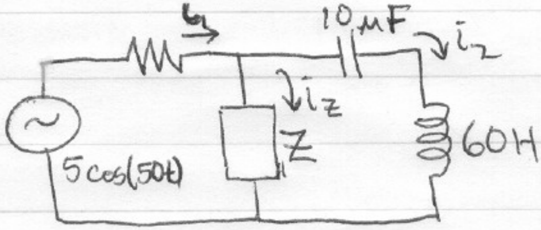
1. a.



Want $Z \parallel \frac{1}{j\omega C} + j\omega L = \infty$

$$\frac{\left(\frac{1}{j\omega C} + j\omega L\right)(Z)}{\frac{1}{j\omega C} + j\omega L + Z} = \infty$$

$$\Rightarrow \frac{1}{j\omega C} + j\omega L + Z = 0$$

$$\Rightarrow \frac{-j}{50\times 10^{-5}} + j\times 50\times 60 + Z = 0$$

$$-2000j + 3000j + Z = 0$$

$$Z = -1000j = \frac{1}{j10^{-3}} = \frac{1}{j\times 50\times 2\times 10^{-5}}$$

so, circuit element is $\boxed{\text{20μF capacitor}}$

b. By KCL $i_z = -i_2$

$$i_z = C\frac{dv_z(t)}{dt} \qquad v_z(t) = 5\cos(50t)$$

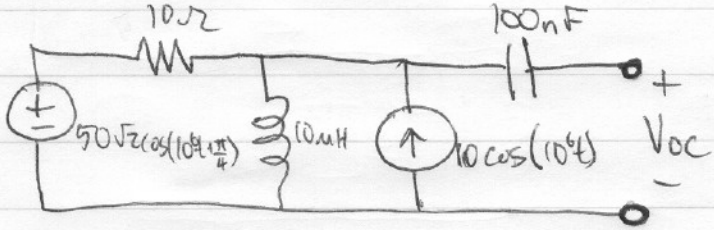
(no current i_1, hence Voltages are equal)

$$i_z = -250\sin(50t) \times 2\times 10^{-5}$$

$$\boxed{i_2 = 500\times 10^{-5}\sin(50t) = 5\sin(50t)\,\text{mA}}$$

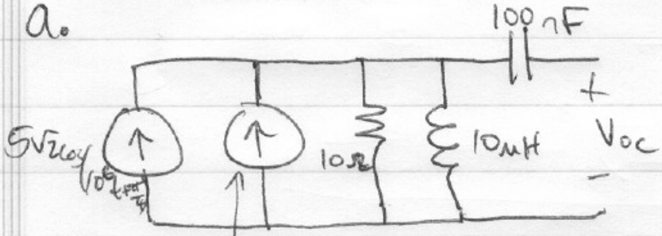
c. Simulation omitted

2.



$$\frac{j100}{10+j10} = \frac{10(j-1)}{2} = \boxed{5\sqrt{2}\angle\frac{\pi}{4}}$$

a.

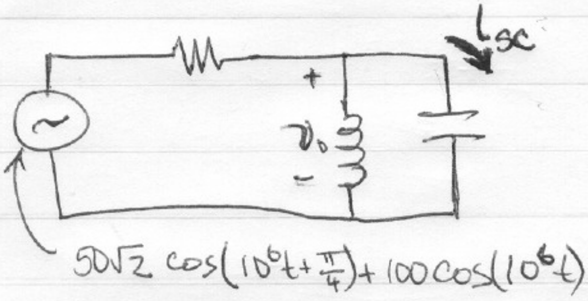


@ 10 M $\frac{\text{rad}}{\text{sec}}$ $\boxed{Z_L = j\omega L = 10j}$

$$V_{oc} = \left(5\sqrt{2}\cos(10^6t+\tfrac{\pi}{4}) + 10\cos(10^6t)\right)(10 \parallel 10j)$$

$$\boxed{V_{oc} = 50\cos(10^6t+\tfrac{\pi}{2}) + 50\sqrt{2}\cos(10^6t+\tfrac{\pi}{4})}$$

b. $I_{sc} =$



$$v_o = v_s\left(\frac{j\omega L \parallel \frac{1}{j\omega C}}{R+\left(j\omega L \parallel \frac{1}{j\omega C}\right)}\right) = v_s$$

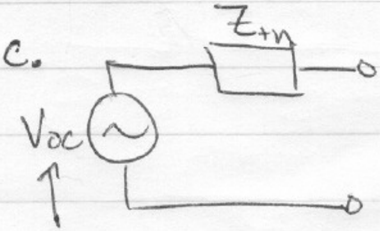
$$j\omega L \parallel \frac{1}{j\omega C} = \frac{\frac{L}{C}}{j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{100}{j(10-10)} = \infty$$

@ 10 M $\frac{\text{rad}}{\text{sec}}$ $Z_C = -Z_L$, so $i_{sc} = -i_L$

$$v_c = v_s \Rightarrow i_c = v_s j\omega C = \frac{v_s j}{10}$$

$$\boxed{i_c = i_{sc} = 5\sqrt{2}\cos(10^6t+\tfrac{3\pi}{4}) + 10\cos(10^6t+\tfrac{\pi}{2})}$$

c.



from a)

$$, Z_{th} = \frac{1}{j\omega C} + \frac{j\omega RL}{j\omega L + R}$$

$$\cancel{= \frac{j\omega L + R + \omega^2 RLC}{j\omega RC - \omega^2 LC}}$$

$$= -j10 + \frac{j100}{j10+10}$$

$$= \frac{100 - j100 + j100}{j10+10} = \frac{100}{j10+10}$$

$$= \frac{10(1-j)}{2} = \boxed{5-5j}$$

d. Yes, this is why Thevenin and Norton equivalents are such useful tools.

e. Yes, since the impedances given are all for $\omega = 10^6 \frac{rad}{sec}$, the circuit will respond as expected.

f. No, the value for Z_{th} was calculated assuming $\omega = 1M \frac{rad}{sec}$. If the frequency is changed, the response will also change.

3. a. The resonant frequency of an LC circuit is where $Z_C = -Z_L$, or to generalize to an RLC ckt, the frequency where $\mathrm{Im}(Z_{RLC}) = 0$. $\cancel{Z_{C}}$

$$Z_C = \frac{1}{j\omega C} = -j\omega L = -Z_L$$

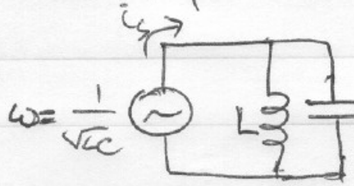
$$\cancel{(j\omega)^2 = +\frac{1}{LC}} \qquad \omega_{res}^2 = \frac{1}{LC}$$

$$\Rightarrow \boxed{\omega_{res} = \frac{1}{\sqrt{LC}}}$$

$\cancel{S}$

$$\cancel{s^2 = \frac{1}{LC} \rightarrow S = \frac{1}{\sqrt{LC}}}$$

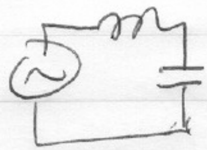
b. If parallel LC:



$Z_{LC} = \infty\,\Omega \Rightarrow i_s = 0$

$\Rightarrow$ just v_s across terminals.

If series LC

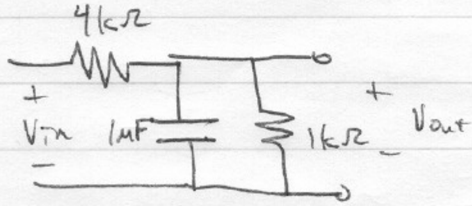


$Z_{LC} = 0\,\Omega \Rightarrow i \rightarrow \infty$ as $t \rightarrow \infty$

c. Since capacitors and inductors can only store energy, not consume it, the source provides power during part of the cycle and consumes it during another part of the cycle providing no net power.

d. Since none of the elements consume power (ideally), no heat is dissipated. The power sloshes back and forth between ckt elements.

4. a.



$$V_{out} = V_{in}\underbrace{\left(\frac{1k\Omega \| 1\mu F}{(1k\Omega\|1\mu F) + 4k\Omega}\right)}_{\text{solve for this}}$$

$$1k\Omega \| 1\mu F = \frac{10^3 \frac{1}{j\omega 10^{-6}}}{10^3 + \frac{1}{j\omega 10^{-6}}} = \frac{10^3}{1 + j\omega 10^{-3}} = \boxed{\frac{10^6}{10^3 + j\omega}}$$

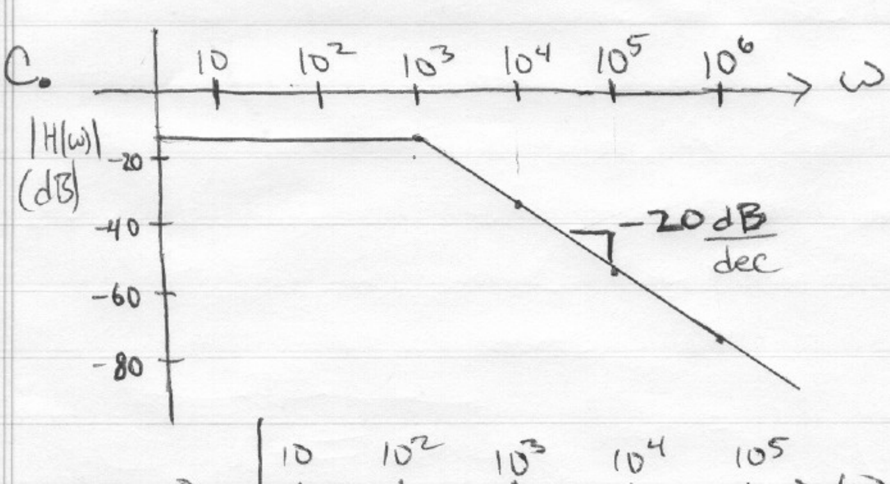
$$\frac{1k \| 1\mu}{(1k\|1\mu) + 4k} = \frac{\frac{10^6}{10^3 + j\omega}}{\frac{10^6}{10^3+j\omega} + 4\times 10^3} = \frac{10^6}{10^6 + 4\times 10^6 + 4j\omega\times 10^3}$$

$$H(j\omega) = \frac{V_{out}}{V_{in}} = \boxed{\frac{10^3}{10^3 + 4(j\omega + 10^3)}} = \frac{1}{5 + j\frac{\omega}{10^3}}$$

b. $$|H(j\omega)| = \frac{10^3}{\sqrt{10^6 * 25 + 16\omega^2}} = \boxed{10^3 \frac{1}{\sqrt{25\times 10^6 + 16\omega^2}}}$$

$$\boxed{\measuredangle H(\omega) = -\tan^{-1}\left(\frac{\omega}{10^3}\right)}$$

c.



d. This is a low pass filter

e. $\omega = 30 \ll 10^3 = \omega_3$, so signal is completely passed with minimal phase shift

$$\Rightarrow \boxed{V_{out} = V_{in} = 5\cos(30t + \pi/4)}$$

f. If load $= 1k\Omega$, then have $R = 1k\Omega \| 1k\Omega = 500\Omega$

so, $\boxed{\omega_c = \frac{1}{R_{new}C} = \boxed{500\,Hz}}$

also, $\boxed{H(0) = \frac{500}{4500} = \frac{1}{9} \text{ instead of } \frac{1}{5}}$

g. For low resistance loads ($R \ll 1k\Omega$), $\omega_c = \frac{1}{R_{load}C}$

so, transfer function changes.

For large resistances, $R \gg 1k\Omega$ $\omega_c = \frac{1}{1k\Omega\, C}$

and $H(0) = \frac{1k \| R_{load}}{1k \| R_{load} + 4k} \approx \frac{1k}{5k} = \frac{1}{5}$ so the transfer function does not change

5. a. The resistor value does not affect the steady state solution, since the impedance of Ls and Cs $= 0$

b. There is not one best solution for this problem. If we choose $R = 0\Omega$, then the s.s. voltage of $V_a = V_s$ and steady state is reached immediately. If we choose $R = \infty$, the source V_s cannot control the oscillator, and steady state is also reached immediately. The last solution is to choose the intermediate value which provides critically damped response to the value $V_a = 0$. You probably can't actually find this value since you would have to solve a third order differential equation.

This part will not be graded due to its ambiguity

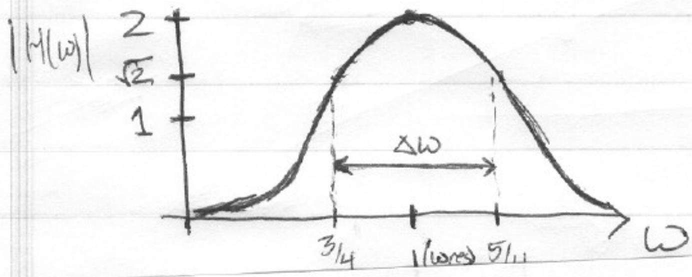
6. a. $H(\omega)=\frac{V}{I}=Z_L \| Z_C \| Z_R=\frac{1}{\frac{1}{j\omega L}+j\omega C+\frac{1}{R}}$

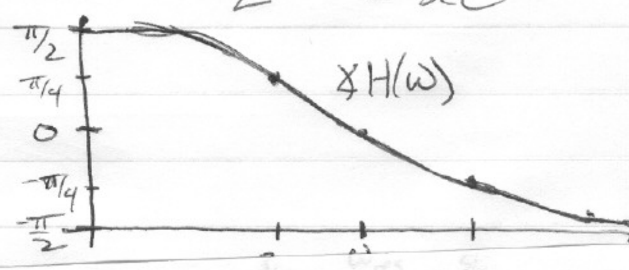
$$=\frac{1}{j(\omega C-\frac{1}{\omega L})+\frac{1}{R}}=\frac{\omega LR}{j\omega^2 LCR-jR\omega L}$$

b. $Q=\frac{\omega_{res}}{\Delta\omega}$ where $\Delta\omega$ is the -3dB Bandwidth

$$=R\sqrt{\frac{C}{L}}=2$$

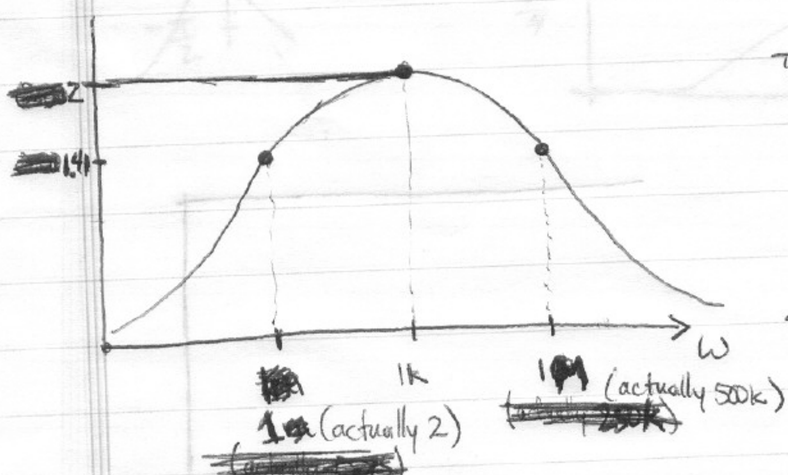
$$\omega_{res}=\frac{1}{\sqrt{LC}}=1\,\frac{rad}{sec} \Rightarrow \Delta\omega=\frac{\omega_{res}}{2}=\frac{1}{2}\,\frac{rad}{sec}$$

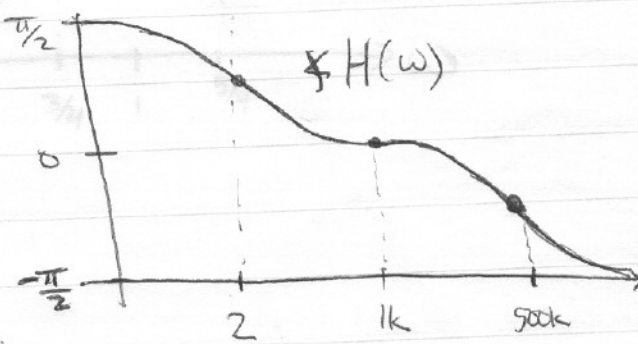




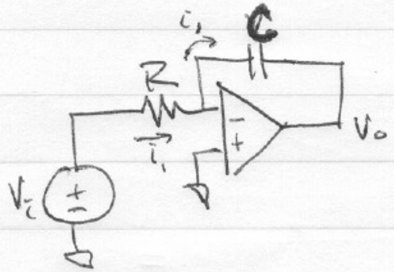
c. $L=1H,\ C=1\mu F,\ R=2\Omega,\quad \omega_{res}=\frac{1}{\sqrt{LC}}=1k\,\frac{rad}{sec}$

$$\Delta\omega=1\times10^{3}Q^{-1}=\frac{1}{2}\times10^{6}=500k\,\frac{rad}{sec}$$





7.

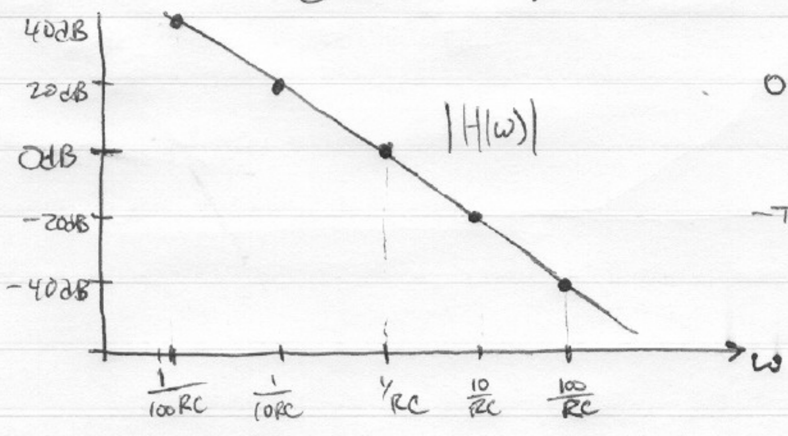


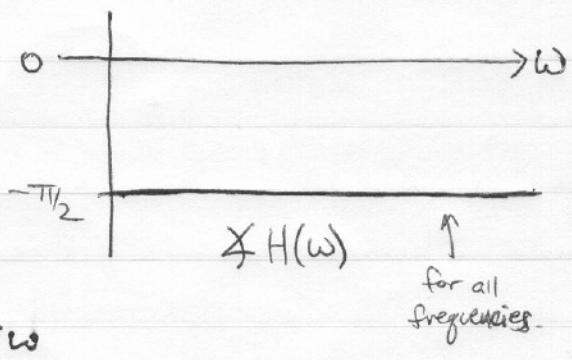
a. By summing pt cnst, $i_1 = \frac{V_i}{R}$, $V_o = -\frac{1}{j\omega C} i_1 = \frac{V_i}{j\omega RC}$

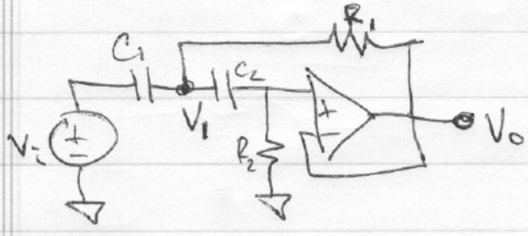
so, $\boxed{H(\omega) = \frac{1}{j\omega RC}}$

b. This filter has a low-pass response. If we consider what integration does, we also see that it tries to smooth out high frequencies. More mathematically, if we input $e^{j\omega t}$, the output will be $\frac{1}{RC}\int e^{j\omega t}\,dt = \frac{1}{j\omega RC} e^{j\omega t}$ and

$$\frac{V_o}{V_i} = \frac{\frac{1}{j\omega RC} e^{j\omega t}}{e^{j\omega t}} = \frac{1}{j\omega RC}$$ which is consistent.





8. 

This is just a Sallen Key highp

a. By summing pt $v^+ = v^- = v_o$

① $\dfrac{V_o - V_1}{R_1} = (V_i - V_1) j\omega C_1 + (V_o - V_1) j\omega C_2$

② $(V_1 - V_o) j\omega C = \dfrac{V_o}{R_2}$

b. $\dfrac{V_o}{V_i} = \dfrac{(j\omega C_1)(j\omega C_2)}{G_1 G_2 + j\omega (C_1 + C_2) G_2 + (j\omega)^2 C_1 C_2}$

as $\omega \to 0$,

$G_1 G_2 \gg j\omega (C_1 + C_2) G_2 + (j\omega)^2 C_1 C_2$

$\Rightarrow \boxed{\dfrac{V_o}{V_i} = \dfrac{(j\omega)^2 C_1 C_2}{G_1 G_2}}$

c. as $\omega \to \infty$

$(j\omega)^2 C_1 C_2 \gg G_1 G_2 + j\omega (C_1 + C_2) G_2$

$\Rightarrow \boxed{\dfrac{V_o}{V_1} = \dfrac{(j\omega)^2 C_1 C_2}{(j\omega)^2 C_1 C_2} = 1}$

d. This is a highpass filter