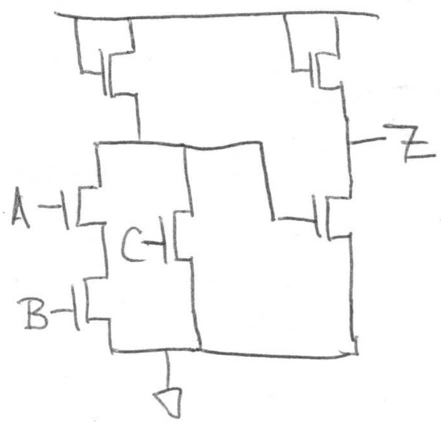
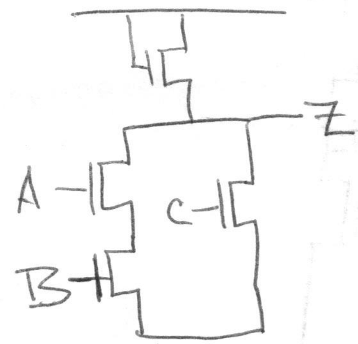
1. (3pts)

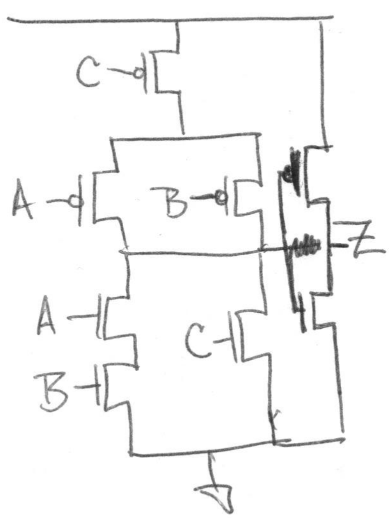
a. AB+C, NMos only



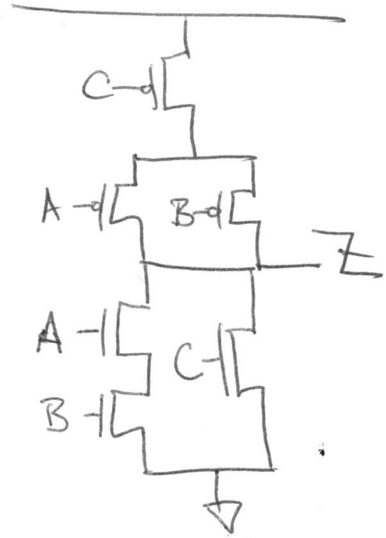
b. $\overline{AB+C}$, NMos only



c. AB+C in CMOS



d. $\overline{AB+C}$ in CMOS



2. a. (3pts) Want path through A&B to be used as compared to path through C because of higher resistance so longer RC, so longer fall time.

So, inputs that meet this requirement

	①	②
A	0→1	1
B	1	0→1
C	0	0

$$-\ln\frac{2.5}{4.5} = \frac{t}{2R_{on}C_L}$$

$$\boxed{t_{fall} = -2R_{on}C_L \ln\left(\frac{2.5}{4.5}\right)}$$

b. Use intuitive method

$$V_{out}(t) = 4.5(\cancel{1-}e^{-t/RC})$$

$$2.5 = 4.5(\cancel{1-}e^{-t/2R_{on}C_L})$$

$$\cancel{t_{fall} = -2R_{on}C_L \ln\left(1-\frac{2.5}{4.5}\right)}$$

c. Due to symmetry in the PMOS network, all low → high transitions will give the same rise time if both A & B PMOS are on

SO,

	①	②	③
A	0	0	1→0
B	0	1→0	0
C	1→0	0	0

d.

$$V_{out}(t) = 0.5 + 4.5\left(1 - e^{-t/RC}\right)$$

$$2.5 = 0.5 + 4.5\left(1 - e^{-t/\frac{3}{2}R_{on}C_L}\right)$$

$$\boxed{t_{rise} = -\frac{3}{2}R_{on}C_L \ln\left(1 - \frac{2}{4.5}\right)}$$

3. a. (5 pts)

$\boxed{Z \text{ is falling}}$

i)

Use intuitive method for RC decay from 3V

$$V_{cgs}(t) = 3e^{-t/1\times10^{-9}}, \quad 0 \le t \le 1\text{ns}$$

ii)

$$V_{cgs}(1\text{ns}) = 3e^{-1\times10^{-9}/1\times10^{-9}} = 3e^{-1} = \boxed{1.104\text{V}}$$

iii)

$$1.5 = 3e^{-t/1\times10^{-9}} \Rightarrow t = \ln(2)(1\times10^{-9}) = \boxed{0.69\text{ns}}$$

iv)

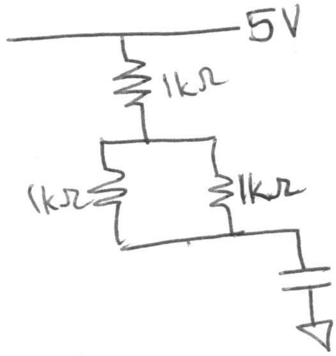
$$P(t) = I_R(t)V(t) = \frac{V(t)^2}{R} = \boxed{9\times10^{-3}e^{-2t\times10^{9}}\text{W}}$$

V) If you believe $E_{cap} = \frac{1}{2}CV^2$, then $E_{diss} = E_{cap,initial} - E_{cap,final}$

$$E_{diss} = \frac{1}{2}CV_{initial}^2 - \frac{1}{2}CV_{final}^2 = \frac{1}{2}C\left(V_i^2 - V_f^2\right)$$

$$= \frac{1}{2}\times10^{-12}\left(3^2 - 1.5^2\right) = \boxed{3.375\text{pJ}}$$

b.



i) $V_{cgs}(1ns)$ will be the same as for part a since the voltage across a capacitor doesn't change instantaneously.

$$\boxed{V_{cgs}(1ns) = 1.104\,V}$$

ii)
$$\boxed{V_{cgs}(t) = 1.104 + (5-1.104)\left(1-e^{-t/1.5\times10^{-9}}\right)}, \quad 1\times10^{-9} < t < 2\times10^{-9}$$

iii) Question should have read "how high does V_{cgs}..."

$$V_{cgs}(1\times10^{-9}) = 1.104 + (3.896)\left(1-e^{-\frac{1}{1.5}}\right) = \boxed{3V}$$

iv) (Your answer should actually be shorter than 1ns!)

$$2.5 = 1.104 + 3.896\left(1-e^{-t/1.5\times10^{-9}}\right)$$

$$-\ln\left(1-\frac{(2.5-1.104)}{3.896}\right)\times1.5\times10^{-9} = \boxed{t = 0.665\,ns}$$

v)
$$P(t) = V_R(t)\,I_R(t) = \frac{V_R(t)^2}{R}$$, where $R = \frac{3}{2}R_{on} = 1.5k\Omega$

$$\boxed{P(t) = \frac{(V_S - V_{cgs}(t))^2}{1.5\times10^3}}$$

vi) You actually <u>don't</u> need to integrate (or at least no like is implied by the problem) for this one either. To do this, you need to understand & trust the method of HW 4, prob. 7.

$$E = \int_0^t i_c(t)(V_{DD}-V_c(t))\,dt$$

$$= \int_0^t (V_{DD}-V_c(t))\,C\frac{dV_c(t)}{dt}\,dt = \int_{V_i}^{V_f}(CV_{DD} - CV_c(t))\,dV_c(t) \;\&\; \Rightarrow \text{next pg.}$$

$= CV_{DD}(V_f - V_i) - \frac{1}{2}C(V_f^2 - V_i^2)$

$= 10^{-12}\times 5(3-1.104) - \frac{1}{2}\times 10^{-12}(3^2-(1.104)^2)$

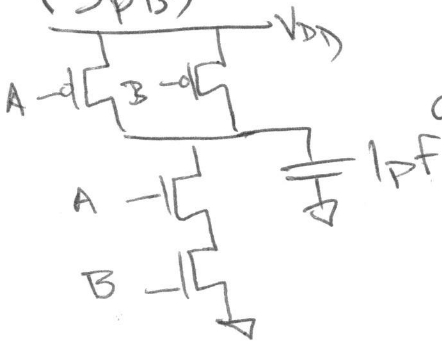
$$\boxed{\begin{array}{ll} \text{In } t_{rise} & E_{diss} = 5.6\,pJ \\ \text{In } t_{low} & E_{diss} = 4.46\,pJ \end{array}}$$

c. $P_{avg} = \frac{E_{cycle}}{T} = \frac{(5.6+3.375)\times10^{-12}}{2\times10^{-9}} = \boxed{4.49\times10^{-3}\,W}$

d. No, since both energies will rise or fall with the squares of V_f, V_i.

e. $\boxed{CV_{DD}^2 f \quad (\text{or } CV_{DD}^2 \frac{1}{T})}$

4. (3pts)



a. $\tau_{dischg} = 2R_{on}C_L = 5\times10^{-9}$

$R_{on} = \frac{5\times10^{-9}}{2\times10^{-12}} = \frac{5}{2}\times10^{3} = \boxed{2.5\,k\Omega}$

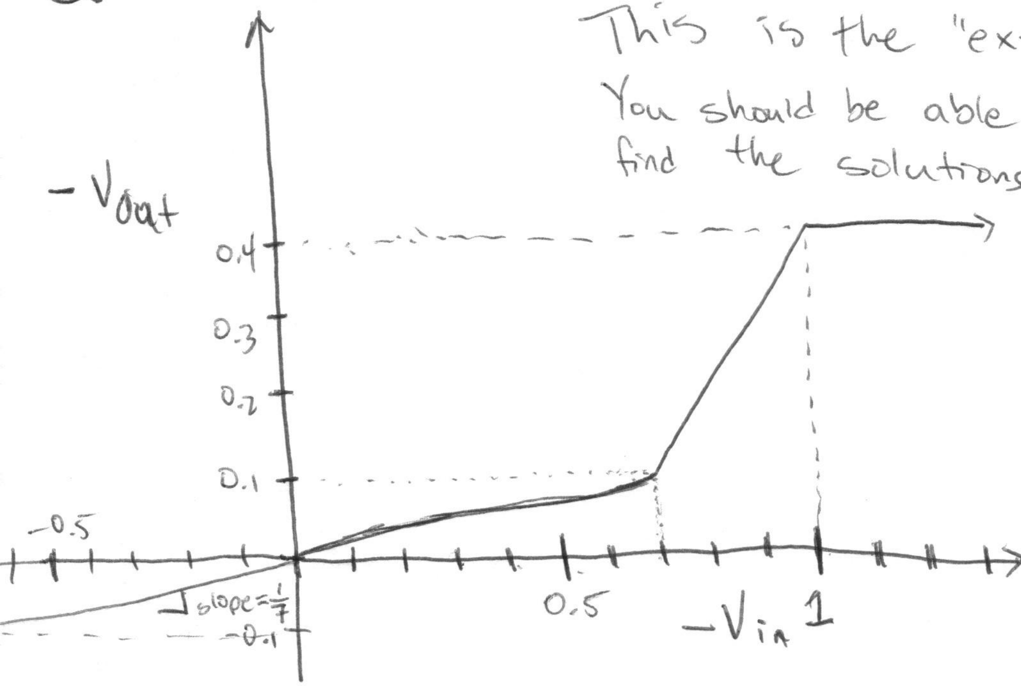
b. For N inputs, $\boxed{\tau_{dischg} = NR_{on}C_L = 2.5\times10^{-9}N}$

c. For charging, worst case R is just R_{on}, so $\boxed{\tau = 2.5\times10^{-9}\,s}$

For best case, $R = \frac{R_{on}}{N} \Rightarrow \boxed{\tau = \frac{2.5\times10^{-9}}{N}}$

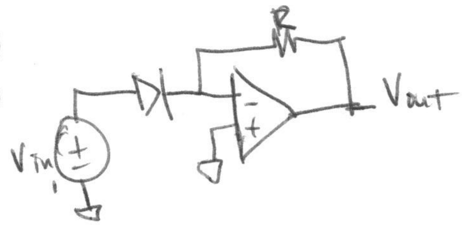
5. (3pts)

This is the "expert" solution. You should be able to use this to find the solutions to the other solutions.



6. (not graded) This is actually not a great model of a diode (or LED), but instead the leads and contact resistance of the diode.

7. (3pts) a. i)



$$i_D = I_S e^{V_{in}/V_T}$$

$$\boxed{V_{out} = -\frac{i_D}{R} = -\frac{I_S e^{V_{in}/V_T}}{R}}$$

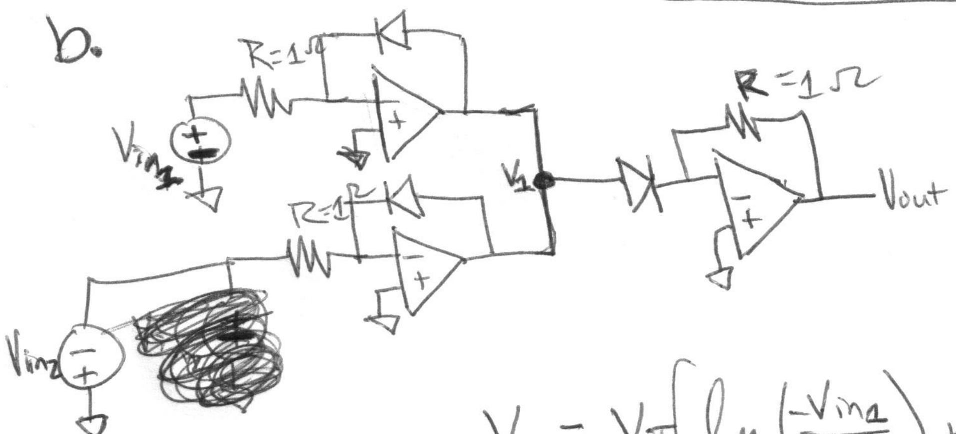
ii)

$$V_{out} = V_D = V_T \ln\left(\frac{I_D}{I_S}\right)$$

$$I_D = -\frac{V_{in}}{R}$$

$$\boxed{V_{out} = V_T \ln\left(\frac{-V_{in}}{RI_S}\right)}$$

b.



$$V_1 = V_T\left[\ln\left(\frac{-V_{in1}}{RI_S}\right) + \ln\left(\frac{+V_{in2}}{RI_S}\right)\right]$$

$$= V_T \ln\left(\frac{-(V_{in1} * V_{in2})}{RI_S}\right)$$

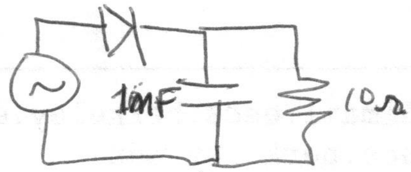
$$V_{out} = -\frac{I_S}{R} e^{\cancel{V_T} \ln\left(\frac{-V_{in1} * V_{in2}}{RI_S}\right)/\cancel{V_T}}$$

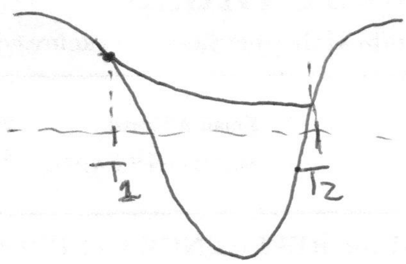
$$= +\frac{\cancel{I_S}}{R}\left(\frac{+V_{in1} * V_{in2}}{R\cancel{I_S}}\right) = \boxed{V_{in1} * V_{in2}} = V_{out}$$

This ckt will function properly for negative values of V_{in1}, positive values of V_{in2}.

8. (3pts)

a.





$V_{out}(T_1) = 10\cos(2\pi * 60 T_1) \approx 1$

at T_2, $V_{out}(T_1)e^{-t/RC} = \cos(2\pi * 60(t - \cancel{T_1}))$ 0

so, $e^{-t/RC} = \cos(2\pi * 60t) * 10$

$\cancel{t = -RC\ln(\cos(2\pi * 60t))}$

~~Use iteration and guess $t = \frac{1}{80}$~~

~~① →~~

nonlinear, cannot be solved by algebraic manipulation, so instead use a calculator solver or iteration.

$$\boxed{T_2 = 0.0132}$$

$$V_{out}(T_2) = 10 * e^{-\frac{0.0132}{0.01}} = \boxed{2.67V}$$

c. For V_{out} to drop no more then 0.1V want that $V_{out}(\frac{1}{60}) = 9.9 = 10e^{-\frac{1}{60}/RC}$

$\Rightarrow -\ln 0.99 = \frac{1}{60RC}$

$\Rightarrow C = -\frac{1}{60\ln(0.99)R} = \boxed{0.166F}$