EE40
Lecture 2
Josh Hug

6/23/2010

Logistical Changes and Notes

- Friday Lunch is now Monday lunch (starting next Monday)
  - Email me by Saturday evening if you'd like to come: JHUG aat eecs.berkeley.edu
- My office hours will be Wednesday and Friday, 11:00-12:00, room TBA
- Google calendar with important dates now online
- Did anybody not get my email sent out Monday (that said no discussion yesterday)?
- Will curate the reading a little more carefully next time

Lab/HW Deadlines and Dates

- Discussions start Friday
- Labs start next Tuesday
- HW0 Due Today
- Homework 1 will be posted by 3PM, due Friday at 5 PM
- **Tuesday homeworks now due at 2PM, not 5PM in Cory 240 HW box**

Summary From Last Time

- ***Current*** = rate of charge flow
- ***Voltage*** = energy per unit charge created by charge separation
- ***Power*** = energy per unit time
- ***Ideal Basic Circuit Element***
  - 2-terminal component that cannot be sub-divided
  - Described mathematically in terms of its terminal voltage and current
- ***Circuit Schematics***
  - Networks of ideal basic circuit elements
  - Equivalent to a set of algebraic equations
  - Solution provides voltage and current through all elements of the circuit

Heating Elements

- Last time we posed a question:
  - Given a fixed voltage, should we pick a thick or thin wire to maximize heat output
  - Note that resistance decreases with wire radius
- Most of you said that we'd want a thin wire to maximize heat output, why is that?
  - Believed that low resistance wire would give the most heat?
  - Didn't believe me that thick wire has low resistance?
  - General intuition?

Intuitive Answer

- I blasted through some equations and said "thicker is better, Q.E.D.", but I'm not sure you guys were convinced, so here's another view
- You can think of a big thick wire as a bunch of small wires connected to a source
  - The thicker the wire, the more little wires
  - Since they are all connected directly to the source, they all have same voltage and current and hence power
  - Adding more wires gives us more total current flow (same voltage), and hence more power

Then Why Don't Toasters and Ovens Have Thicker Elements?

- Thicker elements mean hotter elements
  - Will ultimately reach higher max temperature
  - Will get to maximum faster [see message board after 6 or 7 PM tonight for why]
- Last time, you guys asked "Well if thickness gives you more heat, why aren't toaster elements thicker?"
- The answer is most likely:
  - More burned toast. Nobody likes burned toast.

Toaster Element Design Goals

- Make heating element that can:
  - Can reach a high temperature, but not too high
  - Can reach that temperature quickly
  - Isn’t quickly oxidized into oblivion by high temperature
  - Doesn’t cost very much money
  - Will not melt at desired temperature
- Nichrome is a typical metal alloy in elements:
  - Low oxidation
  - High resistance (so normal gauge wire will not draw too much power and get too hot)
- Size was tweaked to attain desired temperature

Continue the Discussion on BSpace

- Let's get working on some more complicated circuits than this:

Topic 2

Setting Up and Solving Resistive Circuit Models

Circuit Schematics

- Many circuit elements can be approximated as simple ideal two terminal devices or **ideal basic circuit elements**
- These elements can be combined into **circuit schematics**
- Circuit schematics can be converted into algebraic equations
- These algebraic equations can be solved, giving voltage and current through any element of the circuit

Today

- We’ll enumerate the types of ideal basic circuit elements
- We’ll more carefully define a circuit schematic
- We’ll discuss some basic techniques for analyzing circuit schematics
  - Kirchoff’s voltage and current laws
  - Current and voltage divider
  - Node voltage method

Circuit Elements

- There are 5 ideal basic circuit elements (in our course):
  - voltage source
  - current source
  - resistor
  - inductor
  - capacitor

voltage source, current source: ***active elements***, capable of generating electric energy

resistor, inductor, capacitor: ***passive elements***, incapable of generating electric energy

- Many practical systems can be modeled with just sources and resistors
- The basic analytical techniques for solving circuits with inductors and capacitors are the same as those for resistive circuits

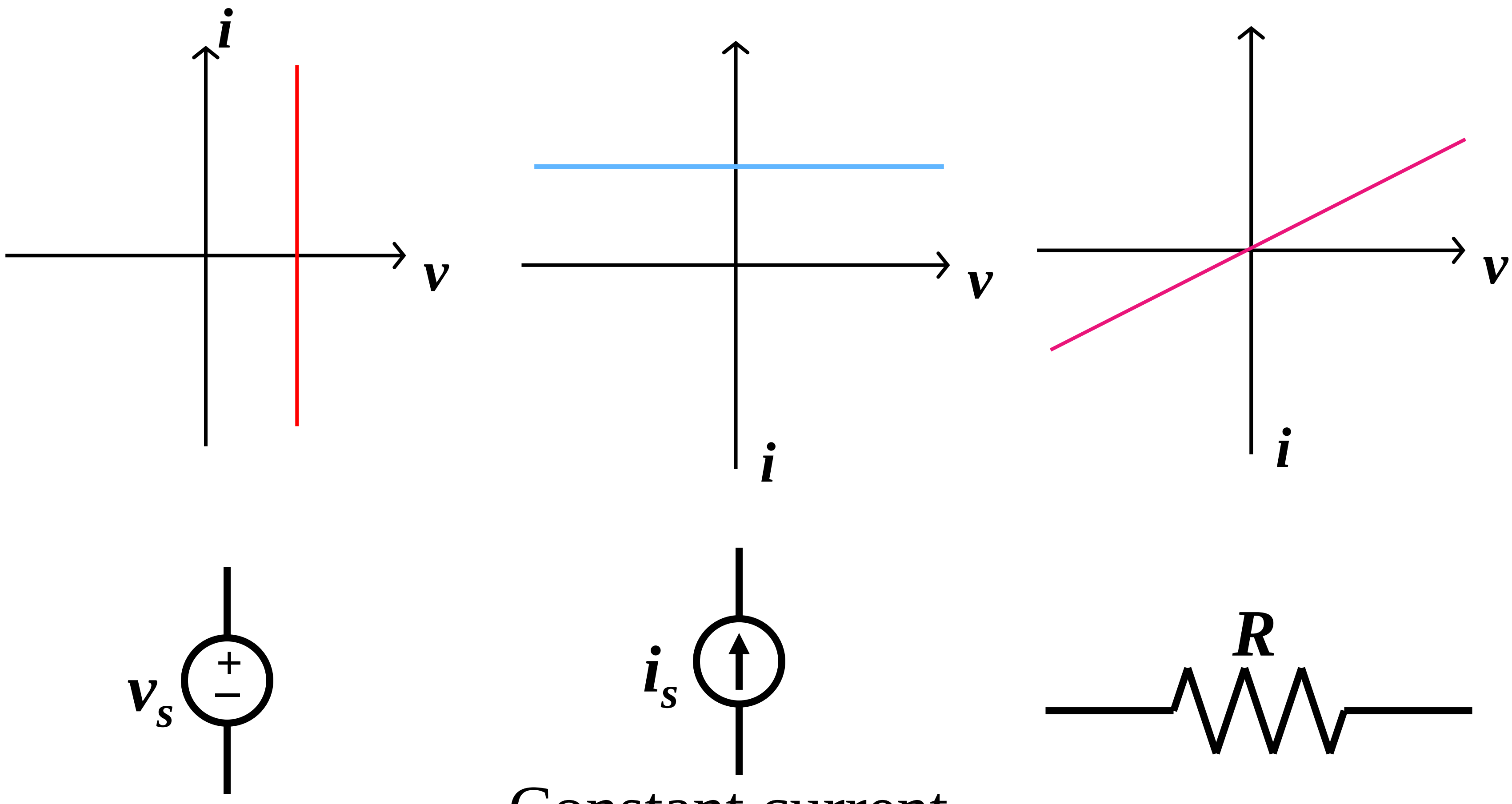
Electrical Sources

- An ***electrical source*** is a device that is capable of converting non-electric energy to electric energy and *vice versa*.

  Examples:
  - battery: chemical ⟺ electric
  - dynamo (generator/motor): mechanical ⟺ electric

→ Electrical sources can either deliver or absorb power

The Big Three



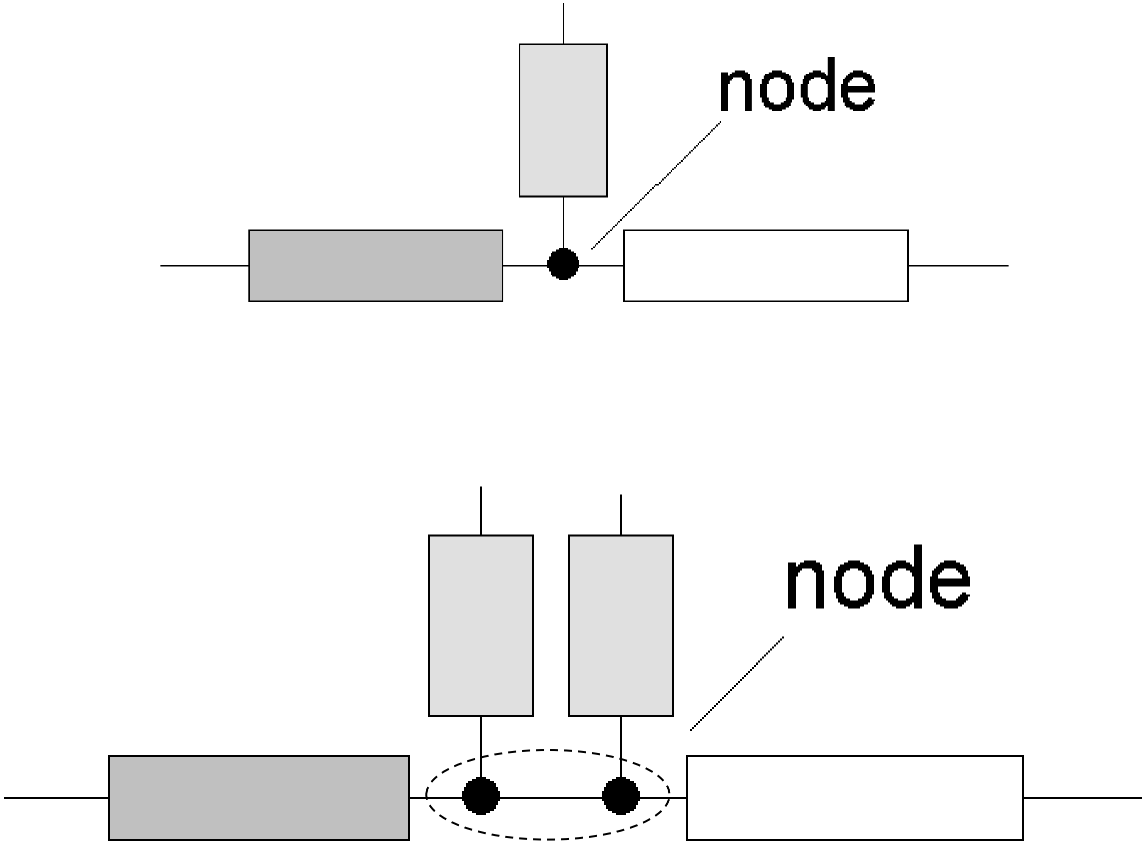
Constant current,
unknown voltage

Circuit Schematics

- A circuit schematic is a diagram showing a set of interconnected circuit elements, e.g.
  - Voltage sources
  - Current sources
  - Resistors
- Each element in the circuit being modeled is represented by a symbol
- Lines connect the symbols, which you can think of as representing zero resistance wires

Terminology: Nodes and Branches

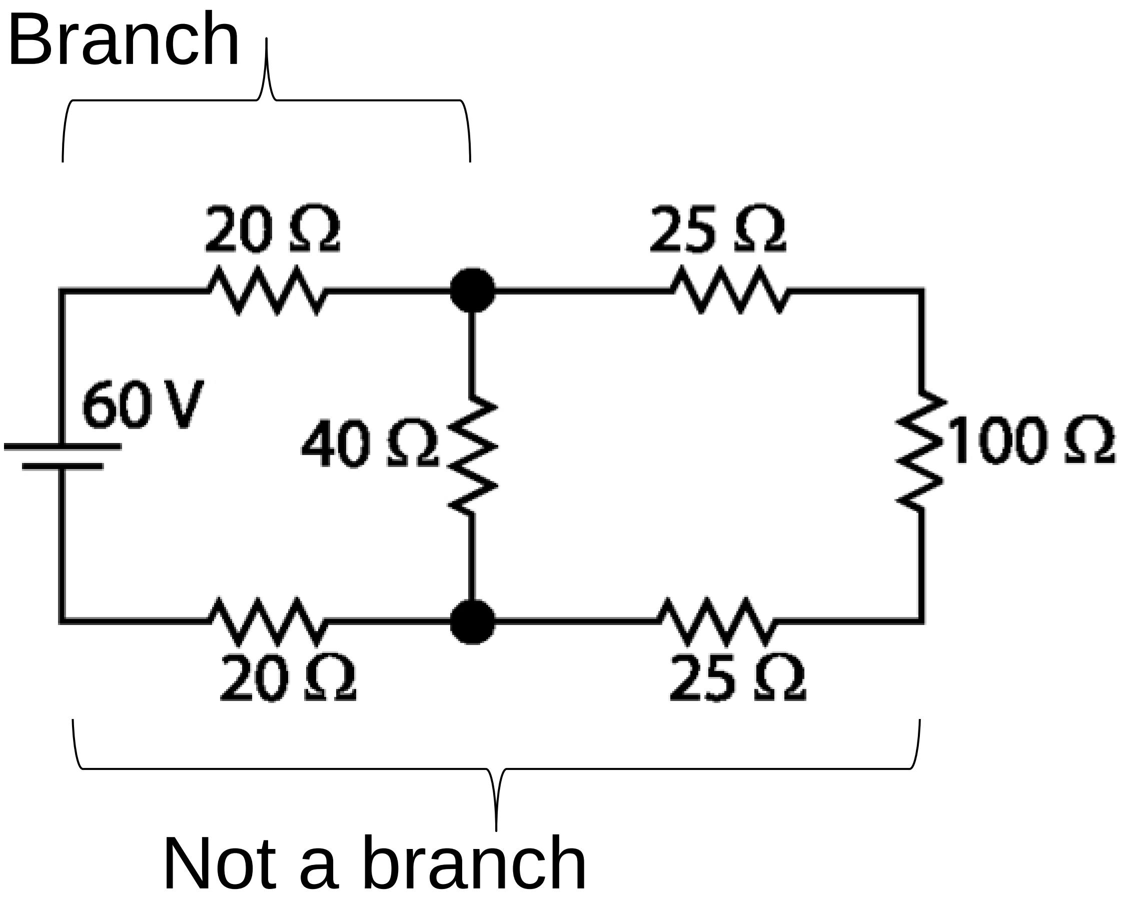
***Node*:** A point where two or more circuit elements are connected – entire wire



Can also think of as the “vertices” of our schematic

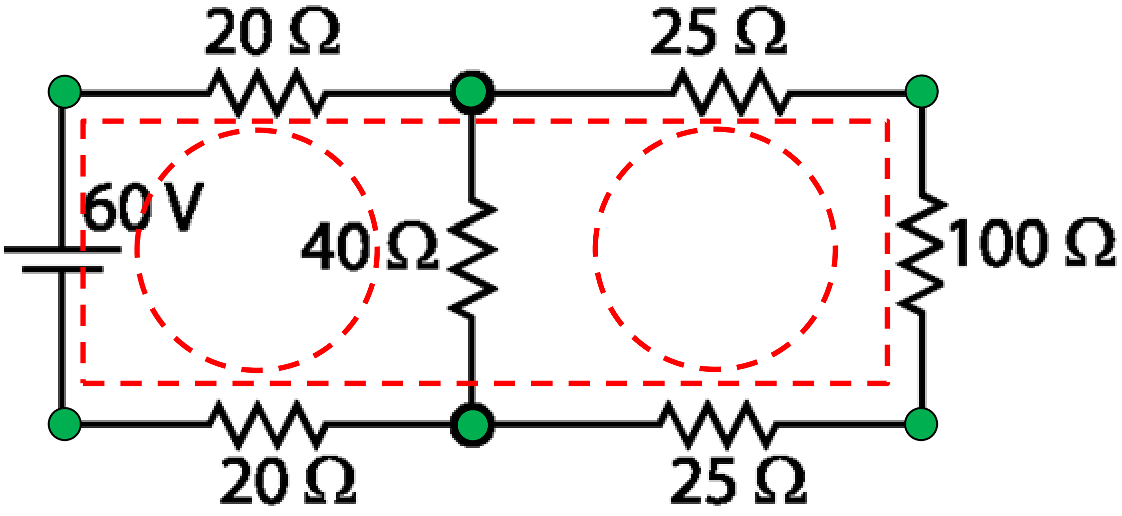
Terminology: Nodes and Branches

***Branch*: A path that connects exactly two nodes**



Terminology: Loops

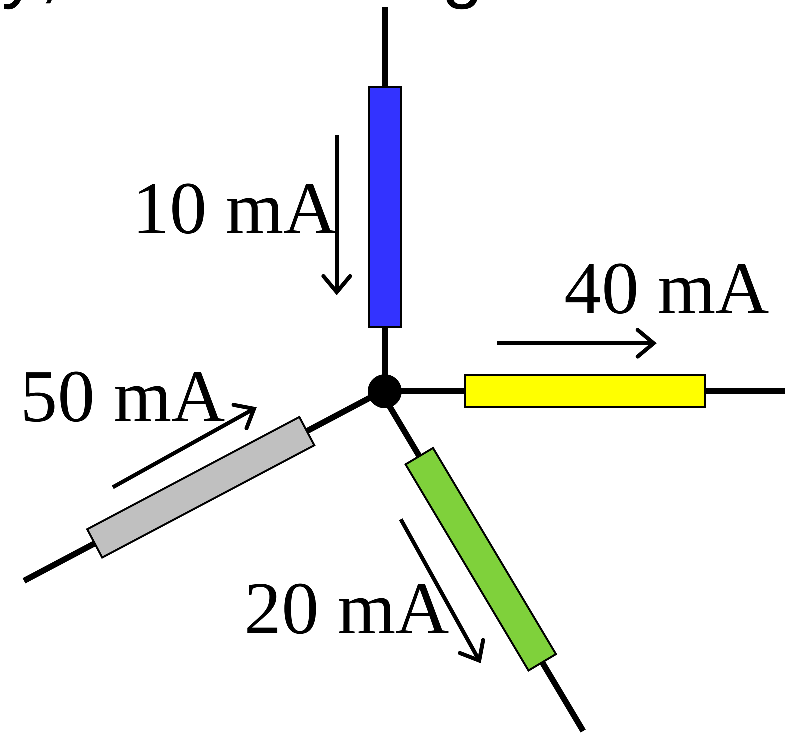
- A ***loop*** is formed by tracing a closed path in a circuit through selected basic circuit elements without passing through any intermediate node more than once
- Example: (# nodes, # branches, # loops)



6 nodes
7 branches
3 loops

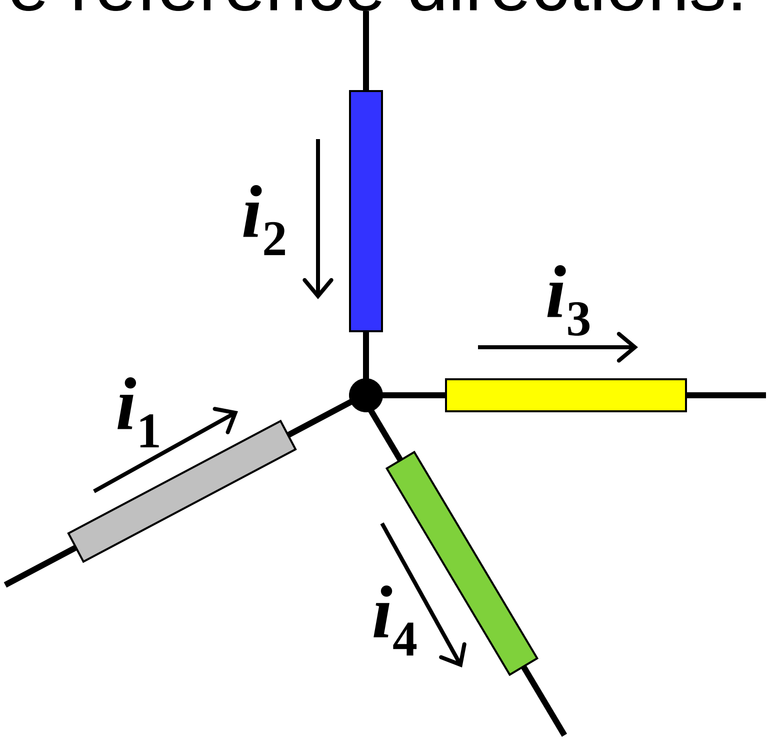
Kirchhoff's Laws

- **<u>Kirchhoff's Current Law (KCL)</u>:**
  - The algebraic sum of all the currents at any node in a circuit equals zero.
  - "What goes in, must come out"
  - Basically, law of charge conservation



Using Kirchhoff's Current Law (KCL)

Often we're considering unknown currents and only have reference directions:



$i_1+i_2=i_3+i_4$

or

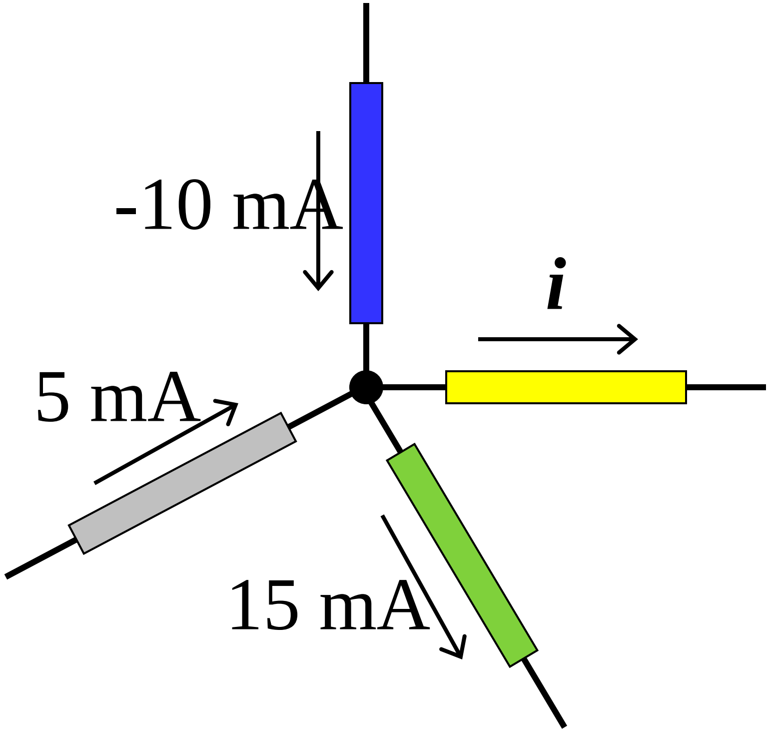
$i_1+i_2-i_3-i_4=0$

or

$-i_1-i_2+i_3+i_4=0$

- Use **reference directions** to determine whether reference currents are said to be "entering" or "leaving" the node – with no concern about actual current directions

KCL Example

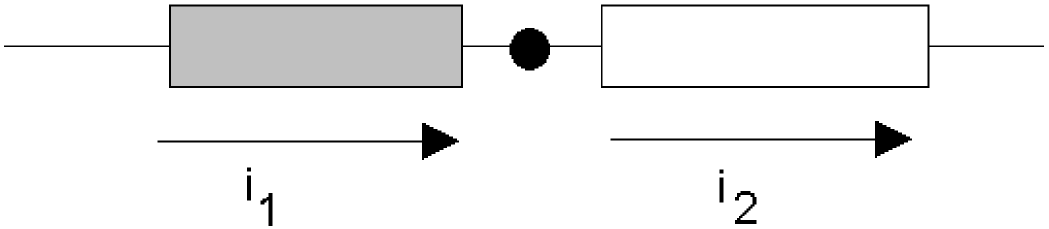


5+(-10)=15+i

i=-20mA

A Major Implication of KCL

- KCL tells us that **all of the elements along a single uninterrupted* path carry the same current**
- We say these elements are connected ***in series***.



Current entering node = Current leaving node

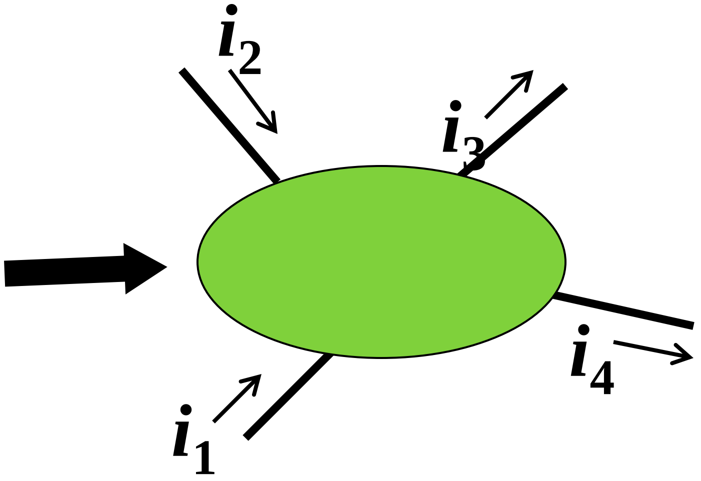
$$i_1 = i_2$$

*: To be precise, by uninterrupted path I mean all branches along the path connected EXACTLY two nodes

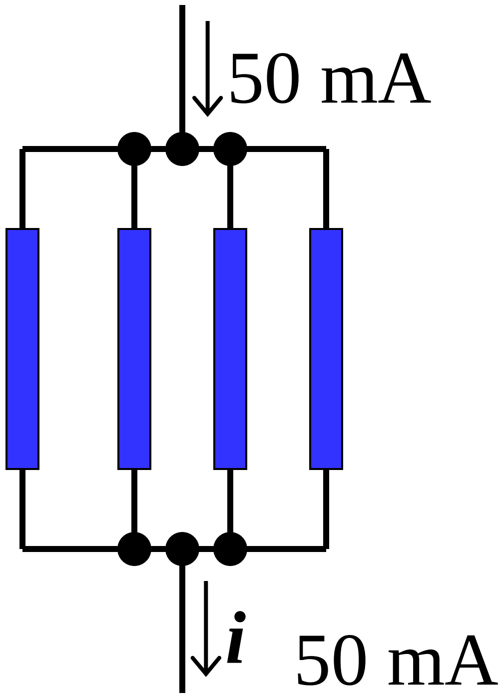
Generalization of KCL

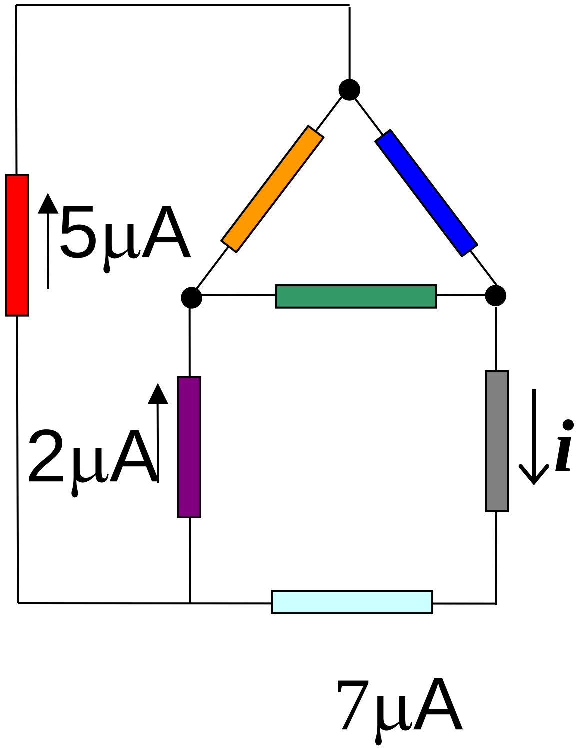
- The sum of currents entering/leaving a **closed surface** is zero. Circuit branches can be inside this surface, *i.e.* the surface can enclose more than one node!

This could be a big chunk of a circuit, *e.g.* a "black box"



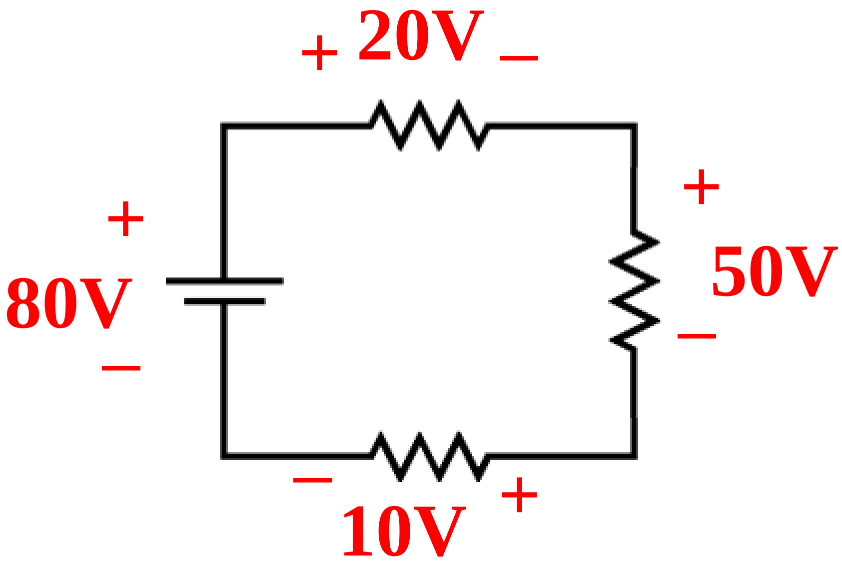
Generalized KCL Examples





Kirchhoff's Laws

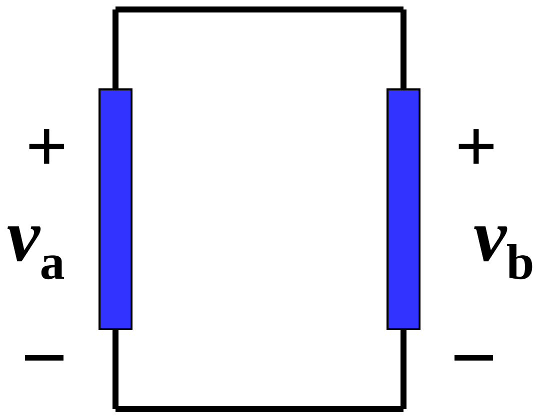
- **Kirchhoff's Voltage Law (KVL):**
  - The algebraic sum of all the voltages around any loop in a circuit equals zero.
  - "What goes up, must come down"



$$80=20+50+10$$

A Major Implication of KVL

- KVL tells us that **any set of elements which are connected at both ends carry the same voltage**.
- We say these elements are connected **in parallel**.

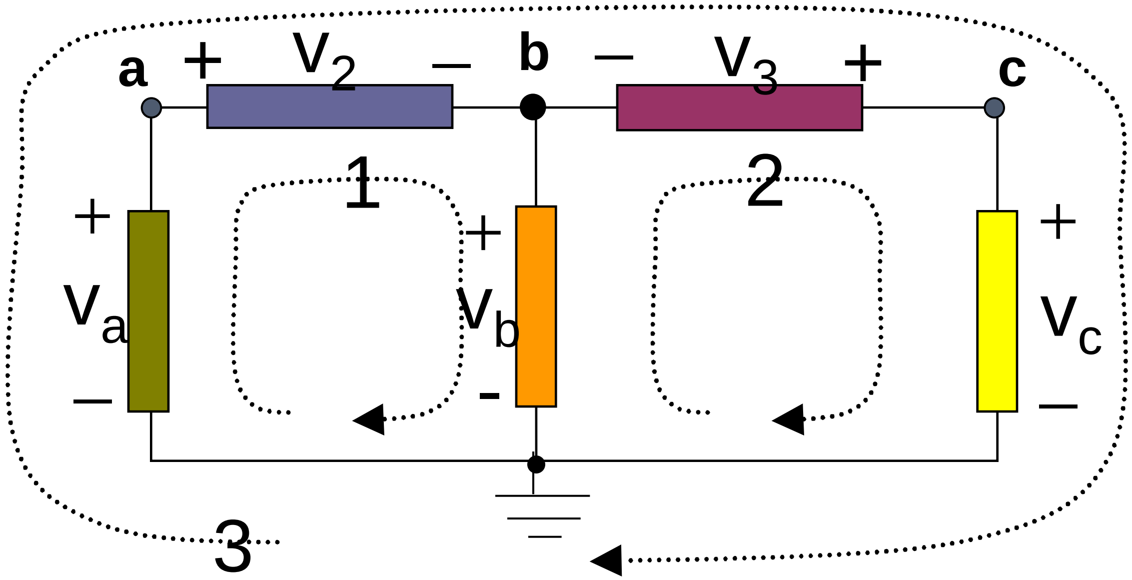


Applying KVL, we have that:

$$v_b - v_a = 0 \Rightarrow v_b = v_a$$

KVL Example

Three closed paths:



Path 1: $V_a = V_2 + V_b$

Path 2: $V_b + V_3 = V_c$

Path 3: $V_a + V_3 = V_2 + V_c$

If you want a mechanical rule:

If you hit a – first, LHS

If you hit a + first, RHS

LHS is left hand side

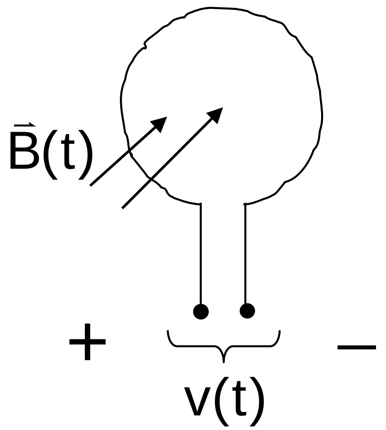
An Underlying Assumption of KVL

- No time-varying magnetic flux through the loop

  Otherwise, there would be an induced voltage (Faraday's Law)

  Voltage around a loop would sum to a nonzero value

- <u>Note</u>: Antennas are designed to "pick up" electromagnetic waves; "regular circuits" often do so undesirably.



How do we deal with antennas (EECS 117A)?

Include a voltage source as the circuit representation of the induced voltage or "noise".

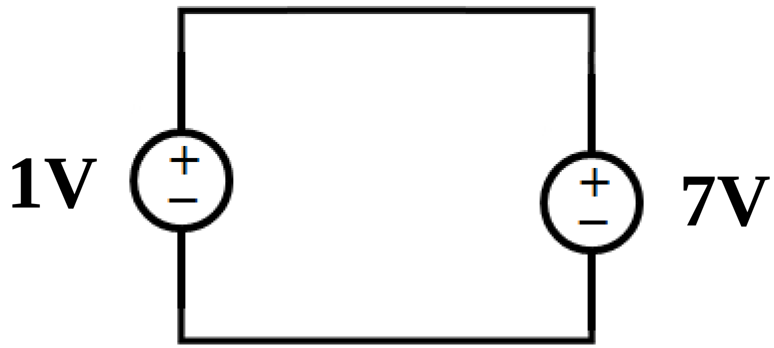
(Use a lumped model rather than a distributed (wave) model.)

Mini-Summary

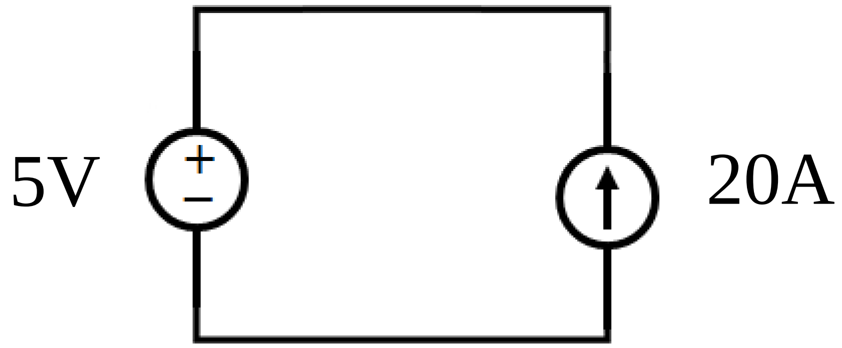
- KCL tells us that **all elements on an uninterrupted path** have the same current.
  - We say they are **"in series"**
- KVL tells us that **a set of elements whose terminals are connected at the same two nodes** have the same voltage
  - We say they are **"in parallel"**

Nonsense Schematics

- Just like equations, it is possible to write nonsense schematics:
  - 1=7
- A schematic is nonsense if it violates KVL or KCL



Verifying KCL and KVL



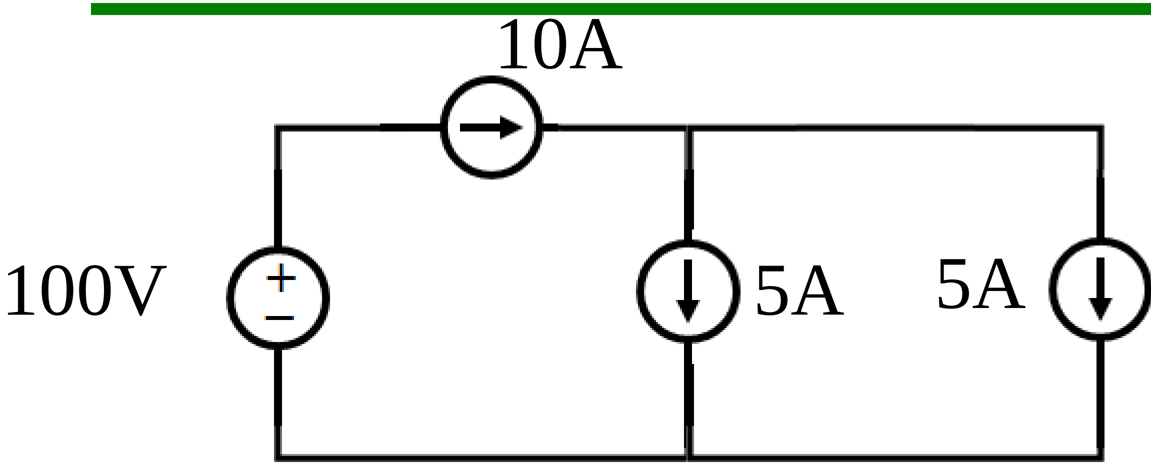
Is this schematic valid? Yes

How much power is consumed/provided by each source?

Voltage source: P_V=5V*20A =100W (consumed)

Current source: P_I=-20A*5V=100W (provided)

Verifying KCL and KVL



Is this valid?

Yes

KCL:

No contradiction

Top left node: I_{100}=10A

Top right node: 10A=5A+5A

Bottom node: 5A+5A=I_{100}

KVL:

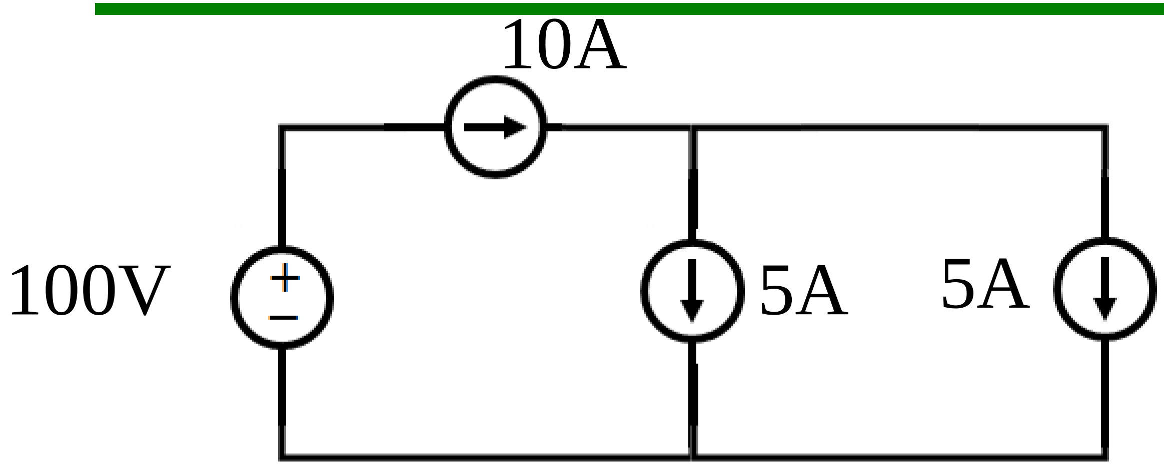
Left loop: 100V=V_{10}+V_5

Right loop: V_5=V_5

Big loop: 100V=V_{10}+V_5

No contradiction

Verifying KCL and KVL



Is this valid?

Yes

KCL:

Top left node: $I_{100}=10A$

KVL:

Left loop: $100V=V_{10}+V_{5}$

2 equations

3 unknowns

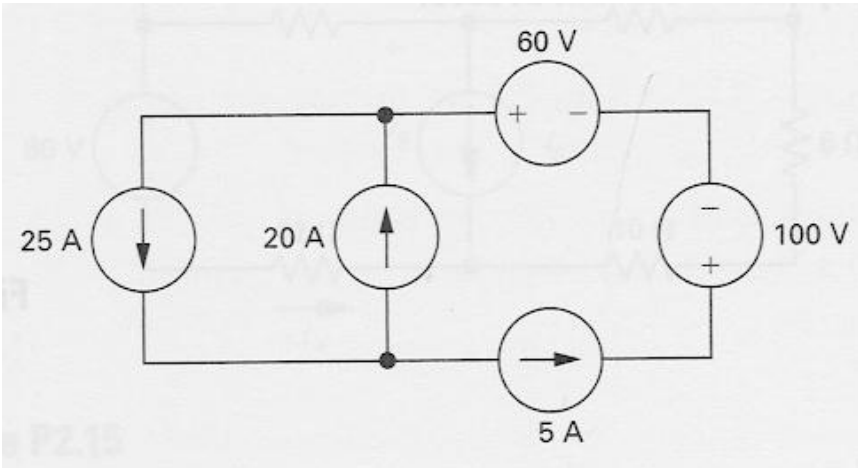
So what are V_{10} and V_{5}?

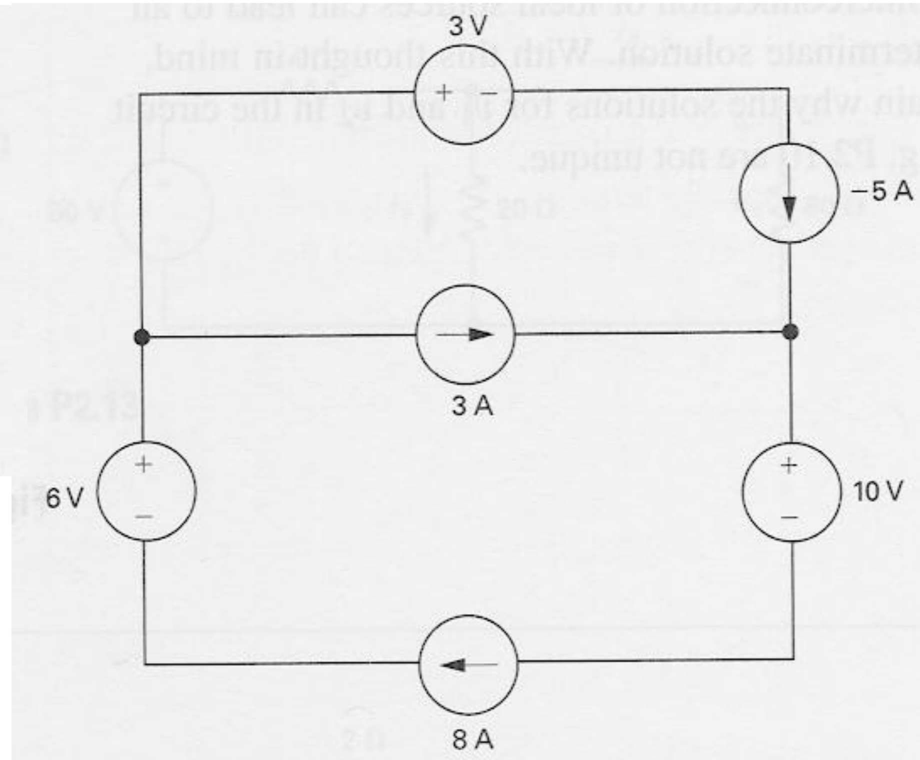
Whatever we want that sums to 100V

Multiple circuit solutions

iClicker #1

- Are these interconnections permissible?





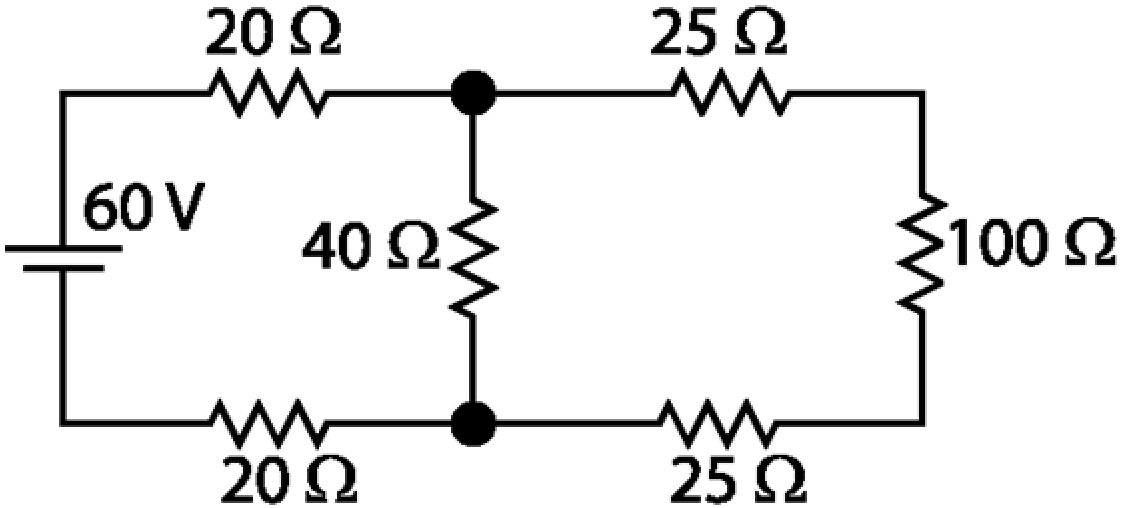
A. Both are bad
B. Left is ok, right is bad
C. Left is bad, right is ok
D. Both are ok

On to Solving Circuits

- Next we'll talk about a general method for solving circuits
  - The book calls this the "basic method"
  - It's a naïve way of solving circuits, and is way more work than you need
    - Basic idea is to write every equation you can think of to write, then solve
  - However, it will build up our intuition for solving circuits, so let's start here

Solving Circuits (naïve way)

- Label every branch with a reference voltage and current
  - If two branches are in parallel, share voltage label
  - If in series, share same current label
- For each branch:
  - Write Ohm's law if resistor
  - Get branch voltage "for free" if known voltage source
  - Get branch current "for free" if known current source



Solving Circuits (naïve way)

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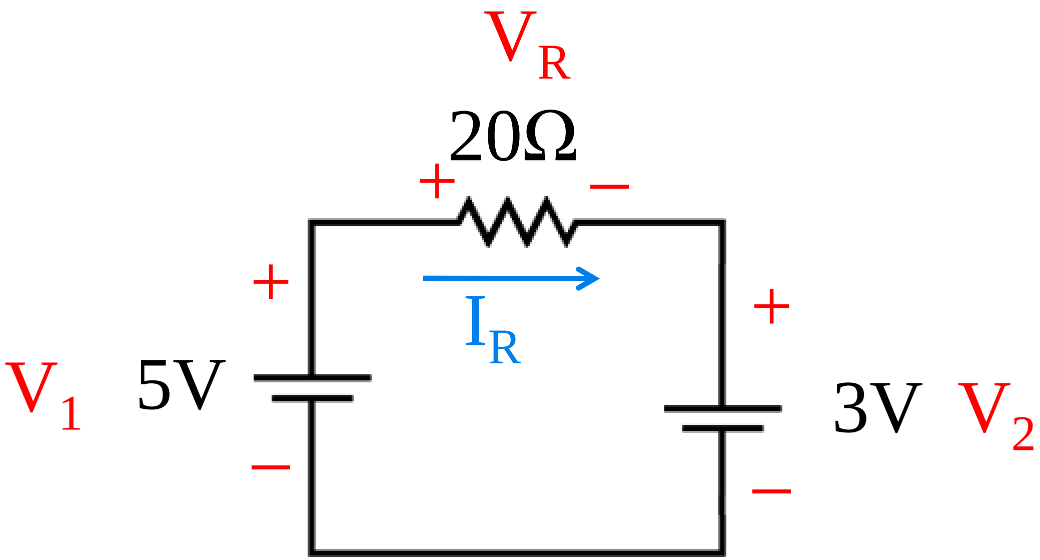
  - If in series, share same current label
- For each branch:
  - Write Ohm's law if resistor
  - Get branch voltage "for free" if known voltage source
  - Get branch current "for free" if known current source
- For each node touching at least 2 reference currents:
  - Write KCL – gives reference current relationships
  - Can omit nodes which contain no new currents
- For each loop:
  - Write KVL – gives reference voltage relationships
  - Can omit loops which contain no new voltages

Could also call this the "kitchen sink" approach

Example: KCL and KVL applied to circuits

- Find the current through the resistor
- Use KVL, we see we can write:



$V_1 = V_R + V_2$

$V_1 = 5V$

$V_2 = 3V$

$I_R = V_R / 20\Omega$

4 equations

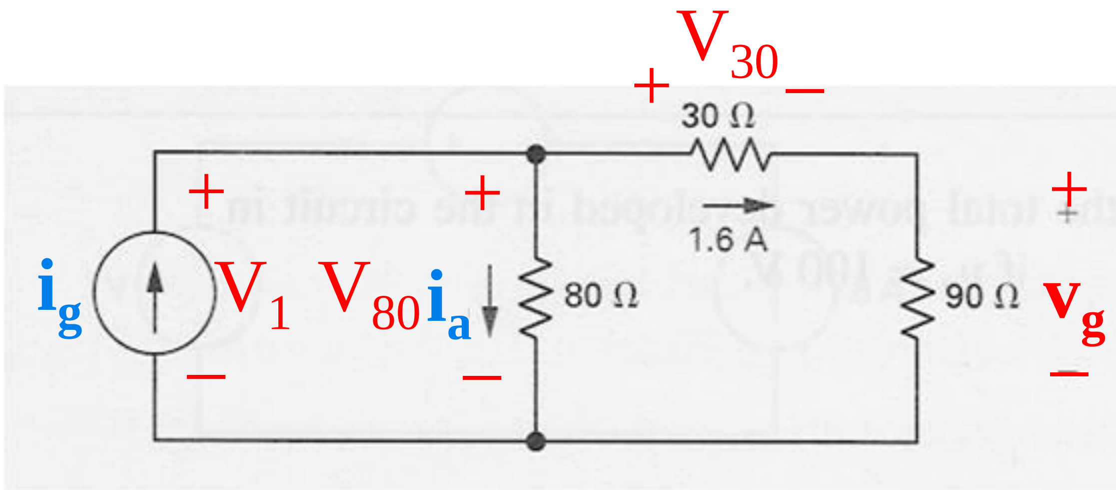
4 "unknowns"

- Now solving, we have:

$5V = V_R + 3V \quad 2V = V_R \qquad I_R = 2V/20\Omega = 0.1$ Amps

Note: We had no node touching 2 ref currents, so no reference current relationships

Bigger example



Branches:

$V_1 = i_a * 80\Omega$

$V_{30} = 1.6A * 30\Omega$

$V_g = 1.6A * 90\ \Omega$

Two nodes which touch two different reference currents:

$i_g = i_a + 1.6$

$i_a + 1.6 = i_g$ [no new currents]

Three loops, but only one needed to touch all voltages:

$V_1 = V_{30} + V_g$

$V_{30} = 48V$

$V_g = 144V$

$V_1 = 192V$

$i_a = 2.4A$

$i_g = 4A$

5 equations

5 unknowns

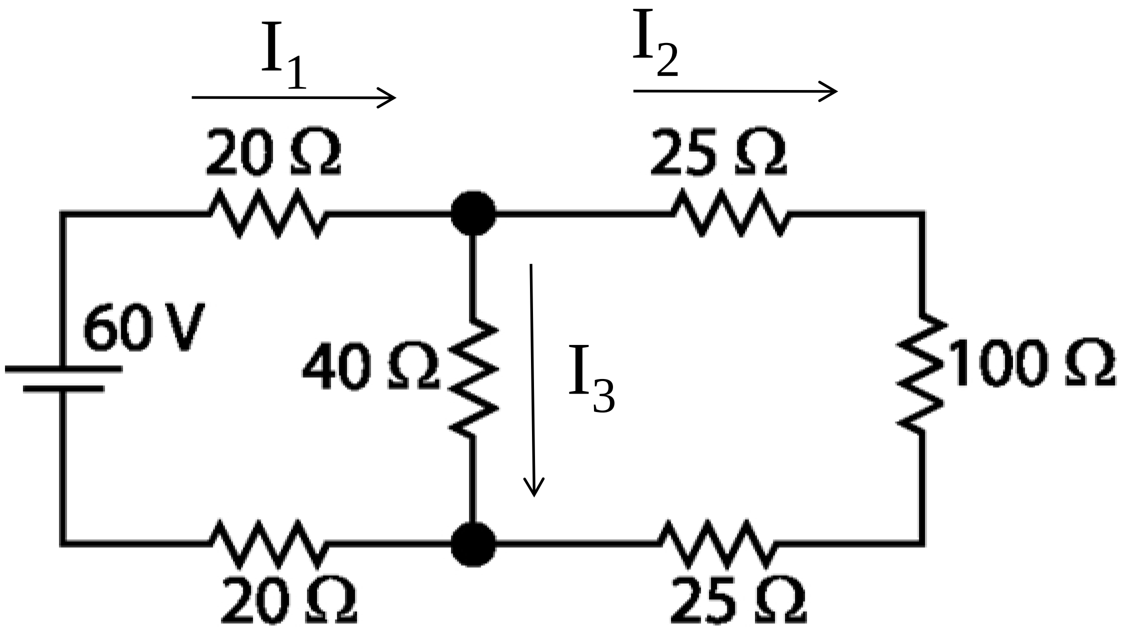
iClicker Proof

- How many KCL and KVL equations will we need to cover every branch voltage and branch current?

2 KVL, 1 KCL

Top node:
$I_1=I_2+I_3$

Bottom node:
$I_3+I_2=I_1$



There are better ways to solve circuits

- The kitchen sink method works, but we can do better
  - Current divider
  - Voltage divider
  - Lumping series and parallel elements together (circuit simplification)
  - Node voltage

Voltage Divider

- Voltage divider
  - Special way to handle N resistors in series
  - Tells you how much voltage each resistor consumes
  - Given a set of N resistors $R_1,\ldots,R_k,\ldots, R_N$ **in series** with total voltage drop **V**, the voltage through R_k is given by

$$V_k = V\frac{R_k}{R_1 + R_2 + \cdots + R_k + \cdots R_n}$$

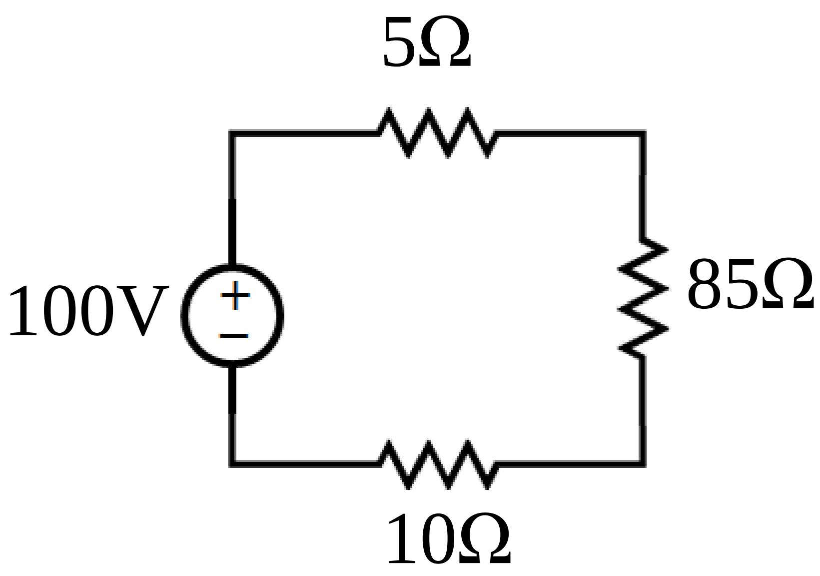
Or more compactly:

$$V_k = \frac{V R_k}{\sum_{i=1}^{N} R_i}$$

Can prove with kitchen sink method (see page 78)

Voltage Divider Example

$$V_k = V\frac{R_k}{R_1 + R_2 + \cdots + R_k + \cdots R_n}$$



$$V_{85} = 100V\frac{85\Omega}{5\Omega + 85\Omega + 10\Omega}$$

$$V_{85} = 100V\frac{85\Omega}{100\Omega}$$

$$V_{85} = 85V$$

And likewise for other resistors

Current Divider

- Current divider
  - Special way to handle N resistors in parallel
  - Tells you how much current each resistor consumes
  - Given a set of N resistors $R_1, \ldots, R_k, \ldots, R_N$ **in parallel** with total current **I** the current through R_k is given by

$$I_k = I \frac{G_k}{G_1 + G_2 + \cdots + G_k + \cdots G_n}$$

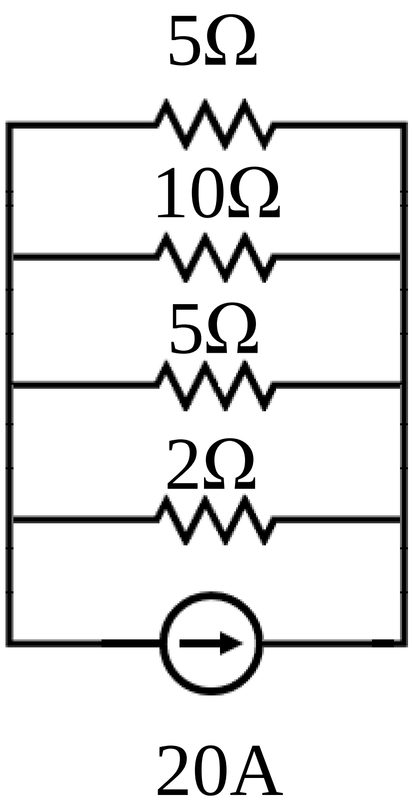
Where: $G_p = \frac{1}{R_p}$

We call G_p the conductance of a resistor, in units of Mhos (℧)

-Sadly, not units of Shidnevacs ()

Can prove with kitchen sink method (see http://www.elsevierdirect.com/companions/9781558607354/casestudies/02~Chapter_2/Example_2_20.pdf)

Current Divider Example



Conductances are:

1/5Ω=0.2℧
1/10Ω=0.1℧
1/5Ω=0.2℧
1/2Ω=0.5℧

Sum of conductances is 1℧ (convenient!)

Current through 5Ω resistor is:

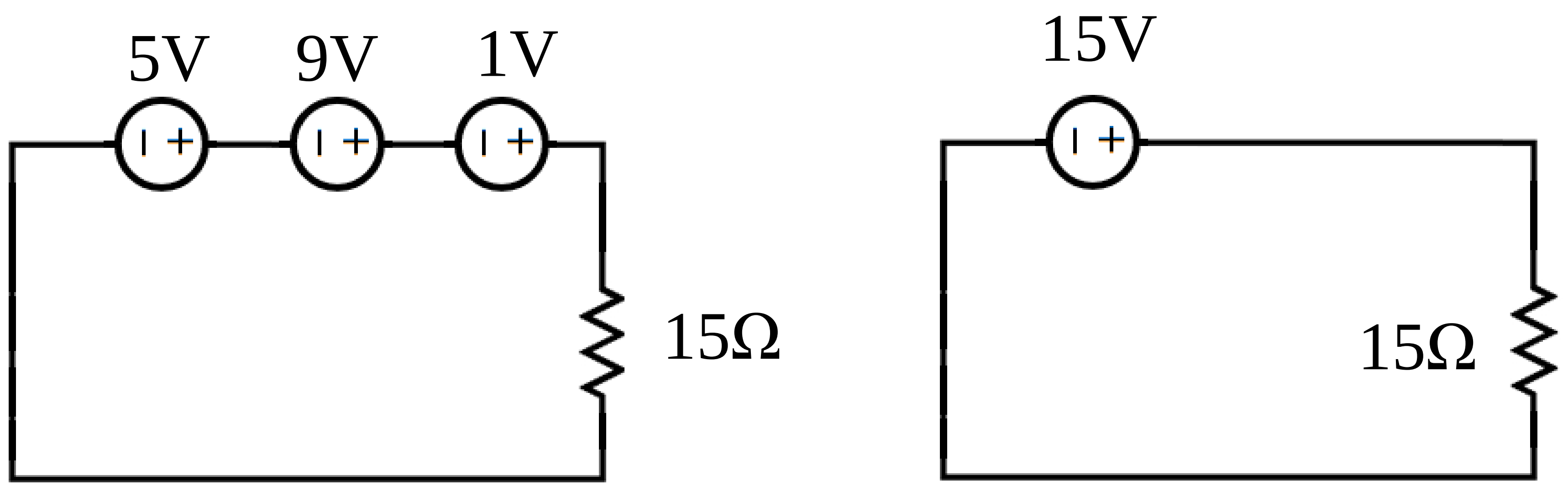
$$I_2 = 20A\frac{0.2}{1} = 4A$$

Circuit Simplification

- Next we'll talk about some tricks for combining multiple circuit elements into a single element
- Many elements in series → One single element
- Many elements in parallel → One single element

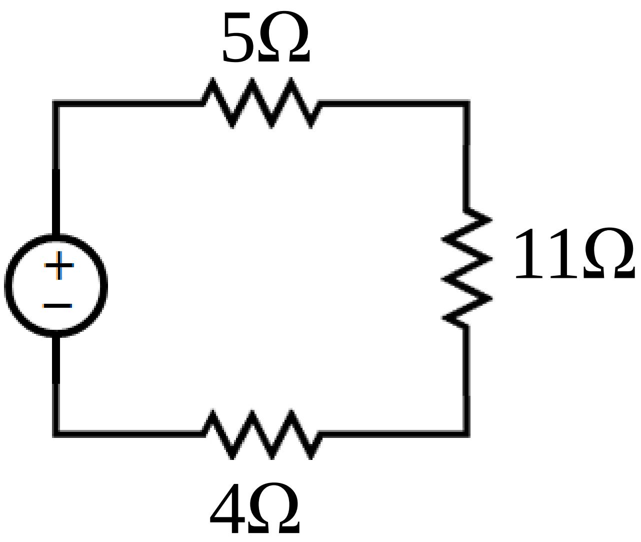
Circuit Simplification Example
Combining Voltage Sources

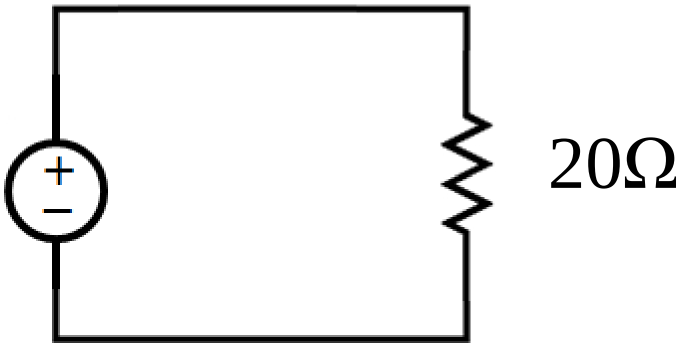
- KVL trivially shows voltage across resistor is 15 V
- Can form equivalent circuit as long as we don't care about individual source behavior
  - For example, if we want power provided by each source, we have to look at the original circuit



Example – Combining Resistances

- Can use kitchen sink method or voltage divider method to show that current provided by the source is equivalent in the two circuits below





Source Combinations

- Voltage sources in series combine additively
- Voltage sources in parallel
  - This is like crossing the streams – "Don't cross the streams"
  - Mathematically nonsensical if the voltage sources are not exactly equal
- Current sources in parallel combine additively
- Current sources in series is bad if not the same current

Resistor Combinations

- Resistors in series combine additively

$$R_{eq} = R_1 + R_2 + \cdots + R_N$$

- Resistors in parallel combine weirdly

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

  - More natural with conductance:

$$G_{eq} = G_1 + G_2 + \cdots + G_n$$

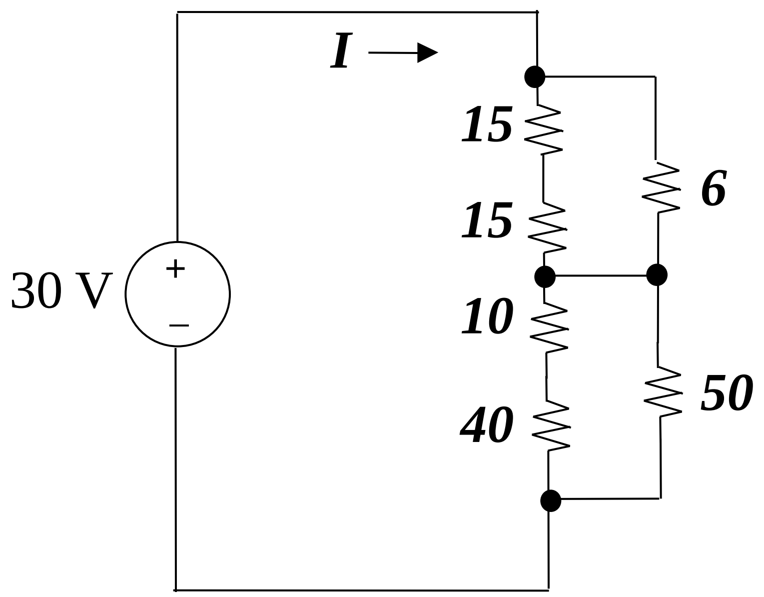
- N resistors in parallel with the same resistance R have equivalent resistance R_{eq}=R/N

Algorithm For Solving By Combining Circuit Elements

- Check circuit diagram
  - If two or more elements of same type in series
    - Combine using series rules
  - If two or more elements of same type in parallel
    - Combine using parallel rules
- If we combined anything, go back to
- If not, then solve using appropriate method (kitchen sink if complicated, divider rule if possible)

Using Equivalent Resistances

Example: Find ***I***



Are there any circuit elements in parallel?

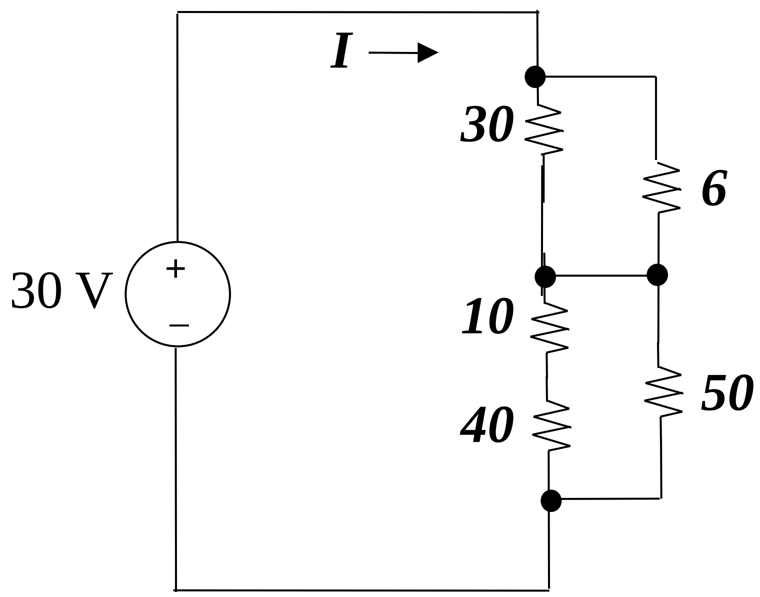
No!

Are there any circuit elements in series?

Yes!

Using Equivalent Resistances

<u>Example</u>: Find ***I***



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Yes!

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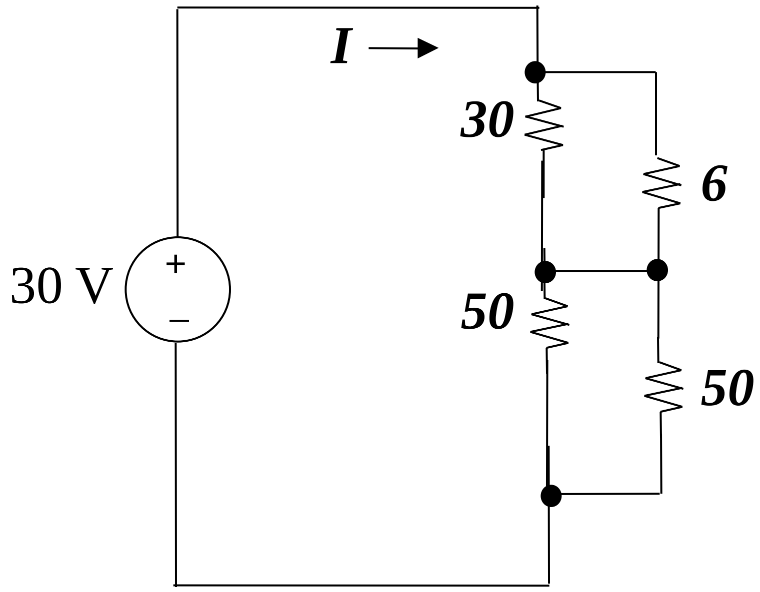
Yes!

Using Equivalent Resistances

Example: Find ***I***



Are there any circuit elements in parallel?

Yes!

Are there any circuit elements in series?

No!

Using Equivalent Resistances

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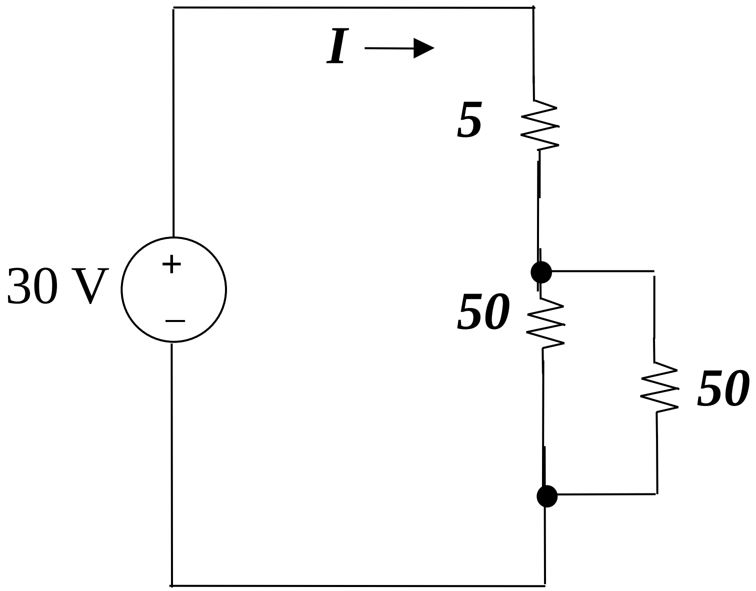

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Using Equivalent Resistances

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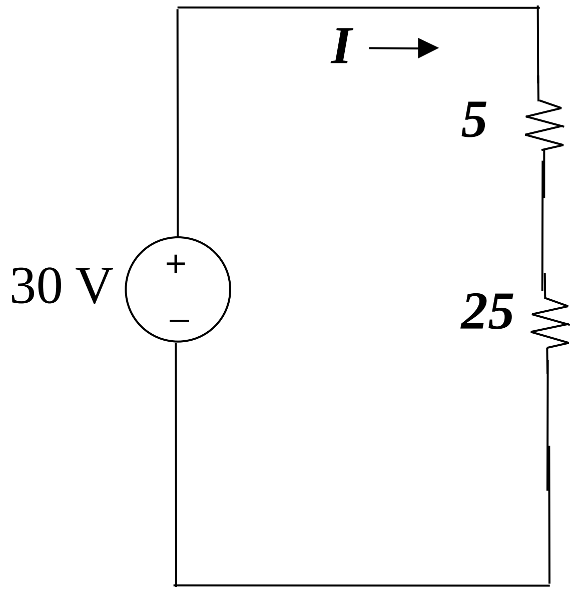

Are there any circuit elements in parallel?

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Are there any circuit elements in series?

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Using Equivalent Resistances

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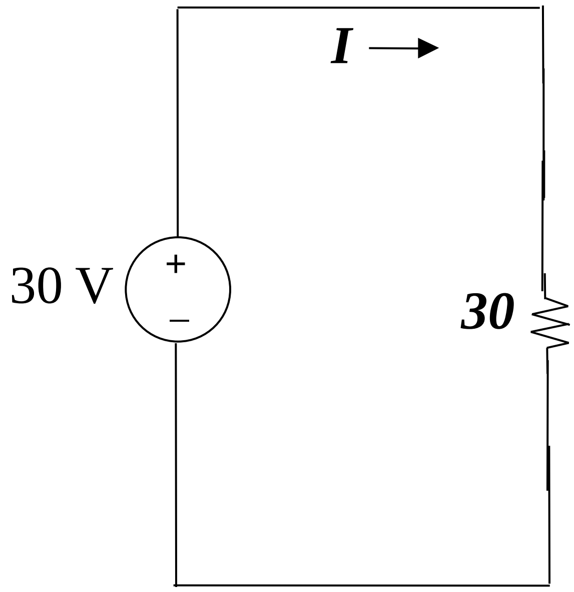


$I=30V/30\Omega=1A$

Are there any circuit elements in parallel?

No!

Are there any circuit elements in series?

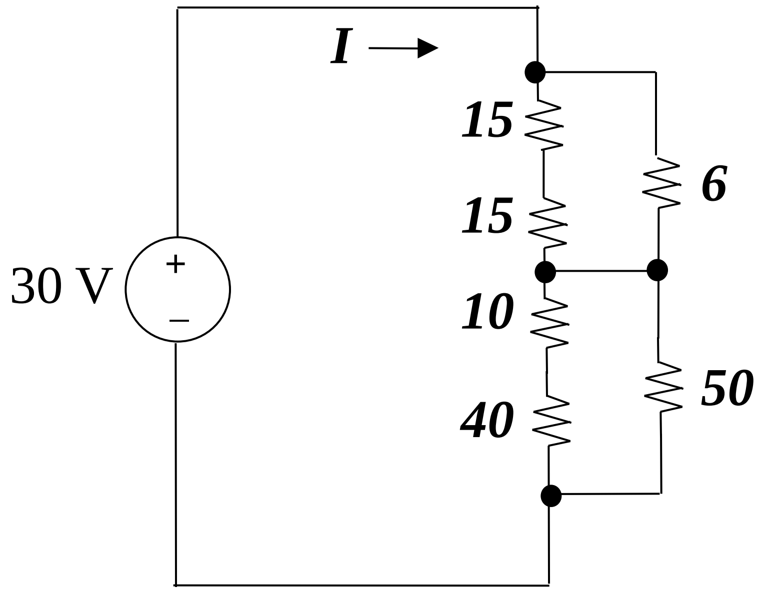
No!

Working Backwards

- Assume we've combined several elements to understand large scale behavior
- Now suppose we want to know something about one of those circuit elements that we've combined
  - For example, current through a resistor that has been combined into equivalent resistance
- We undo our combinations step by step
  - At each step, use voltage and current divider tricks
  - Only undo enough so that we get the data we want

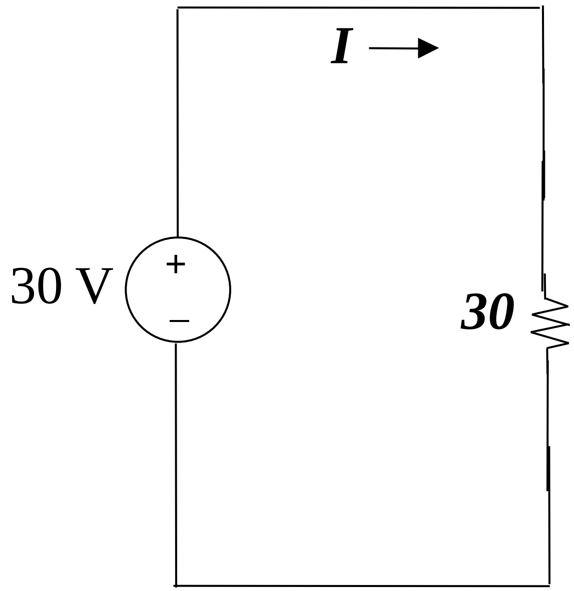
Working Backwards Example

- Suppose we want to know the voltage across the 40Ω Resistor



Using Equivalent Resistances

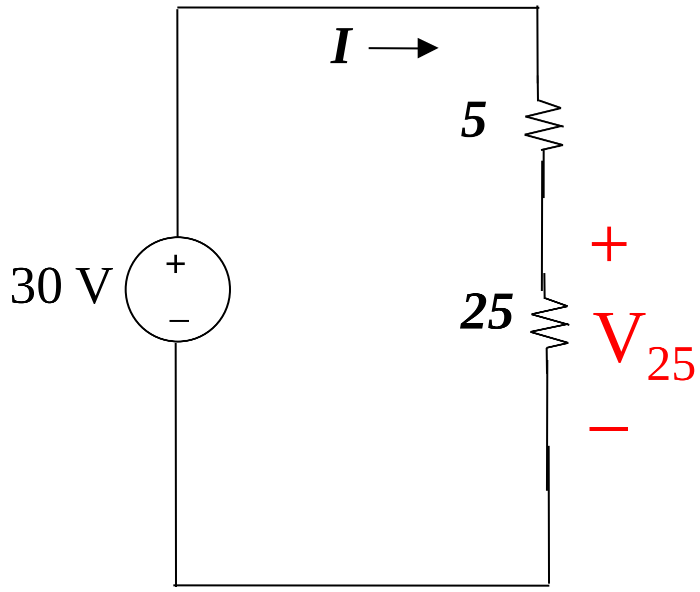
I=1 Amp



Starting from here…

Using Equivalent Resistances

I=1 Amp



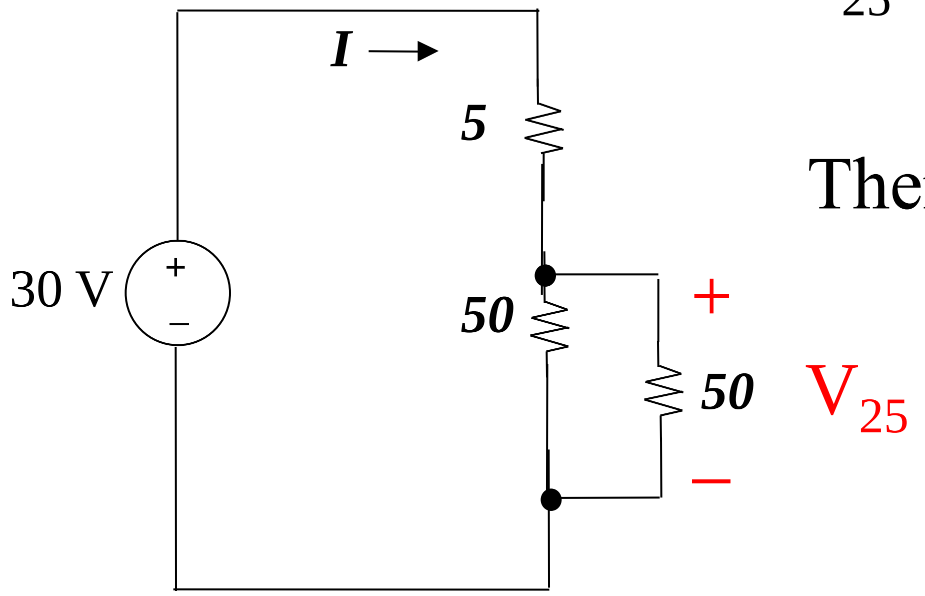
We back up one step…

V_{25}=I*25Ω=25V

Then another…

Using Equivalent Resistances

I=1 Amp



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Using Equivalent Resistances

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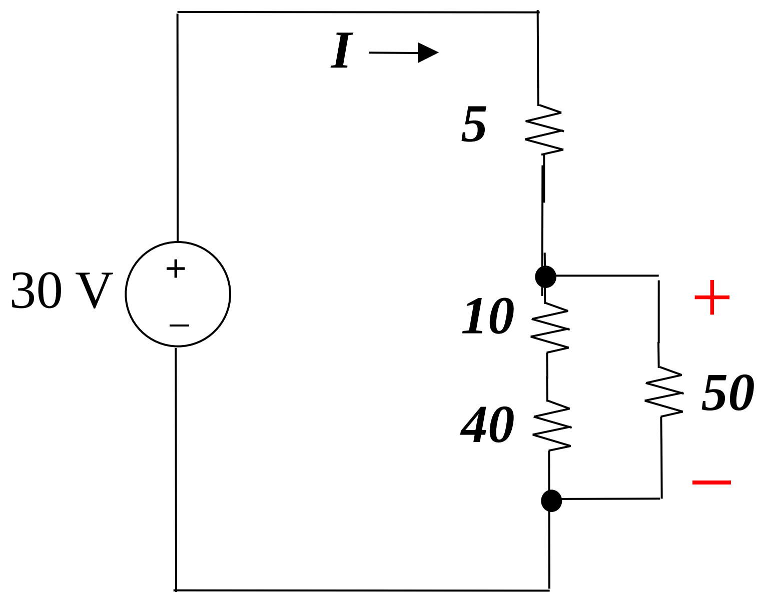


We back up one step…

V_{25}=I*25Ω=25V

Then another…

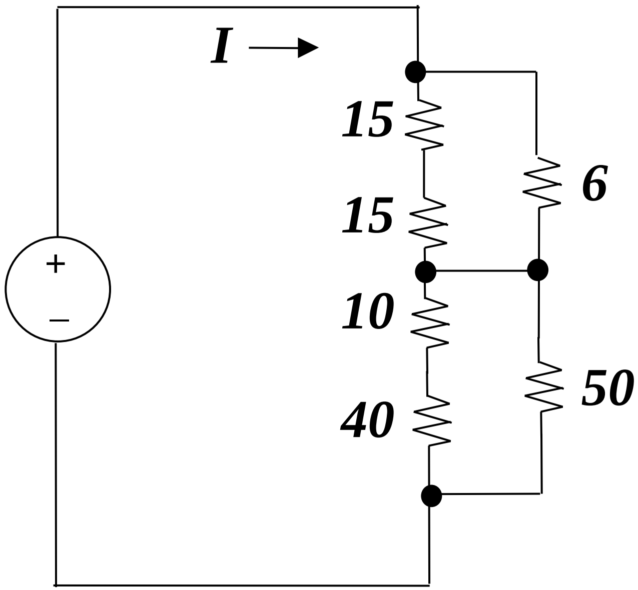
Then one more…

Now we can use the voltage divider rule, and get

$$V_{40} = \frac{40\Omega}{10\Omega + 40\Omega} 25V \qquad V_{40} = 20V$$

Using Equivalent Resistances

I=1 Amp



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Then another…

Then one more…

Now we can use the voltage divider rule, and get

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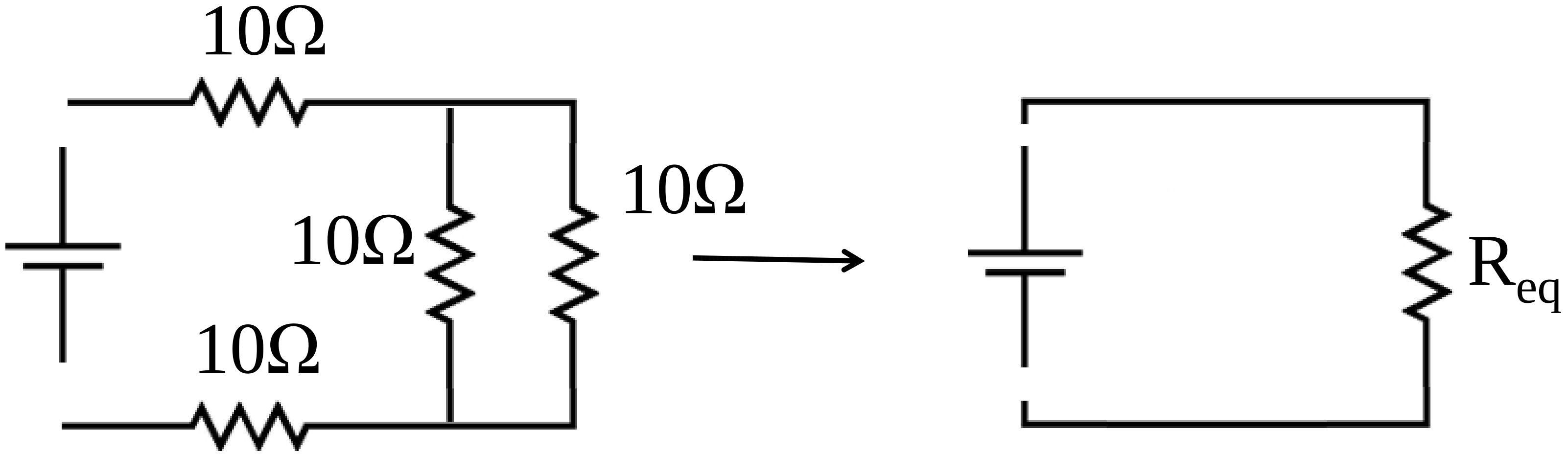
Equivalent Resistance Between Two Terminals

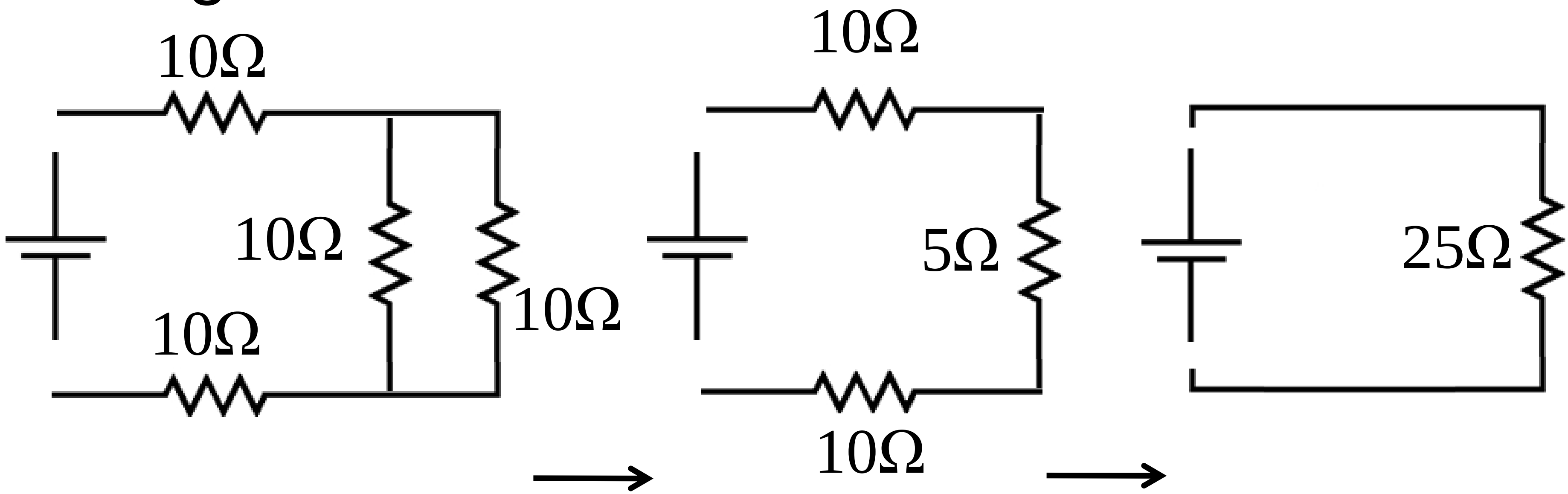
- We often want to find the equivalent resistance of a network of resistors with no source attached



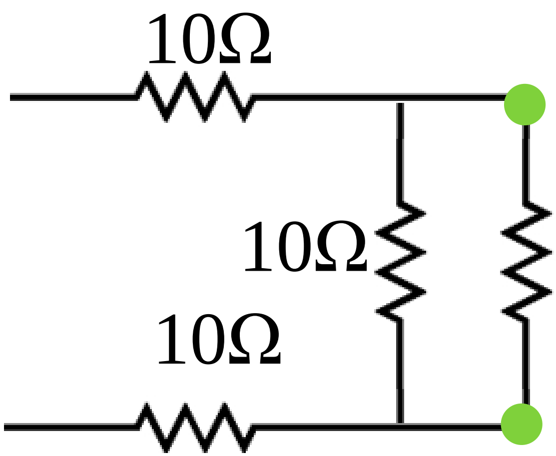
- Tells us the resistance that a hypothetical source would "see" if it were connected
  - e.g. In this example, the resistance that provides the correct source current

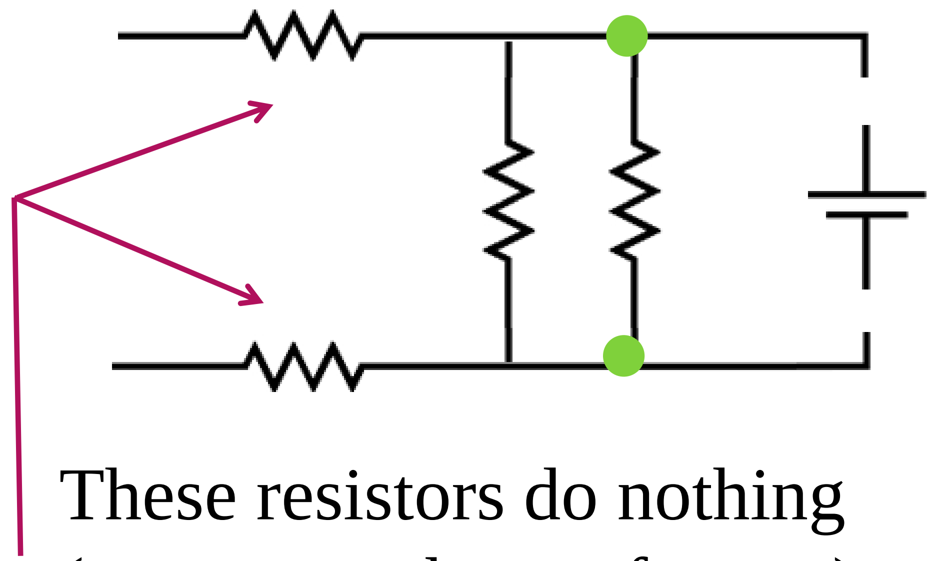
Equivalent Resistance Between Two Terminals

- Pretend there is a source of some kind between the circuits
- Perform the parallel/series combination algorithm as before

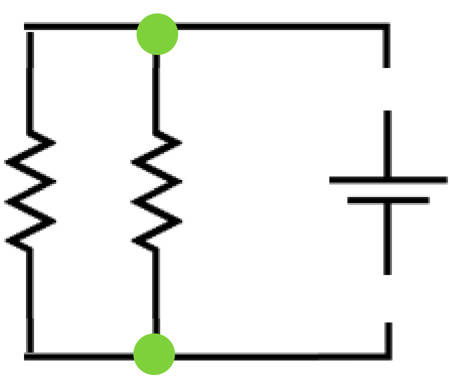


Can Pick Other Pairs of Terminals

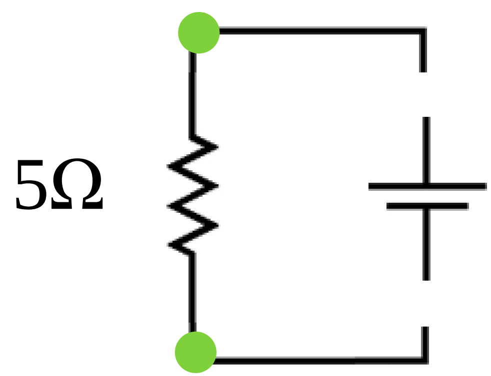




These resistors do nothing
(except maybe confuse us)



Combine these
parallel resistors



There are better ways to solve circuits

- The kitchen sink method works, but we can do better
  - Current divider
  - Voltage divider
  - Lumping series and parallel elements together (circuit simplification)
  - **Node voltage**

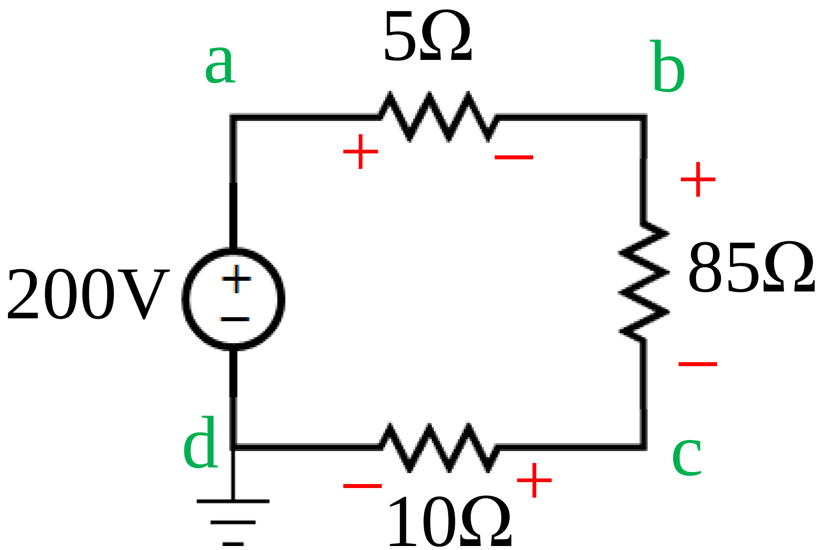
The Node Voltage Technique

- We’ll next talk about a general technique that will let you convert a circuit schematic with N nodes into a set of N-1 equations
- These equations will allow you to solve for every single voltage and current
- Works on any circuit, linear or nonlinear!
- Much more efficient than the kitchen sink

Definition: Node Voltage and Ground Node

- Remember that voltages are always defined in terms of TWO points in a circuit
- It is convenient to label one node in our circuit the "Ground Node"
  - Any node can be "ground", it doesn't matter which one you pick
- Once we have chosen a ground node, we say that each node has a "node voltage", which is the voltage between that node and the arbitrary ground node
- Gives each node a universal single valued voltage level

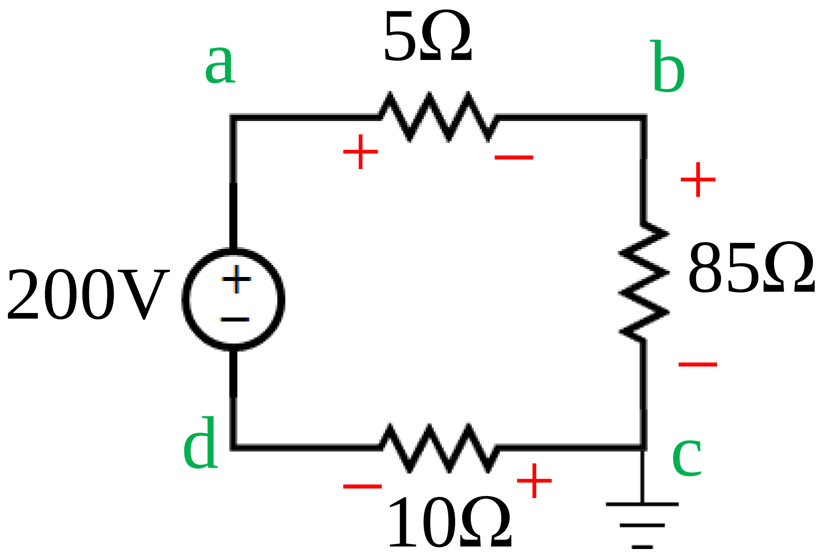
Node Voltage Example



V_5=10V
V_{85}=170V
V_{10}=20V

- Pick a ground, say the bottom left node.
- Label nodes a, b, c, d. Node voltages are:
  - V_d=voltage between node d and d=0V
  - V_c=voltage between node c and d=V_{10}=20V
  - V_b=voltage between node b and d=V_{85}+V_{10}=190V
  - V_a=voltage between node a and d=200V

iClicker #4: Node Voltages



V_5=10V
V_{85}=170V
V_{10}=20V

What is V_a?

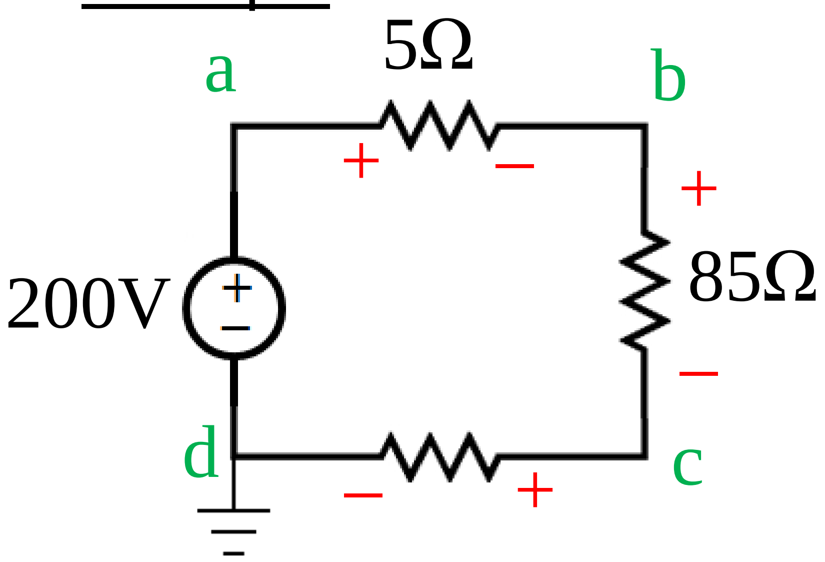
A. 200V
B. 20V
C. 160V
D. 180V

$V_a = V_5 + V_{85} = 180V$

Relationship: Node and Branch Voltages

- Node voltages are useful because:
  - The branch voltage across a circuit element is simply the difference between the node voltages at its terminals
  - It is easier to find node voltages than branch voltages

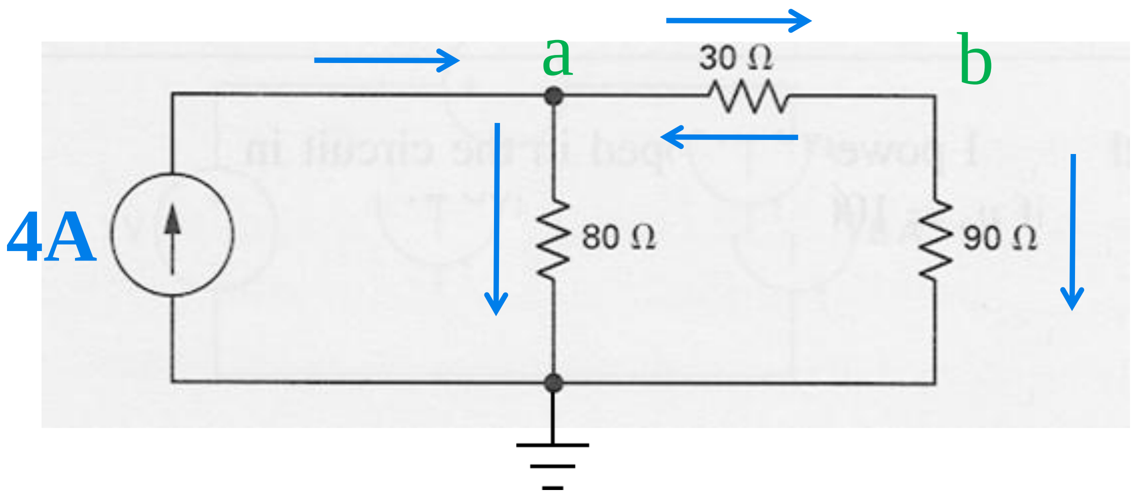
Example:



V_d=V
V_c=20V
V_b=190V
V_a=200V

$V_{85}=V_b-V_c$=190V-20V=170V

Why are Node Voltages Easier to Find?



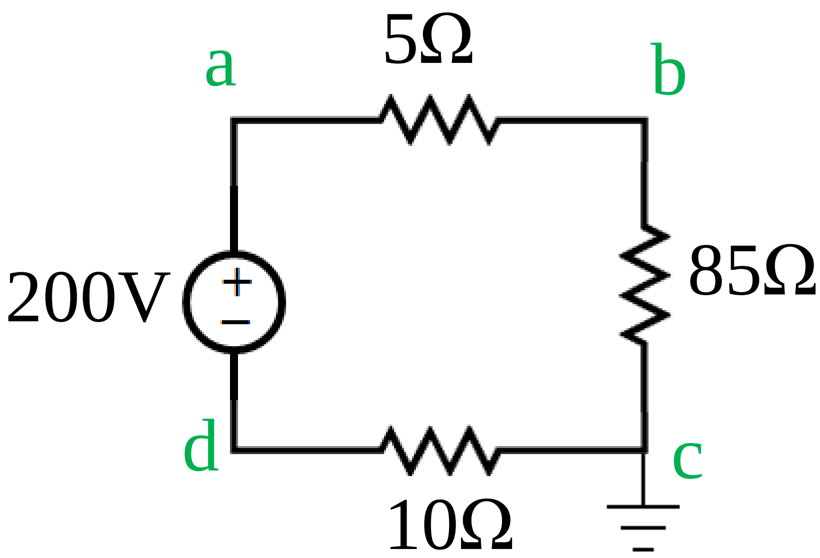
- KCL is easy to write in terms of node voltages
- For example, at node a:
  - $4A = V_a/80\Omega + (V_a - V_b)/30\Omega$
- And at node b:
  - $(V_b - V_a)/30\Omega = V_b/90\Omega$
- Well look, two equations, two unknowns. We're done.
- Better than 5 equations, 5 unknowns with kitchen sink method

(Almost) The Node Voltage Method

- Assign a ground node
- For every node except the ground node, write the equation given by KCL in terms of the node voltages
  - Be very careful about reference directions
- This gives you a set of N-1 linearly independent algebraic equations in N-1 unknowns
  - Solvable using whatever technique you choose

What about Voltage Sources?

- Suppose we have the circuit below



- When we try to write KCL at node a, what happens?
- How do we get around this?
  - Write fixed node voltage relationship:

$$V_a = V_d + 200$$

Full Node Voltage Method

- Assign a ground node
- For every node (except the ground node):
  - If there is no voltage source connected to that node, then write the equation given by KCL in terms of the node voltages
  - **If there is a voltage source connecting two nodes, write down the simple equation giving the difference between the node voltages**
  - Be very careful about reference directions (comes with practice)
- This gives you a set of N-1 linearly independent algebraic equations in N-1 unknowns
- Solvable using whatever technique you choose

More Examples Next Time!

Next Class

- Node voltage practice and examples
- Why we are bothering to understand so deeply the intricacies of purely resistive networks
  - Things we can build other than the most complicated possible toaster
- How we actually go about measuring voltages and currents
- More circuit tricks
  - Superposition
  - Source transformations

Quick iClicker Question

- How was my pacing today?
    A. Way too slow
    B. A little too slow
    C. Pretty good
    D. Too fast
    E. Way too fast

Extra Slides

Summary (part one)

- There are five basic circuit elements
  - Voltage Sources
  - Current Sources
  - Resistors
  - Capacitors
  - Inductors
- Circuit schematics are a set of interconnect ideal basic circuit elements
- A connection point between elements is a node, and a path that connects two nodes is a branch
- A loop is a path around a circuit which starts and ends at the same node without going through any circuit element twice

Summary (part two)

- Kirchoff’s current law states that the sum of the currents entering a node is zero
- Kirchoff’s voltage law states that the sum of the voltages around a loop is zero
- From these laws, we can derive rules for combining multiple sources or resistors into a single equivalent source or resistor
- The current and voltage divider rules are simple tricks to solve simple circuits
- The node voltage technique provides a general framework for solving any circuit using the elements we’ve used so far

Short Circuit and Open Circuit

Wire ("short circuit"):

- $R = 0$ → **no voltage difference exists**
  (all points on the wire are at the same potential)
- Current can flow, as determined by the circuit

Air ("open circuit"):

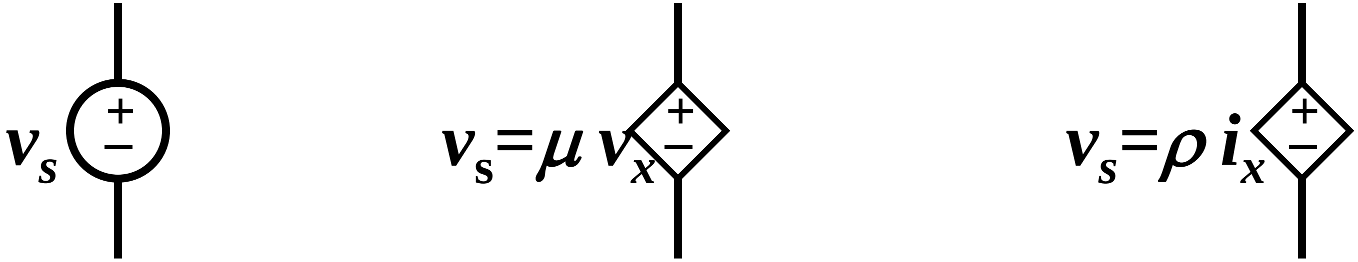
- $R = \infty$ → **no current flows**
- Voltage difference can exist, as determined by the circuit

Figure 2.10 Circuit symbols. (a) Short circuit. (b) Open circuit. (c) Switch.

Ideal Voltage Source

- Circuit element that maintains a prescribed voltage across its terminals, **regardless of the current flowing in those terminals**.
  - Voltage is known, but current is determined by the circuit to which the source is connected.
- The voltage can be either **independent or dependent** on a voltage or current elsewhere in the circuit, and can be constant or time-varying.

Circuit symbols:



independent **voltage-controlled** **current-controlled**

Ideal Current Source

- Circuit element that maintains a prescribed current through its terminals, **regardless of the voltage across those terminals**.
  - Current is known, but voltage is determined by the circuit to which the source is connected.
- The current can be either **independent or dependent** on a voltage or current elsewhere in the circuit, and can be constant or time-varying.

Circuit symbols: