EE40 Su 2010 Homework #6, due Friday 7/23/10 by 5 PM in the Homework box in 240 Cory

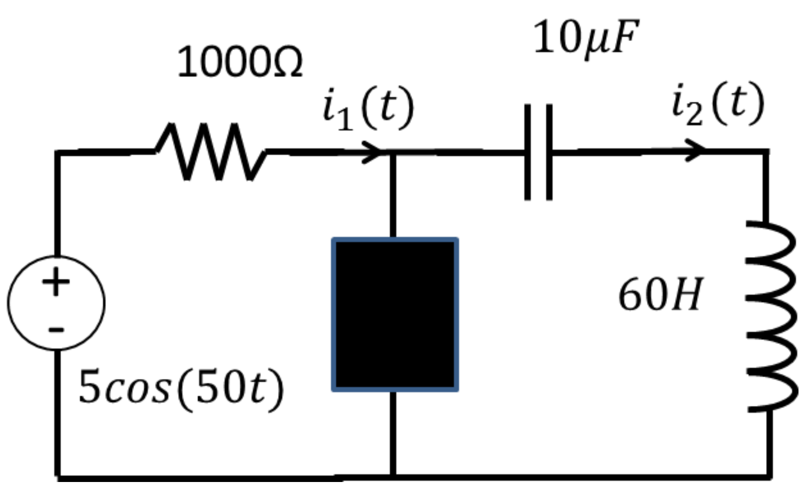
[ok there are 3 more problems, but one of them is really short but useful (integrator problem)]

1. On the previous homework, we saw that two passive components with memory could act like a short circuit if their equivalent series impedances sum to zero.

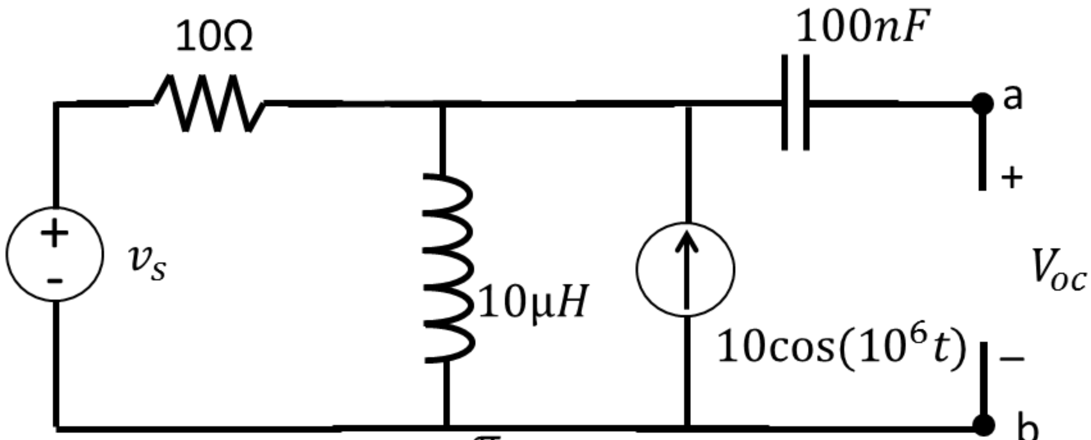
a. For the circuit below, find a passive circuit element (that you'll put in the black box) such that the current I_1 is zero in steady state. What element did you choose? What is the equivalent impedance of the right half of the circuit after you add your passive circuit element (i.e. what is the equivalent of impedance of your element in parallel with the sum of the capacitor and inductor)?

b. For the element you chose, is $i_2(t)$ zero as well? If so, why? If not, what is $i_2(t)$? [Hint: You know the voltage across the circuit element you chose!]

c. **Extra (not for a grade):** Use the Falstad circuit simulator to visually explore what's going on here. Reminder that frequency in the Falstad simulator will be $\frac{50}{2\pi}$.

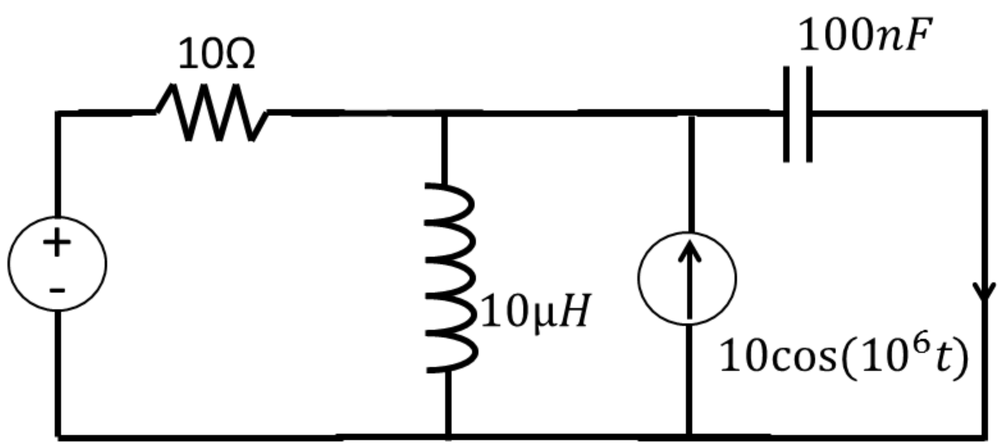


2. Consider the circuit below



$$v_s = 50\sqrt{2}cos(10^6t + \frac{\pi}{4})$$

a. Find the open circuit voltage $V_{oc}(t)$ between terminals a and b. [Your rectangular form should come out nice, the polar format will not be as nice]
b. Find the short circuit current $I_{sc}(t)$ [Your rectangular form should come out nice]



$$v_s = 50\sqrt{2}cos(10^6t + \frac{\pi}{4})$$

c. Draw the Thevenin equivalent circuit $V_{th}(t) = V_{oc}(t)$, and where Z_{eq} is $V_{oc}(t)/I_{sc}(t)$ [The rectangular form for Z should come out nice].
d. Would the Thevenin equivalent circuit provide the same behavior as the original circuit if we attach a passive circuit element? (i.e. if we connect a resistor, capacitor or inductor across terminals a and b in the circuit up above and measure the voltage across that component, do we get the same voltage that we'd get if we attached the same resistor, capacitor, or inductor to the Thevenin equivalent circuit)

e. Would the Thevenin equivalent circuit provide the same behavior as the original circuit if we attach a source with frequency $\omega = 10^6$?
f. Would the Thevenin equivalent circuit provide the same behavior as the original circuit if we attach a source with frequency $\omega = 10^3$?
g. **Extra (not for a grade):** Try using the usual method for directly finding the Thevenin resistance on part a: Set the sources to zero, and replace all of the components with their equivalent impedance with $\omega = 10^6$. Note that you don't get the same Z_{th}? Explain why this procedure doesn't work (hint consider the Z_{th} you'd get if the phase of the source were different)

3. In class and in the homework, we discussed the operation of an undriven LC circuit.

Notably, we found that it oscillates forever at a constant frequency, known as the resonant or natural frequency.

a. What is the resonant frequency of an LC circuit in terms of L and C?
b. What happens if we connect an AC source to an LC circuit with the same frequency as the resonant frequency? (Hint you can solve this with phasors and some intuition, maybe backed up by the Falstad circuit simulator)
c. Is the source providing net power?
d. How much heat would this circuit emit? [Note we cannot actually build one of these things because there will always be some resistance somewhere in the circuit] If none, where does the power go?
e. Optional [not for a grade]: Put this circuit in the Falstad circuit simulator to explore what happens. Try varying the AC source frequency so that it is slightly off from the resonant frequency. Observe what happens.

4. Consider the passive filter circuit in figure P6.60

a. Find the transfer function $H(j\omega) = \frac{V_{out}}{V_{in}}$
b. Find the magnitude and phase ($|H(j\omega)|$ and $\angle H(j\omega)$) of the transfer function. j should not appear in your answer. In all other problems you do in this class, unless stated otherwise, you can just leave the $j\omega$ in the expression for the magnitude (like we did in class when doing Bode plots, for example)
c. Draw the Bode magnitude and phase plots by hand.
d. What kind of filter is this? (band pass, low pass, high pass, all pass, etc)
e. Give the output of the circuit for the signal $V_{in} = 5\cos(30t + \pi/4)$.

f. How does the transfer function change when you attach a 1000Ω resistor as a load? Does the filter type change? Does the cutoff frequency change?

g. Remember on homework 1 when you built a circuit such that $v_O = \frac{v_{in}}{1000}$? In that case, the circuit worked for high resistance loads, but not for low resistance loads. In this case, will the circuit work properly for very high resistances? Very low resistances? [by work properly, I mean will the transfer function change?]

h. Extra problem (not for a grade): What is the largest/smallest resistor (based on your answer in part g) such that the circuit function is not significantly changed (use your judgment for what significantly changed means)

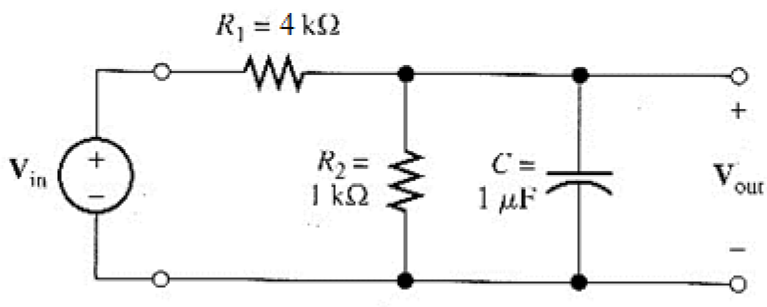
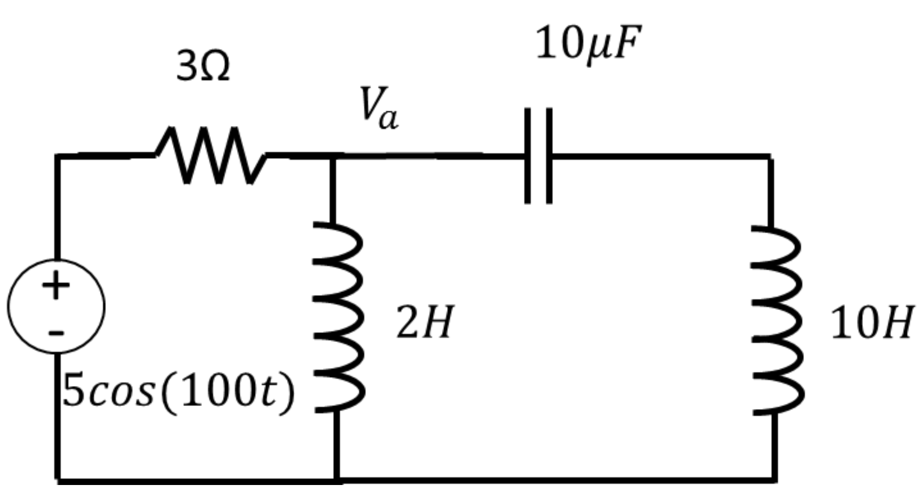


Figure P6.60

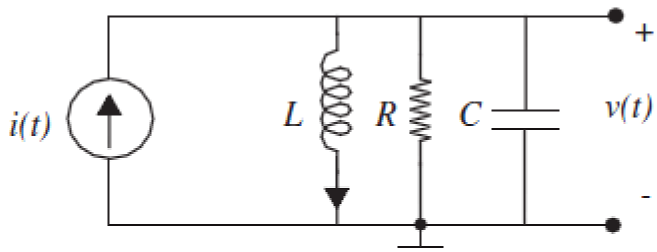
5. Consider the circuit below (also featured in HW 5). On HW5, you found that the voltage at the node V_a decays to zero in steady state.

a. Does the value of the resistor affect the steady state (particular) solution?

b. If we wanted to ensure that our circuit reached steady state as fast as possible, what sort of resistor value should we choose? Why?



6. Consider the circuit below

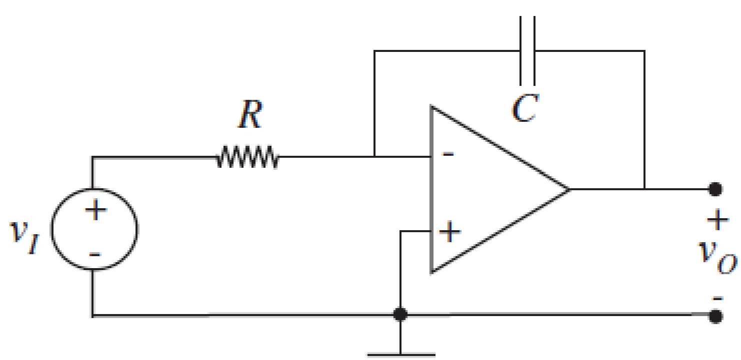


a. Find the transfer function $H(j\omega) = \frac{\hat{V}}{\hat{I}}$
b. Plot the Bode Magnitude and Phase plots assuming $L = 1H, C = 1F, R = 2\Omega$
c. Now plot the Bode Magnitude and Phase plots assuming $L = 1H, C = 100\mu F, R = 2\Omega$. How does the break frequency change? How does the steepness of the magnitude of the filter around the break frequency change? [Hint consider how much higher or lower the asymptotes intersect than the actual value of the magnitude at the break frequency]

Extra (not for a grade):

Show that the steepness of the filter increases as $Q = R\sqrt{\frac{C}{L}}$ increases. This is often called the "quality factor" in filter design. Different circuit configurations will have different quality factors.

7. Consider the circuit below



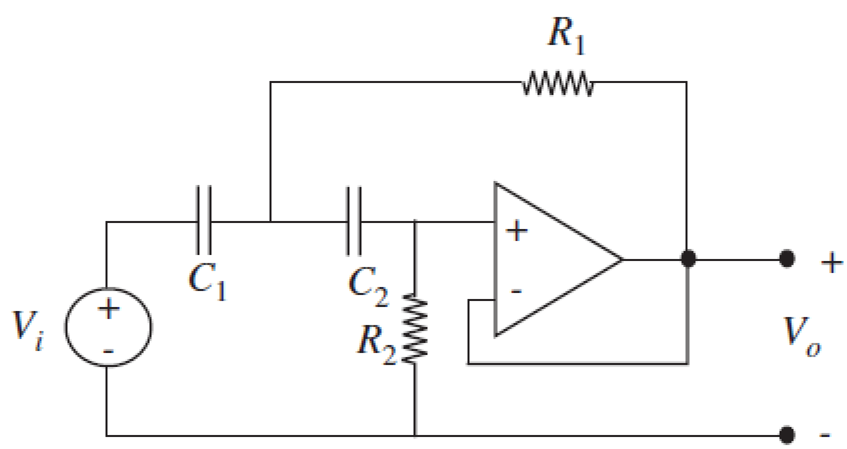
a. Assume the summing point constrain works. Find the transfer function $H(j\omega) = \frac{\hat{V}_o}{\hat{V}_{in}}$

b. On the previous homework, you found that this circuit is an integrator. Is your answer from that homework consistent with your answer from part a? Explain why or why not.

c. Plot the Bode Magnitude and Phase plots [note my Bode plot handout will work, but this problem is so easy to plot that it might be confusing. If you're stuck, move on to something else and come back to it and it will hit you]

d. **Extra (not for a grade):** Prove that the summing point constraint (pg 861 in the book may be helpful)

8. Consider the circuit below



a. Write a set of equations, which, if solved, would give you the transfer function.

b. Assuming that these equations, when solved, yield

$$V_o/V_i = \frac{(j\omega C_1)(j\omega C_2)}{G_1 G_2 + j\omega(C_1 + C_2)G_2 + (j\omega)^2 C_1 C_2}$$

Find an expression for the low-frequency asymptote of V_o/V_i. (Zero is not an acceptable answer.)

c. Find an expression for the high-frequency asymptote of V_o/V_i. (Zero is not an acceptable answer.)

d. Is this a low pass filter or a high pass filter or a bandpass filter or a bandstop filter?

Extra Problems **[not for a grade]**:

1. Find the Thevenin equivalent circuit between terminals a and b. Note that all of the elements in this circuit have already been replaced by their phasor equivalents (so ω is undefined)

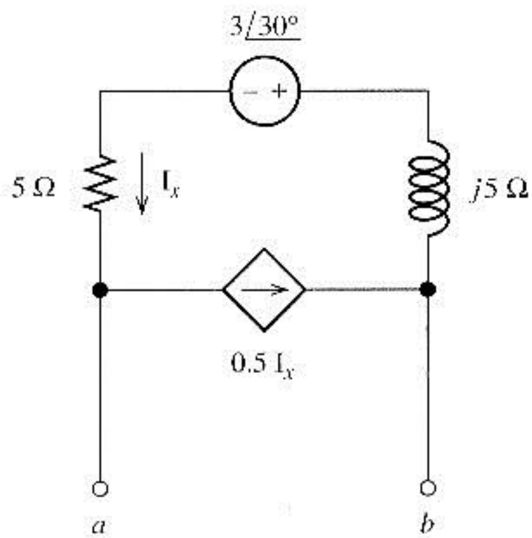
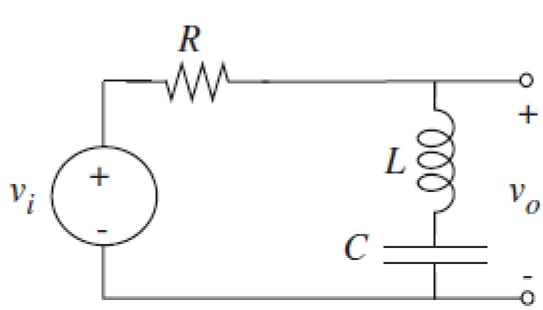


Figure P5.91

2. Is it possible to build a 2nd order low pass filter with only resistors and capacitors? (A 2nd order filter is a filter where the magnitude drops with the square of frequency)

3. Using op-amps, resistors, and capacitors, build a filter where the high frequency asymptote is proportional to w^3. Ultra extra: Plot the whole bode plot of your circuit.

4.



a. Find the transfer function $H(j\omega) = \frac{\hat{V}_o}{\hat{V}_{in}}$

b. Plot the Bode Magnitude and Phase plots assuming $L = 1H, C = 1F, R = 2\Omega$

c. Now plot the Bode Magnitude and Phase plots assuming $L = 1H, C = 100\mu F, R = 2\Omega$. How does the break frequency change?

d. How does the steepness of the filter around the break frequency change? Show that the steepness of the filter increases as $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$ increases. This is often called the "quality factor" in filter design.