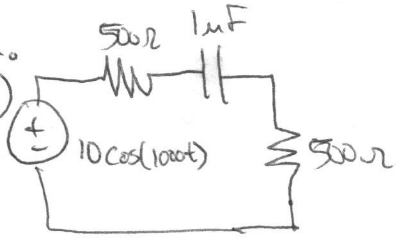
1. a.

i)



$$\tilde{V}_L = \tilde{V}_S\left(\frac{500}{1000+\frac{1}{j10^{-3}}}\right) = \tilde{V}_S\left(\frac{500}{1000(1-j)}\right) = 0.25(1+j)\tilde{V}_S$$

$$= \boxed{2.5\sqrt{2}\,e^{j\pi/4}}$$

$$\tilde{I}_L = \frac{\tilde{V}_S}{1000(1-j)} = 0.005(1+j) = \boxed{5\times10^{-3}\sqrt{2}\,e^{j\pi/4}}$$

ii) $p(t) = v(t)\,i(t)$

$$= 2.5\sqrt{2}\cos(1000t + \pi/4) * 0.005\sqrt{2}\cos(1000t+\pi/4)$$

$$\boxed{p(t) = 0.025\cos^2(1000t + \pi/4)}$$

iii) <u>Method 1 RMS</u>

$$V_{RMS} = \frac{2.5\sqrt{2}}{\sqrt{2}} = 2.5, \quad I_{RMS} = \frac{0.005\sqrt{2}}{\sqrt{2}} = 0.005$$

$$P_{AVG} = V_{RMS}I_{RMS} = 0.0125\,W = \boxed{12.5\,mW}$$

<u>Method 2 Integration</u>

$$P_{avg} = \frac{\int_0^T 0.025\cos^2(1000t)\,dt}{T}, \quad T = \frac{1}{1000}$$

$$= 25\int_0^{\frac{1}{1000}} \frac{1}{2} + \underbrace{\frac{1}{2}\cos(2000t)}_{\to 0}\,dt$$

$$= 25\int_0^{\frac{1}{1000}} \frac{1}{2}\,dt = 25\left(\frac{1}{2}\times\frac{1}{1000}\right) = \boxed{12.5\,mW}$$

iv) $P_{avg} = \frac{1}{2}|V||I|\cos\theta, \quad \theta = \frac{\pi}{4} - \frac{\pi}{4} = 0$

$$= \frac{1}{2}(2.5\sqrt{2})(5\times10^{-3}\sqrt{2}) = \boxed{12.5\,mW}$$

v) $P_{avg} = \frac{1}{2}\mathrm{Re}(\tilde{V}\tilde{I}^*) = \frac{1}{2}\mathrm{Re}\left(2.5\sqrt{2}\,\cancel{e^{j\pi/4}} * 5\times10^{-3}\sqrt{2}\,\cancel{e^{-j\pi/4}}\right)$

$$= \frac{1}{2}\mathrm{Re}(25\times10^{-3}) = \boxed{12.5\,mW}$$

b. i)

$E = \frac{1}{2}LI^2$, so if maximize I, maximize E.

$$I_{max} = \frac{V_{s\,max}}{500 + j10^3 - j10^3} = \frac{V_{s\,max}}{500} = \frac{10}{500} = \boxed{0.02A}$$

$$E_{max} = \frac{1}{2}LI_{max}^2 = \frac{1}{2}(1H)(0.02A)^2 = \boxed{0.2mJ}$$

ii) $P_{Avg} = \frac{1}{2}|V||I|\cos\theta$, $\theta = \frac{\pi}{2}$ across an inductor

$$= \frac{1}{2}|V||I|\underbrace{\cos\frac{\pi}{2}}_{0} = \boxed{0\,W}$$

iii) $P_{reactive} = \frac{1}{2}Im(VI^*) = \frac{1}{2}|\tilde{V}||\tilde{I}|$

$$V = \tilde{V}_s \frac{j\omega L}{500} = j\tilde{V}_s \times 2 = \boxed{j20V}$$

$$I = \frac{V}{j\omega L} = \frac{j20}{j10^3} = \boxed{0.02A}$$

$$P_{reactive} = \frac{1}{2}Im(j0.4) = \boxed{0.2W}$$

iv) $P_{avg} = \frac{1}{2}|V_R||I| = \frac{1}{2}(10)(0.02) = \boxed{0.1W}$

$$V_R = IR = (0.02)(500) = 10V$$

c. To draw max average power, want to cancel (resonate out) reactive components and match resistive ones:

so, $Z_L = Z_s^* = (500 - 1000j)^* = 500 + 1000j$

$$P_{Avg} = \frac{1}{2}(5V)(0.01A) = \boxed{25mW}$$

d. To maximize power with a purely resistive load want $|Z_s| = R_L$

$$|Z_s| = |500 - j1000| = \sqrt{500^2 + 1000^2} = \boxed{1118\,\Omega}$$

$$P_L = \frac{1}{2}|I||V|$$

$$\cancel{|V|} \quad \tilde{V} = \tilde{V}_s \frac{1118}{1618 + j10^3} = \tilde{V}_s \frac{(1118)(1618 - j10^3)}{1618^2 + 10^6}$$

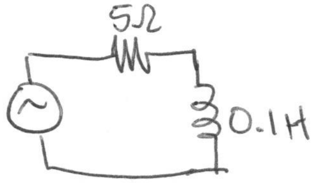
$$= V_s \frac{1.8\times10^6 - j1.1\times10^6}{3.6\times10^6} = 10(0.5 - j0.3)$$

$$|V| = \cancel{0.58} \; 5.8\,V$$

$$I = \frac{V}{1118} \Rightarrow |I| = \frac{|V|}{1118} \Rightarrow P_{L_{avg}} = \frac{1}{2}\frac{|V|^2}{1118} = \boxed{15\,mW}$$

e. To maximize power delivered by a fixed voltage source need to minimize resistance (impedance), or maximize current. So, $\boxed{\text{inductor from port b}}$ that resonates out capacitor is the correct choice. (1H)

2. a.



$$Z_L = j \times 377 \times 0.1 = j37.7$$

$$\tilde{V} = V_S = 110V$$

$$\tilde{I} = \frac{\tilde{V}}{Z} = \frac{110}{5 + j37.7} = \frac{550 - j4147}{1446.29} = 0.38 - j2.87$$

$$\tilde{I}^* = 0.38 + j2.87$$

$$\Rightarrow P_{Avg} = \frac{1}{2}\mathrm{Re}(110(0.38 + j2.87)) = \frac{1}{2}(110 \times 0.38) = \boxed{20.9\,W}$$

$$P_{Reactive} = \frac{1}{2}\mathrm{Im}(110(0.38 + j2.87)) = \frac{1}{2}(110 \times 2.87) = \boxed{157.85\,W}$$

b. $\tilde{I}^* = 100 \times (0.38 + j2.87) = \boxed{38 + j287}$

$$P_{Avg} = \frac{1}{2}\mathrm{Re}(110 \times (38 + j287)) = \boxed{2090\,W}$$

$$P_{Reactive} = \frac{1}{2}\mathrm{Im}(110 \times (38 + j287)) = \boxed{15785\,W}$$

c. i) Choose capacitance sith.

$$Z_c = -j\frac{37.7}{100} = \frac{1}{j377 \times C}$$

$$C = \frac{100}{377 \times 37.7} = \boxed{7\,mF}$$

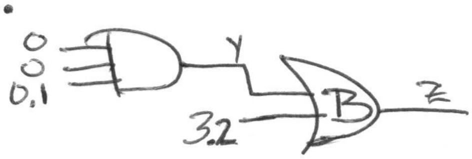
ii)

$$Z_{motors} = 5/100\ \Omega$$

$$\tilde{I} = \frac{110}{5/100} = 2200A$$

$$\Rightarrow P_{Avg} = \frac{1}{2}\tilde{V}\tilde{I} = \frac{1}{2} \times 110 \times 2200 = \boxed{121\,kW}$$

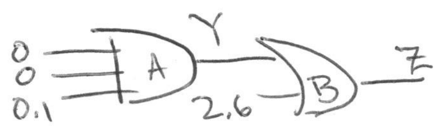
3. a.



Y should output a "low" so, $\boxed{0V \le Y \le 0.5V}$

B has 1 input low, 1 input high so, Z should output a high, or $\boxed{4.5V \le Z \le 5V}$

b.



Y is unchanged, so $\boxed{0 < Y < 0.5}$

B has one input low, one input undefined, so the output of B is undefined, or

$$\boxed{0V \le Z \le 5V}$$

c.

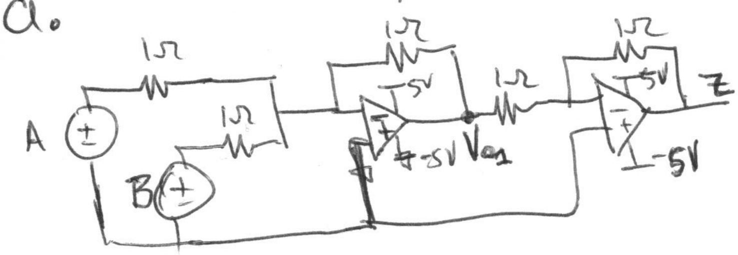
A has three inputs high, so

$$\boxed{4.5V \le Y \le 5V}$$

If B has one high input and one low input, the output should be a high, so

$$\boxed{4.5V \le Z \le 5V}$$

4.
a. If $A = 0.9V$, $B = 0.9V$, then $V_{o1} = (-1)(0.9 + 0.9) = -1.8V$.



Then, $Z = (-1)(-1.8) = 1.8V$, which is in the undefined region for our "static discipline",

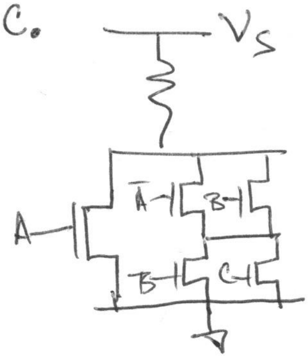
b. Not Graded

5.
a.

A	B	C	Z
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

$$Z = \overline{A + \bar{A}B + BC}$$
$$= \boxed{\overline{A + B(\bar{A} + C)}}$$

b. The pull-up resistor gives us a path to V_{DD} so that if the PDN network is off, the output will be pulled high. If the resistor were made comparable to the on resistance of a transistor, then when one transistor is on, $V_{out} = V_{DD}\left(\frac{R_{on}}{R_{pup}+R_{on}}\right) \approx \frac{V_{DD}}{2}$ which is not a valid high or low output.

c. 

A	B	C	Z
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

$\boxed{Z = \overline{ABC}}$

This is NAND3 !

6. a. Does not meet static discipline if

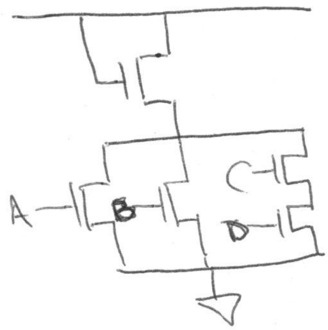
$$V_o = 1 = 5\left(\frac{R_{on}*N}{R_{on}*N+R}\right) = \frac{N}{5}R_{on}+\frac{R}{5} = R_{on}*N$$

$$\frac{R_{on}}{5} + \frac{R}{5}*\frac{1}{N} = R_{on}$$

$$\frac{4}{5}R_{ON} = \frac{R}{5N} \qquad 4\frac{R}{R_{ON}} = N = \frac{100}{4} = N = \boxed{25}$$

b. If all inputs are on, then the resistance from Z to ground $= \frac{R_{ON}}{n}$, so the current is maximized, and power is $\frac{V_{DD}^2}{R+\frac{R_{on}}{n}}$ which goes to $\frac{V_{DD}^2}{R}$ as $n \to \infty$.

7. $Z = \overline{A + B + CD}$



$$V_L = V_{DD}\left(\frac{R_{N,largest}}{R_{P,on} + R_{N,on\ largest}}\right)$$

Typically for digital ckts, we set L to the minimum value always

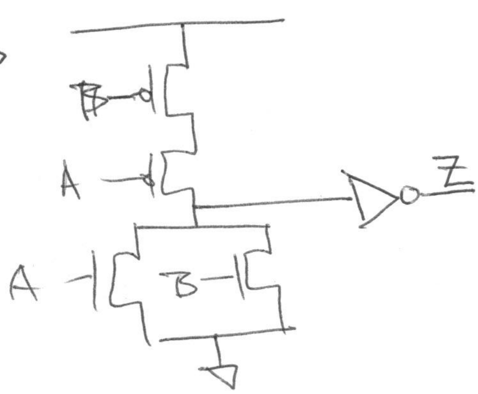
$$\frac{V_L}{V_{DD}} = \frac{R_{N,on}}{R_{P,on} + R_{N,on}}$$

$$\frac{V_L}{V_{DD}} = \frac{2\frac{1}{W_{NC}}}{2\frac{1}{W_{NC}} + \frac{1}{W_{N_{diode\text{-}conn}}}}$$

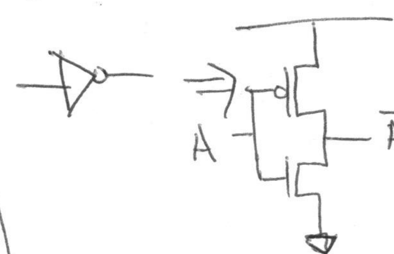
$$2\frac{1}{W_{CN}}\left(1 - \frac{V_L}{V_{DD}}\right) = \frac{V_L}{V_{DD}}\frac{1}{W_{N_{diode\text{-}conn}}}$$

$$\boxed{\frac{W_{N_{diode\text{-}conn}}}{W_{CN}} = \frac{V_L}{2V_{DD}}\left(\frac{1}{1 - \frac{V_L}{V_{DD}}}\right)}$$

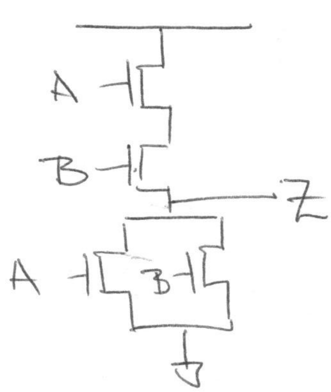
8. a. i. $A + B$ $\Rightarrow$



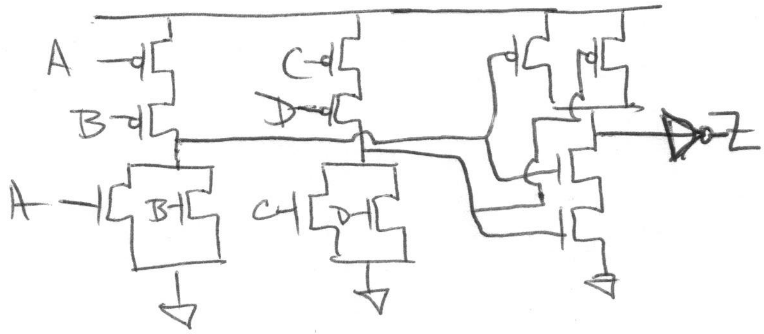
Note:



ii. $\overline{A + B}$

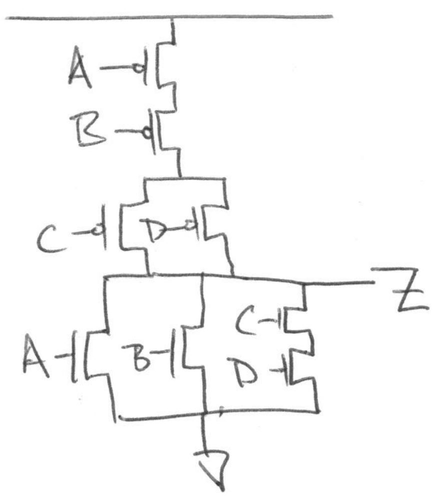


iii. $(A+B)(C+D) \Rightarrow AC+AD+BC+BD \Rightarrow A(C+D)+B(C+D)$



iv. Same as iii, but no inverter. $\left(\overline{(A+B)(C+D)}\right)$

v. $\overline{A+B+CD}$



b. No, with static CMOS, in steady-state the output will always be either VDD or GND, and this is not affected by the on resistance.

9. a. CMOS <u>does not</u> have static power dissipation (i.e. 0W)

b. As was shown in HW 4, prob. 7, $E_{diss} = \frac{1}{2}CV_{DD}^2$

so, $E_{diss} = \frac{1}{2}(30\times10^{-9})(5)^2 = \boxed{375nJ}$

10. <u>CMOS</u>

<u>Advantages</u>

- No static power consumption
- Very narrow forbidden region (large noise margins)
- Output will always go to VDD or GND

<u>Disadvantages</u>

- Possibly considered slower than NMOS (not part of this class)

<u>NMOS only</u>

<u>Advantages</u>

- Possibly faster than CMOS

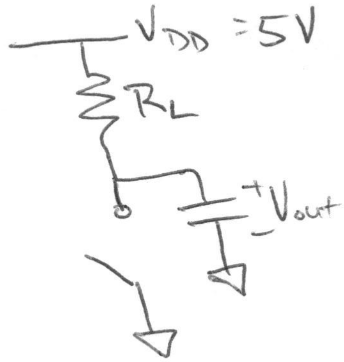
<u>Disadvantages</u>

- Static power in low output
- possibly wide forbidden region
- output cannot reach GND.

11. i. First define rise and fall time.

t_{rise} is time for output beginning at 0V to reach V_H. t_{fall} is time from V_{DD} to V_L.

Ckt for rise time



So, solve differential equation (intuitive method)

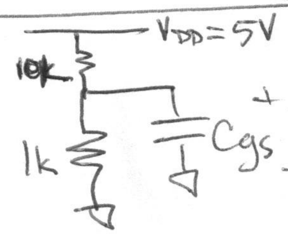
$$V_{out}(t) = V_{DD}\left(1 - e^{-t/R_L C_{gs}}\right)$$

When $V_{out} = 3V$ have reached t_{rise}.

$$3 = 5\left(1 - e^{-t/10^{-5}}\right)$$

$$10^{-5}\ln(\ ) = \boxed{t_{rise} = 5.1\mu s}$$

Ckt for fall time



$$C\frac{dV_{out}}{dt} + \frac{V_{out}}{1k} = \frac{V_{DD} - V_{out}}{10k}$$

$$\frac{dV_{out}}{dt} = \frac{V_{DD}}{(C)10k} - \frac{1.1V_{out}}{(C)1k}$$

$$V_{out}(t) = \left(V_{DD} - V_{DD}\left(\frac{1}{11}\right)\right)e^{-t/\frac{10^{-6}}{1.1}} + V_{DD}($$

$$1 = \left(5 - \frac{5}{11}\right)e^{-t/(10^{-6}/1.1)} + \frac{5}{11}$$

$$-\ln\left(\left(\frac{6}{11}\right)/\left(\frac{50}{11}\right)\right) = t\left(\frac{1.1}{10^{-6}}\right)$$

$$\boxed{t = 1.9\mu s}$$

(not for a grade)

iii. Propagation delay is usually defined as the amount of time for the output to reach $\frac{V_{DD}}{2}$ after the input reaches $\frac{V_{DD}}{2}$. We usually say NMOS/PMOS switch at $\frac{V_{DD}}{2}$ for a simplistic model.

This means we have two propagation delays t_{pHL} & t_{pLH}

t_{pHL}: output high to low

t_{pLH}: output low to high

The books definition would say $t_p = t_{rise} = \boxed{5.1\mu s}$ (actually $\max\{t_{rise}, t_{fall}\}$)

Most digital designers instead say

$t_p = t$ s.th $V_{out} = \frac{V_{DD}}{2}$ in a transition, so

$$t_{pHL} \Rightarrow 2.5 = \left(5 - \frac{5}{11}\right) e^{-t/\left(\frac{10^{-6}}{11}\right)} + \frac{5}{11}$$

$$\Rightarrow \boxed{\cancel{t_{pHL} = 0.02\mu s}} \quad \boxed{t_{pHL} = 0.7\mu s}$$

$$t_{pLH} \Rightarrow 2.5 = 5\left(1 - e^{-t/10^{-5}}\right)$$

$$\ln(2)10^{-5} = \boxed{t_{pLH} = 6.93\mu s}$$

These are all valid answers

12.

$$\frac{j\omega}{1+j\omega} = \frac{j\frac{\omega}{\omega_c}}{1+j\frac{\omega}{\omega_c}} \text{ with } \omega_c = 1\frac{rad}{sec} \quad \text{so, HPF}$$