1. (3pts)

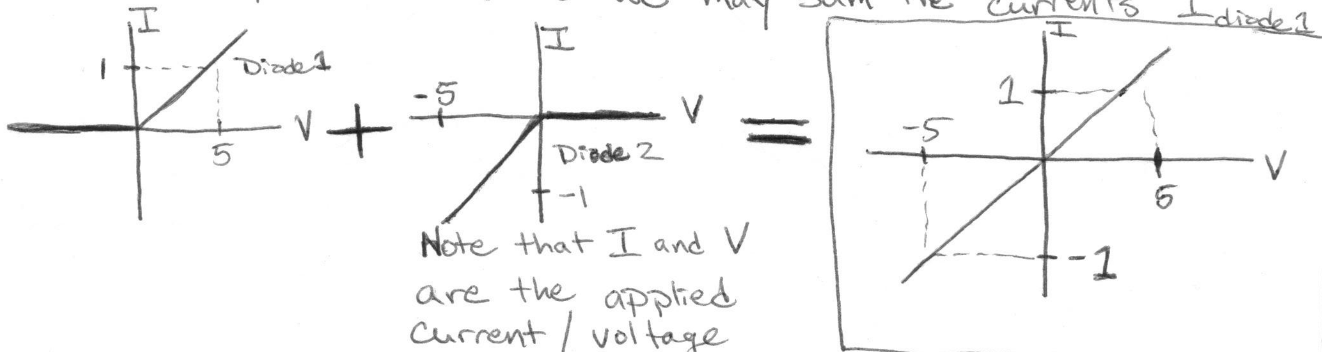
Since the two linear diodes are in parallel, the voltage across them is equal and their currents add. We can think of the I-V characteristic as a function $I(V)$. We can then say:

$$I(V) = I_{diode\,1}(V) + I_{diode\,2}(V)$$

Since the diodes are facing in opposite directions,

$$I_{diode\,2}(V) = -I_{diode\,1}(-V).$$

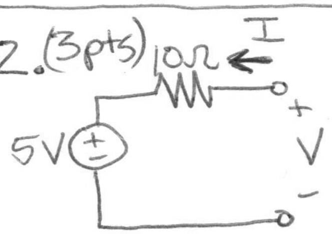
Currents in parallel add so we may sum the currents $I_{diode\,1}$ and $I_{diode\,2}$.



Note that I and V are the applied current / voltage in the circuit, not the passive reference direction current/votage for the element.

This is the I-V characteristic of a 5Ω resistor!

2. (3pts)

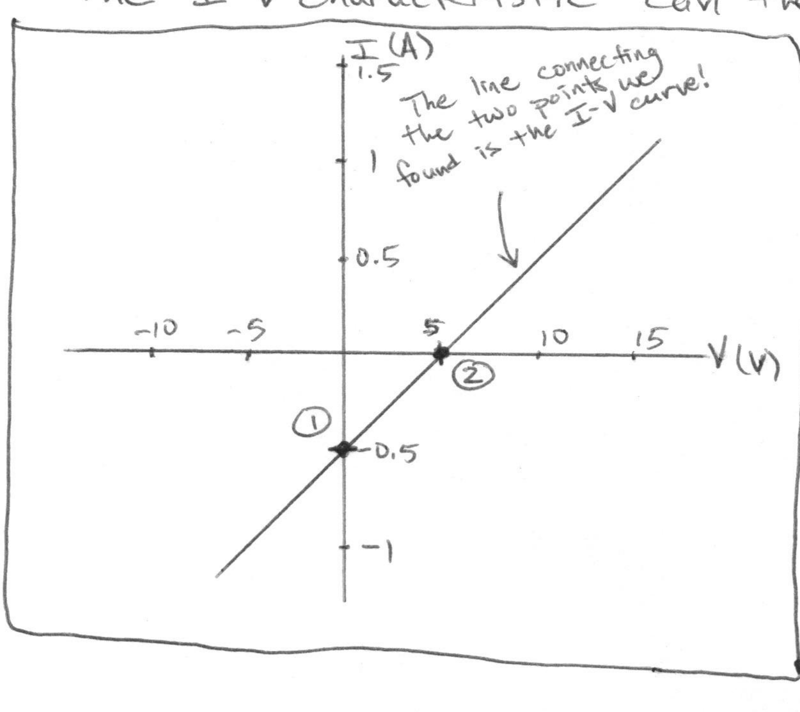


Since both elements are linear, their I-V characteristics will also be linear. If we can find two points on the I-V curve we are done.

① If we short the input terminals, $V = 0V$ and $I = -\frac{5V}{10\Omega} = -0.5A$.

② If we leave the terminals open circuited then $V = 5V$ and $I = 0A$.

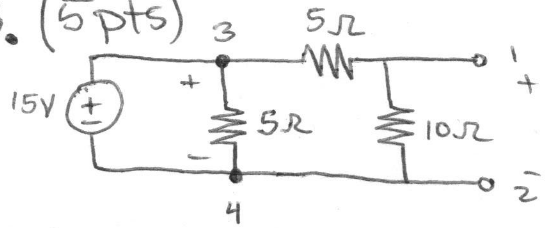
The I-V characteristic can then be plotted as:



Alternatively, we can write the I-V characteristic as a function $I(V) = I_0 + \frac{1}{R}V$ where $I_0 = I(0)$ and $\frac{1}{R}$ = the slope of the line. When $V=0$, $I = -0.5A$ as we found above. The slope of the line is $\frac{1}{R} = 0.125$

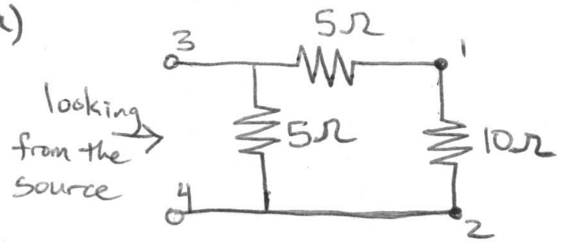
so, $\boxed{I(V) = -0.5 + 0.1V}$

3. (5pts)

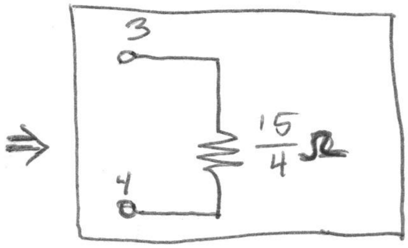


Note: To me, the way the voltage source was placed in the original schematic was weird, so I redrew the schematic with the source moved. If a schematic looks confusing, redraw it so you can see what is going on better.

a)

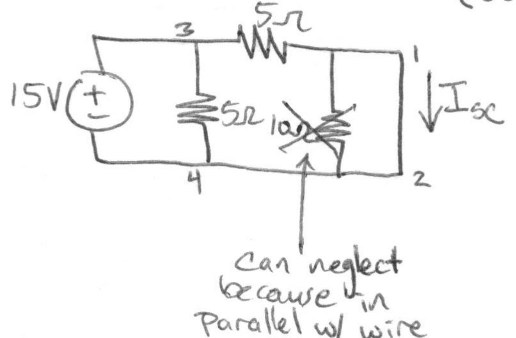


$\Rightarrow$



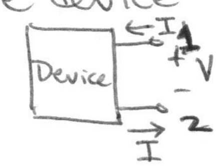
The current is $\dfrac{15V}{\left(\frac{15}{4}\Omega\right)} = \boxed{4A}$

b) Find I when V=0 (output is short circuit).



$$I_{sc} = \frac{V_3 - 0}{5\Omega} = \frac{15}{5} = 3A$$

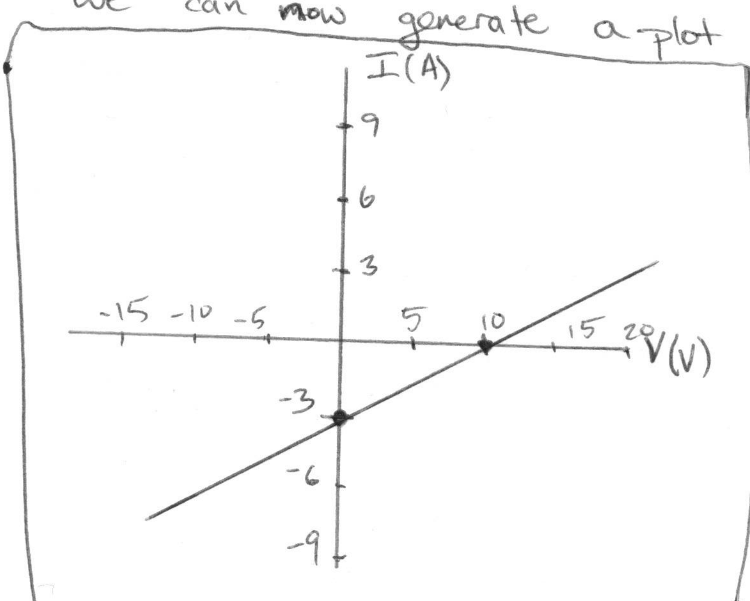
However, the current of the "device" flows in the opposite direction so $I(V{=}0) = -3A$



Find V when I=0 (output is open circuit).

This is a voltage divider of the 5Ω & 10Ω resistors that are in series. $V_{12} = V_{10\Omega} = V_{source}\left(\frac{10\Omega}{5\Omega + 10\Omega}\right) = 15\left(\frac{2}{3}\right) = 10V.$

We can now generate a plot or equation for the I-V curve



OR With the two points we can use the point-slope formula

$(V_1, I_1) = (10, 0)$, $(V_0, I_0) = (0, -3)$

$$\text{slope} = \frac{(I_1 - I_0)}{(V_1 - V_0)} = \frac{I - I_0}{V - V_0}$$

$$I - I_0 = \left(\frac{0 - (-3)}{10 - 0}\right)(V - 0)$$

$$\boxed{I(V) = -3 + \frac{3}{10}V}$$

c) To find the Thévenin equivalent

① Find R_{th}: zero sources

shorthand for $\frac{(5)(10)}{5+10}$ parallel combination.

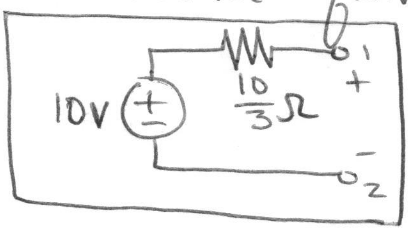
parallel w/ wire

$\leftarrow R_{th} = 5\Omega \| 10\Omega = \frac{10}{3}\Omega$

② Find V_{oc}: We found this in part b: $V_{oc} = 10V = V_{th}$

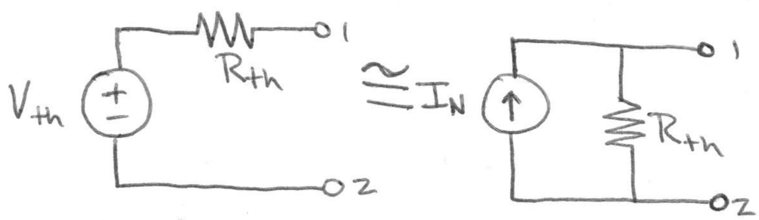
Note: We also could have used I_{sc} from part b, but if we didn't know it, finding R_{th} would be easier than finding I_{sc}.

So the Thévenin equivalent is



This has exactly the same I-V characteristic as the original circuit. This is the beauty of Thévenin and Norton. You can replace any linear circuit with its unique Norton or Thévenin equivalent when solving problems. We call Norton and Thévenin canonical representations.

d) Norton and Thevenin equivalents are duals of eachother. The following use of Ohm's law relates them

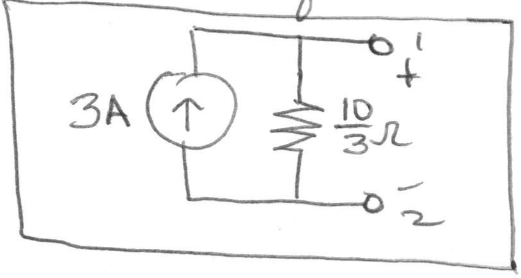


$$V_{th} = I_N R_{th}$$

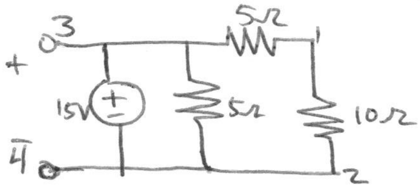
To find the Norton equivalent

$$I_N = \frac{V_{th}}{R_{th}} = \frac{10}{\left(\frac{10}{3}\right)} = 3A$$

So the Norton equivalent is



e) If we redraw the circuit



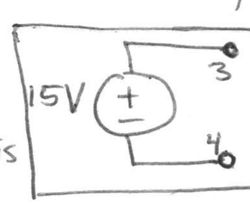
(You should verify that all the nodes match)

$V_{oc} = V_s = 15V = V_{th}$

$R_{th} \Rightarrow$

all in parallel with wire $\Rightarrow R_{th} = 0\Omega$

The Thévenin equivalent is



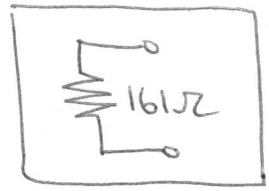
4. We only need two points on either of the I-V characteristics
(3pts) since the data is linear (you should verify it is linear)

For device #1 choose $V=0V, I=0A$ and $V=1V, I=6.2mA$

Using $I(V)=I_0+\frac{1}{R}V$ (from prob. 2)

$\underbrace{I_0=I(0)=0}_{\text{no sources}}$, $I(V)=\frac{1}{R}V$ $\quad 6.2\times10^{-3}=\frac{1}{R}(1)\Rightarrow R=161\Omega$

so the device is equivalent to



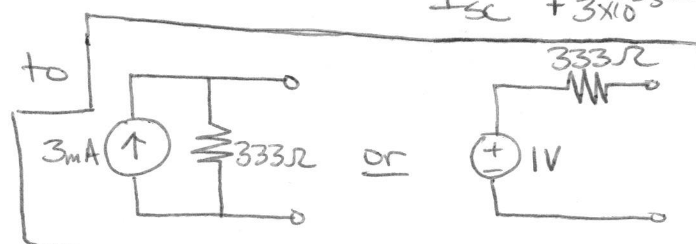
Note, there are infinitely many equivalent devices. This is the simplest one.

For device #2

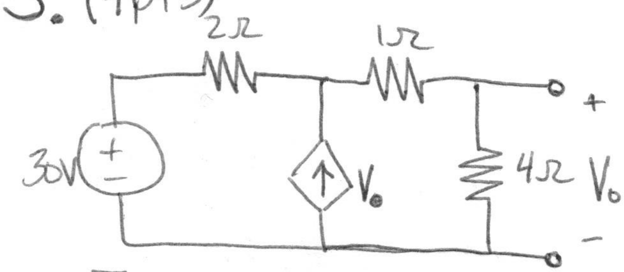
$V=0V, I=-3mA$; $V=1V, I=0mA$

$I_0=-3mA=I_{sc}$ $\quad V(I=0)=1V=V_{oc}\Rightarrow R_{th}=\frac{V_{oc}}{I_{sc}}=\frac{1}{+3\times10^{-3}}=333\Omega$

so the device is equivalent to



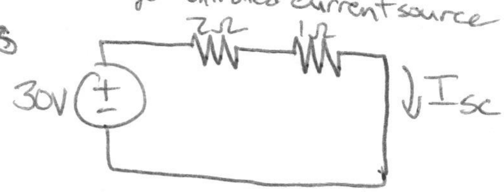
5. (4pts)



Find I_{sc}

When shorted $V_o=0V\Rightarrow I_{vccs}=0$ so the circuit can be redrawn as

voltage controlled current source



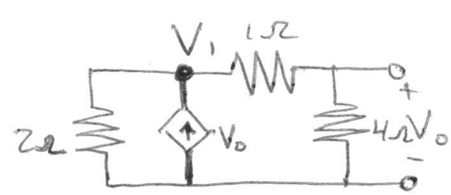
$I_{sc}=\frac{30V}{3\Omega}=10A$

Find R_{th}

Zero all independent sources first

Let $V_o=1V$

$I_{4\Omega}=\frac{1}{4}A$



$\frac{V_1}{2}-V_o=\frac{V_o-V_1}{1}$

$\frac{3}{2}V_1=2V_o$

$V_1=\frac{4}{3}V_o=\frac{4}{3}V$

$I_{1\Omega}=\frac{\frac{4}{3}-1}{1}=\frac{1}{3}A$

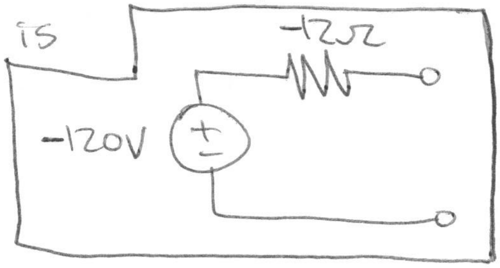
$I=-\frac{1}{3}A+\frac{1}{4}A=-\frac{1}{12}A$

$R_{eq}=\frac{1}{-\frac{1}{12}}=-12\Omega$

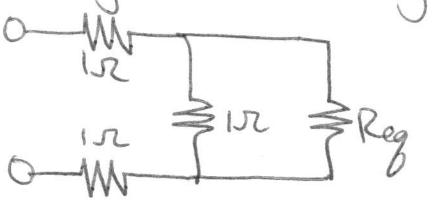
continued on next page

the thévenin voltage can be found as

$V_{th} = I_{sc} R_{th} = 10 \cdot (-12) = -120\,V$, so the thévenin equivalent circuit is



6. Using the hint given
(3pts)



$$R_{eq} = 2 + \frac{R_{eq}}{1 + R_{eq}}$$

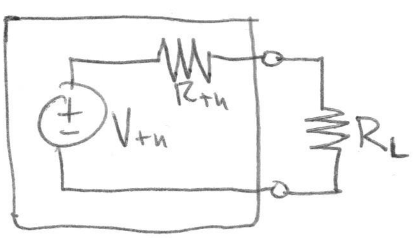
$$R_{eq} + R_{eq}^2 = 2 + 2R_{eq} + R_{eq}$$

$$R_{eq}^2 - 2R_{eq} - 2 = 0$$

$R_{eq} \geq 0$

$$R_{eq} = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm \sqrt{12}}{2} = \frac{2 + \sqrt{12}}{2} = \boxed{(1+\sqrt{3})\Omega}$$

7. This is a very important result that is used
(3pts) extensively when building communications circuits. It is also useful in many other applications.



$$P_L = \frac{V_L^2}{R_L} = \frac{V_{th}^2 \left(\frac{R_L}{R_{th}+R_L}\right)^2}{R_L} = \boxed{\frac{V_{th}^2 R_L}{R_{th}^2 + 2R_{th}R_L + R_L^2}}$$

$$\cancel{\frac{dP_L}{dR_L} = \frac{V_{th}^2 - V_{th}^2 R_L}{}}$$

$$\frac{d}{dR_L} P_L = \frac{V_{th}^2 (R_{th}^2 + 2R_{th}R_L + R_L^2) - V_{th}^2 R_L (2R_{th} + 2R_L)}{(R_{th}+R_L)^4}$$

to maximize power

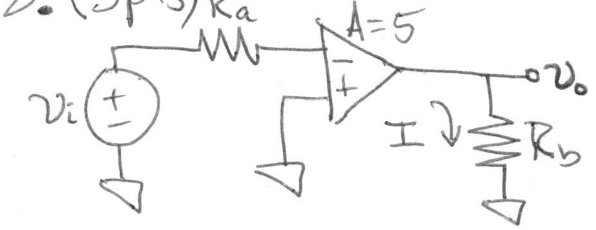
$$0 = \frac{V_{th}^2 R_{th}^2 - V_{th}^2 R_L^2}{(R_{th}+R_L)^4}$$

$$V_{th}^2 R_{th}^2 = V_{th}^2 R_L^2$$

$$R_{th}^2 = R_L^2 \implies R_{th} = R_L$$

The maximum power is delivered to the load when the load resistance is equal to the source resistance!

8. (3pts)

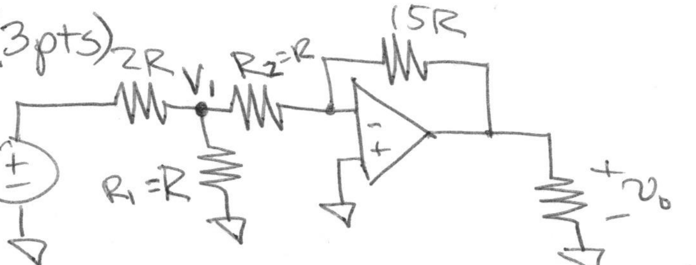


Summing point constraint cannot be applied for this problem because $A \ll \infty$ and there is no negative feedback.

However, the input resistance of the opamp is still infinite, so no current goes into the input terminals of the opamp. Hence, $v^- = v_i$ and $v_o = A(v^+ - v^-)$
$= 5(0 - v_i) = -5v_i$.

I can be found from $I = \frac{v_o}{R_b} = \boxed{\frac{-5v_i}{R_b}}$

9. (3pts)



Have negative fb so summing pt. constraint applies. $v^+ = 0V \Rightarrow v^- = 0V$. Need to find the current through R_2.

Since $R_1 = R_2 = R$ the current from the source will divide evenly between R_1 & R_2.

$I_{in} = \frac{v_{in}}{2R + (R \| R)} = \frac{v_{in}}{\frac{5}{2}R}$, $I_{R_2} = \frac{v_{in}}{5R}$

$v_o = -I_{R_2}(15R) = -\frac{15R}{5R} v_{in} = -3v_{in}$

so, $\boxed{\frac{v_o}{v_{in}} = -3}$

Extra Problems

1.

No, this circuit does not have a thévenin equivalent. The concept of Thévenin or Norton equivalent circuits only applies for networks consisting of only linear elements. Since the dependent source is nonlinear, we cannot find a Thévenin (or Norton) equivalent.

2. As the strength (gain) of the dependent source increases, the resistance (and V_{th}) of the Thévenin equivalent will become positive.