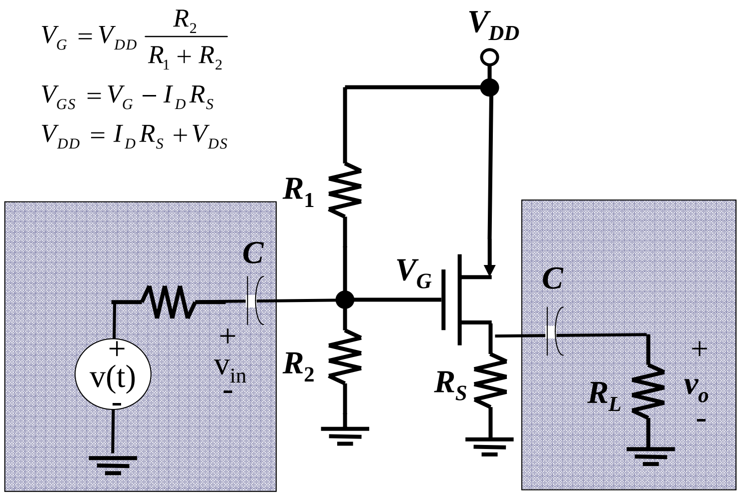


Step 1: find Q point

$$V_G = V_{DD}\frac{R_2}{R_1 + R_2}$$

$$V_{GS} = V_G - I_D R_S$$

$$V_{DD} = I_D R_S + V_{DS}$$



Load line:

From load lines, we get I_D → and hence g_m and r_d

Small Signal Model:

$$R_L' = \frac{1}{r_d^{-1} + R_S^{-1} + R_L^{-1}}$$

$$v_{gs} = v_{in} - v_o$$

$$v_o = g_m v_{gs} R_L'$$

$$v_{in} = v_{gs}(1 + g_m R_L')$$

$$A_v = \frac{v_o}{v_{in}} = \frac{g_m R_L'}{1 + g_m R_L'}$$

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{R_1 R_2}{R_1 + R_2}$$

For output impedance R_{out}:

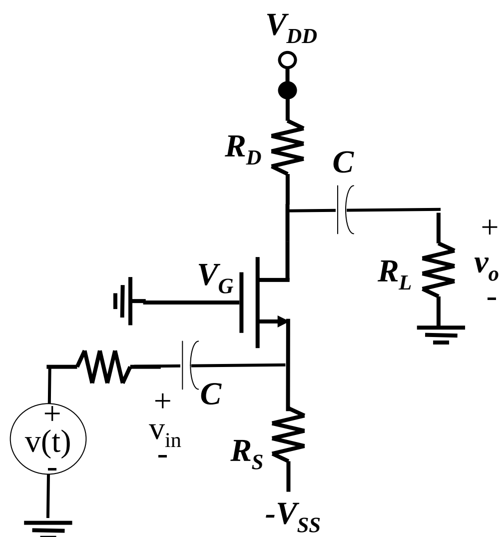
1. Turn off all independent sources.
2. Take away R_L
3. Add V_x and find i_x

$$v_x = v_s, v_g = 0, v_{gs} = -v_x$$

$$R_s' = \frac{r_d R_s}{r_d + R_s}, i_x = \frac{v_x}{R_s'} - g_m(-v_x) = v_x\left(R_s'^{-1} + g_m\right)$$

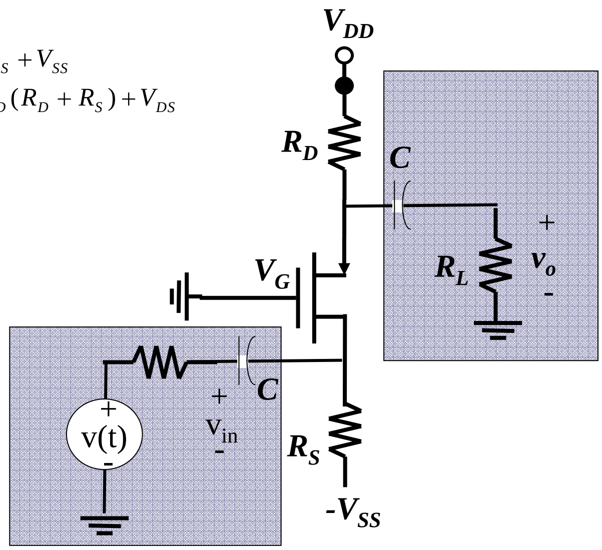
$$R_{out} = \frac{1}{g_m + r_d^{-1} + R_s^{-1}}$$

Common Gate Amplifier



Step 1: find Q point

$$V_{GS} = 0 - I_D R_S + V_{SS}$$
$$V_{DD} + V_{SS} = I_D (R_D + R_S) + V_{DS}$$



Load line

The onıy difference in all three circuits are the intercepts at the axes.
Again from load lines, we get I_D → and hence g_m and r_d

Small Signal Model:

$$R_L' = \frac{1}{R_L^{-1} + R_D^{-1}}$$
$$v_{gs} = -v_{in}$$
$$v_o = -g_m v_{gs} R_L'$$
$$A_v = \frac{v_o}{v_{in}} = g_m R_L'$$
$$i_{in} = -(g_m v_{gs} + \frac{v_{gs}}{R_s})$$
$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{1}{g_m + R_s^{-1}}$$

For output impedance R_{out}:

1. Turn off all independent sources.
2. Take away R_L
3. Add V_x and find i_x

$$R' = \frac{RR_s}{R + R_s}$$
$$i_x = \frac{v_x}{R_D} + g_m v_{gs} = v_x\left(R_s'^{-1} + g_m\right)$$
$$v_{gs} = -g_m v_{gs} R',\ \ but\ \ g_m R' \neq 1 \therefore v_{gs} = 0$$
$$R_{out} = R_D$$

Review on Bode Plots

- Transfer function= H(f) = V_{out}/V_{in}
- It is a complex number and typically a function of frequency.

$$\mathbf{H}(f) = \frac{\mathbf{V_{out}}}{\mathbf{V_{in}}} = \frac{V_{out}}{V_{in}} \angle\left(\theta_{out} - \theta_{in}\right)$$
$$\mathbf{H(f)} = \left|\mathbf{H(f)}\right| \angle\theta\left(f\right)$$

$\left|\mathbf{H(f)}\right|$ is the magnitude

$\theta\left(f\right)$ is the phase

Both are functions of frequency f.

Review on Bode Plots

$$\mathbf{H}(f) = \mathbf{H}_1(f) \cdot \mathbf{H}_2(f)$$

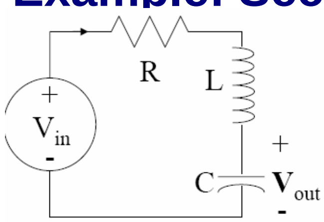
$$|\mathbf{H(f)}| = |\mathbf{H}_1\mathbf{(f)}| \cdot |\mathbf{H}_2\mathbf{(f)}|$$

$$\theta(f) = \theta_1(f) + \theta_2(f)$$

$$Y = 20\log|\mathbf{H(f)}| = 20\log|\mathbf{H}_1\mathbf{(f)}| + 20\log|\mathbf{H}_2\mathbf{(f)}|$$

- Bode plots include magnitude vs. plot and phase vs. frequency plots
  - Magnitude plots are log-log (dB vs. log Hz) → Y vs. log f
  - Phase are linear-log (angle in degrees or radians vs. log frequency)
- Factor of 10 increase in frequency = 1 decade
- Factor of 2 increase in frequency = 1 octave
- If H(f) can be factored into H_1(f) and H_2(f), it will be easier to plot bode plots for each and then sum them up.

Example: Second Order Circuit



$$\mathbf{H(f)} = \frac{\mathbf{V}_c}{\mathbf{V}} = \frac{1/(j\omega C)}{1/(j\omega C) + R + j\omega L} = \frac{1}{1 - \omega^2 LC + j\omega RC}$$

$$= \frac{1}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}} \angle\left[-\tan^{-1}\left(\frac{\omega RC}{1-\omega^2 LC}\right)\right]$$

In plotting the magnitude Bode plot, we should evaluate the extreme cases: ω=0 and infinity

In addition, we should examine 0+ (small positive), and the frequency at which $|\mathbf{H(f)}|$ is maximum or minimum.

Examine $|\mathbf{H(f)}|$, we can easily see that max occurs at $\omega_0 = \left(\frac{1}{\sqrt{LC}}\right)$

$$\max|\mathbf{H(f)}| = \left(\frac{1}{\omega_0 RC}\right) = \frac{\sqrt{LC}}{RC}$$

Recall in Lec. 9 when we talked about 2nd order circuit, we had a resonance frequency, which is exactly $\omega_0 = \left(\frac{1}{\sqrt{LC}}\right)$

Magnitude Response

$$|\mathbf{H(f)}| = \frac{1}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}$$

$\omega = 0,\ |\mathbf{H(f)}| = 1,\ Y = 20\log(1) = 0\ dB$

$0 < \omega < \omega_0,\ \omega^2 LC < 1,\ as\ \omega\ increases, |\mathbf{H(f)}|\ decreases.$

$At\ \omega = \omega_0, 1-\omega^2 LC = 0, |\mathbf{H(f)}| = \frac{\sqrt{L}}{R\sqrt{C}} = \frac{1}{2\zeta}$

$Y = 20\log\frac{1}{2\zeta} = 20\left[\log\frac{1}{2} + \log\frac{1}{\zeta}\right]$

Recall, $\zeta = 1 \rightarrow critically\ damped,\ Y(\omega_0) = -3dB$

$\zeta > 1 \rightarrow over\ damped,\ Y(\omega_0) < -3dB$

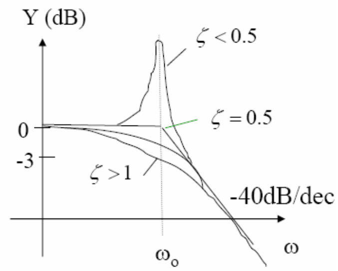
$\zeta < 1 \rightarrow under\ damped,\ Y(\omega_0) > -3dB$

$\zeta < 0.5 \rightarrow under\ damped,\ Y(\omega_0) > 0dB$

$For\ \omega >> \omega_0, As\ \omega\ increases, |\mathbf{H(f)}| \sim \frac{\omega_0^2}{\omega^2}$

$Y\ reduces\ by\ 40dB\ per\ 10x\ increse$ in $\omega, i.e.$

$slope\ in\ bode\ magnitude\ plot = -40dB/dec$



Phase Response

$\omega = 0,\ |\mathbf{H(f)}| = 1,\ \theta = 0$

$\omega = 0+,\ |\mathbf{H(f)}| = 1,\ \theta = negative\ small\ value$

$As\ \omega\ increases, \theta\ becomes\ more\ negative.$

$at\ \omega = \omega_0, 1-\omega^2 LC = 0, \theta = \frac{\pi}{2}\ or\ -\frac{\pi}{2}$

$at\ \omega = \omega_0 - \delta, 1-\omega^2 LC > 0, \theta \rightarrow -\frac{\pi}{2}$

$at\ \omega = \omega_0 + \delta, 1-\omega^2 LC < 0, \theta \rightarrow \frac{\pi}{2}$

$For\ \omega >> \omega_0, As\ \omega\ increases, \theta\ is\ positive\ but\ decreases.$

$as\ \omega \rightarrow \infty, \theta \rightarrow 0^\circ$