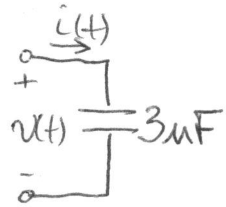
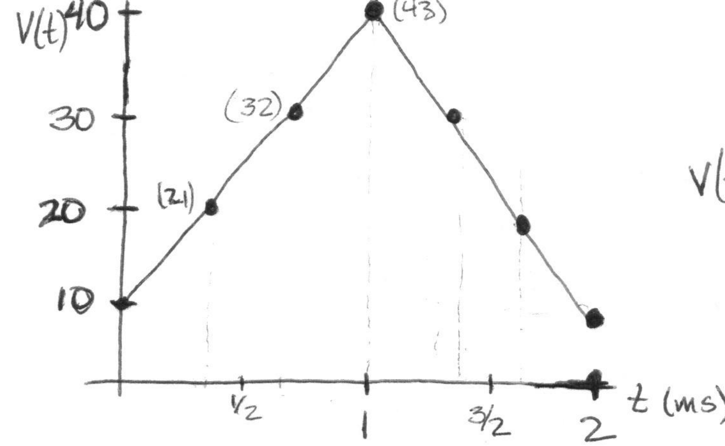
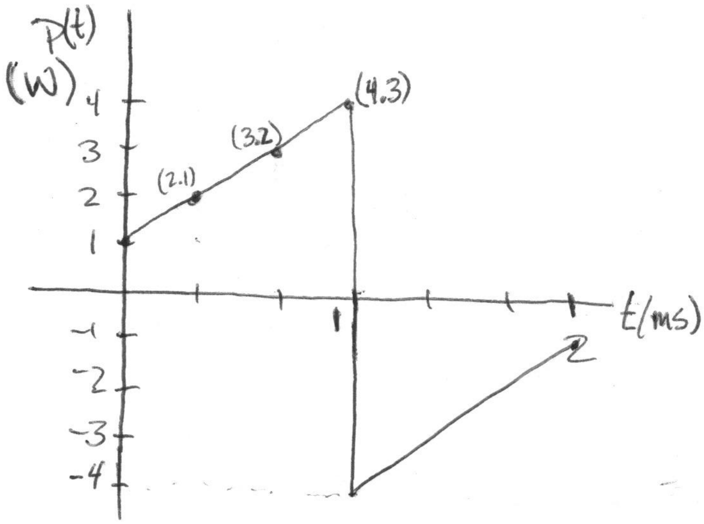
1.

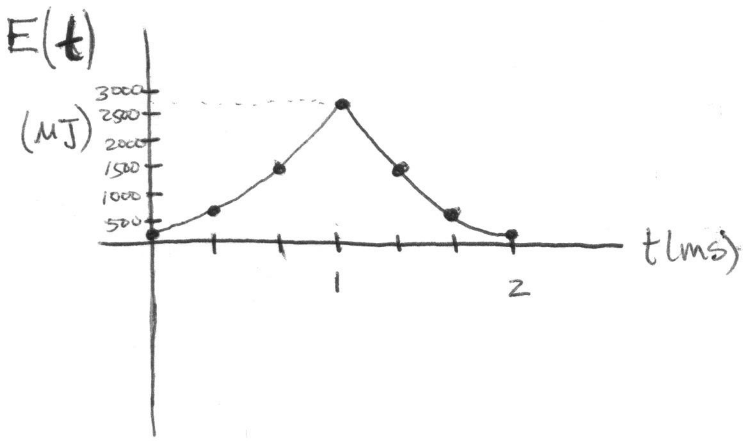



$$V(t) = \int_0^t \frac{i(t)}{C} dt + V_0$$




$$p(t) = i(t)v(t)$$




$$E_{initial} = \frac{1}{2} C V_{initial}^2 = 150 \mu J$$

$$E(t) = \int_0^t p(t)\,dt + E_{initial}$$

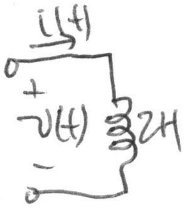
or

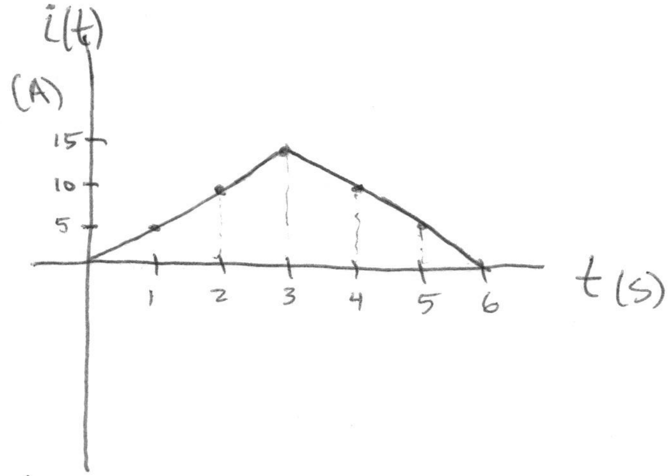
$E(21V) = 662 \mu J$

$E(32V) = 1.54 mJ$

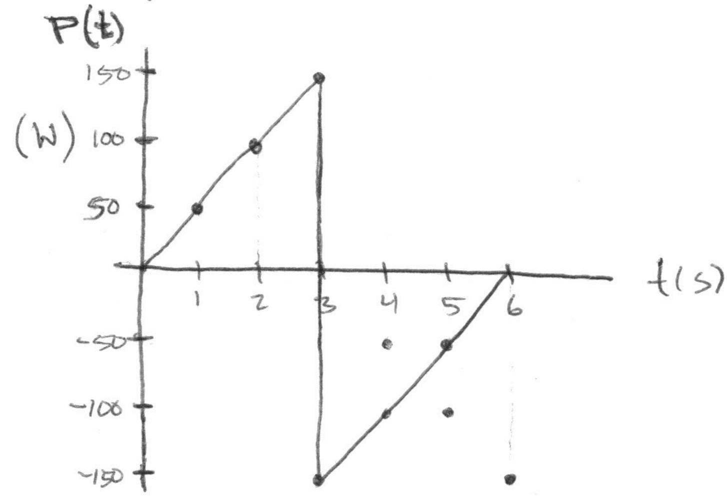
$E(43V) = 2.77 mJ$

2.

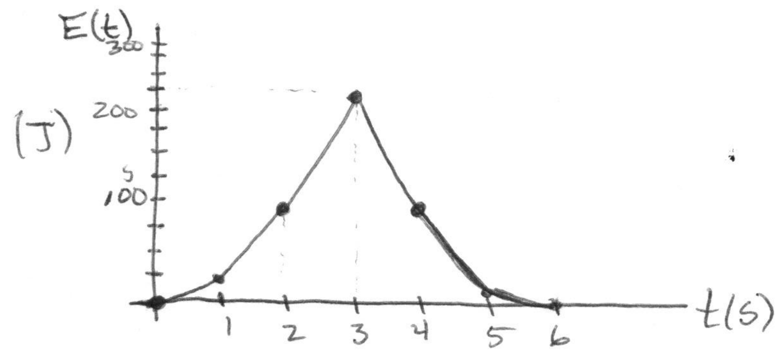



$$i_L(t) = \int \frac{1}{L} v_L(t)\,dt$$




$$p(t) = i(t)\,v(t)$$




$$E(t) = \int_0^t p(t)\,dt$$

3.


a.

$$\frac{v_0 - v_c(t)}{1k\Omega} = C\frac{dv_c(t)}{dt} + \frac{v_c(t)}{1k\Omega}$$

$$\boxed{\frac{dv_c(t)}{dt} = \frac{v_0 - 2v_c(t)}{(1k\Omega)(1mF)}}$$

b. If $v_0 = 0$ for $t < 0$, then no energy is being put into the circuit, the capacitor will discharge and hence $\boxed{v_c(0) = 0V}$

c. $$\frac{dv_c(t)}{dt} = -\frac{2}{1\times10^{-3}} v_c(t) + \frac{v_0}{10^{-3}}$$

guess: $v_c(t) = Ae^{-t/\tau} + B$

$$\frac{dv_c(t)}{dt} = -\frac{A}{\tau}e^{-t/\tau} = -\frac{1}{\tau}v_c(t) + B$$

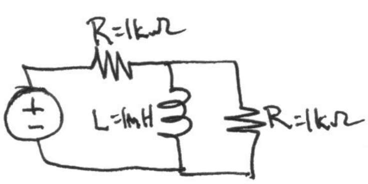
so, $\boxed{\tau = 5\times10^{-4}}$ and $\boxed{B = \frac{v_0}{10^{-3}}}$

$v_c(0) = 0 = Ae^0 + B$

$$\boxed{A = -B = -\frac{v_0}{10^{-3}}}$$

$$\boxed{v_c(t) = v_0\times10^3\left(1 - e^{-t/5\times10^{-4}}\right)}$$

4.




a. $$\frac{v_0 - v_L}{R} = \frac{v_L}{R} + i_L, \quad v_L(t) = L\frac{di_L(t)}{dt}$$

$$\frac{di_L(t)}{dt} = \left(\frac{v_0}{R} - i_L\right)\frac{R}{2L}$$

(If you don't try to write the inductor current as an integral of the voltage, you don't need to use the non-obvious fact that was given)

b. As for 3, if there is no input voltage for $t<0$, then the energy stored in the inductor will be zero and hence $\boxed{i_L(0) = 0A}$.

c. As for 3, guess: $i_L(t) = Ae^{-t/\tau} + B$

$$\frac{di_L(t)}{dt} = -\frac{1}{\tau}Ae^{-t/\tau} = \frac{v_0}{2L} - \frac{R}{2L}\left(Ae^{-t/\tau} + B\right)$$

$$\boxed{\tau = \frac{2L}{R}, \quad B = \frac{v_0}{2L}}$$

$i_L(0) = 0 = A + B$

$$\boxed{A = -B = -\frac{v_0}{2L}}, \quad \boxed{i_L(t) = \frac{v_0}{2L}\left(1 - e^{-t/2\times10^{-6}}\right)}$$

5. a) state var.: $v_c(t)$

state eqn: $\frac{dv_c(t)}{dt} = \frac{i_o}{C} - \frac{v_c(t)}{RC}$

b) state var: $i_L(t)$

State eqn: $\frac{di_L(t)}{dt} = \frac{i_L(t)R - V_o}{L}$

c) ~~stat~~ state var: $v_c(t)$

State eqn: D.E. from 3a)

d) state var: $i_L(t)$

State eqn: D.E. from 4a)

6. Easy way (note that this is not very obvious and in general you do need to solve the ODE)

Resonant frequency of the LC tank: $\frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4\times10^{-8}}} = 5000 \frac{\text{rad}}{\text{sec}}$

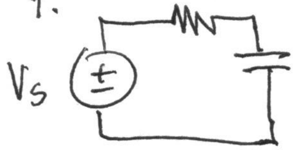
$i_L(t) = 0.1 \cos(\underbrace{5000t}_{\omega_{res}})$

Since the current of the inductor oscillates at the resonant frequency of the tank ckt, no energy is being injected by the current source, hence $\boxed{i(t) = 0A.}$ Then by KCL, $\boxed{i_c(t) = -i_L(t)}$

and $v_L(t) = L\frac{di_L(t)}{dt} = (4\times10^{-3})(-500\sin(5000t))$

$$\boxed{v(t) = -2\sin(5000t)}$$

7.




$$E_{cap}(\infty) = \int_0^\infty p(t)\,dt = \int_0^\infty v(t)\,i(t)\,dt = \int_0^\infty v_c(t)\, C\frac{dv_c(t)}{dt}\,dt$$

$$= C\int_0^{V_s} v_c(t)\,dv_c = C\left.\frac{v_c^2}{2}\right|_0^{V_s} = \boxed{\frac{1}{2}CV_s^2}$$

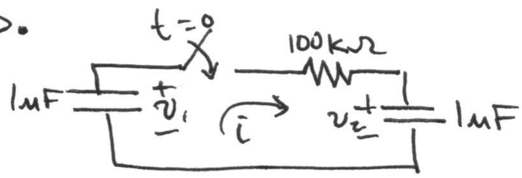
$$E_{res} = E_{source} - E_{cap}$$

$$E_{source} = \int_0^\infty p(t)\,dt = V_s\int_0^\infty i(t)\,dt = V_s\left(V_s C\right) = CV_s^2$$

So, $\boxed{E_{res} = CV_s^2 - \frac{1}{2}CV_s^2 = \frac{1}{2}CV_s^2 = E_{cap}.}$

No, nothing can be changed to increase the ratio of energy stored. This is the theoretical maximum.

8.




By KCL

$$C\dot{V}_1 = -C\dot{V}_2 \Rightarrow \dot{V}_1 = -\dot{V}_2$$

By KVL

$$V_1 - IR - V_2 = 0$$

$$\dot{V}_1 - \dot{V}_2 = \dot{I}R$$

$$-2\dot{V}_2 = \dot{I}R \quad , \quad I = C\dot{V}_2 \Rightarrow \dot{V}_2 = \frac{I}{C}$$

$$\frac{-2I}{RC} = \dot{I}$$

$$I(t) = Ae^{-t/\tau} \quad , \quad \tau = \frac{RC}{2}$$

$$I(0) = \frac{100-0}{100k} = 1mA = A$$

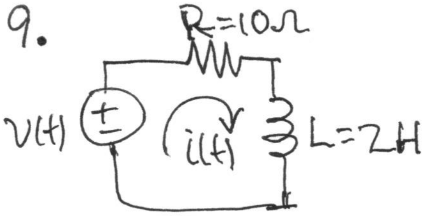
So, $\boxed{I(t) = 10^{-3}e^{-\frac{2t}{RC}}}$

When $t=\infty$, no voltage difference between $V_1$ & $V_2$

$$Q_{initial} = C_1 V_{initial} = \underbrace{(C_1+C_2)}_{2C_1} V_{final}$$

$$\boxed{V_{final} = \frac{100V}{2} = 50V = V_2\Big|_{t\to\infty}}$$

9.




$$I_L(t) = \frac{V(t) - v_L(t)}{R} \quad , \quad V_L(t) = L\frac{dI_L(t)}{dt}$$

$$I = \frac{V}{R} - \frac{L}{R}\dot{I}_L$$

<u>homogeneous</u>

$V = 0$

$$\boxed{I_{hom}(t) = \overbrace{A}^{=0} e^{-t/(L/R)} = Ae^{-t/\frac{2}{10}} = 0}$$

$I_{particular}(t) = A' + Bt$ (as suggested by hint)

$$I_L(t) = \frac{10t}{R} - \frac{L}{R}\dot{I}_L(t)$$

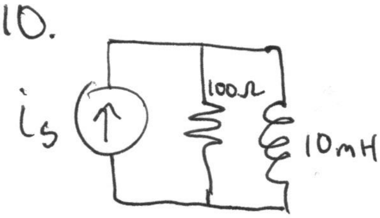
$A' + Bt = t - 0.2B$

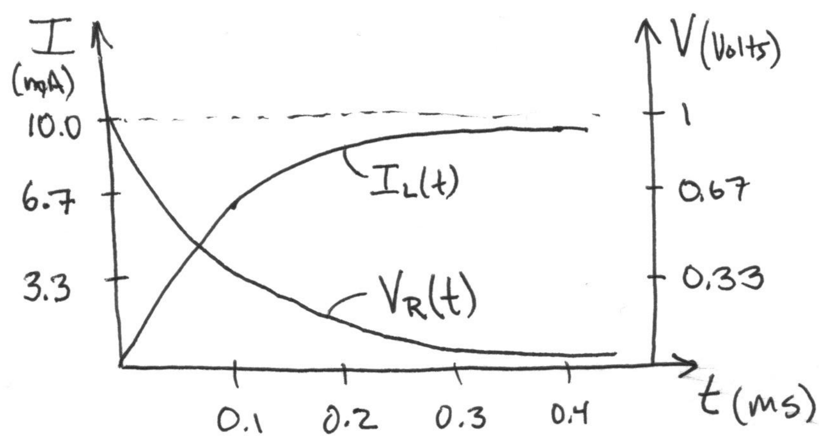
$\Rightarrow B = 1$ $\quad\quad \Rightarrow I_p = -0.2 + t$

$\Rightarrow A' = -0.2$

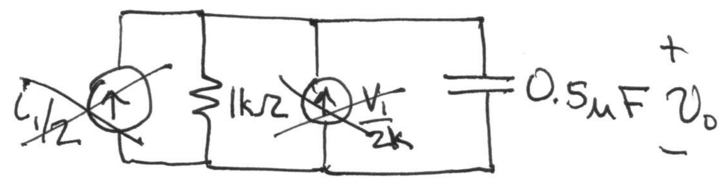
$$\boxed{I(t) = t - 0.2}$$

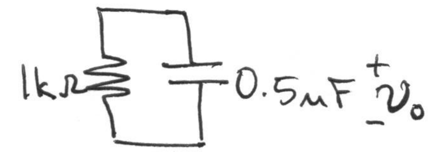
10.



11. Simplify the circuit



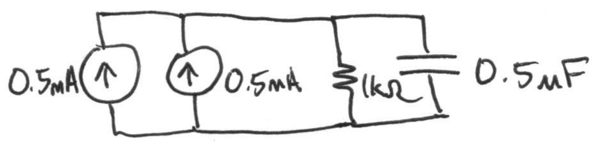




$$\boxed{v_o(t) = e^{-\frac{t\times10^3}{0.5}}}$$

(meets initial cond.)

12. Using simplified ckt above and $V_1(t>0) = 1V$

$I_1(t>0) = 1mA$




$\tau = RC = 0.5ms$

$V_o(\infty) = 1V$, $V_o(0) = 0V$

so, $\boxed{V_o(t) = \left(1 - e^{-\frac{10^3 t}{0.5}}\right)}$

13. a. While the capacitor charges, the inductor current is zero. This is because the inductor is open circuited, hence current cannot flow.

b. The capacitor voltage charges to 5V, the voltage of the source.

c. The resonant frequency is $\frac{1}{2\pi\sqrt{LC}} = \boxed{41\,Hz}$

d. Inductor voltage and capacitor current are $\frac{\pi}{2}$ out of phase,

hence $\left(\frac{1}{f}\right)\frac{1}{4} = \frac{T}{4} = \boxed{6ms}$

e. To make current oscillate twice as fast, want

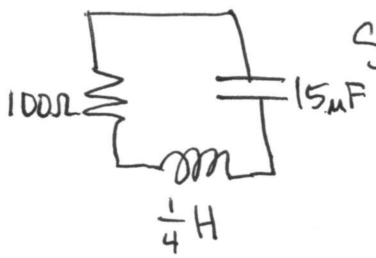
$f_{res} = \frac{1}{2\pi\sqrt{LC}}$ to inc. by factor of 2 $\Rightarrow \sqrt{L}$ dec. by 2x $\Rightarrow$ L dec by 4x

$\Rightarrow L_{new} = \frac{L}{4} = \boxed{0.25H}$

f. The peak voltage across the capacitor does not change since this value is dictated by the 5V voltage source. The curren through the inductor increases by two so that the max energy in the ckt is the same, or $E = \frac{1}{2}LI^2 = \frac{1}{2}L_{new}I_{new}^2$

$\Rightarrow \boxed{I_{new} = 2I}$ (assuming $L_{new} = \frac{L}{4}$)

g.




Series RLC

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = 10\sqrt{\frac{1}{60}} = 1.29 > 1$$

so, the circuit is $\boxed{\text{underdamped.}}$

h. With much larger R, $Q < 1 \Rightarrow$ circuit is $\boxed{\text{overdamped}}$ the circuit has a very slow rising transient to the steady-state value.

i. Fastest decay when critically damped, so

$$Q = 1 = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\boxed{R = \sqrt{\frac{L}{C}} = 129\,\Omega}$$

14. a.

b. Same as prob. 13. Initially all energy in capacitor so,

$$\boxed{\begin{array}{l} v_c(t) = 2\cos(\omega t) \\ i_L(t) = -0.2\sin(\omega t) \end{array}}$$

$$\frac{1}{2}CV^2 = \frac{1}{2}I^2L$$

$$I = \sqrt{\frac{C}{L}V^2}$$

$$\boxed{= 0.2\,A}$$

$$\boxed{\begin{array}{l} \omega = \frac{1}{\sqrt{LC}} \\ = 10\ \frac{\text{rad}}{\text{sec}} \end{array}}$$