# EE40 Lecture 2 Josh Hug

### 6/23/2010

# **Logistical Changes and Notes**

- Friday Lunch is now Monday lunch (starting next Monday)
  - Email me by Saturday evening if you'd like to come: JHUG aat eecs.berkeley.edu
- My office hours will be Wednesday and Friday, 11:00-12:00, room TBA
- Google calendar with important dates now online
- Did anybody not get my email sent out Monday (that said no discussion yesterday)?
- Will curate the reading a little more carefully next time

## Lab/HW Deadlines and Dates

- Discussions start Friday
- Labs start next Tuesday
- HW0 Due Today
- Homework 1 will be posted by 3PM, due Friday at 5 PM
- Tuesday homeworks now due at 2PM, not 5PM in Cory 240 HW box

## **Summary From Last Time**

- *Current* = rate of charge flow
- **Voltage** = energy per unit charge created by charge separation
- *Power* = energy per unit time
- Ideal Basic Circuit Element
  - 2-terminal component that cannot be sub-divided
  - Described mathematically in terms of its terminal voltage and current

### Circuit Schematics

- Networks of ideal basic circuit elements
- Equivalent to a set of algebraic equations
- Solution provides voltage and current through all
   EE40 Summer 2010

# **Heating Elements**

- Last time we posed a question:
  - Given a fixed voltage, should we pick a thick or thin wire to maximize heat output
  - Note that resistance decreases with wire radius
- Most of you said that we'd want a thin wire to maximize heat output, why is that?
  - Believed that low resistance wire would give the most heat?
  - Didn't believe me that thick wire has low resistance?
  - General intuition?

# **Intuitive Answer**

- I blasted through some equations and said "thicker is better, Q.E.D.", but I'm not sure you guys were convinced, so here's another view
- You can think of a big thick wire as a bunch of small wires connected to a source
  - The thicker the wire, the more little wires
  - Since they are all connected directly to the source, they all have same voltage and current and hence power
- Adding more wires gives us more total current flow (same voltage), and hence more power

#### Then Why Don't Toasters and Ovens Have Thicker Elements?

- Thicker elements mean hotter elements
  - Will ultimately reach higher max temperature
  - Will get to maximum faster [see message board after 6 or 7 PM tonight for why]
- Last time, you guys asked "Well if thickness gives you more heat, why aren't toaster elements thicker?"
- The answer is most likely:
  - More burned toast. Nobody likes burned toast.

## **Toaster Element Design Goals**

- Make heating element that can:
  - Can reach a high temperature, but not too high
  - Can reach that temperature quickly
  - Isn't quickly oxidized into oblivion by high temperature
  - Doesn't cost very much money
  - Will not melt at desired temperature
- Nichrome is a typical metal alloy in elements:
  - Low oxidation
  - High resistance (so normal gauge wire will not draw too much power and get too hot)
- Size was tweaked to attain desired temperature

## **Continue the Discussion on BSpace**

• Let's get working on some more complicated circuits than this:



## **Topic 2**

# Setting Up and Solving Resistive Circuit Models

# **Circuit Schematics**

- Many circuit elements can be approximated as simple ideal two terminal devices or ideal basic circuit elements
- These elements can be combined into circuit schematics
- Circuit schematics can be converted into algebraic equations
- These algebraic equations can be solved, giving voltage and current through any element of the circuit

# Today

- We'll enumerate the types of ideal basic circuit elements
- We'll more carefully define a circuit schematic
- We'll discuss some basic techniques for analyzing circuit schematics
  - Kirchoff's voltage and current laws
  - Current and voltage divider
  - Node voltage method

# **Circuit Elements**

- There are 5 ideal basic circuit elements (in our course):
  - voltage source
  - current source
  - resistor
  - inductor
  - capacitor

*active elements*, capable of generating electric energy

*passive elements*, incapable of generating electric energy

- Many practical systems can be modeled with just sources and resistors
- The basic analytical techniques for solving circuits with inductors and capacitors are the same as those for resistive circuits

## **Electrical Sources**

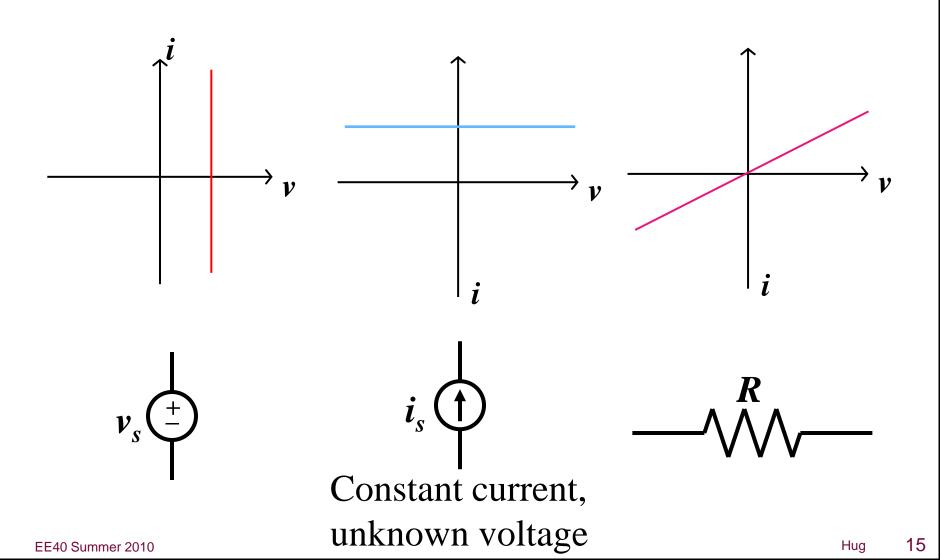
 An *electrical source* is a device that is capable of converting non-electric energy to electric energy and *vice versa*.

Examples:

- battery: chemical + electric
- dynamo (generator/motor): mechanical + electric

 $\rightarrow$  Electrical sources can either deliver or absorb power

# **The Big Three**

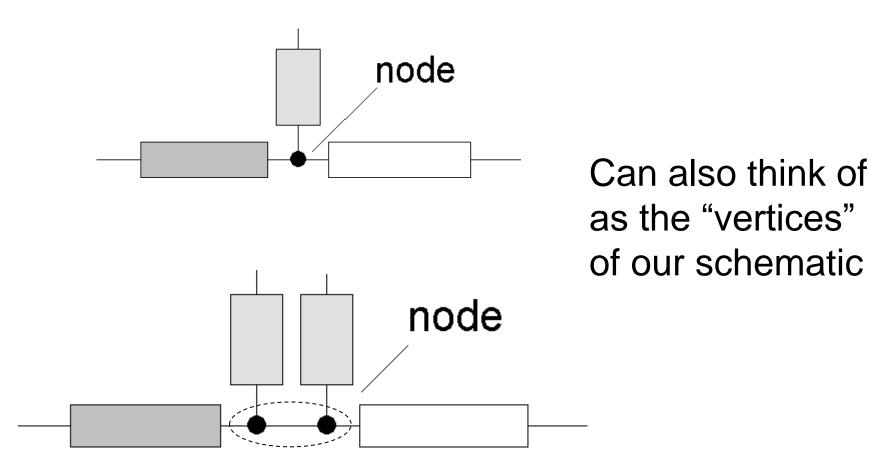


# **Circuit Schematics**

- A circuit schematic is a diagram showing a set of interconnected circuit elements, e.g.
  - Voltage sources
  - Current sources
  - Resistors
- Each element in the circuit being modeled is represented by a symbol
- Lines connect the symbols, which you can think of as representing zero resistance wires

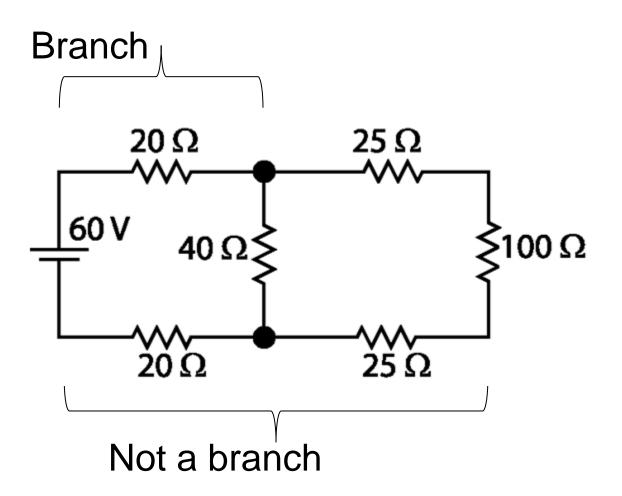
## **Terminology: Nodes and Branches**

**Node:** A point where two or more circuit elements are connected – entire wire



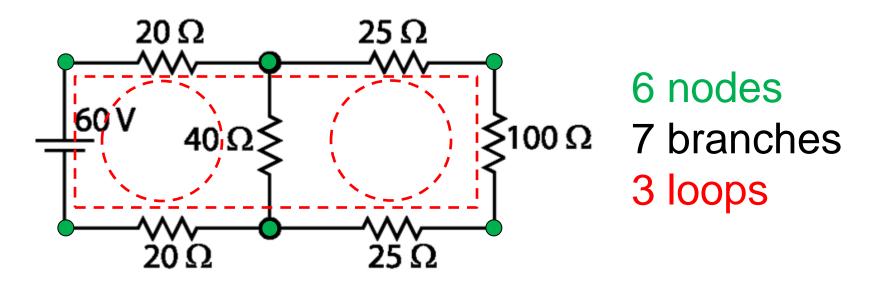
## **Terminology: Nodes and Branches**

**Branch:** A path that connects exactly two nodes



# **Terminology: Loops**

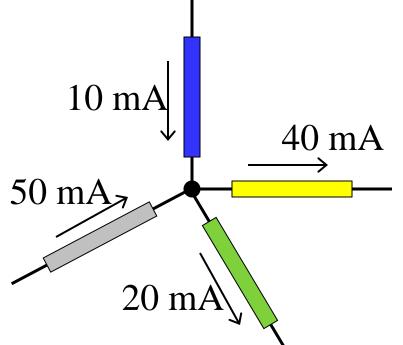
- A *loop* is formed by tracing a closed path in a circuit through selected basic circuit elements without passing through any intermediate node more than once
- Example: (# nodes, # branches, # loops)



## **Kirchhoff's Laws**

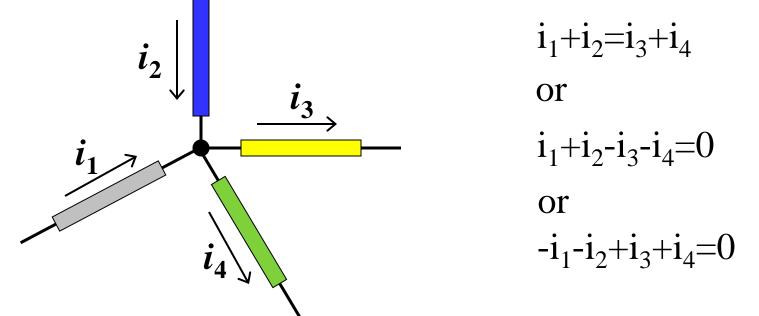
### Kirchhoff's Current Law (KCL):

- The algebraic sum of all the currents at any node in a circuit equals zero.
- "What goes in, must come out"
- Basically, law of charge conservation



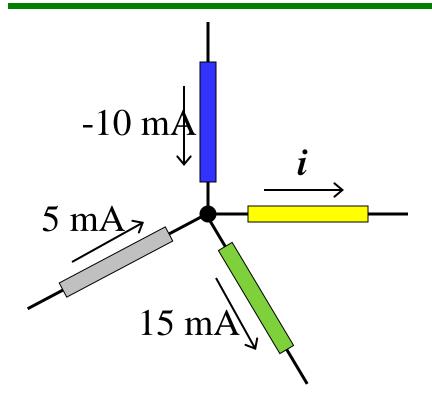
### Using Kirchhoff's Current Law (KCL)

Often we're considering unknown currents and only have reference directions:



 Use reference directions to determine whether reference currents are said to be "entering" or "leaving" the node – with no concern about actual current directions

## **KCL Example**

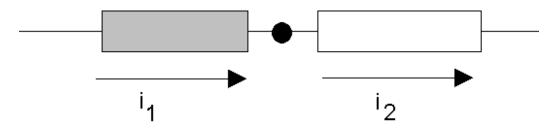


#### 5+(-10)=15+i

#### i=10mA

# **A Major Implication of KCL**

- KCL tells us that all of the elements along a single uninterrupted<sup>\*</sup> path carry the same current
- We say these elements are connected *in series*.



Current entering node = Current leaving node

$$i_1 = i_2$$

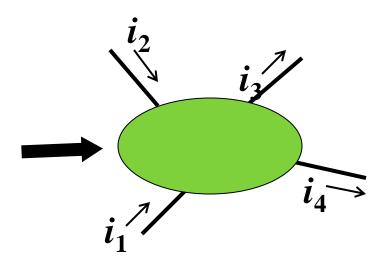
\*: To be precise, by uninterrupted path I mean all branches along the path connected EXACTLY two nodes

EE40 Summer 2010

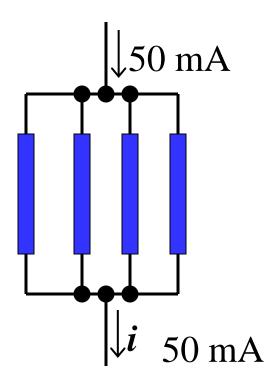
## **Generalization of KCL**

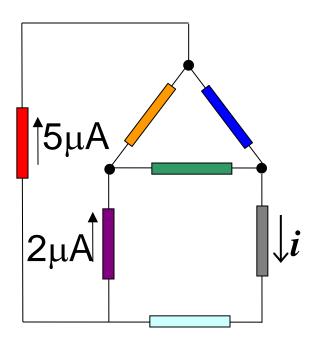
 The sum of currents entering/leaving a closed surface is zero. Circuit branches can be inside this surface, *i.e.* the surface can enclose more than one node!

This could be a big chunk of a circuit, *e.g.* a "black box"



### **Generalized KCL Examples**

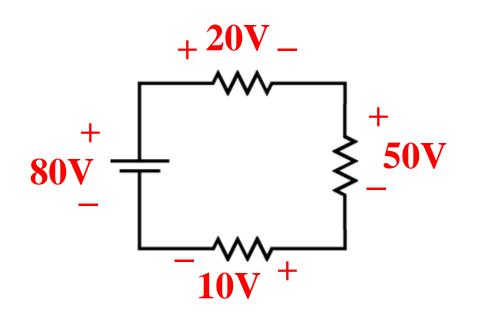




7μΑ

## **Kirchhoff's Laws**

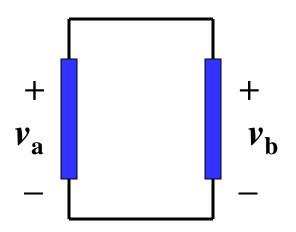
- Kirchhoff's Voltage Law (KVL):
  - The algebraic sum of all the voltages around any loop in a circuit equals zero.
  - "What goes up, must come down"



80=20+50+10

# **A Major Implication of KVL**

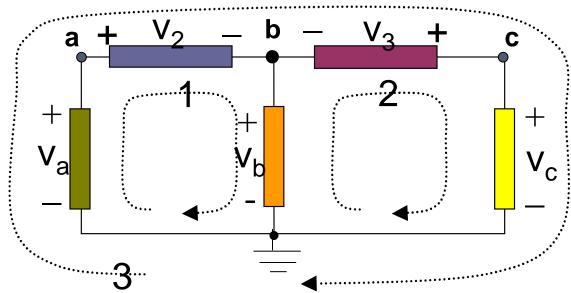
- KVL tells us that any set of elements which are connected at both ends carry the same voltage.
- We say these elements are connected in parallel.



Applying KVL, we have that:  $v_b - v_a = 0 \Rightarrow v_b = v_a$ 

### **KVL Example**

#### Three closed paths:



Path 1: $V_a = V_2 + V_b$ If you want a mechanical rule:Path 2: $V_b + V_3 = V_c$ If you hit a - first, LHSPath 3: $V_a + V_3 = V_2 + V_c$ 

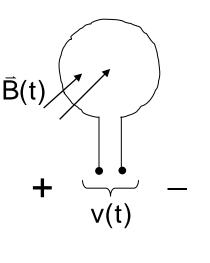
LHS is left hand side

## An Underlying Assumption of KVL

- No time-varying magnetic flux through the loop Otherwise, there would be an induced voltage (Faraday's Law) Voltage around a loop would sum to a nonzero value
- <u>Note</u>: Antennas are designed to "pick up" electromagnetic waves; "regular circuits" often do so undesirably.

How do we deal with antennas (EECS 117A)? Include a voltage source as the circuit representation of the induced voltage or "noise".

(Use a lumped model rather than a distributed (wave) model.)



- KCL tells us that all elements on an uninterrupted path have the same current.
  - We say they are "in series"
- KVL tells us that a set of elements whose terminals are connected at the same two nodes have the same voltage

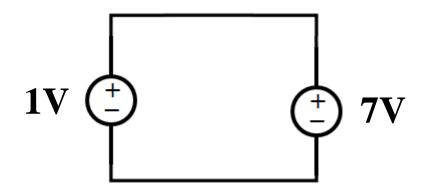
- We say they are "in parallel"

### **Nonsense Schematics**

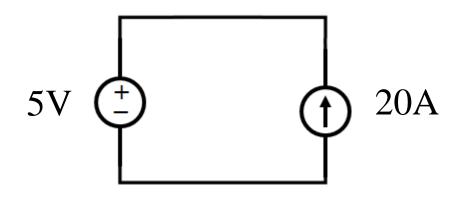
• Just like equations, it is possible to write nonsense schematics:

- 1=7

 A schematic is nonsense if it violates KVL or KCL



# Verifying KCL and KVL



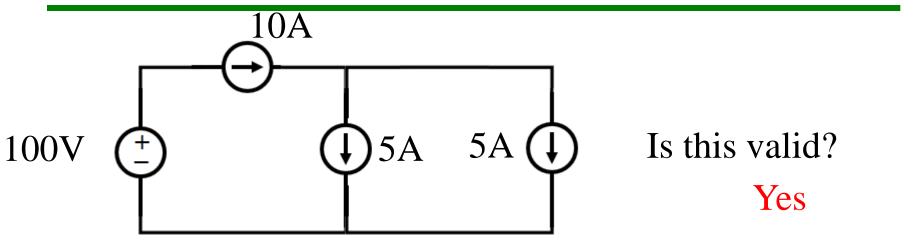
Is this schematic valid? Yes

How much power is consumed/provided by each source?

**Voltage source**:  $P_V = 5V * 20A = 100W$  (consumed)

**Current source**: P<sub>I</sub>=-20A\*5V=100W (provided)

# Verifying KCL and KVL



#### KCL:

No contradiction

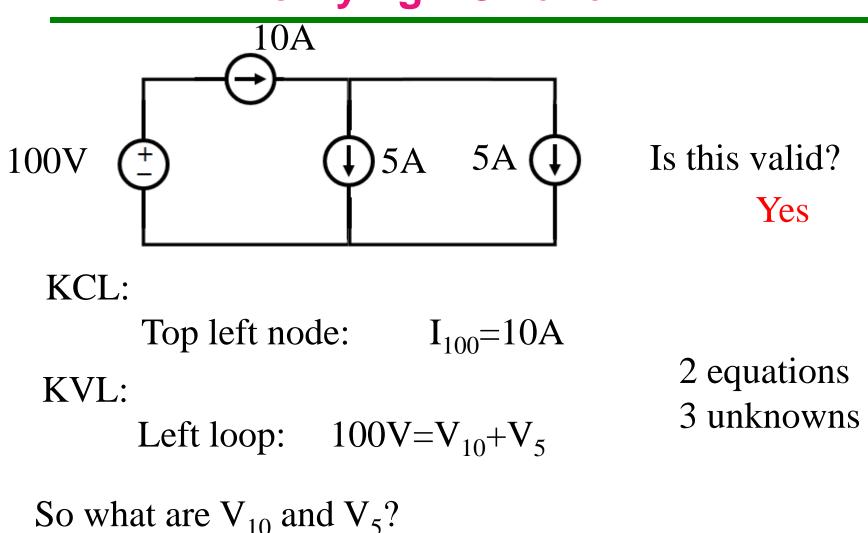
Top left node:  $I_{100}=10A$ Top right node: 10A=5A+5ABottom node:  $5A+5A=I_{100}$ KVL: Left loop:  $100V=V_{10}+V_5$ 

Big loop:  $100V = V_{10} + V_5$ 

Right loop:  $V_5 = V_5$ 

#### No contradiction

# Verifying KCL and KVL

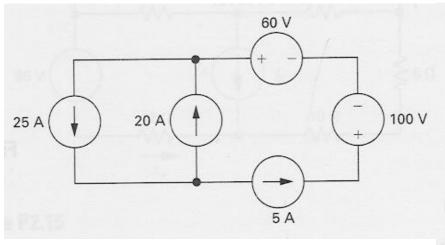


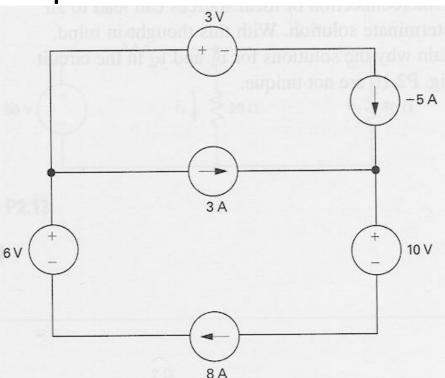
Whatever we want that sums to 100V

Multiple circuit solutions

## iClicker #1

• Are these interconnections permissible?





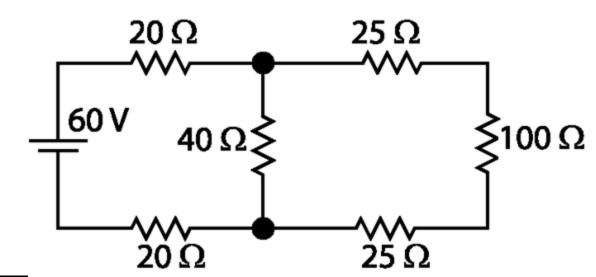
A. Both are badB. Left is ok, right is badC. Left is bad, right is okD. Both are ok

# **On to Solving Circuits**

- Next we'll talk about a general method for solving circuits
  - The book calls this the "basic method"
  - It's a naïve way of solving circuits, and is way more work than you need
    - Basic idea is to write every equation you can think of to write, then solve
  - However, it will build up our intuition for solving circuits, so let's start here

# Solving Circuits (naïve way)

- Label every branch with a reference voltage and current
  - If two branches are in parallel, share voltage label
  - If in series, share same current label
- For each branch:
  - Write Ohm's law if resistor
  - Get branch voltage "for free" if known voltage source
  - Get branch current "for free" if known current source



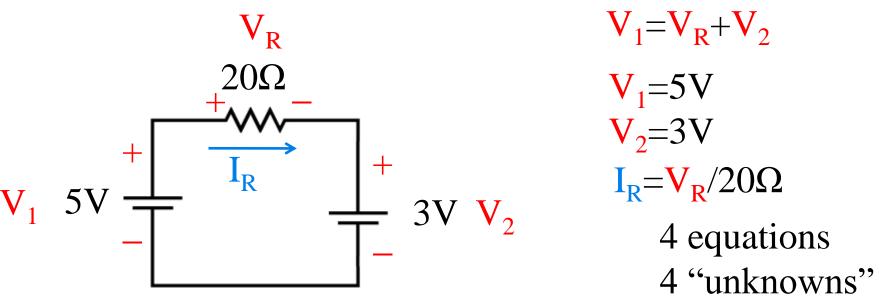
# Solving Circuits (naïve way)

- Label every branch with a reference voltage and current
  - If two branches are in parallel, share voltage label
  - If in series, share same current label
- For each branch:
  - Write Ohm's law if resistor
  - Get branch voltage "for free" if known voltage source
  - Get branch current "for free" if known current source
- For each node touching at least 2 reference currents:
  - Write KCL gives reference current relationships
  - Can omit nodes which contain no new currents
- For each loop:
  - Write KVL gives reference voltage relationships
  - Can omit loops which contain no new voltages

Could also call this the "kitchen sink" approach

### **Example: KCL and KVL applied to circuits**

- Find the current through the resistor
- Use KVL, we see we can write:

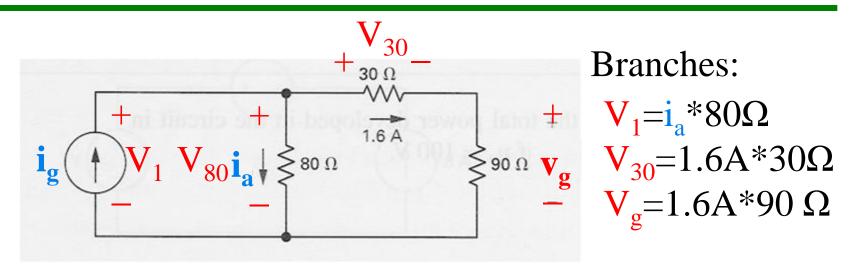


• Now solving, we have:

 $5V = V_R + 3V \quad 2V = V_R \quad I_R = 2V/20\Omega = 0.1 \text{ Amps}$ 

Note: We had no node touching 2 ref currents, so no reference current relationships

# **Bigger example**



Two nodes which touch two different reference currents:

$$i_g = i_a + 1.6$$
  
 $i_a + 1.6 = i_g$  [no new currents]

Three loops, but only one needed to touch all voltages:

$$V_{1} = V_{30} + V_{g}$$

$$V_{30} = 48V$$

$$i_{a} = 2.4A$$

$$V_{g} = 144V$$

$$i_{g} = 4A$$

5 equations5 unknowns

## iClicker #2

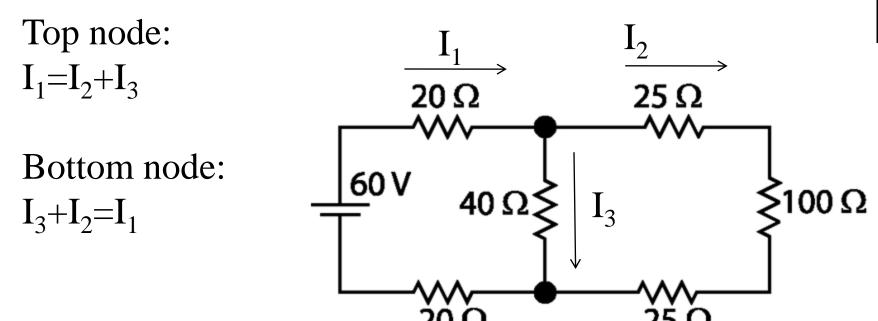
 How many KCL and KVL equations will we need to cover every branch voltage and branch current?

A. 2 KVL loops, 2 KCL nodes B. 3 KVL loops, 2 KCL nodes C. 2 KVL loops, 4 KCL nodes D. 3 KVL loops, 4 KCL nodes **E.** None of these 20 Ω 25 Ω I am the worst 60 V 40 Ω 2 KVL, 1 KCL

100 Ω

# **iClicker Proof**

- How many KCL and KVL equations will we need to cover every branch voltage and branch current?
  - 2 KVL, 1 KCL



EE40 Summer 2010

### There are better ways to solve circuits

- The kitchen sink method works, but we can do better
  - Current divider
  - Voltage divider
  - Lumping series and parallel elements together (circuit simplification)
  - Node voltage

# **Voltage Divider**

- Voltage divider
  - Special way to handle N resistors in series
  - Tells you how much voltage each resistor consumes
  - Given a set of N resistors R<sub>1</sub>,...,R<sub>k</sub>,..., R<sub>N</sub> in series with total voltage drop V, the voltage through R<sub>k</sub> is given by

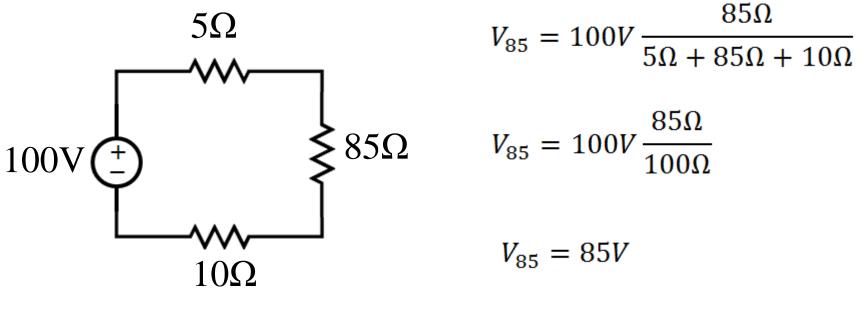
$$V_k = V \frac{R_k}{R_1 + R_2 + \dots + R_k + \dots + R_n}$$

Or more compactly:  $V_k = \frac{VR_k}{\sum_{i=1}^N R_i}$ 

Can prove with kitchen sink method (see page 78)

#### **Voltage Divider Example**

$$V_k = V \frac{R_k}{R_1 + R_2 + \dots + R_k + \dots + R_n}$$



And likewise for other resistors

# **Current Divider**

- Current divider
  - Special way to handle N resistors in parallel
  - Tells you how much current each resistor consumes
  - Given a set of N resistors R<sub>1</sub>,...,R<sub>k</sub>,..., R<sub>N</sub> in parallel with total current I the current through R<sub>k</sub> is given by

$$I_k = I \frac{G_k}{G_1 + G_2 + \dots + G_k + \dots + G_n} \qquad \text{Where:} \quad G_p = \frac{1}{R_p}$$

We call G<sub>p</sub> the conductance of a resistor, in units of Mhos (℧)
 Sadly, not units of Shidnevacs (🏹)

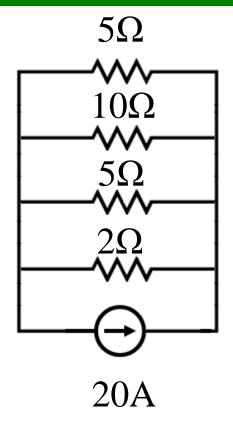
Can prove with kitchen sink method (see

http://www.elsevierdirect.com/companions/9781558607354/casestudies/02~Chapter\_2/Example\_2\_20.pdf)

EE40 Summer 2010

Hug 46

## **Current Divider Example**



Conductances are:

1/5Ω=0.2℧ 1/10Ω=0.1℧ 1/5Ω=0.2℧ 1/2Ω=0.5℧

Sum of conductances is 1<sup>°</sup>U (convenient!)

Current through  $5\Omega$  resistor is:

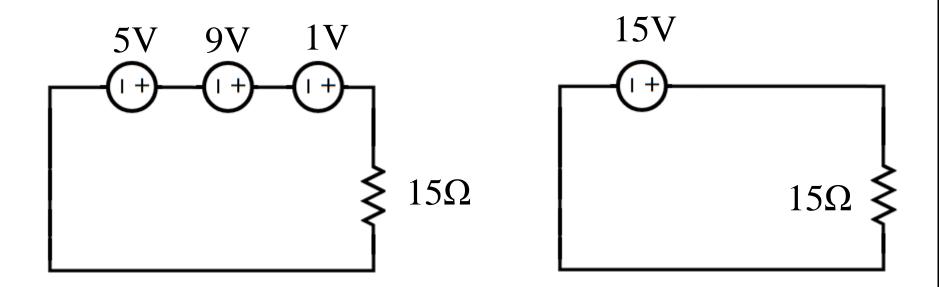
$$I_2 = 20A \frac{0.2}{1} = 4A$$

# **Circuit Simplification**

- Next we'll talk about some tricks for combining multiple circuit elements into a single element
- Many elements in series → One single element
- Many elements in parallel → One single element

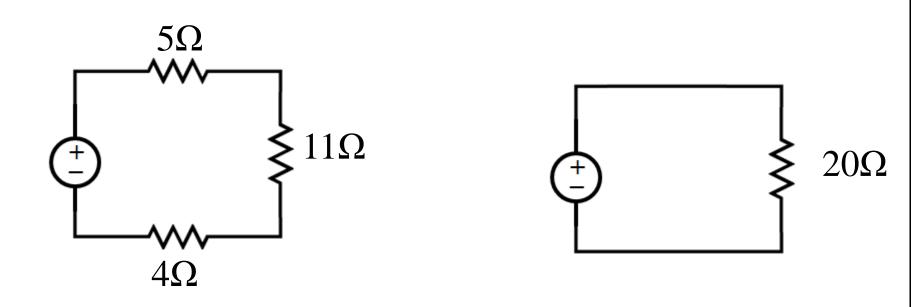
#### Circuit Simplification Example Combining Voltage Sources

- KVL trivially shows voltage across resistor is 15 V
- Can form equivalent circuit as long as we don't care about individual source behavior
  - For example, if we want power provided by each source, we have to look at the original circuit



## **Example – Combining Resistances**

 Can use kitchen sink method or voltage divider method to show that current provided by the source is equivalent in the two circuits below



# **Source Combinations**

- Voltage sources in series combine additively
- Voltage sources in parallel
  - This is like crossing the streams "Don't cross the streams"
  - Mathematically nonsensical if the voltage sources are not exactly equal
- Current sources in parallel combine additively
- Current sources in series is bad if not the same current

## **Resistor Combinations**

Resistors in series combine additively

$$R_{eq} = R_1 + R_2 + \dots + R_N$$

Resistors in parallel combine weirdly

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$

– More natural with conductance:

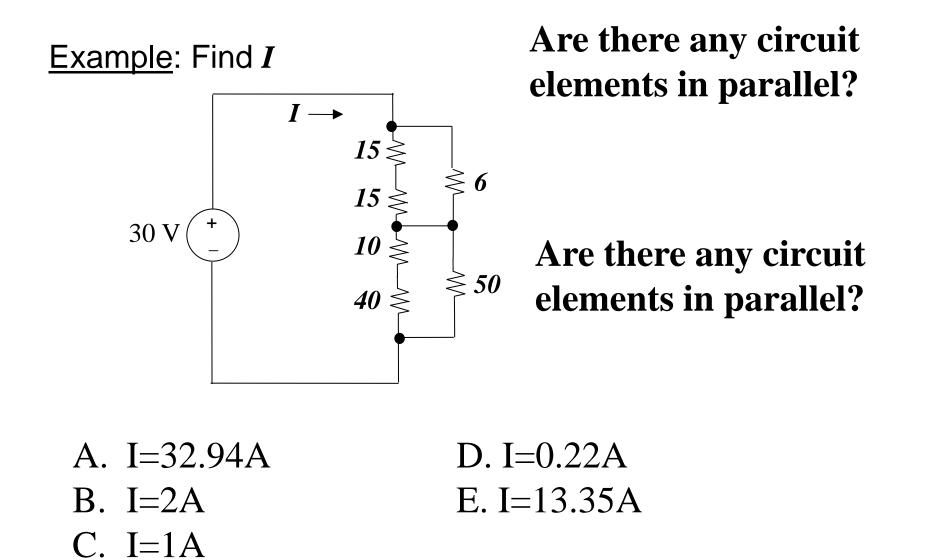
$$G_{eq} = G_1 + G_2 + \dots + G_n$$

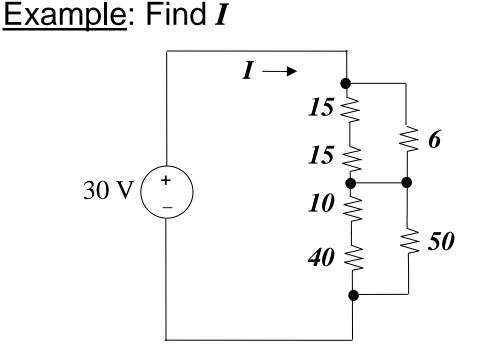
• N resistors in parallel with the same resistance R have equivalent resistance  $R_{eq}=R/N$ 

#### **Algorithm For Solving By Combining Circuit Elements**

- Check circuit diagram
  - If two or more elements of same type in series
    - Combine using series rules
  - If two or more elements of same type in parallel
    - Combine using parallel rules
- If we combined anything, go back to
- If not, then solve using appropriate method (kitchen sink if complicated, divider rule if possible)

# iClicker #3: Using Equivalent Resistances

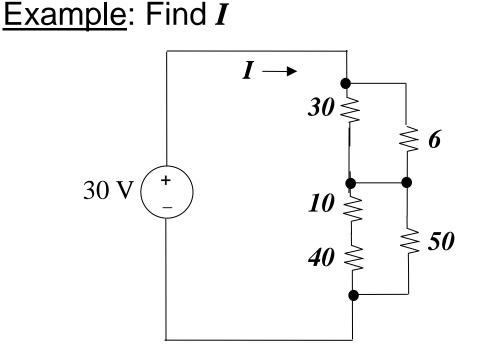




# Are there any circuit elements in parallel?

No!

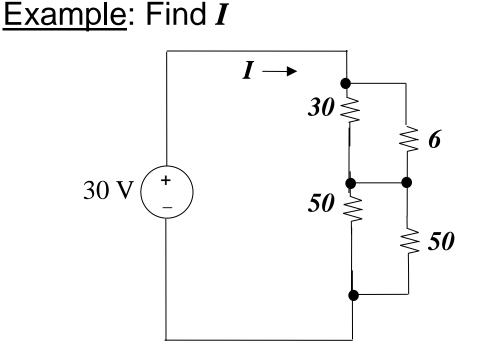
# Are there any circuit elements in parallel?



# Are there any circuit elements in parallel?

Yes!

# Are there any circuit elements in parallel?

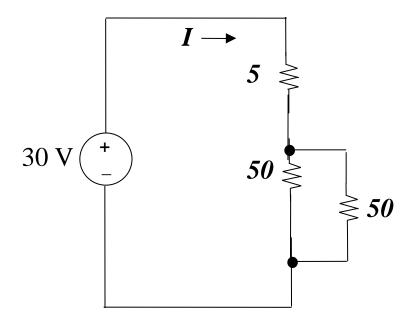


# Are there any circuit elements in parallel?

No!

# Are there any circuit elements in parallel?

#### Example: Find I

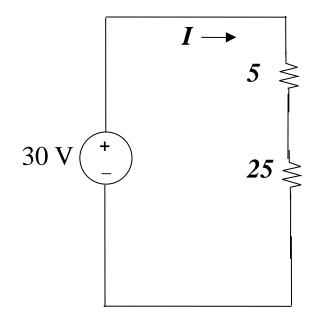


# Are there any circuit elements in parallel?

No!

# Are there any circuit elements in parallel?

#### Example: Find I



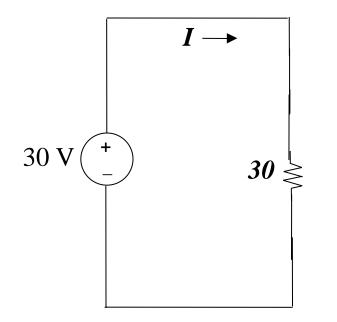
# Are there any circuit elements in parallel?

No!

# Are there any circuit elements in parallel?

No!

#### Example: Find I



# Are there any circuit elements in parallel?

No!

# Are there any circuit elements in parallel?

No!

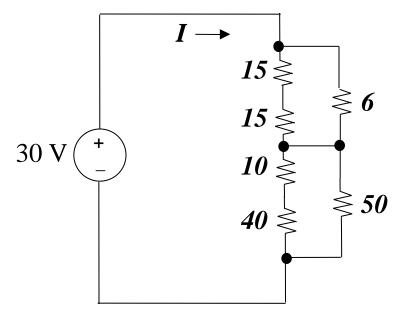
#### $I=30V/30\Omega=1A$

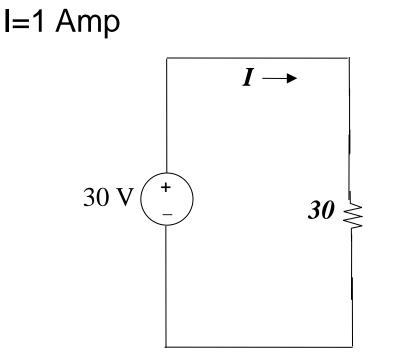
# **Working Backwards**

- Assume we've combined several elements to understand large scale behavior
- Now suppose we want to know something about one of those circuit elements that we've combined
  - For example, current through a resistor that has been combined into equivalent resistance
- We undo our combinations step by step
  - At each step, use voltage and current divider tricks
  - Only undo enough so that we get the data we want

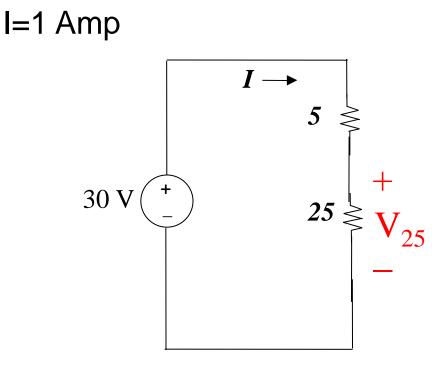
### **Working Backwards Example**

Suppose we want to know the voltage across the 40Ω Resistor



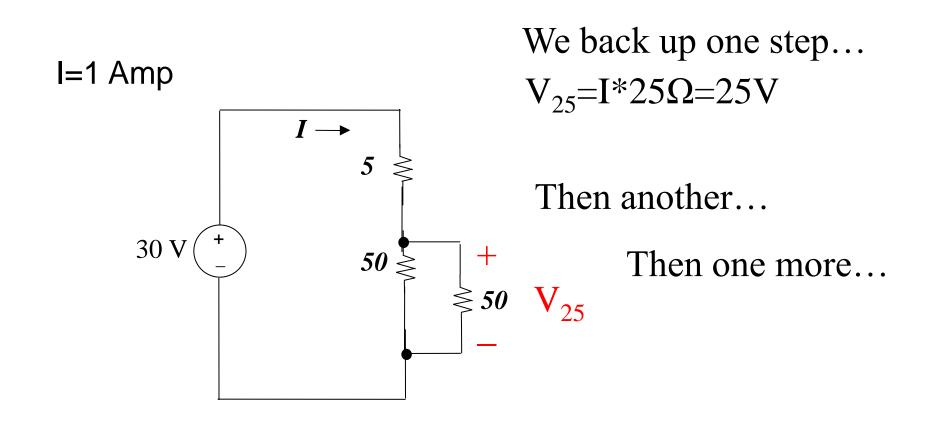


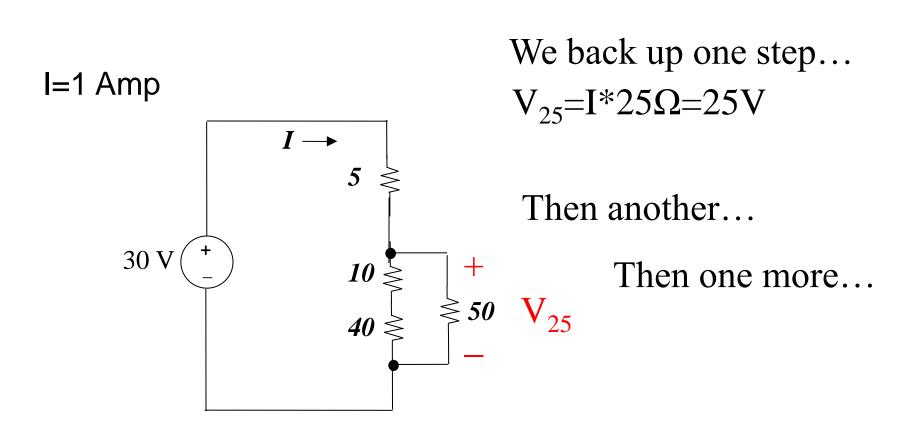
#### Starting from here...



We back up one step...  $V_{25}=I*25\Omega=25V$ 

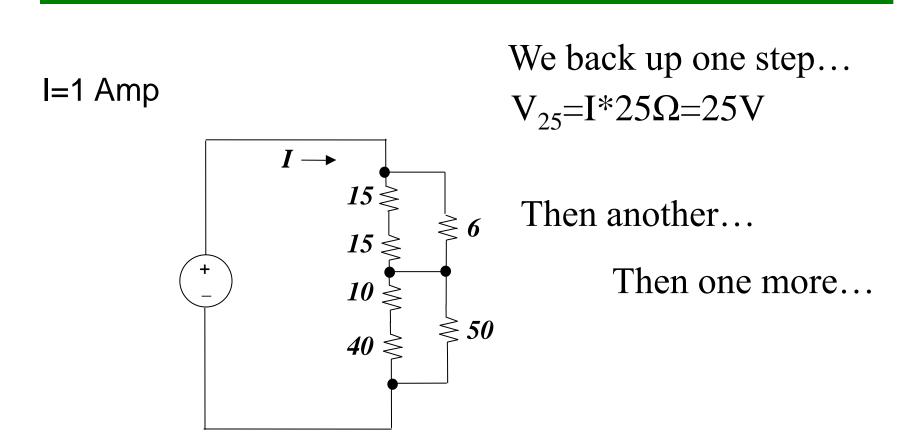
Then another...





Now we can use the voltage divider rule, and get

$$V_{40} = \frac{40\Omega}{10\Omega + 40\Omega} 25V \qquad V_{40} = 20V$$



Now we can use the voltage divider rule, and get

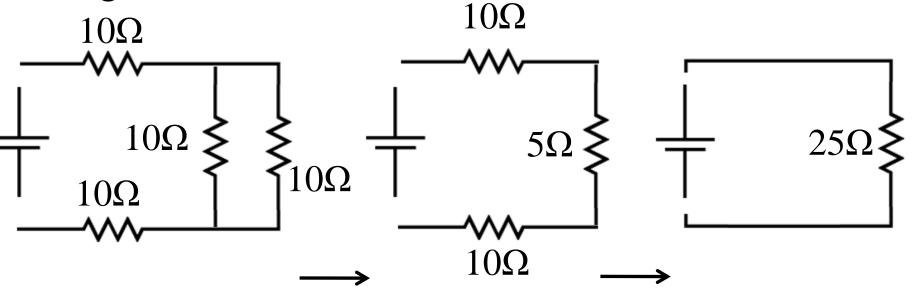
$$V_{40} = \frac{40\Omega}{10\Omega + 40\Omega} 25V \qquad V_{40} = 20V$$

#### **Equivalent Resistance Between Two Terminals**

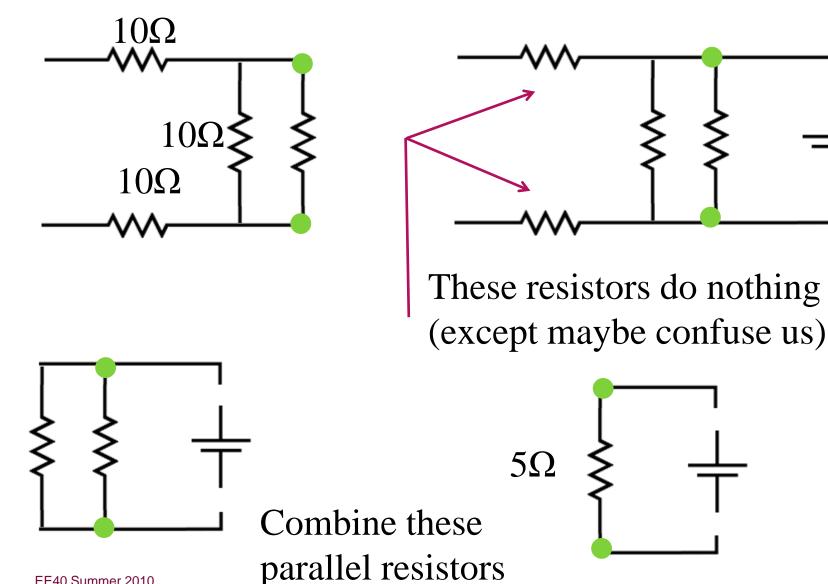
- We often want to find the equivalent resistance of a network of resistors with no source attached  $\downarrow 10\Omega$  $\downarrow 10\Omega$
- Tells us the resistance that a hypothetical source would "see" if it were connected
  - e.g. In this example, the resistance that provides the correct source current

#### **Equivalent Resistance Between Two Terminals**

- Pretend there is a source of some kind between the circuits
- Perform the parallel/series combination algorithm as before



# **Can Pick Other Pairs of Terminals**



## There are better ways to solve circuits

- The kitchen sink method works, but we can do better
  - Current divider
  - Voltage divider
  - Lumping series and parallel elements together (circuit simplification)
  - Node voltage

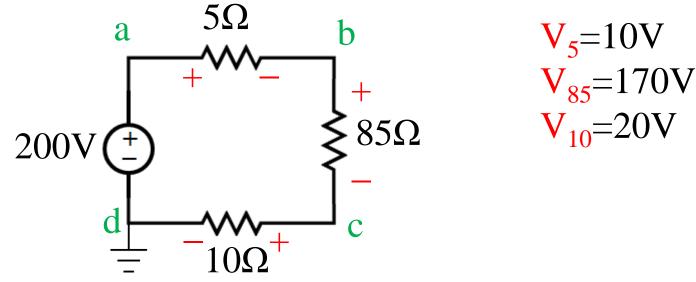
## **The Node Voltage Technique**

- We'll next talk about a general technique that will let you convert a circuit schematic with N nodes into a set of N-1 equations
- These equations will allow you to solve for every single voltage and current
- Works on any circuit, linear or nonlinear!
- Much more efficient than the kitchen sink

#### **Definition: Node Voltage and Ground Node**

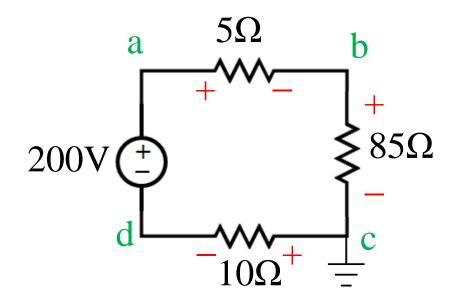
- Remember that voltages are always defined in terms of TWO points in a circuit
- It is convenient to label one node in our circuit the "Ground Node"
  - Any node can be "ground", it doesn't matter which one you pick
- Once we have chosen a ground node, we say that each node has a "node voltage", which is the voltage between that node and the arbitrary ground node
- Gives each node a universal single valued voltage level

#### Node Voltage Example



- Pick a ground, say the bottom left node.
- Label nodes a, b, c, d. Node voltages are:
  - $-V_d$ =voltage between node d and d=0V
  - $-V_c$ =voltage between node c and d= $V_{10}$ =20V
  - $V_b$ =voltage between node b and d= $V_{85}+V_{10}$ =190V
  - V<sub>a</sub>=voltage between node a and d=200V

#### iClicker #4: Node Voltages



$$V_5 = 10V$$
  
 $V_{85} = 170V$   
 $V_{10} = 20V$ 

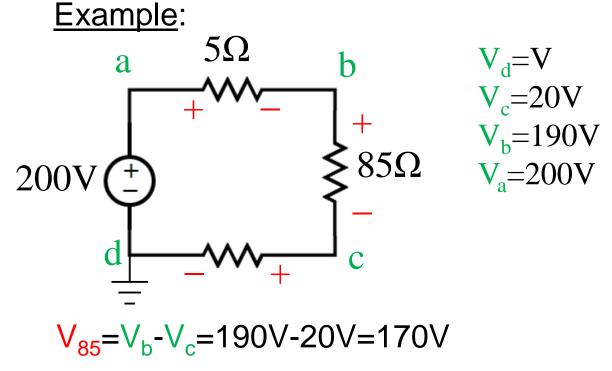
What is  $V_a$ ?

- A. 200V
- B. 20V
- C. 160V
- D. 180V

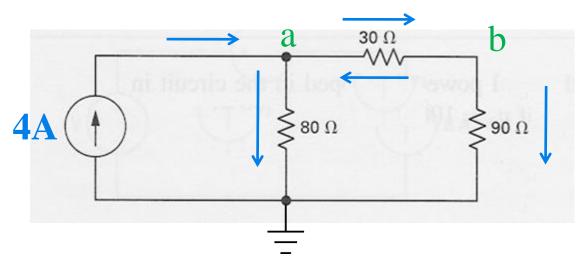
$$V_a = V_5 + V_{85} = 180V$$

### **Relationship: Node and Branch Voltages**

- Node voltages are useful because:
  - The branch voltage across a circuit element is simply the difference between the node voltages at its terminals
  - It is easier to find node voltages than branch voltages



# Why are Node Voltages Easier to Find?



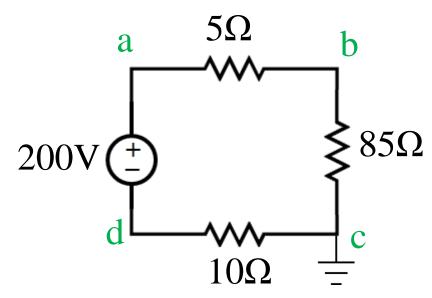
- KCL is easy to write in terms of node voltages
- For example, at node a:
  - $4A = V_a / 80\Omega + (V_a V_b) / 30\Omega$
- And at node b:
  - $(V_{b}-V_{a})/30\Omega=V_{b}/90\Omega$
- Well look, two equations, two unknowns. We're done.
- Better than 5 equations, 5 unknowns with kitchen sink method EE40 Summer 2010

# (Almost) The Node Voltage Method

- Assign a ground node
- For every node except the ground node, write the equation given by KCL in terms of the node voltages
  - Be very careful about reference directions
- This gives you a set of N-1 linearly independent algebraic equations in N-1 unknowns
  - Solvable using whatever technique you choose

#### What about Voltage Sources?

Suppose we have the circuit below



- When we try to write KCL at node a, what happens?
- How do we get around this?

– Write fixed node voltage relationship:

$$V_a = V_d + 200$$

# **Full Node Voltage Method**

- Assign a ground node
- For every node (except the ground node):
  - If there is no voltage source connected to that node, then write the equation given by KCL in terms of the node voltages
  - If there is a voltage source connecting two nodes, write down the simple equation giving the difference between the node voltages
  - Be very careful about reference directions (comes with practice)
- This gives you a set of N-1 linearly independent algebraic equations in N-1 unknowns
- Solvable using whatever technique you choose **More Examples Next Time!** EE40 Summer 2010

#### **Next Class**

- Node voltage practice and examples
- Why we are bothering to understand so deeply the intricacies of purely resistive networks
  - Things we can build other than the most complicated possible toaster
- How we actually go about measuring voltages and currents
- More circuit tricks
  - Superposition
  - Source transformations

#### **Quick iClicker Question**

- How was my pacing today?
  - A. Way too slow
  - B. A little too slow
  - C. Pretty good
  - D. Too fast
  - E. Way too fast

#### **Extra Slides**

# Summary (part one)

- There are five basic circuit elements
  - Voltage Sources
  - Current Sources
  - Resistors
  - Capacitors
  - Inductors
- Circuit schematics are a set of interconnect ideal basic circuit elements
- A connection point between elements is a node, and a path that connects two nodes is a branch
- A loop is a path around a circuit which starts and ends at the same node without going through any circuit element twice

# Summary (part two)

- Kirchoff's current law states that the sum of the currents entering a node is zero
- Kirchoff's voltage law states that the sum of the voltages around a loop is zero
- From these laws, we can derive rules for combining multiple sources or resistors into a single equivalent source or resistor
- The current and voltage divider rules are simple tricks to solve simple circuits
- The node voltage technique provides a general framework for solving any circuit using the elements we've used so far

## **Short Circuit and Open Circuit**

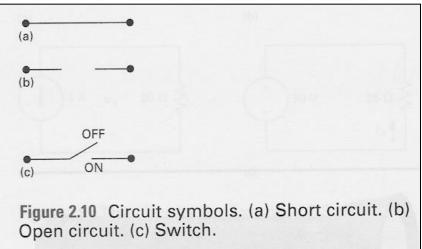
#### <u>Wire</u> ("short circuit"):

- *R* = 0 → no voltage difference exists

   (all points on the wire are at the same potential)
- Current can flow, as determined by the circuit

# <u>Air</u> ("open circuit"):

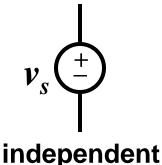
- $R = \infty \rightarrow$  no current flows
- Voltage difference can exist, as determined by the circuit



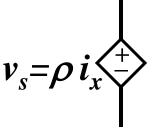
# **Ideal Voltage Source**

- Circuit element that maintains a prescribed voltage across its terminals, regardless of the current flowing in those terminals.
  - Voltage is known, but current is determined by the circuit to which the source is connected.
- The voltage can be either **independent or dependent** on a voltage or current elsewhere in the circuit, and can be constant or time-varying.

Circuit symbols:



voltage-controlled

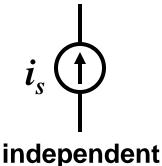


current-controlled

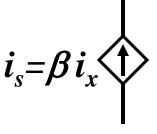
### **Ideal Current Source**

- Circuit element that maintains a prescribed current through its terminals, regardless of the voltage across those terminals.
  - Current is known, but voltage is determined by the circuit to which the source is connected.
- The current can be either **independent or dependent** on a voltage or current elsewhere in the circuit, and can be constant or time-varying.

Circuit symbols:



voltage-controlled



current-controlled