EE40
Lecture 12
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7/21/2010

Logistical Things

- HW6 due Friday at 5PM (also short)
- Midterm next Wednesday 7/28
  - Focus is heavily on HW4, 5, 6, and Labs P1, 4, 5
  - Will reuse concepts from HW 1,2,3

Filtering

- For the past couple of lectures, we've discussed using **phasors** and **impedances** to solve circuits
- Usually, we've assumed we have some single frequency source, and found the resulting output
- Last time in lecture, we showed that we could apply two different frequencies at one time using superposition
  - Each was scaled and shifted by different amounts

Transfer Functions

- $\hat{V}_{out} = H(j\omega)\hat{V}_{in}$, e.g. $\hat{V}_{out} = \frac{1}{1+0.1jw}\hat{V}_{in}$
- Maps system input signal to system output signal
  - Plug an input voltage $A\cos(\omega t + \phi)$ into $\hat{V}_I$
  - Get an output voltage $A|H(j\omega)|\cos\left(\omega t + \phi + \angle H(j\omega)\right)$
    - Output is scaled and shifted in time
      - Scaling and shifting depend on frequency
    - Frequency is unchanged (linear system)
- Tells you how system will respond to any frequency, a.k.a. frequency response

Using a Transfer Function

- $\hat{V}_{out} = H(j\omega)\hat{V}_{in} = \frac{1}{1+0.1jw}\hat{V}_{in}$
- Suppose $v_{in}(t)$ is $3\cos(\mathbf{10}t + \frac{\pi}{2})$
  - $\hat{V}_I = 3\angle\frac{\pi}{2}$
  - $H(j10) = \frac{1}{1+j}$

$|H(j10)| = 1/\sqrt{2}$

$\angle H(j10) = -ArcTan[1/1]$
$= -\pi/4$

- Output phasor $\hat{V}_{out}$ is just $\hat{V}_{in} \times H(j10)$
  - $\hat{V}_{out} = \frac{3}{\sqrt{2}}\angle(\frac{\pi}{2} - \frac{\pi}{4})$
  - $v_{out}(t) = \frac{3}{\sqrt{2}}\cos(10t + \frac{\pi}{2} - \frac{\pi}{4})$

Using a Transfer Function

- $\hat{V}_{out} = H(j\omega)\hat{V}_{in} = \frac{1}{1+0.1jw}\hat{V}_{in}$
- Suppose $v_i(t)$ is $3\cos(\mathbf{50}t + \frac{\pi}{4})$
  - $\hat{V}_I = 3\angle\frac{\pi}{4}$
  - $H(j50) = \frac{1}{1+5j}$

$|H(j50)| = 1/\sqrt{26}$

$\angle H(j50) = -ArcTan[5/1]$

$= -1.37$

- Output phasor $\hat{V}_C$ is just $\hat{V}_I \times H(j50)$
  - $\hat{V}_C = \frac{3}{\sqrt{26}}\angle(\frac{\pi}{4} - 1.37)$
  - $v_c(t) = \frac{3}{\sqrt{26}}\cos(50t + \frac{\pi}{4} - 1.37)$

Transfer Function

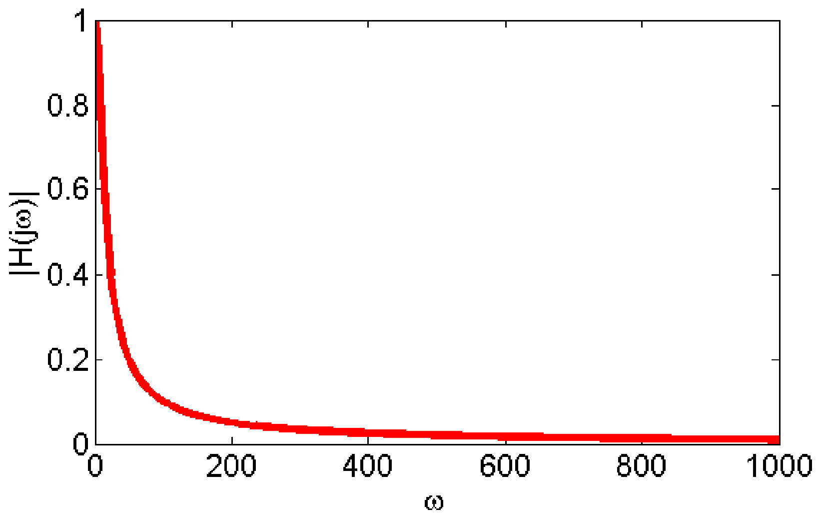
- $\hat{V}_{out} = H(j\omega)\hat{V}_{in} = |H(j\omega)|\angle H(j\omega)\hat{V}_{in}$
- For each frequency, different:
  - Scaling [magnitude]
  - Delay [phase shift]
- It is useful to graphically depict the magnitude and phase of the transfer function

Bode Magnitude Plot

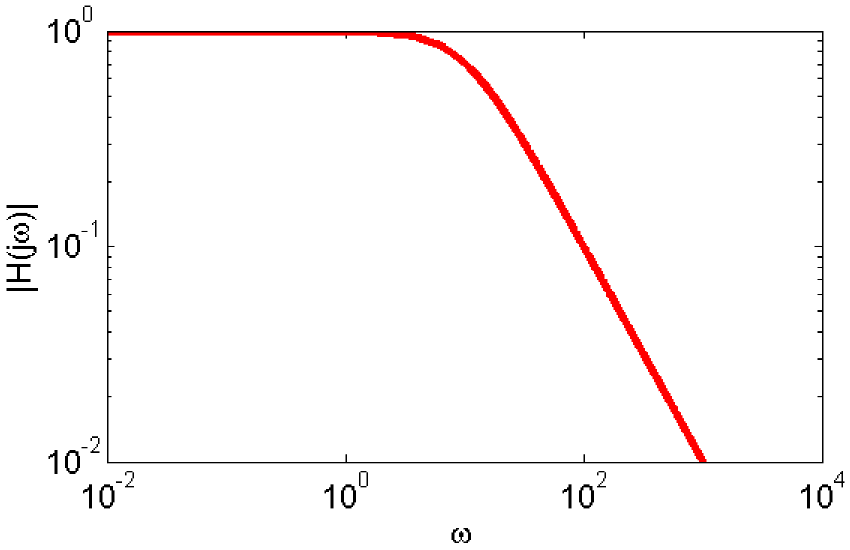
- $\hat{V}_C = \frac{1}{1+0.1jw} \hat{V}_I = H(j\omega)\hat{V}_I$

  $|H(j\omega)| = 1/\sqrt{1 + 0.01\omega^2}$

- Magnitude plot is just a plot of $|H(j\omega)|$ as a function of ω

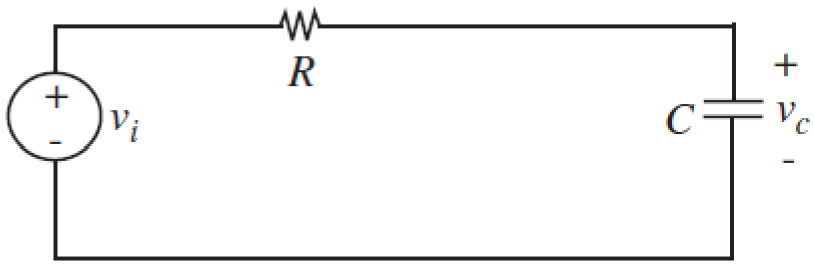


Linear Scale



Log Scale

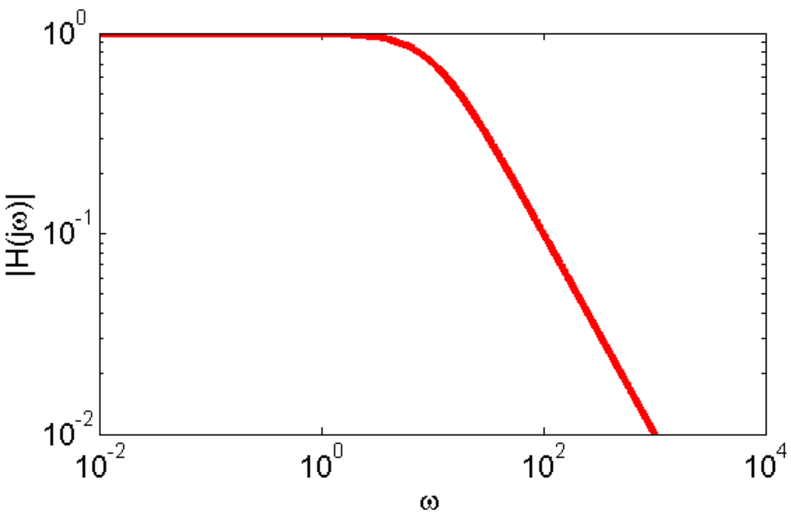
Bode Magnitude Plot in Context of Circuit



$$v_I = V_i cos(\omega t), \qquad t > 0$$

$$R = 10{,}000\Omega$$
$$C = 1\mu F$$
$$V_i = 5V$$

$$\hat{V}_C = \frac{1}{1 + 0.1jw}\hat{V}_I = H(j\omega)\hat{V}_I$$



$$|H(j\omega)| = 1/\sqrt{1 + 0.01\omega^2}$$

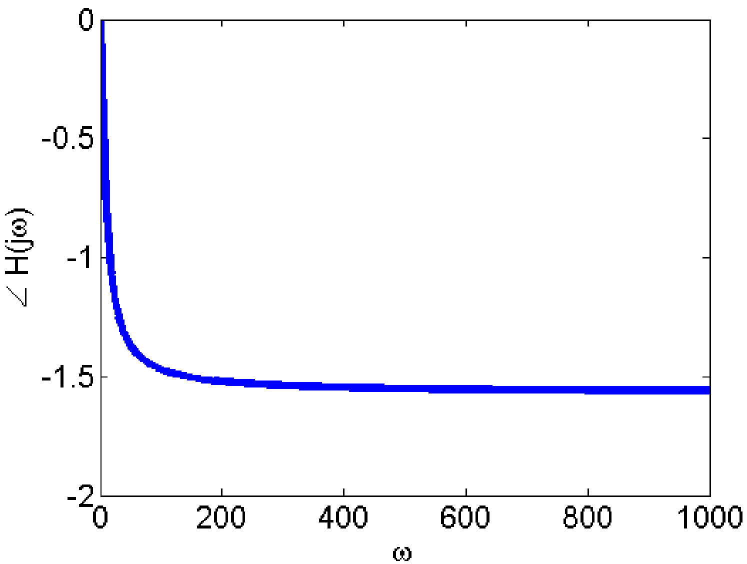
All frequencies below $w_c = 10$ get through pretty well. Above, that increasingly attenuated

Bode Phase Plot

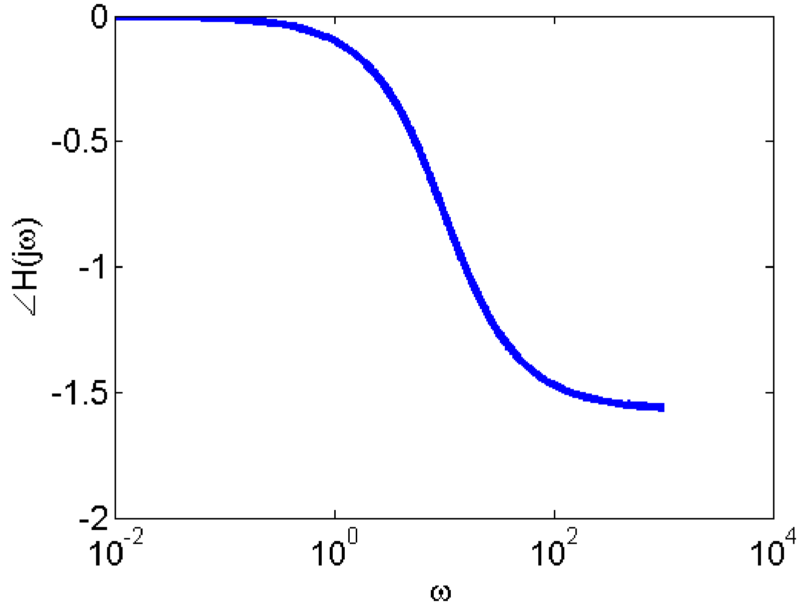
- $\hat{V}_C = \frac{1}{1+0.1jw}\hat{V}_I = H(j\omega)\hat{V}_I$

  $\angle H(j\omega) = -ArcTan[0.1\omega/1]$

- Phase plot is just a plot of $\angle H(j\omega)$ as a function of ω

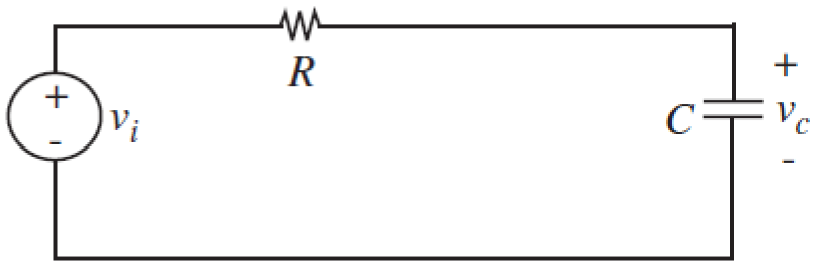


Linear Scale



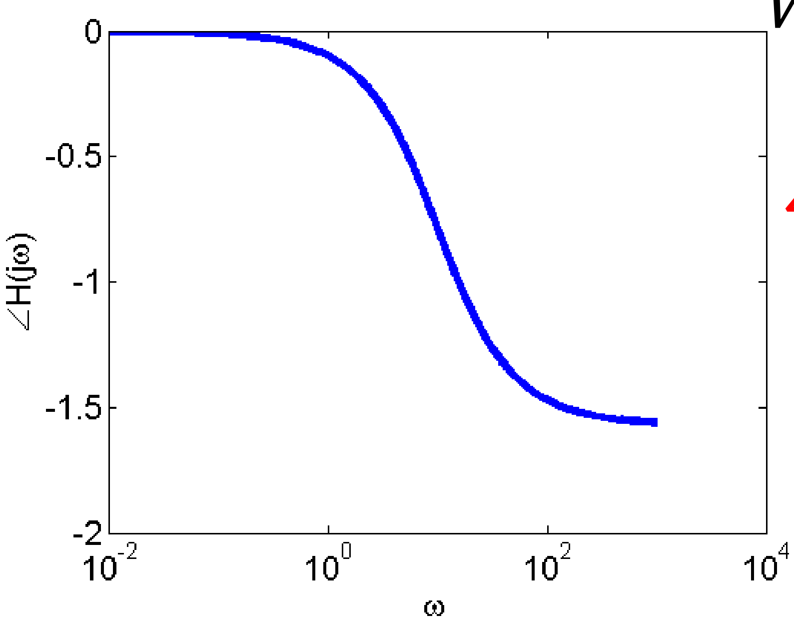
Semilog Scale

Bode Phase Plot in Context of Circuit



$$v_I = V_i cos(\omega t), \qquad t > 0$$

$$R = 10{,}000\Omega$$
$$C = 1\mu F$$
$$V_i = 5V$$



$$\hat{V}_C = \frac{1}{1 + 0.1jw} \hat{V}_I = H(j\omega)\hat{V}_I$$

$$\angle H(j\omega) = -ArcTan[0.1\omega/1]$$

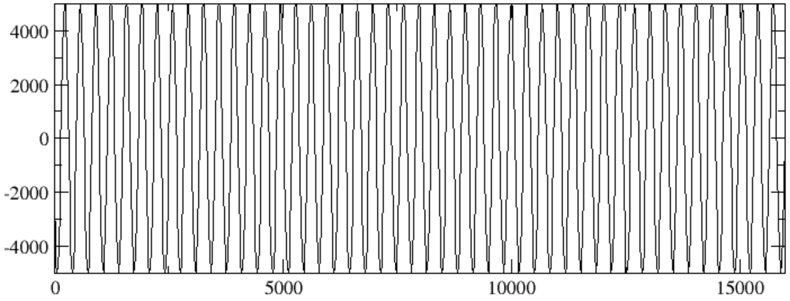
All frequencies below $w_c = 10$ move in time with the source, above that, v_c gets out of phase

Multiple Frequencies

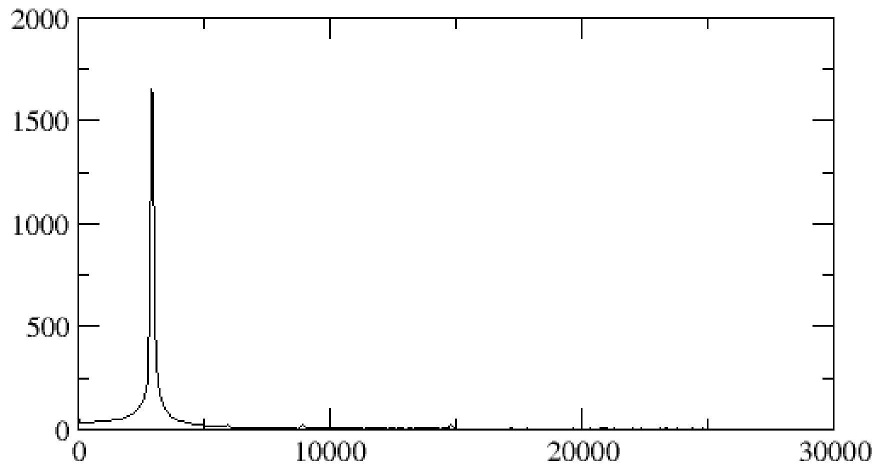
- Real signals are often a combination of a continuum of many frequencies
  - Radio antenna input
  - Microphone input
- Intuitively:
  - Thunder contains a bunch of low frequency sounds
  - Boiling kettles contains a bunch of high frequency sounds
- There is a mathematically well defined idea of what it means for a signal to "contain many frequencies"

Time vs. Frequency Domain

- The Fourier Transform takes a function $f(t)$ and converts it into a function $\mathcal{F}(\omega)$
- Every signal can be made out of a sum of an infinite number of sinusoids
- Fourier transform tells you how much of each one to include



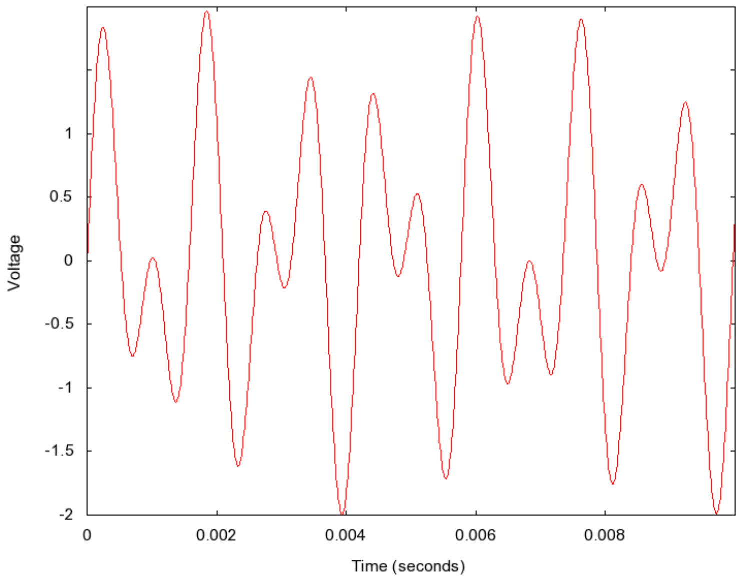
$$f(t) = \sin(3000t)$$

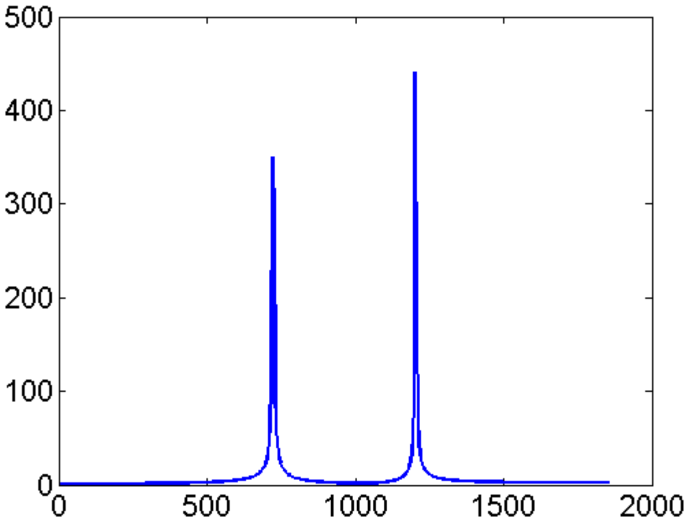


$$\mathcal{F}(\omega) \approx \delta(\omega - 3000)$$

Multiple Frequencies

- The "1" button on a phone is a combination of a 697 Hz tone and a 1209 Hz tone



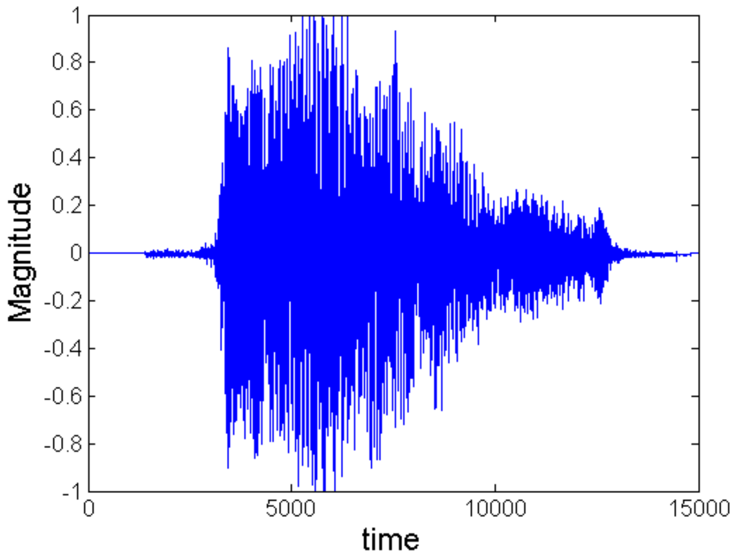


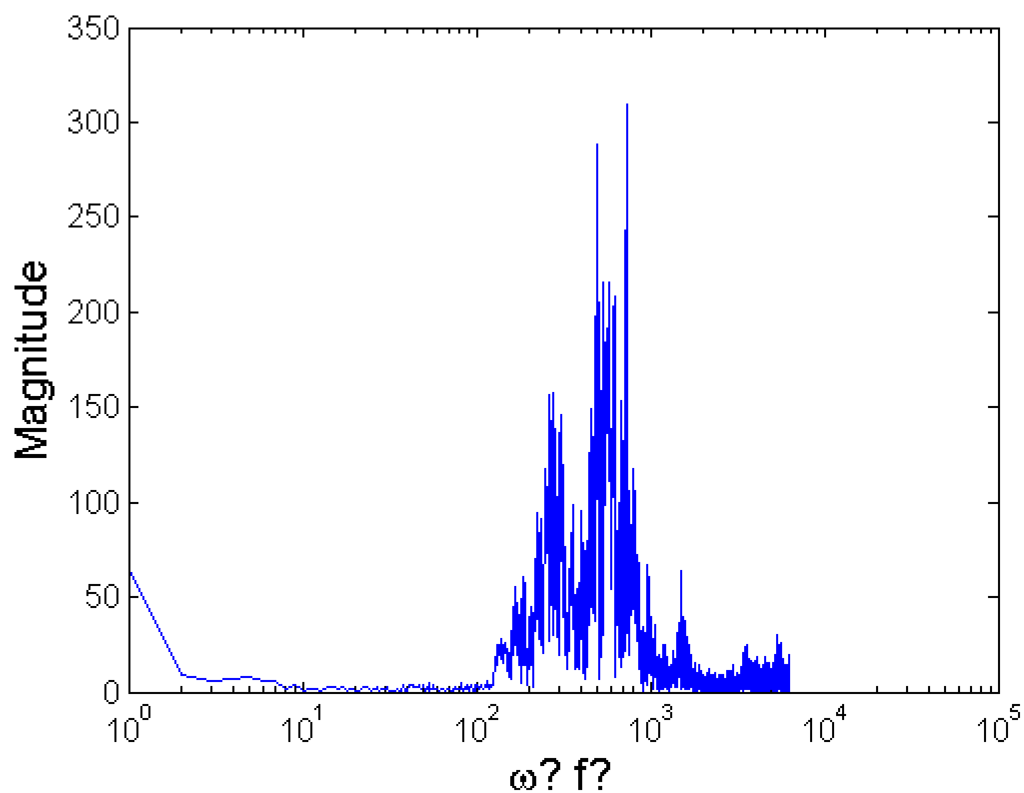
$$f(t) = \sin(2\pi 697t) + \sin(2\pi 1209t)$$

$$\mathcal{F}(\omega) \approx \delta(f - 692) + \delta(f - 1209)$$

Multiple Frequencies

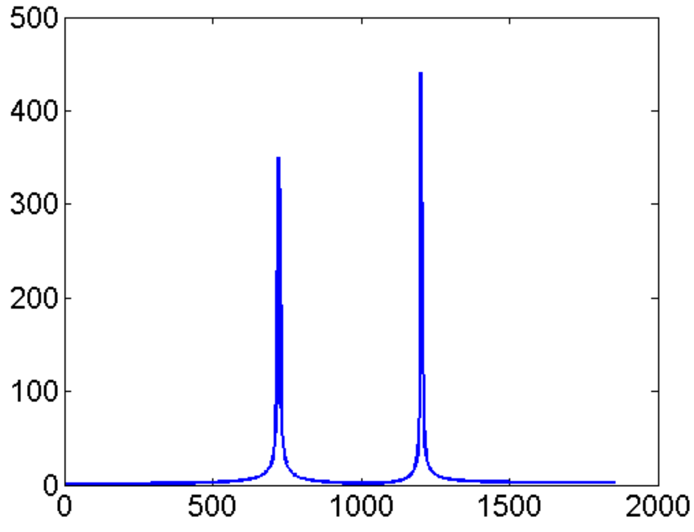
- Bill and Ted saying the word “bogus” is a more complex set of frequencies

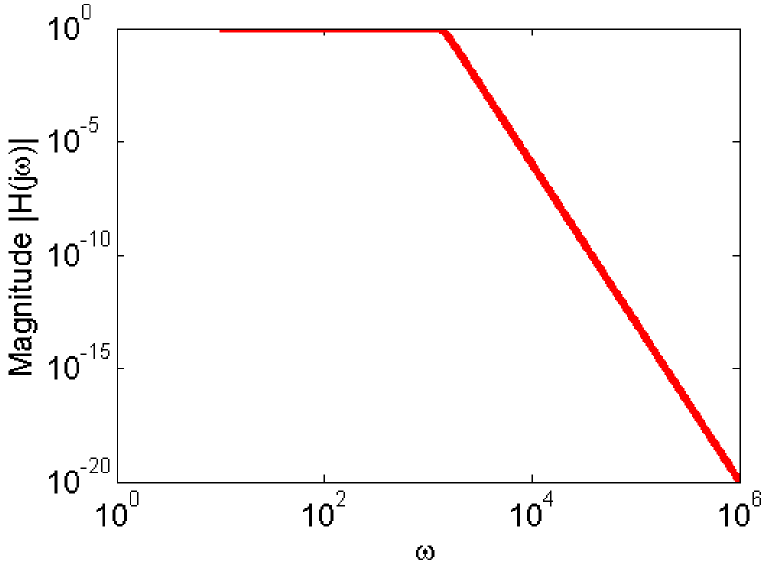




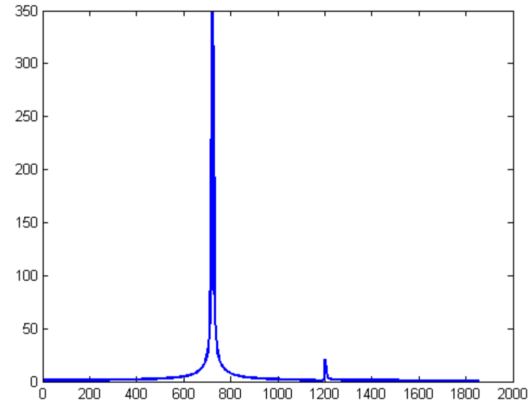
Filtering Example

- If we apply a filter with the frequency response on the right to the signal on the left

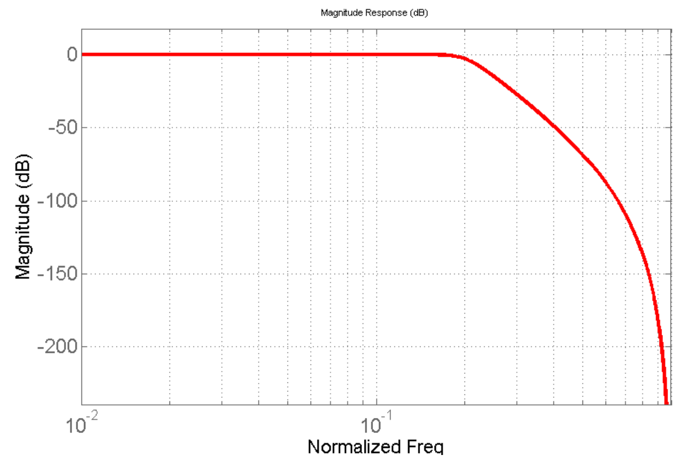


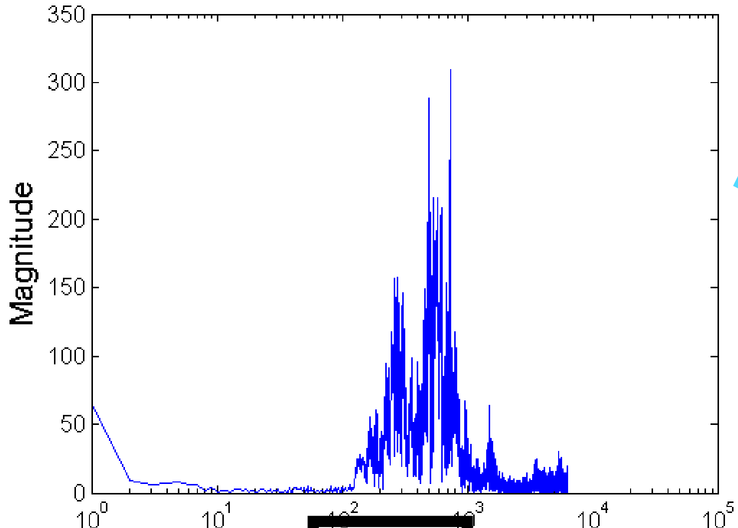


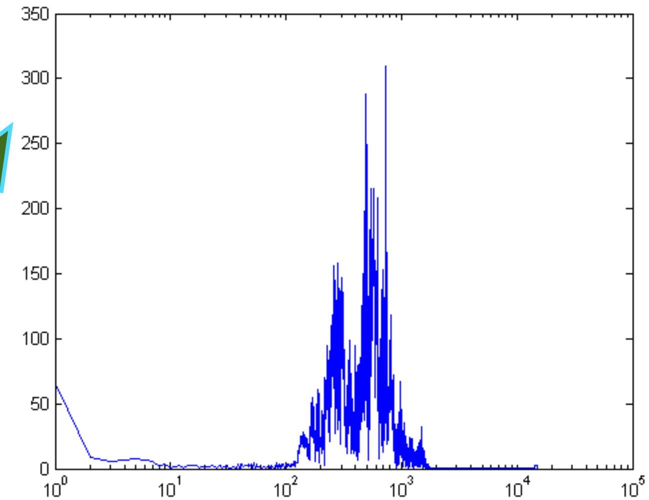
Then we'll get:



More complex filtering

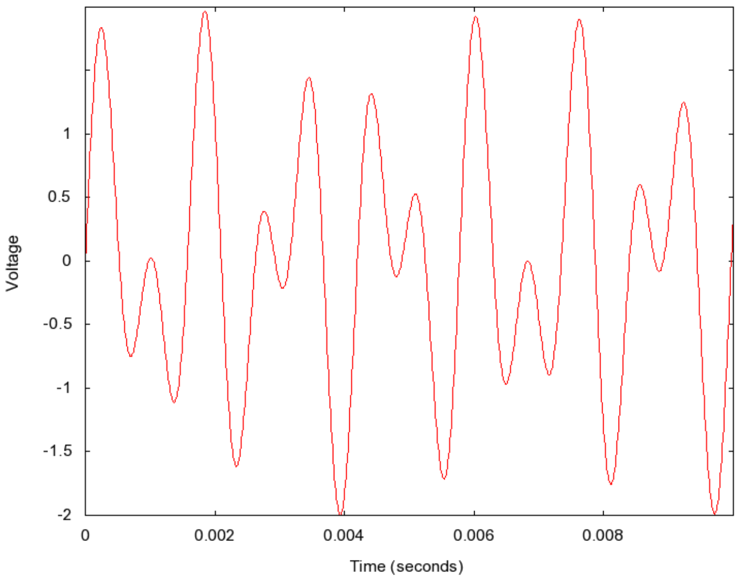


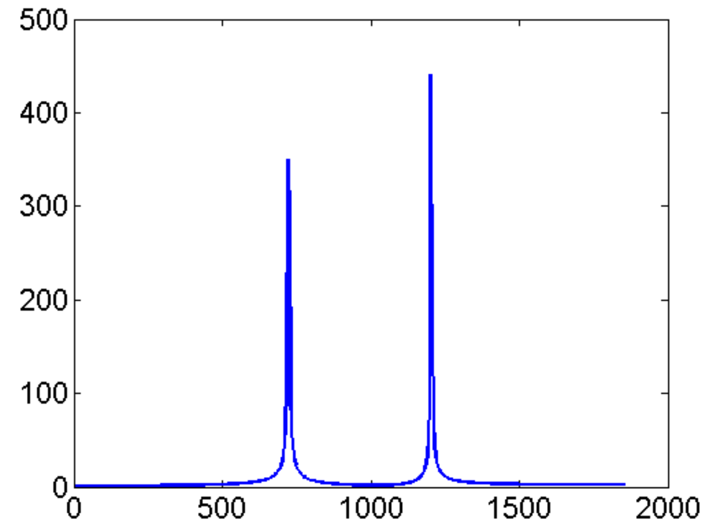




Each frequency individually scaled

Phase Effects





Original “1 button” tone

If we shift the phase of the larger sine, we get

Magnitude and Phase Demo

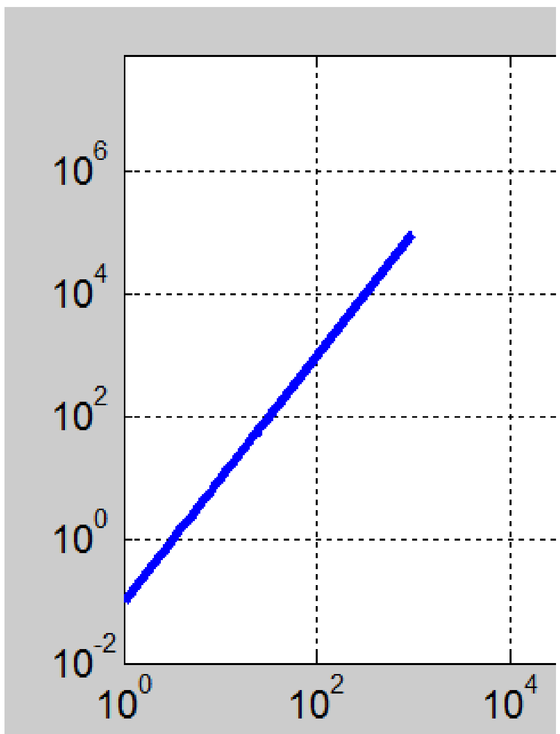
- Let’s try the ever risky live demo

Bode Plots

- Hopefully I've convinced you that magnitude and phase plots are useful
- Now, the goal will be to draw them straight from the transfer function
- First, some reminders on loglog plots

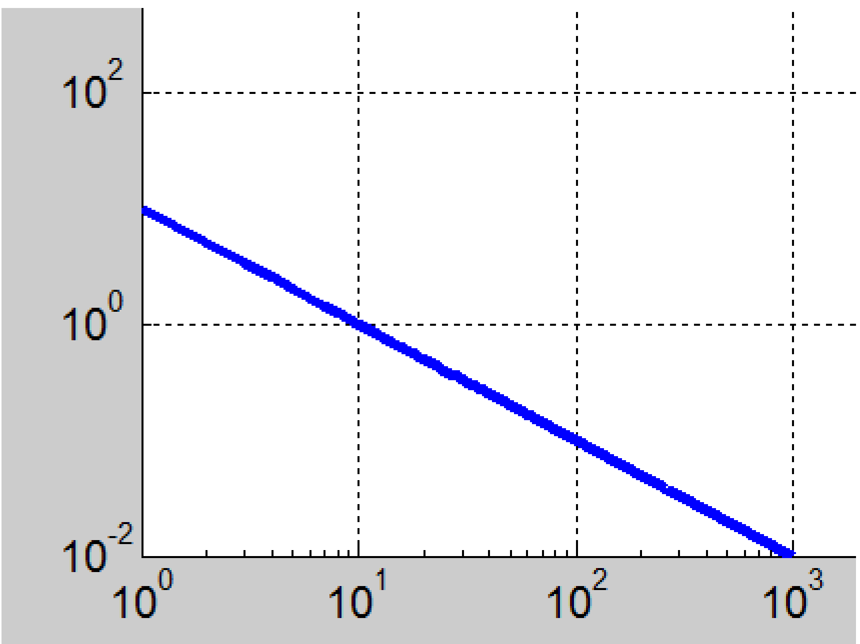
Loglog Plots

- On a loglog plot, $f(x) = Ax^n$ looks like a straight line w/slope n and y-offset A, because:
  - $\log(y) = n log(x) + \log(A)$



$f(x) = 0.1x^2$

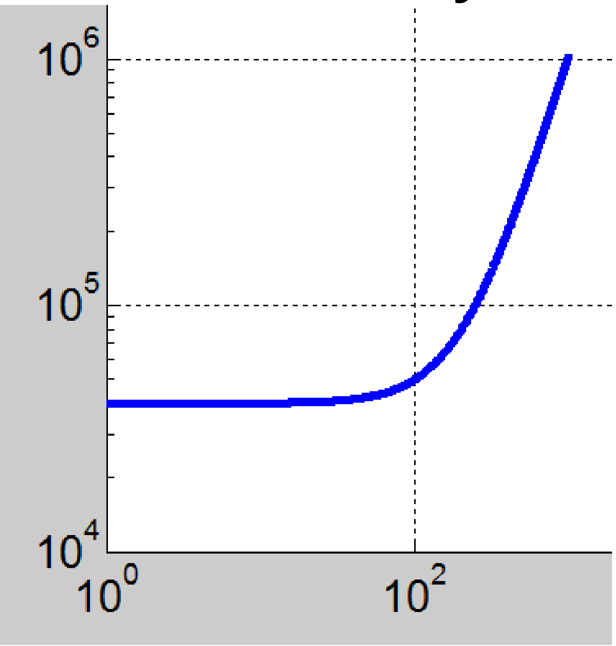
Y-offset – intercept at $x = 10^0$



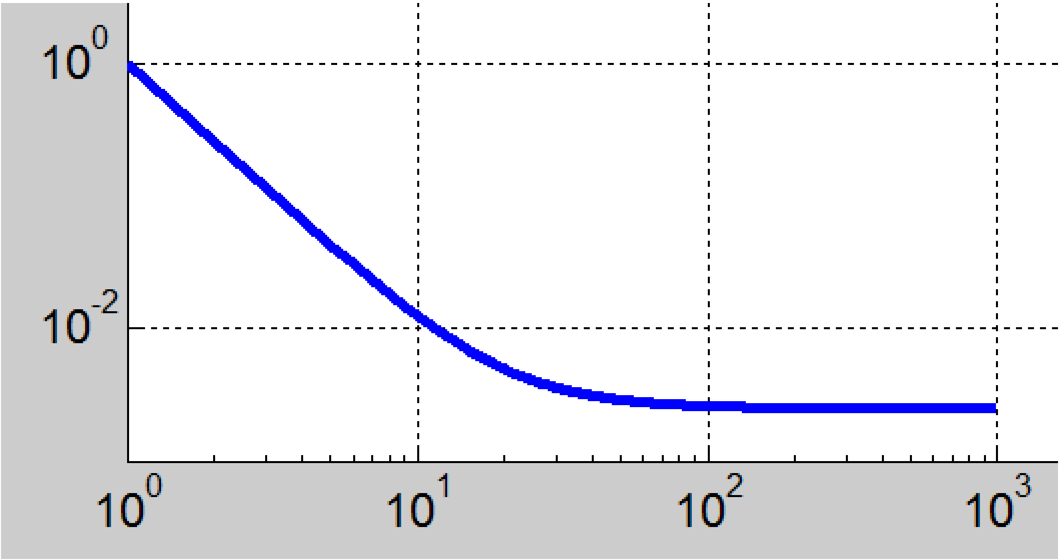
$f(x) = 10x^{-1}$

Loglog Plots

- On a loglog plot, $f(x) = x^n + k^n$ will:
  - Be flat for values of $x^n < k^n$
  - Be a straight line of slope n for values of $x^n > k^n$
  - Have a y-offset of $1 + k^n$



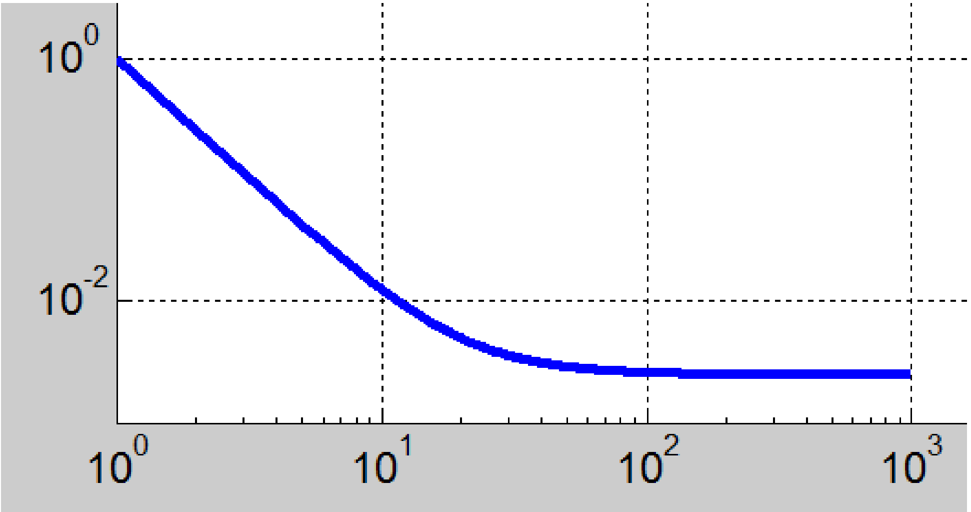
$f(x) = x^2 + 200^2$



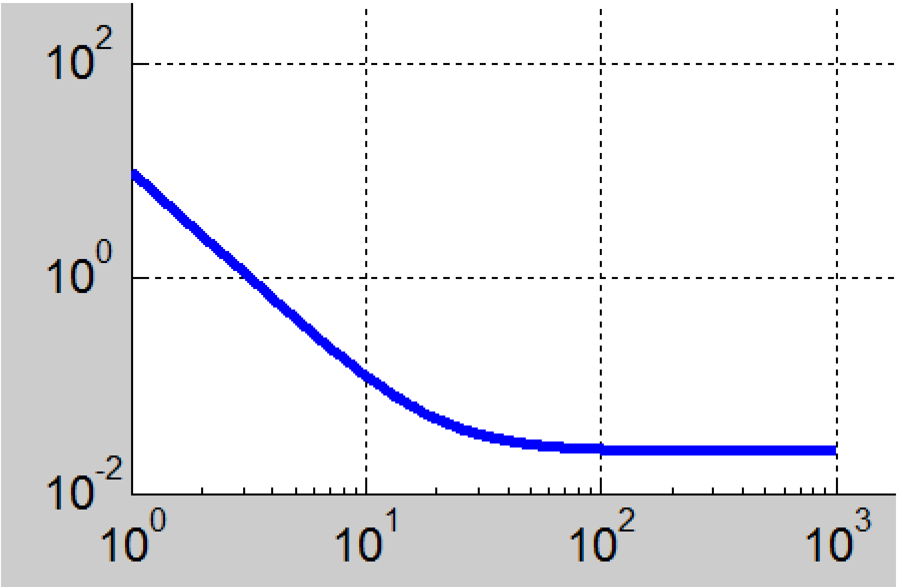
$f(x) = x^{-2} + 20^{-2}$

Loglog Plots

- On a loglog plot, $f(x) = A(x^n + k^n)$ will:
  - Be flat for values of $x^n < k^n$
  - Be a straight line of slope n for values of $x^n > k^n$
  - Have an y-offset of Ak^n



$$f(x) = x^{-2} + 20^{-2}$$



$$f(x) = 10(x^{-2} + 20^{-2})$$

Manual Bode Plots

- On board, using handout

2^{nd} Order Filter Example

- Also on board

2nd order Bode Plots

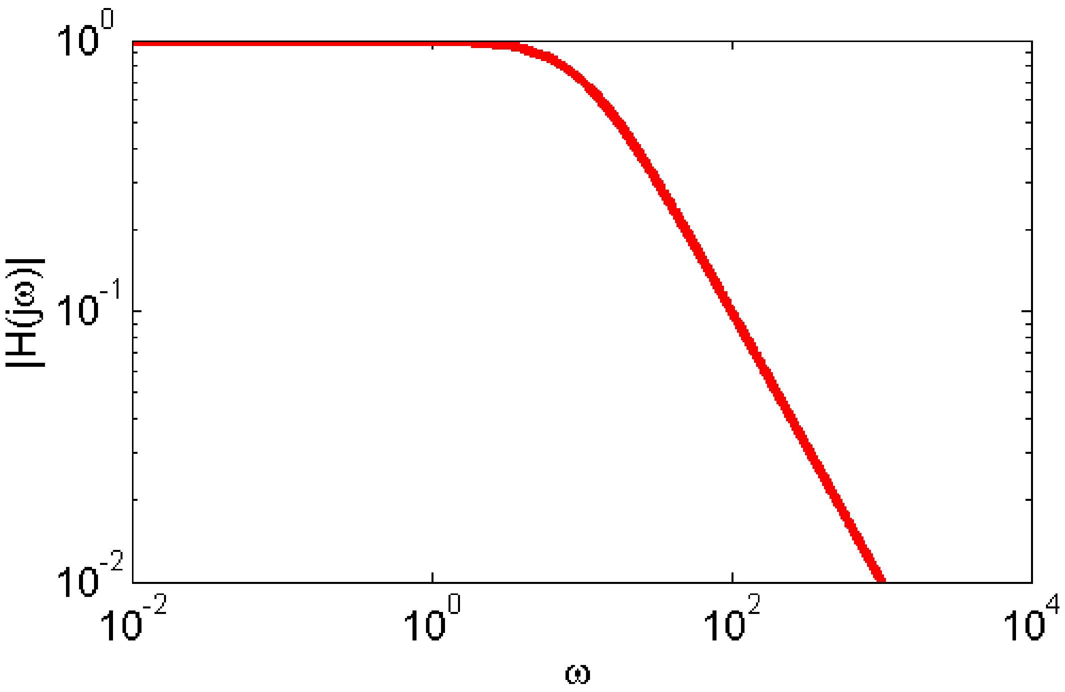
- Also on board
- This is where we stopped in class

Active filter example

- On board

Magnitude Plot Units

- So far, we've been plotting our Bode plots on where 10^0 represents a signal getting through perfectly



Bel and Decibel (dB)

- A **bel** (symbol **B**) is a unit of measure of ratios of power levels, i.e. relative power levels.
  - **B = log$_{10}$(P_1/P_2)** where P_1 and P_2 are power levels.
  - The bel is a logarithmic measure
  - Zero bels corresponds to a ratio of 1:1
  - One bel corresponds to a ratio of 10:1
  - Three bels corresponds to a ratio of 1000:1
- The bel is too large for everyday use, so the **decibel (dB)**, equal to 0.1B, is more commonly used.
  - 1dB = 10 $\log_{10}(P_1/P_2)$
  - 0 dB corresponds to a ratio of 1:1
  - 10 dB corresponds to a ratio of 10:1
  - -10 dB corresponds to a ratio of 1:10
- dB are used to measure
  - Electric power, filter magnitude

Logarithmic Measure for Power

- To express a power in terms of decibels, one starts by choosing a reference power, $P_{reference}$, and writing

  Power P in decibels = $10 \log_{10}(P/P_{reference})$

- Exercise:
  - Express a power of 50 mW in decibels relative to 1 watt.
  - $P\ (dB) = 10 \log_{10} (50 \times 10^{-3}) = -13$ dB

- Use logarithmic scale to express power ratios varying over a large range

dB: $$10 \log\left(\frac{P_1}{P_2}\right)\ (\mathrm{dB})$$

Note: dB is not a unit for a physical quantity since power ratio is unitless. It is just a notation to remind us we are in the log scale.

Decibels for measuring transfer function magnitude?

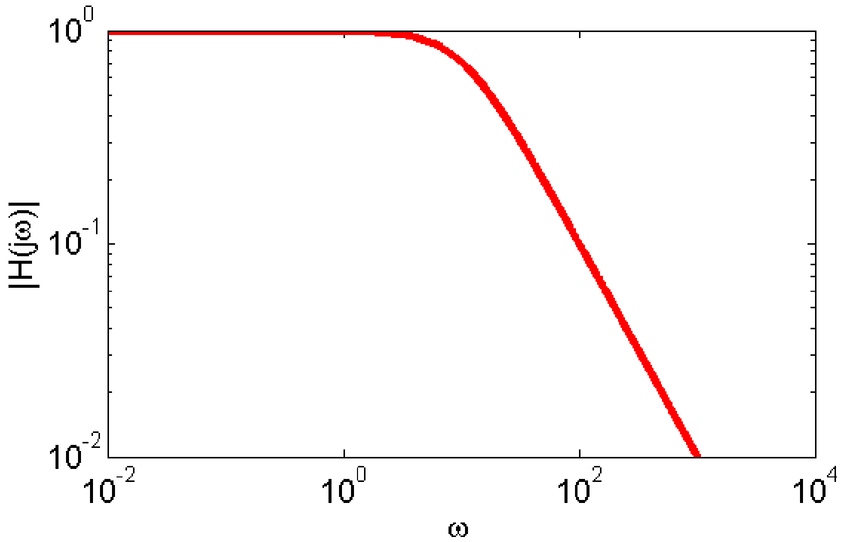
- Decibels provide a measure of relative power
- However, we said they also can be used for transfer functions
- The key is in realizing that $P \propto V^2 \propto I^2$
- Thus, if a voltage gets reduced by a factor of 100, we'd say the power ratio between the output and in the input would be

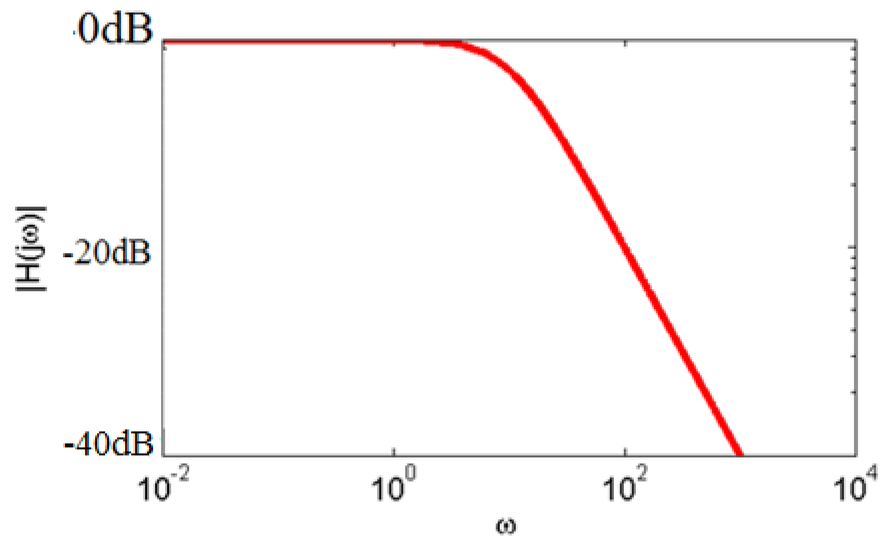
$$- \; 10\log_{10}\left(\frac{V_{out}^2}{V_{in}^2}\right) = 10\log_{10}\left(\left(\frac{V_{out}}{V_{in}}\right)^2\right) = 20\log_{10}\left(\frac{V_{out}}{V_{in}}\right)$$

Transfer Function in dB

- Given a transfer function magnitude $|H(j\omega)|$
- We can convert this transfer function into dB by:
- $|H(j\omega)|_{in\ dB} = 20\log_{10}(|H(j\omega|)$
- There's not that much to this, but it's common in datasheets

Example





$$|H(j\omega)|_{in\ dB} = 20\log_{10}(|H(j\omega|)$$