The iPhone 4 integrates the antenna into the case

EE40 Lecture 3 Josh Hug

6/25/2010

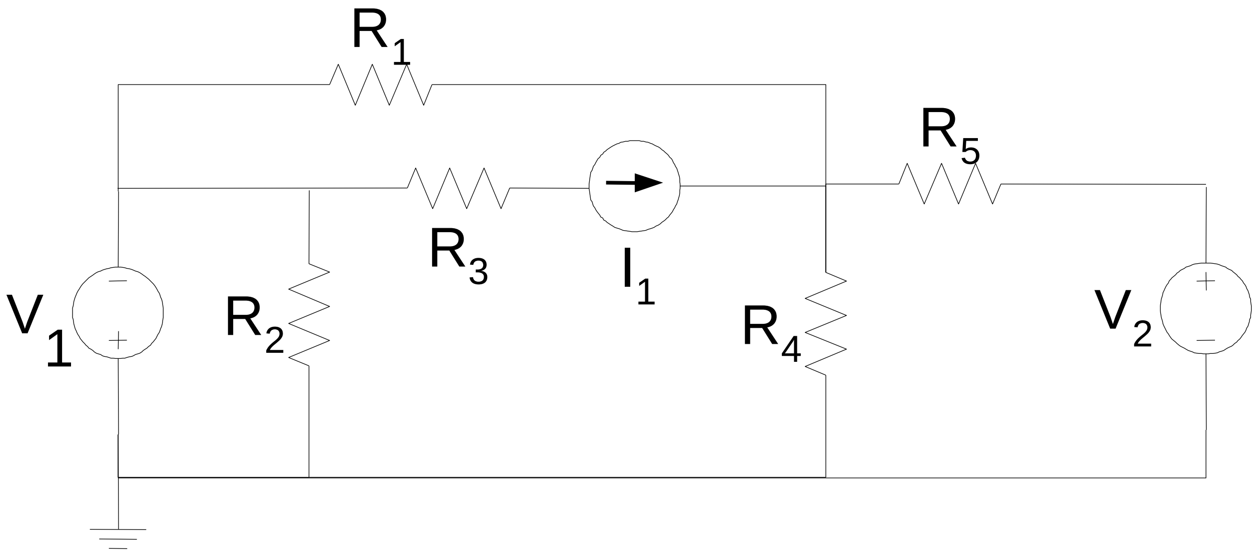
"Users [are] reporting a drop in signal strength when **the phone is held**." -BBC

"If you ever experience this on your iPhone 4, avoid gripping it in the lower left corner in a way that covers both sides of the black strip in the metal band, or simply use one of many available cases."-Apple

Logistical Notes

- Office Hours – Room reservation has been put in, but no word from the people yet. I’ve got someone looking into it
- HW1 due today at 5 PM in the box in 240 Cory
- No i>Clicker today [couldn’t get hardware today]

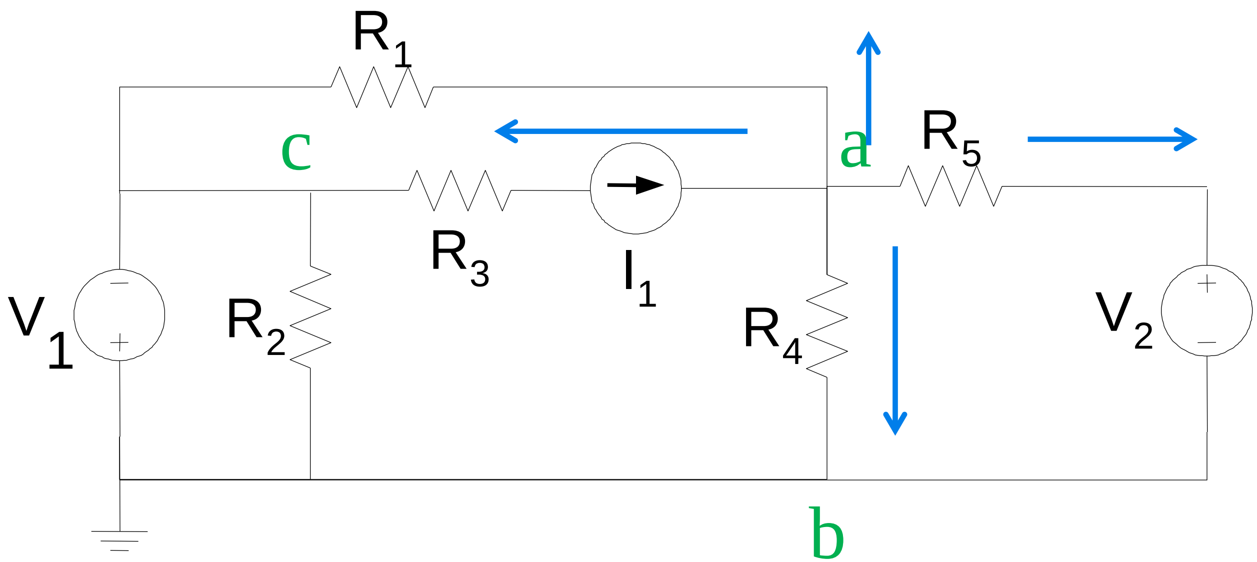
Nodal Analysis: Example



Using the basic method

- 5 unknowns
- 2 KCL equations
- 3 KVL equations

Nodal Analysis: Example



One equation, one unknown

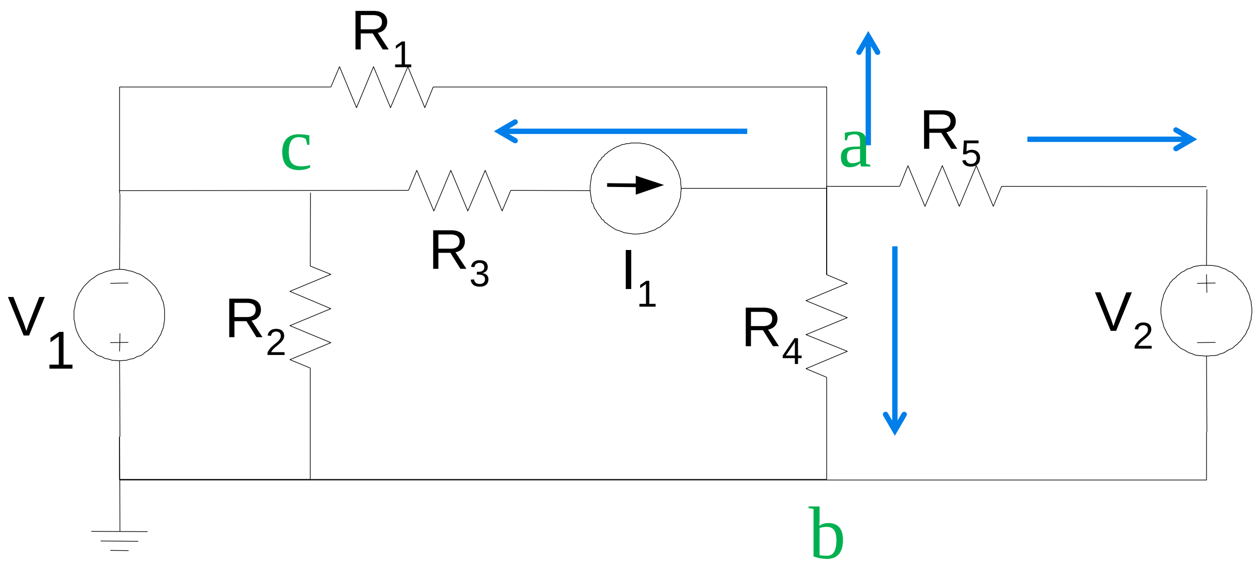
With Node Voltage:

$$\frac{V_a - V_2}{R_5} + \frac{V_a}{R_4} - I_1 + \frac{V_a + V_1}{R_1} = 0$$

$$G_5(V_a - V_2) + G_4 V_a - I_1 + G_1(V_a + V_1) = 0$$

$$V_a(G_5 + G_4 + G_1) = G_5 V_2 + I_1 - G_1 V_1$$

Nodal Analysis: Example



$$V_a(G_5 + G_4 + G_1) = G_5 V_2 + I_1 - G_1 V_1$$

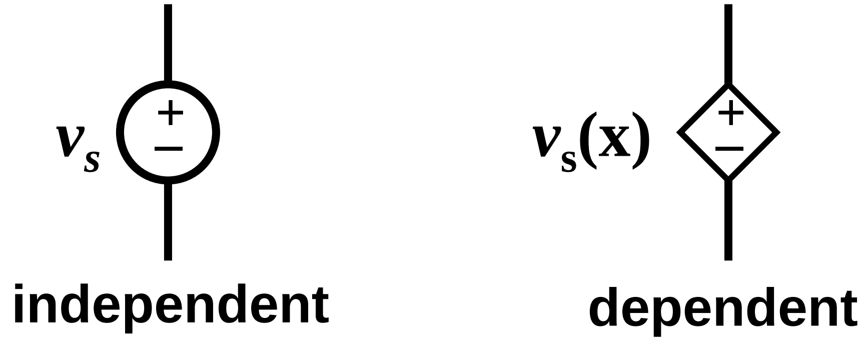
$$V_a = \frac{G_5 V_2 + I_1 - G_1 V_1}{G_5 + G_4 + G_1}$$

$$V_a = \frac{R_4(I_1 R_1 R_5 - R_5 V_1 + R_1 V_2)}{R_4 R_5 + R_1(R_4 + R_5\ )}$$

It's fine to leave your answer in terms of conductances on HW and exams

Dependent Sources

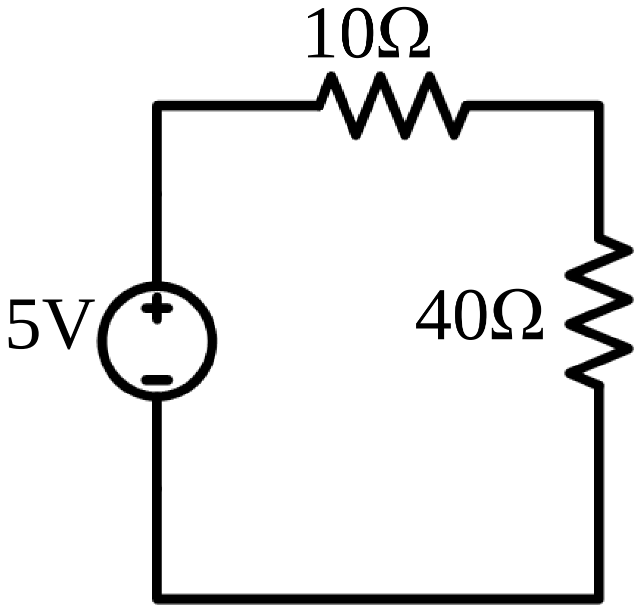
- In practice, we'll want to use controllable sources
  - Called "dependent sources" since their output is dependent on something external to the source itself

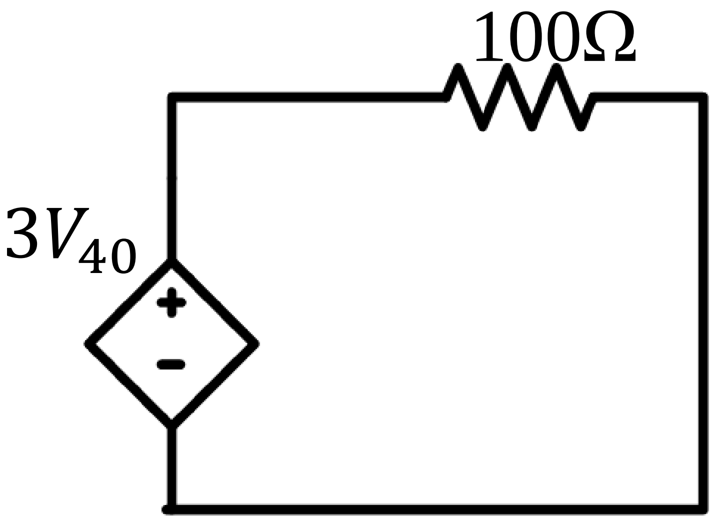


- In theory, a dependent source could be a function of anything in the universe
  - Intensity of light incident on the source
  - Number of fish within 3 miles

Dependent Sources

- Since we're building electrical circuits, dependent sources have been developed which are functions of other electrical quantities





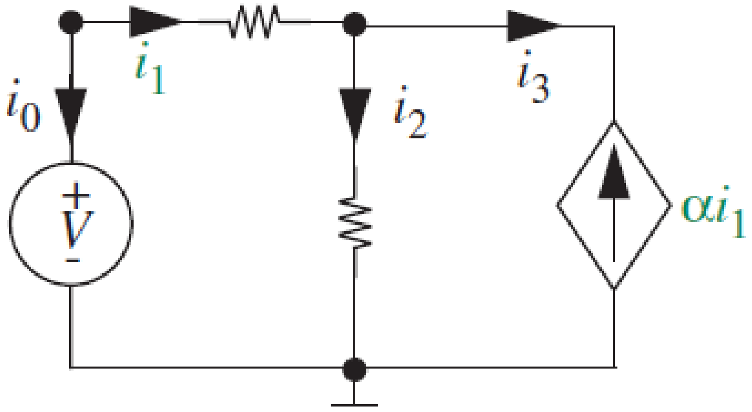
$$I_{100} = \frac{3V_{40}}{100\Omega} = \frac{3 \times 4V}{100\Omega} = 0.12A$$

Dependent Sources

- Dependent sources allow us to decouple the controller from the controlled
  - Acceleration of the engine affect by gas pedal
  - Gas pedal not affected by engine acceleration
- This is in contrast to our circuits so far where everything is connected

Dependent Sources With Feedback

- Dependent sources can be coupled to their controller
- This is useful for when the controller needs feedback from the thing being controlled
- Can be a little tricky to analyze

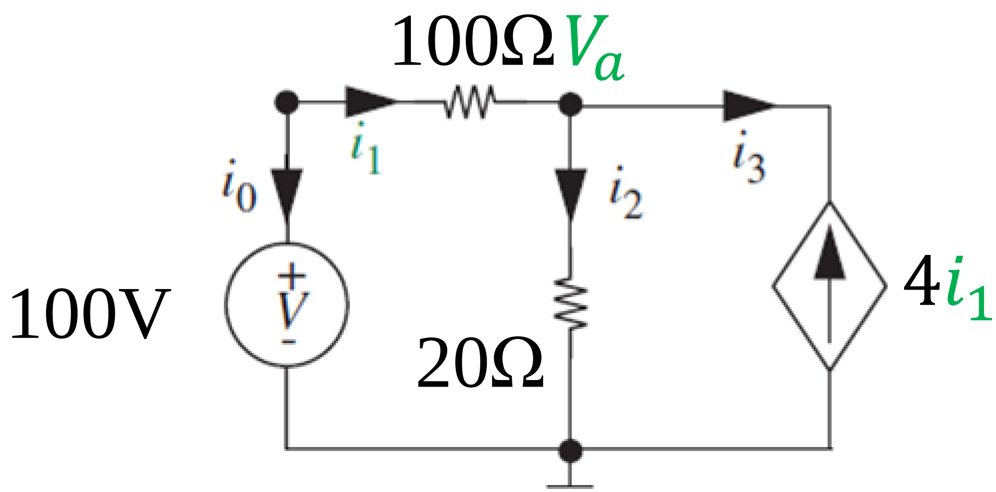


Node Voltage With Dependent Source

$$\frac{V_a - 100}{100} + \frac{V_a}{20} - 4i_1 = 0$$

$$i_1 = -\frac{V_a - 100}{100}$$

- There are two ways to proceed:
  - **Direct substitution (almost always better)**
  - Indirect substitution (tough part of the reading)



Direct Substitution Method for Dependent Sources

$$\frac{V_a - 100}{100} + \frac{V_a}{20} - 4i_1 = 0$$

$$i_1 = -\frac{V_a - 100}{100}$$

$$\frac{V_a - 100}{100} + \frac{V_a}{20} + 4\frac{V_a - 100}{100} = 0$$

$$V_a - 100 + 5V_a + 4V_a - 400 = 0$$

$$10V_a = 500$$

$$V_a = 50$$

Node Voltage With Dependent Source

$$\frac{V_a - 100}{100} + \frac{V_a}{20} - 4i_1 = 0$$

$$i_1 = -\frac{V_a - 100}{100}$$

- There are two ways to proceed:
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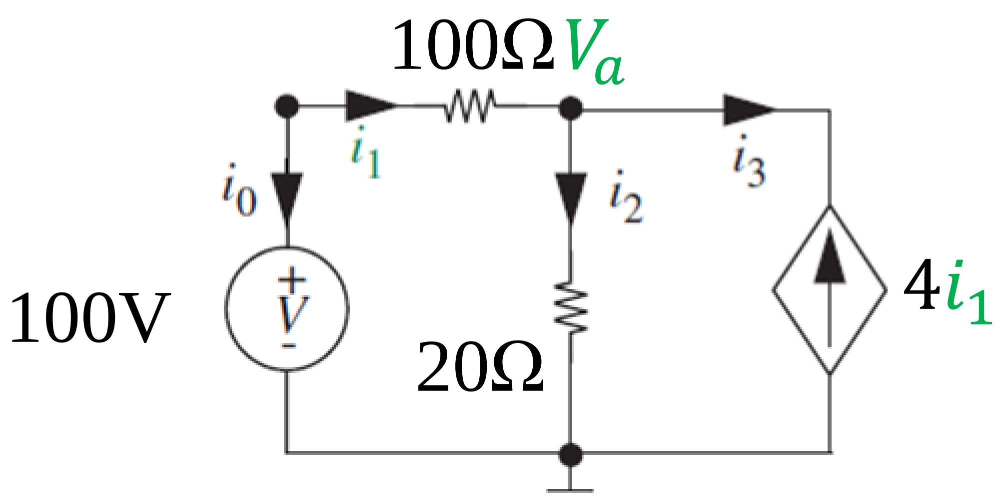


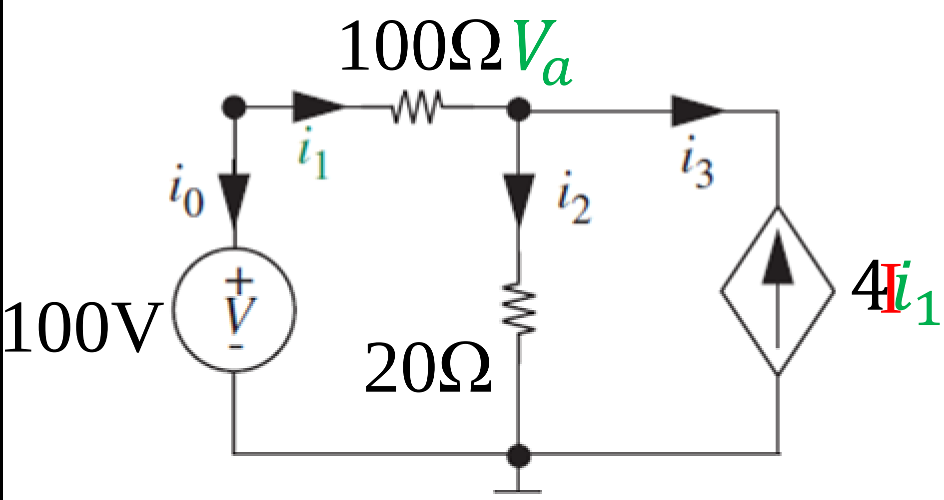
?

$$i = g(i) \quad ?$$

$$I = f(i) = f(g(i))$$

?

Indirect Substitution Method for Dependent Sources



$$\frac{V_a - 100}{100} + \frac{V_a}{20} - I = 0$$

Node voltage vs. dummy source

$$V_a = \frac{100 + 100I}{6}$$

Node voltage vs. controlling current

$$i_1 = \frac{100 - V_a}{100}$$

Real source vs. dummy source

$$\frac{I}{4} = i_1$$

$$\frac{I}{4} = \frac{100 - \frac{100 + 100I}{6}}{100}$$

$$I = 2A$$

Summary So Far

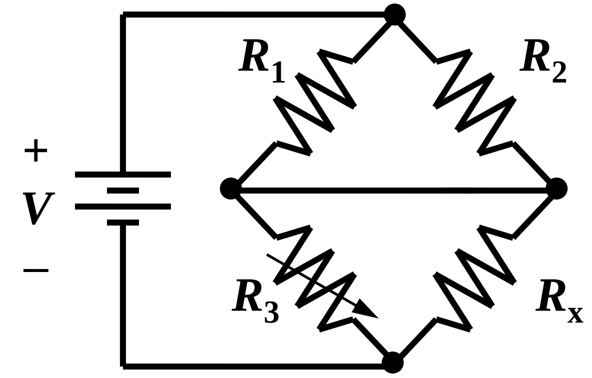
- Dependent sources model controllable electrical sources
- Node voltage can be used with dependent sources
  - Controlling input can be seemingly difficult to deal with, e.g. $\frac{V_a - 100}{100} + \frac{V_a}{20} - 4i_1 = 0$
  - Direct Substitution: Replace controlling input with expression in terms of
    - The node voltages you're trying to solve for
    - Known currents if controlling input driven by current source
  - Indirect Substitution: Treat dependent source as a new variable and solve (better in rare cases)

Useful Resistive Circuits

- Wheatstone Bridge
  – Used for measuring unknown resistances
- Strain Gauge
  – Used for measuring weight

Wheatstone Bridge

- Named for Charles Wheatstone
- Used for measuring resistance of an unknown resistor
- Parts:
  - – Known resistors R_1 and R_2
  - – Adjustable resistor R_3
  - – Unknown resistance R_x



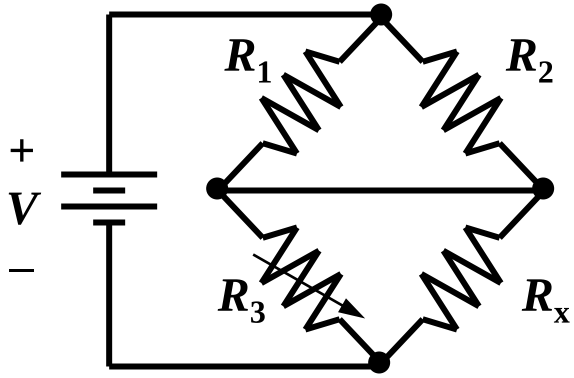
- Basic concept:
  - – If $\frac{R_1}{R_3} = \frac{R_2}{R_x}$, then no current will flow in the middle branch

Invented 1833 by Samuel Christie
Hit the charts when remixed by Wheatstone in 1843

Finding the value of R_x

- Adjust R_3 until there is no current in the detector

Then, $$R_x = \frac{R_2}{R_1} R_3$$



Derivation:

KCL => $i_1 = i_3$ and $i_2 = i_x$

KVL => $i_3 R_3 = i_x R_x$ and $i_1 R_1 = i_2 R_2$

$$i_1 R_3 = i_2 R_x$$

$$\frac{R_3}{R_1} = \frac{R_x}{R_2}$$

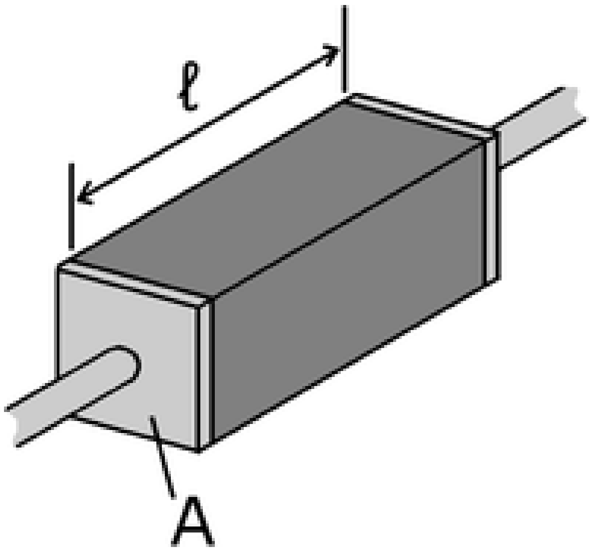
Strain Gauge Intuition

- Resistance is a function of wire length and area
- Weight stretches a wire, changing its shape
- Can theoretically get weight of a load by seeing how resistance varies when a load is added

Resistivity

- Wire resistance: $R = \rho \frac{l}{A}$
- ρ is resistivity, measured in Ω·m
- Can think of as how tightly molecular lattice holds on to electrons

Material	ρ
Copper	1.68×10^{-8}
Aluminum	2.82×10^{-8}
Nichrome	1.1×10^{-6}
Glass	10^{10}



Wire Gauge

Gauge	Diameter [mm]	Area [mm^2]
10	2.58	5.26
14	1.62	2.08
16	1.29	1.31

Resistance of 30m, 16 gauge extension cord?

$$1.68 \times 10^{-8} \Omega m \frac{30\text{m}}{1.31 \times 10^{-6} m^2}$$

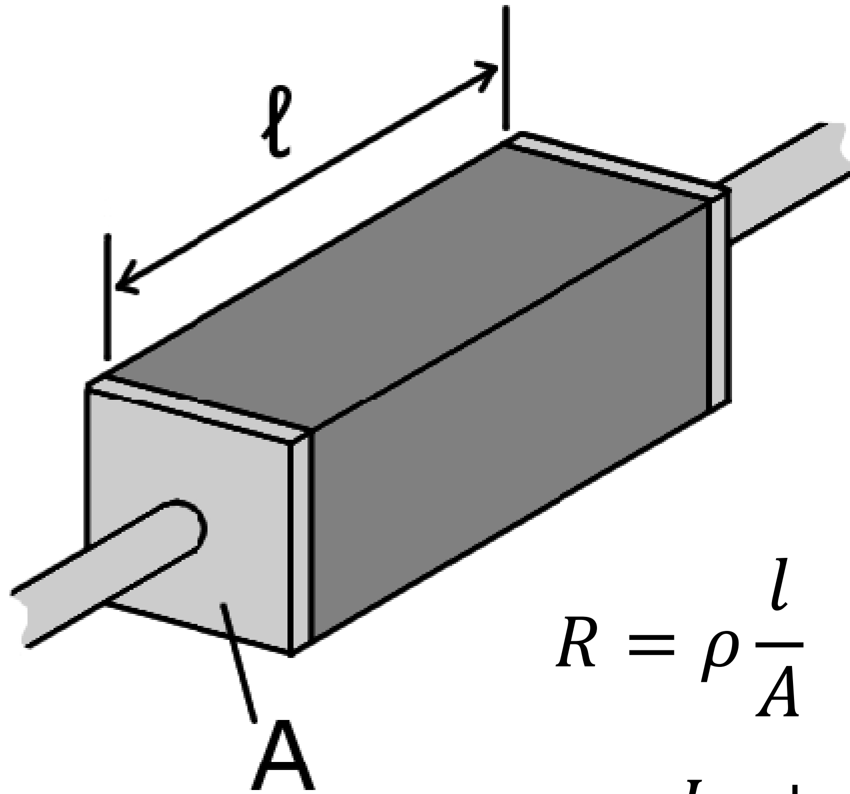
$$0.384\Omega$$

If carrying 10 amps, how much power dissipated?

$$P = VI = I^2 R = 38.4\, W$$

Basic Principle

- Pull on resistor:
  - $L=L_0+\Delta L$
  - $A=A_0-\Delta A$
  - $V=LA$ [constant]
- Length wins the battle to control resistance
- $R=R_0+\Delta R$



$$R = \rho \frac{l}{A}$$

Define strain $\varepsilon = \frac{\Delta L}{L_0}$

$$R = R_0 + \Delta R = \rho \frac{L_0 + \Delta L}{A_0 - \Delta A}$$

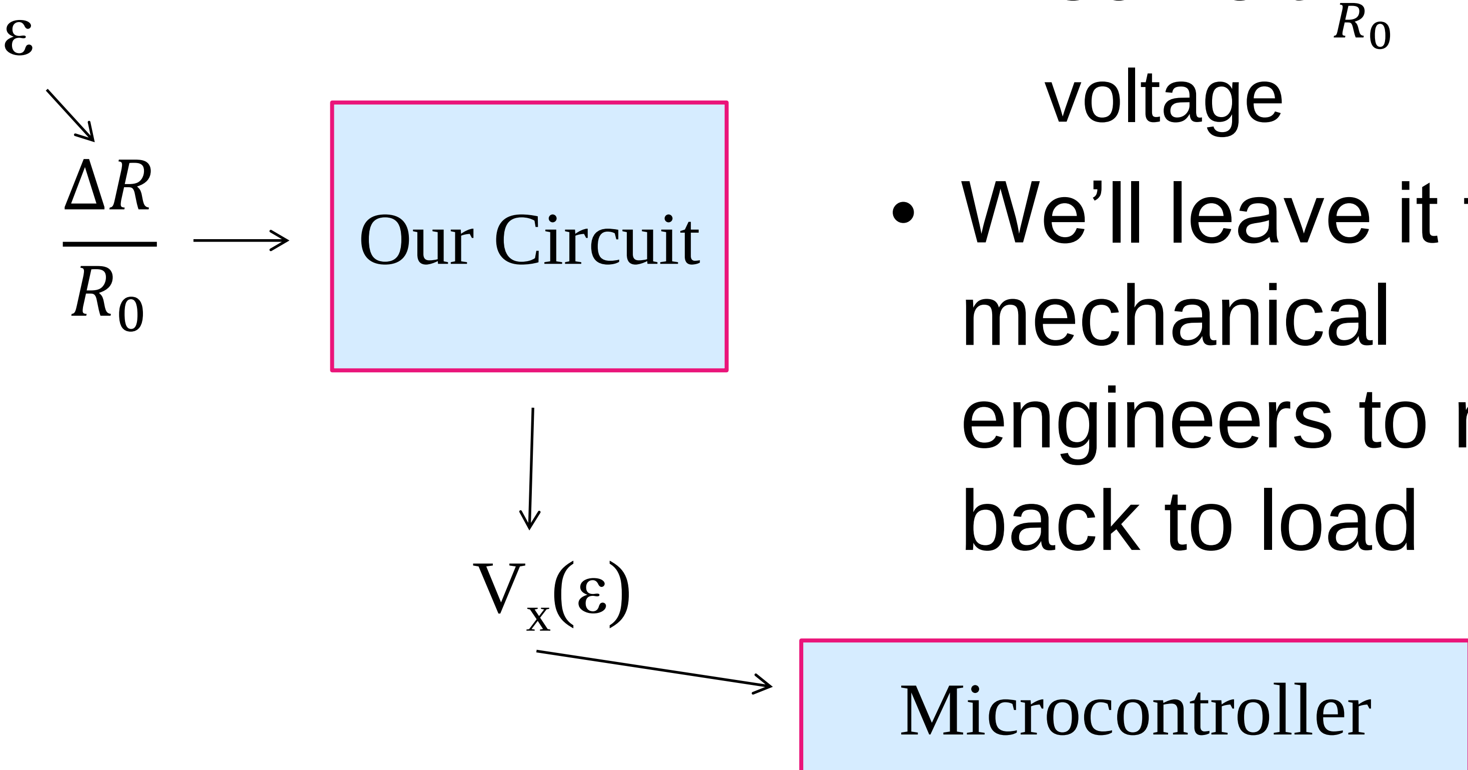
$$\frac{\Delta R}{R_0} = GF \times \varepsilon$$

Where the Gauge Factor relates to how length/area change. GF~2

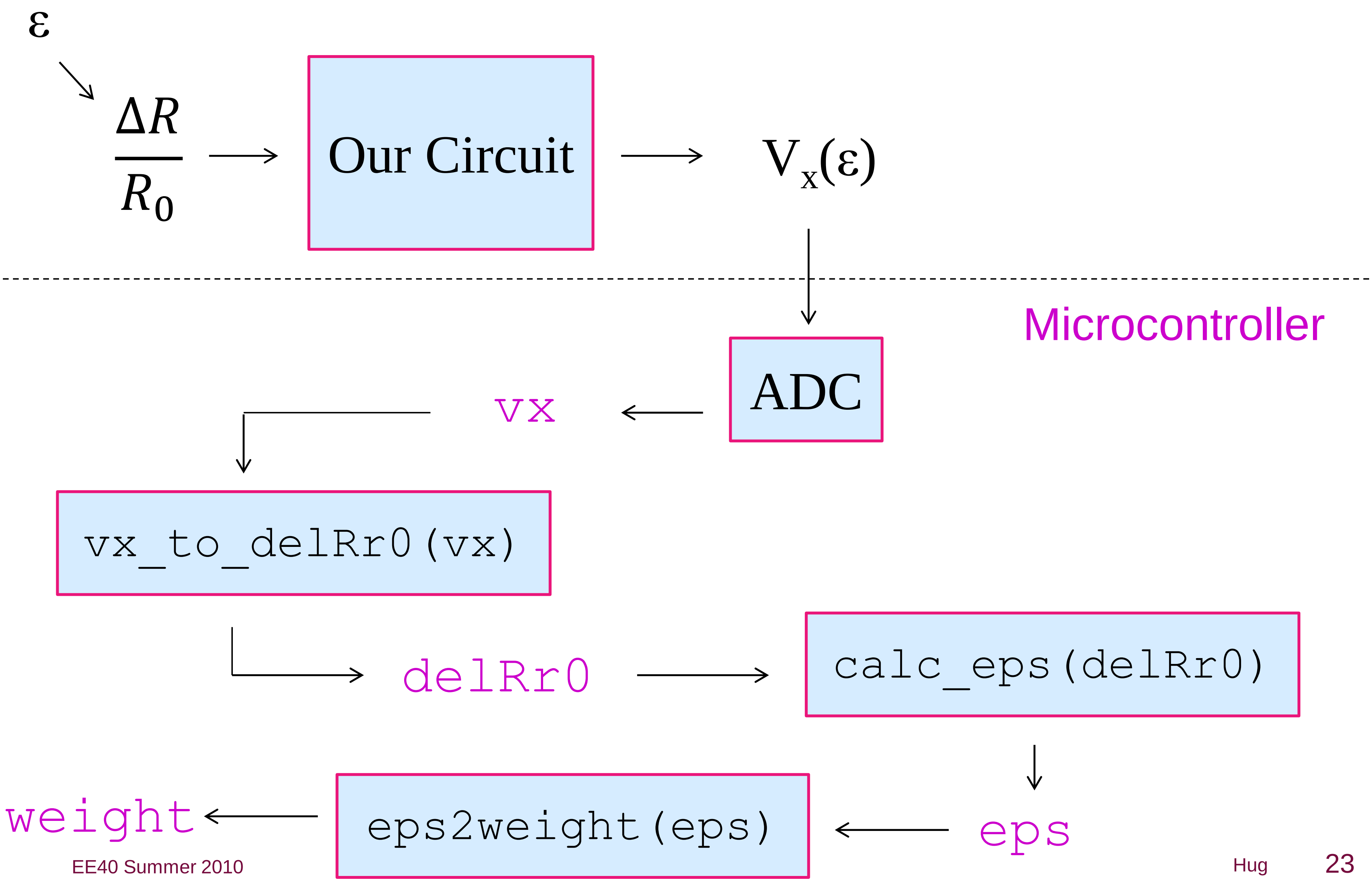
Using Strain to Measure Weight

$$\frac{\Delta R}{R_0} = GF \times \varepsilon$$

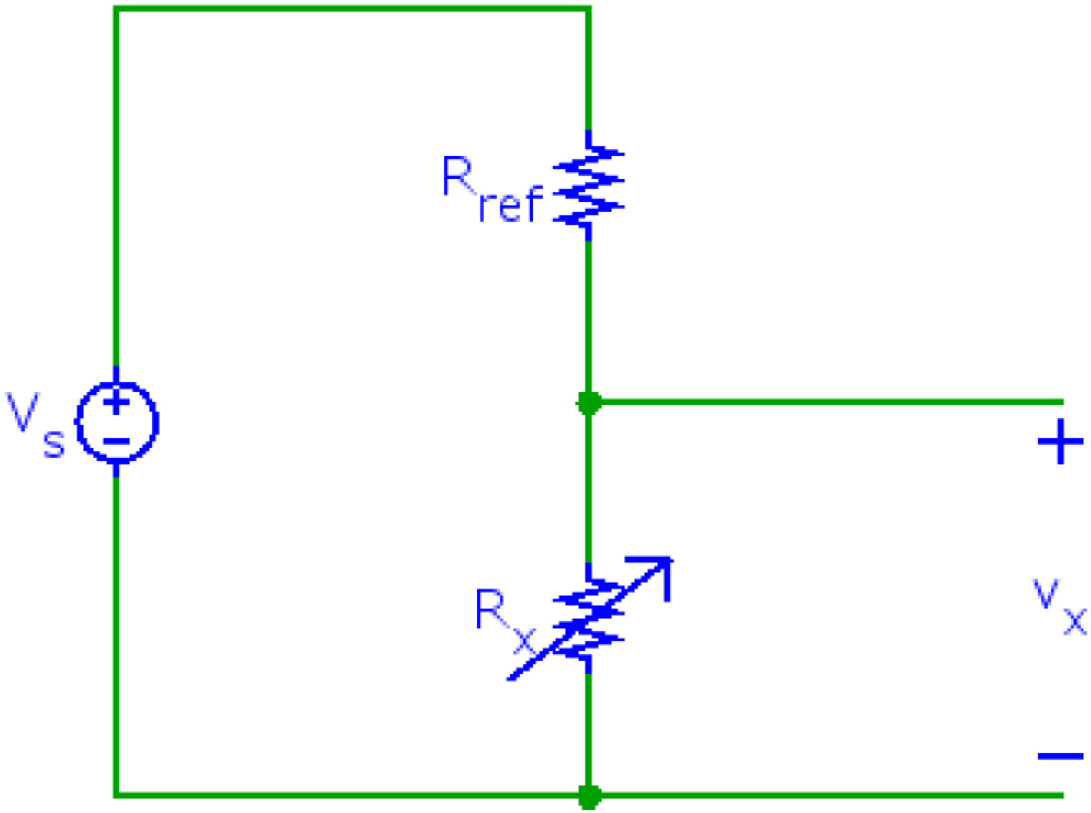
- We'll attempt to design a circuit which can measure ε
  - Convert $\frac{\Delta R}{R_0}$ into a voltage
- We'll leave it to the mechanical engineers to map ε back to load



Using Strain to Measure Weight

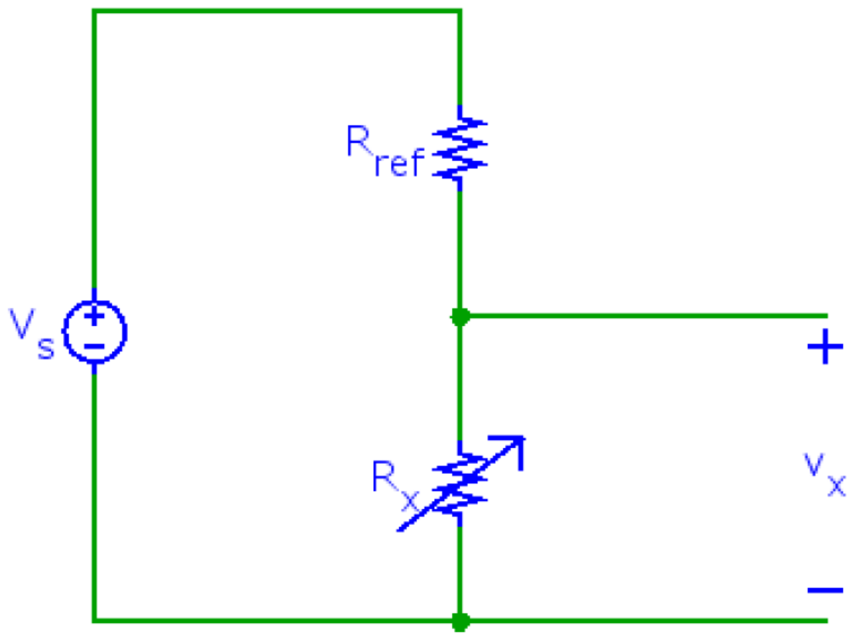


One Possible Design



- Here, R_x is a variable resistor, where R_x is dependent on strain
- As strain varies, so will v_x

One Possible Design



$$v_x = V_s \frac{R_o + \Delta R}{R_o + \Delta R + R_{\mathrm{ref}}}$$

$$\cong V_s \frac{R_o + \Delta R}{R_o + R_{\mathrm{ref}}}$$

v_x is what μController sees

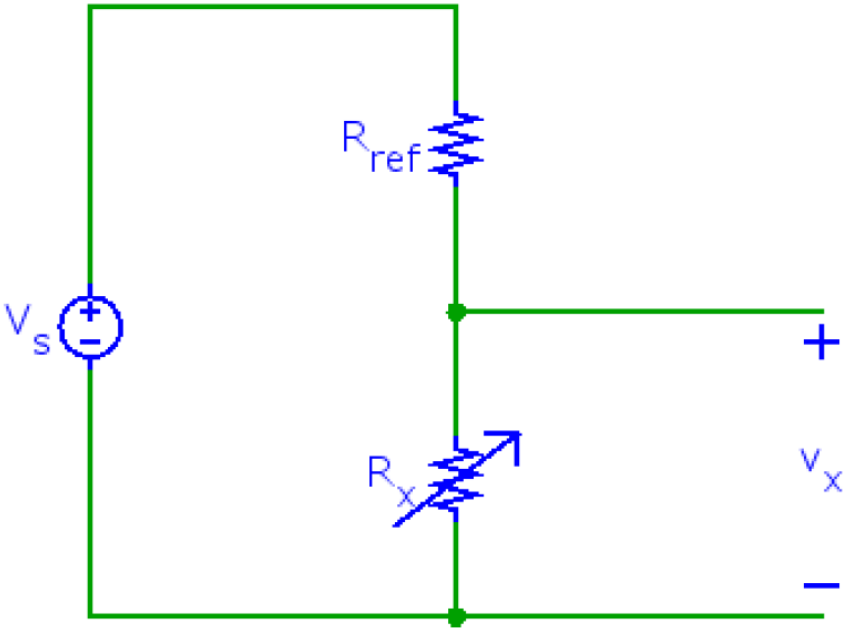
$$\Delta R = \frac{-R_0 V_s + R_0 V_x + R_{ref} V_x}{V_s}$$

$$\varepsilon = \frac{\Delta R}{R_0} \times \frac{1}{GF}$$

$$weight = f(\varepsilon)$$

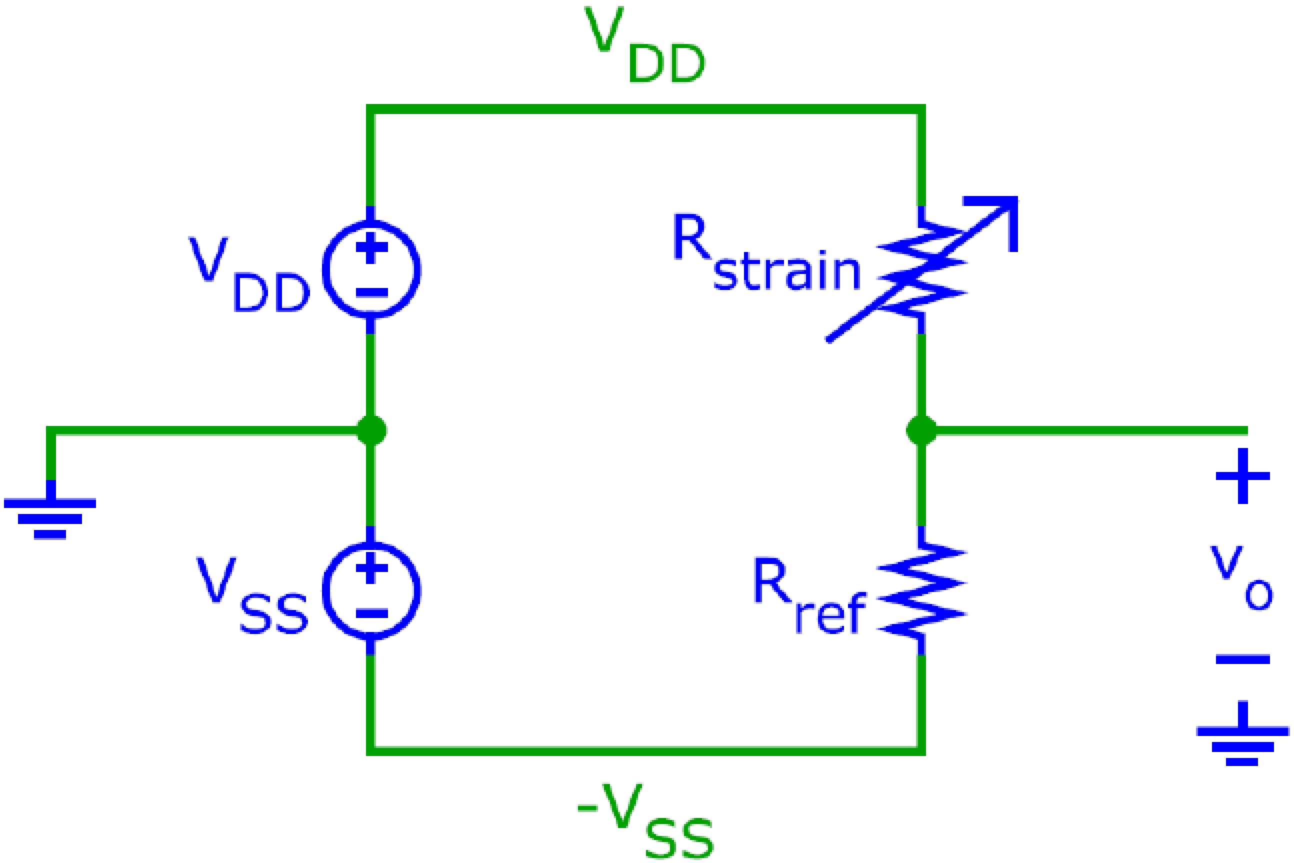
μController calculates ε from known quantities V_x, V_s, R_0, R_{ref}, GF, and then weight from ε

Better Circuits



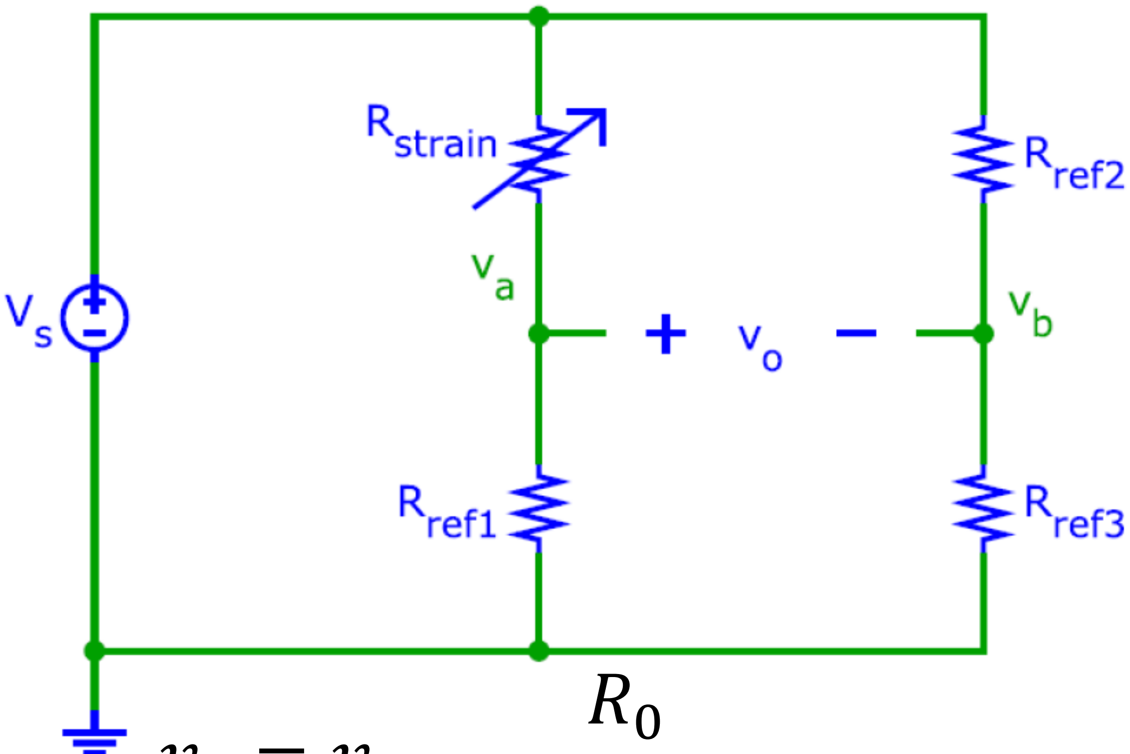
- This will work, but:
  - $V_x \neq 0$ for $\varepsilon = 0$
  - As source voltage varies (e.g. battery gets old), calibration changes – “zero drift”

Improvement #1: Half Bridge



Works, but requires balanced sources

Improvement #2: Full Bridge



$R_{strain} = R_o + \Delta R$

$R_{ref1} = R_{ref2} = R_{ref3} = R_o$

Want v_0

Use voltage divider

$$v_a = v_s \frac{R_0}{R_0 + \Delta R + R_0}$$

$$v_b = v_s \frac{R_0}{R_0 + R_0} = \frac{v_s}{2}$$

Works, but requires resistors with value equal to R_0

Strain Gauge Summary

- We can map strain (weight) to resistance
- Simplest design (voltage divider) works, but is subject to zero-drift
- More complex circuits give different design tradeoffs
  - Wheatstone-bridge provides arguably the best design
- We will explore these tradeoffs in lab on Wednesday

Useful Resistive Circuit Summary

- The Wheatstone bridge (and other designs) provide us with a way to measure an unknown resistance
- There are resistors which vary with many useful parameters, e.g.
  - Incident light
  - Temperature
  - Strain
- And then… there are always toasters

Back to Circuit Analysis

- Next we'll discuss a few more circuit analysis concepts
  - Superposition
  - Equivalent Resistance
    - Deeper explanation of equivalent resistance
    - For circuits with dependent sources
  - Thevenin/Norton Equivalent Circuits
  - Simulation

Superposition

- Principle of Superposition:
- In any **linear** circuit containing multiple **independent** sources, the current or voltage at any point in the network may be calculated as the **algebraic sum** of the individual contributions of **each source acting alone**.
- A linear circuit is one constructed only of linear elements (linear resistors, and linear capacitors and inductors, linear dependent sources) and independent sources.
- Linear means I-V characteristic of all parts are straight when plotted

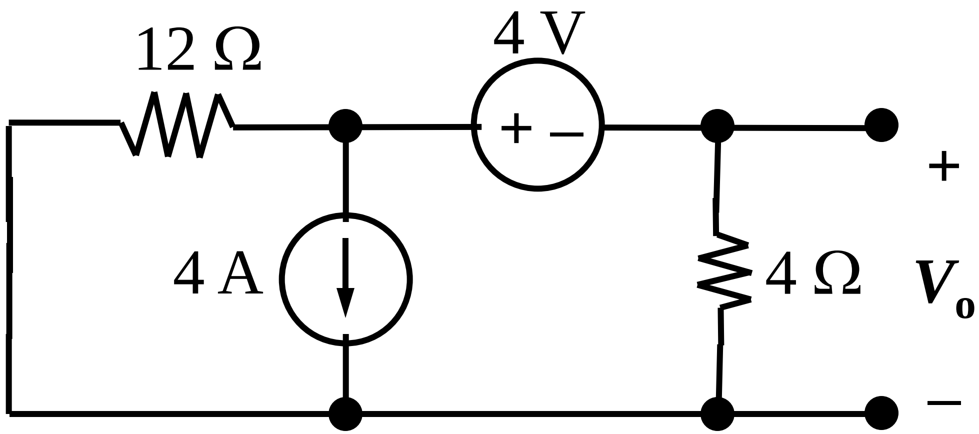
Superposition

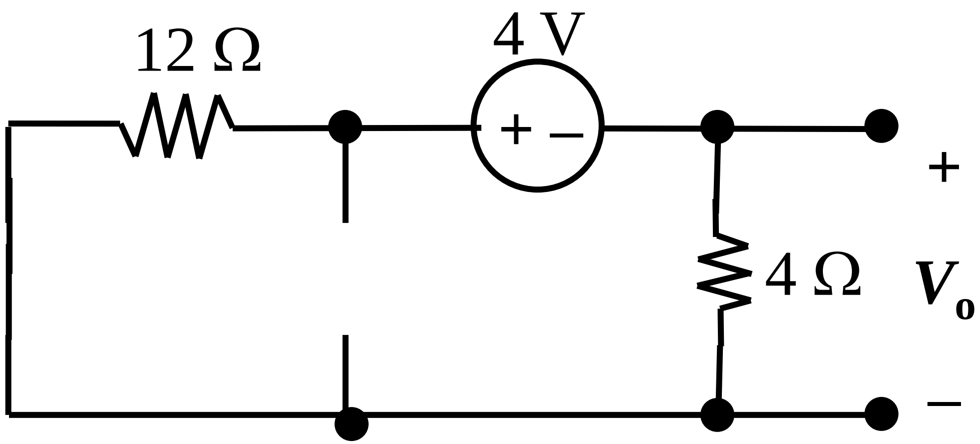
Procedure:

1. Determine contribution due to one independent source
   - Set all other sources to 0:
     - Replace independent voltage source by short circuit
     - independent current source by open circuit
2. Repeat for each independent source
3. Sum individual contributions to obtain desired voltage or current

Easy Example

- Find V_o

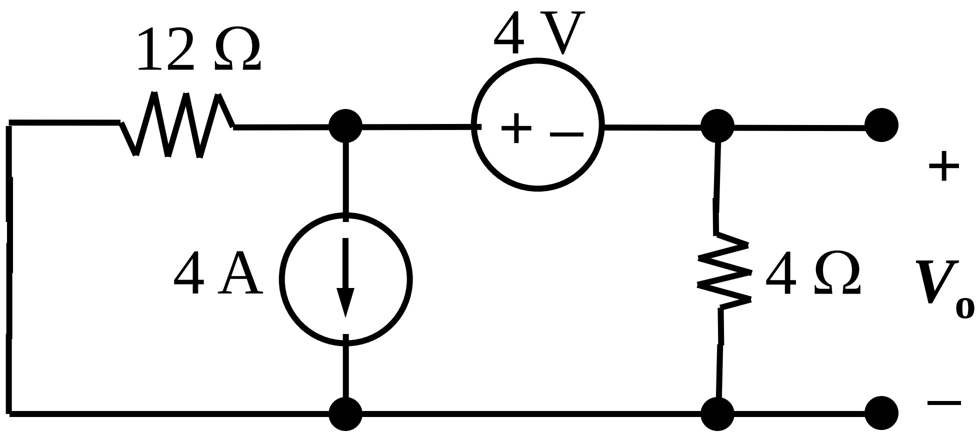


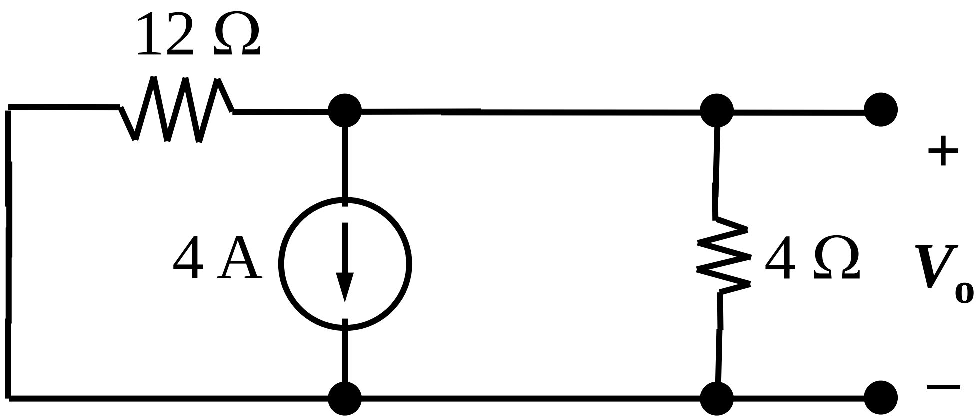


Voltage Divider: -1V

Easy Example

- Find V_o

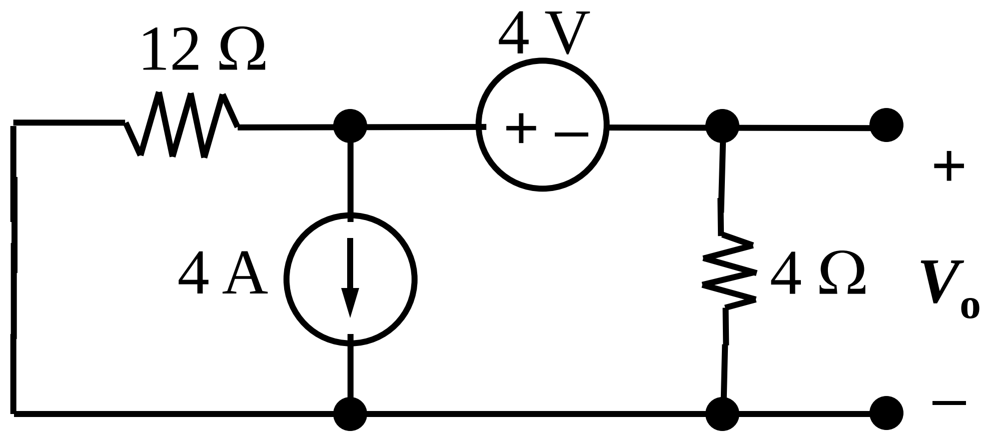




Current Divider: -(3A*4Ω)=-12V

Easy Example

- Find V_o

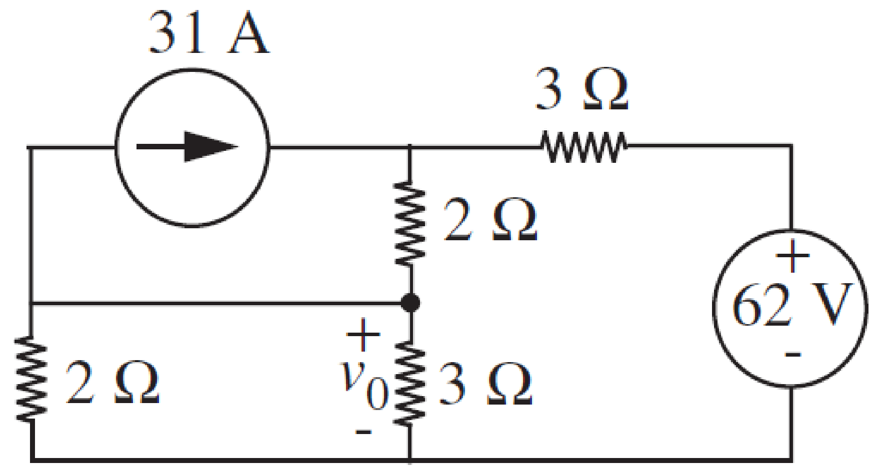


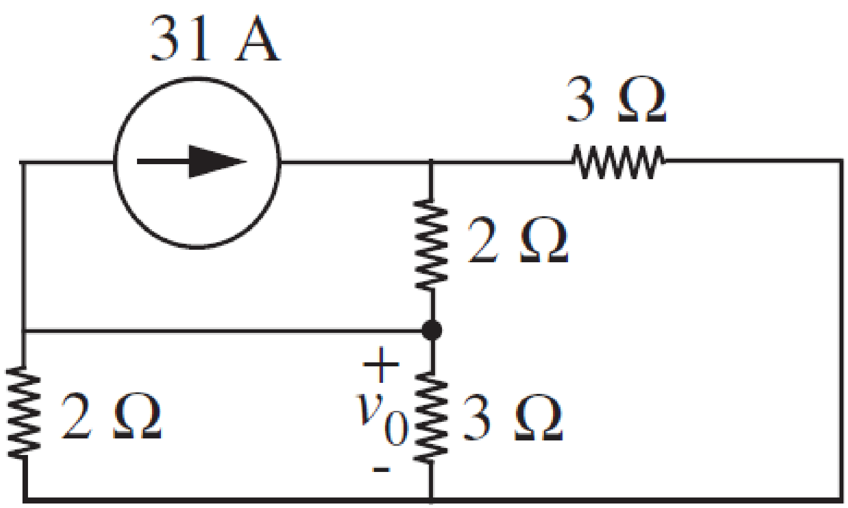
V_0=-12V-1V=-13V

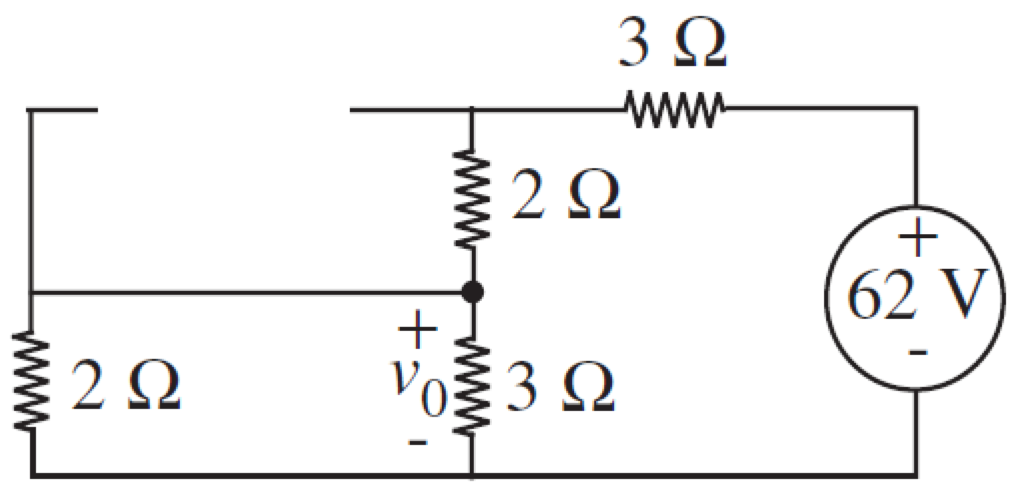
Due to current source

Due to voltage source

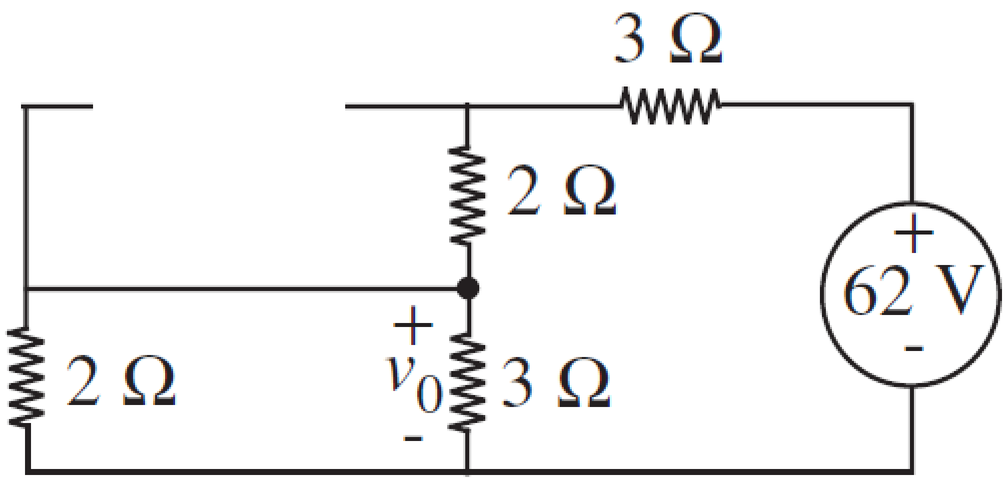
Hard Example







Example



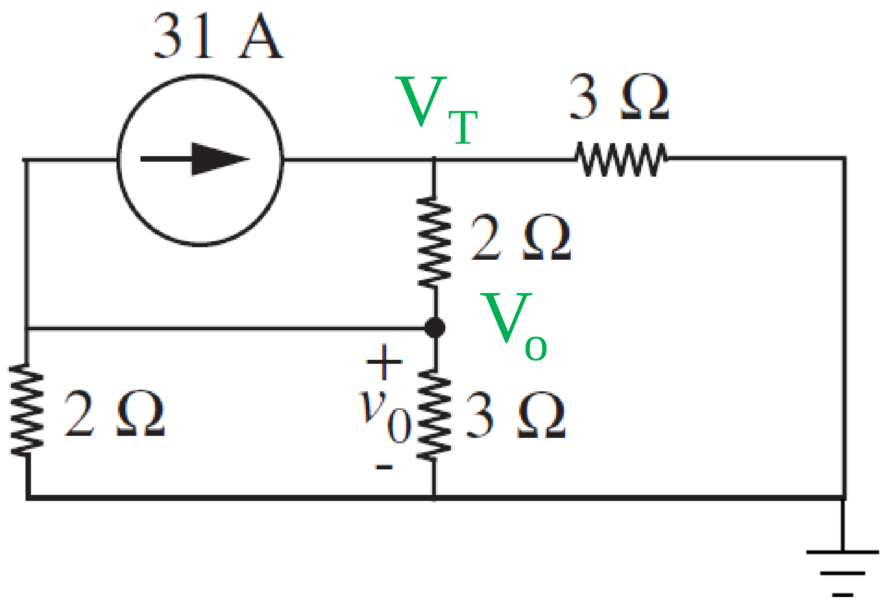
Equivalent resistance:

$3+2+(3||2) = 5+(3*2)/(3+2)=6.2V$

Current is 10A

50V loss through top 5Ω, leaving 12V across v_0

Example



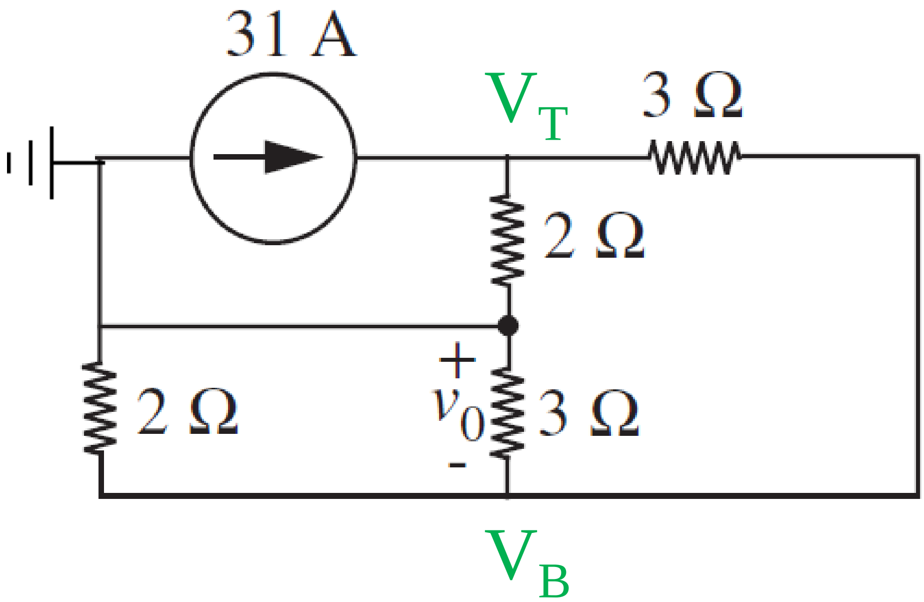
$$\frac{V_o}{3} + \frac{V_o}{2} + 31 + \frac{V_o - V_T}{2} = 0$$

$$-31 + \frac{V_T - V_o}{2} + \frac{V_T}{3} = 0$$

This will work…

But algebra is easier if we pick a better ground

Example

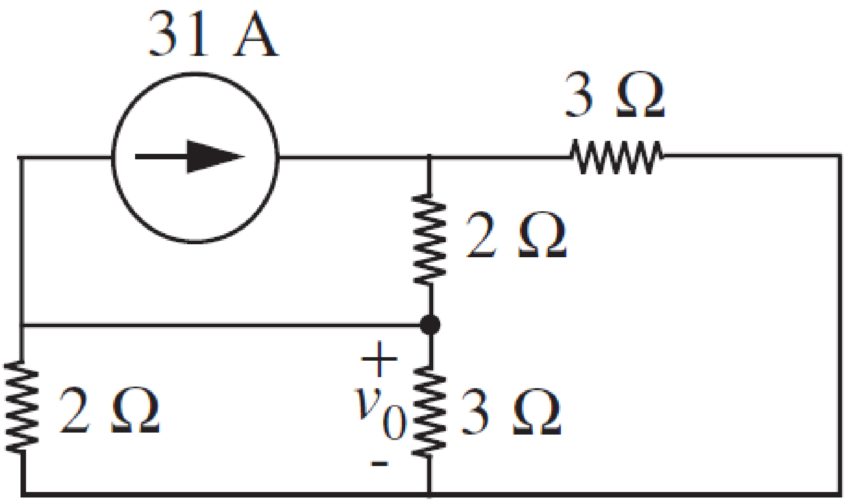


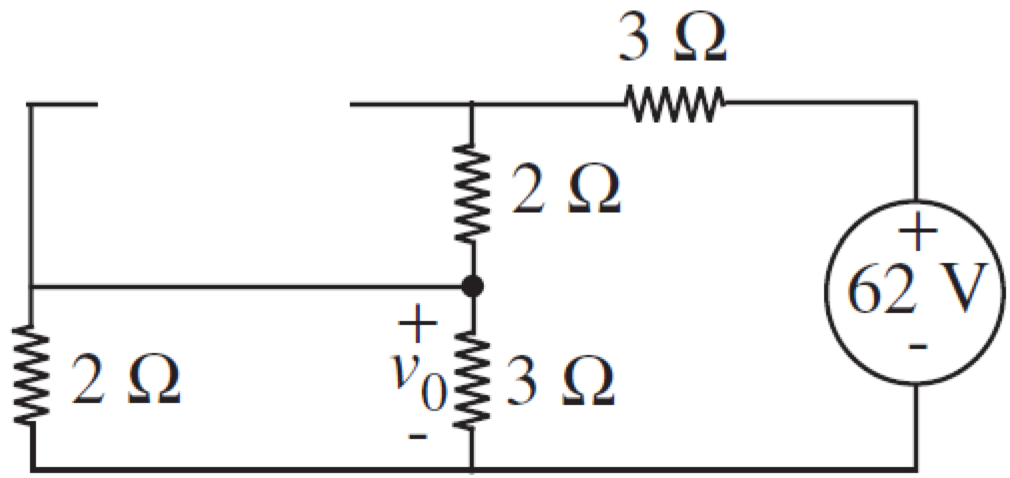
$$\frac{V_B}{2} + \frac{V_B}{3} + \frac{V_B - V_T}{3} = 0 \longrightarrow V_B = \frac{2}{7} V_T$$

$$-31 + \frac{V_T}{2} + \frac{(V_T - V_B)}{3} = 0 \longrightarrow V_B = 12V$$

$$V_o = -12V$$

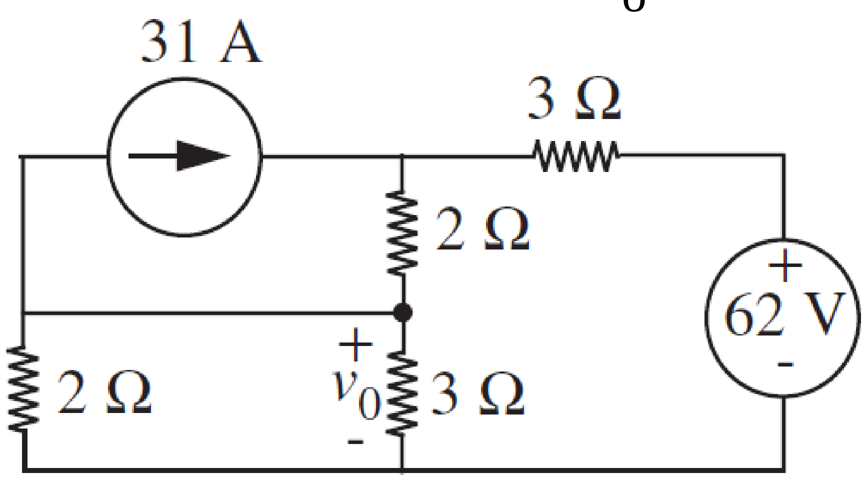
Example





$V_o = -12V$

$V_o = 12V$



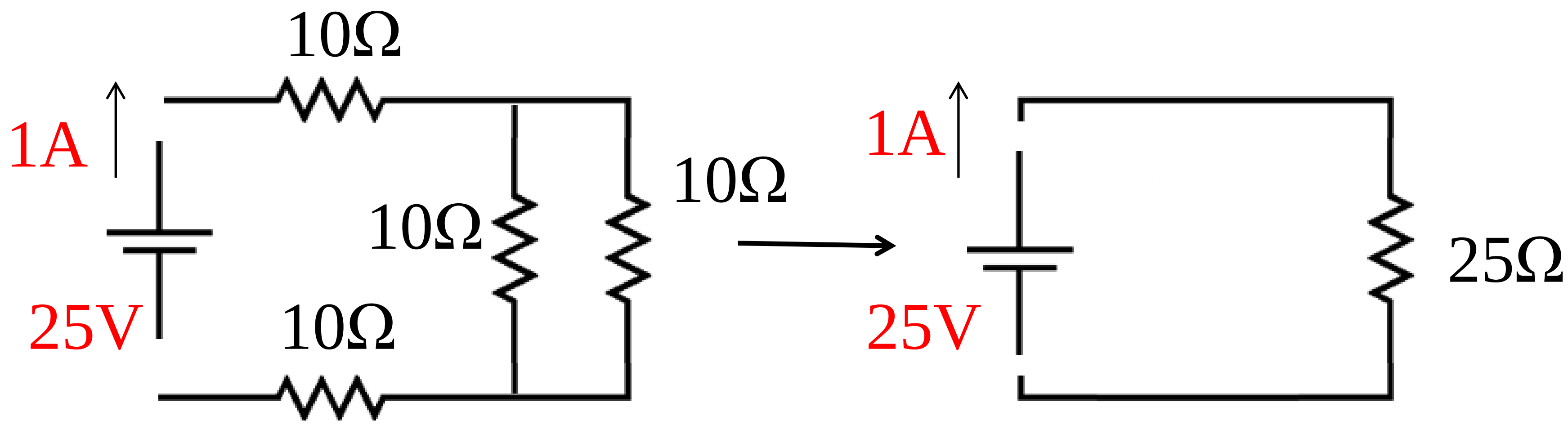
$$V_o = -12V + 12V = 0V$$

Note on Dependent Sources

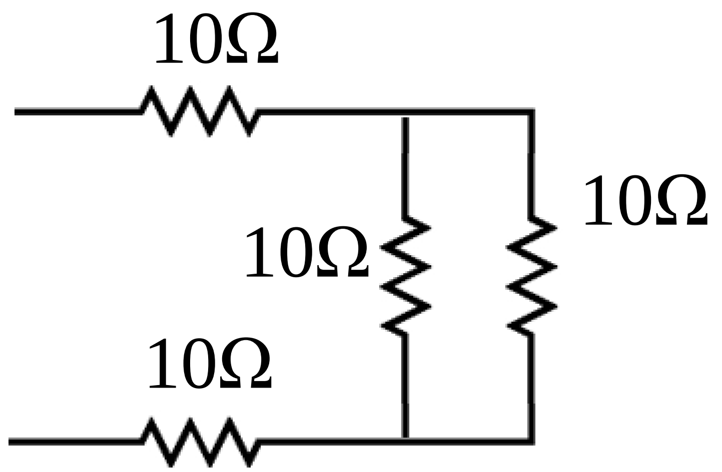
- You can use superposition in circuits with dependent sources
- However, DON'T remove the dependent sources! Just leave them there.

Equivalent Resistance Review

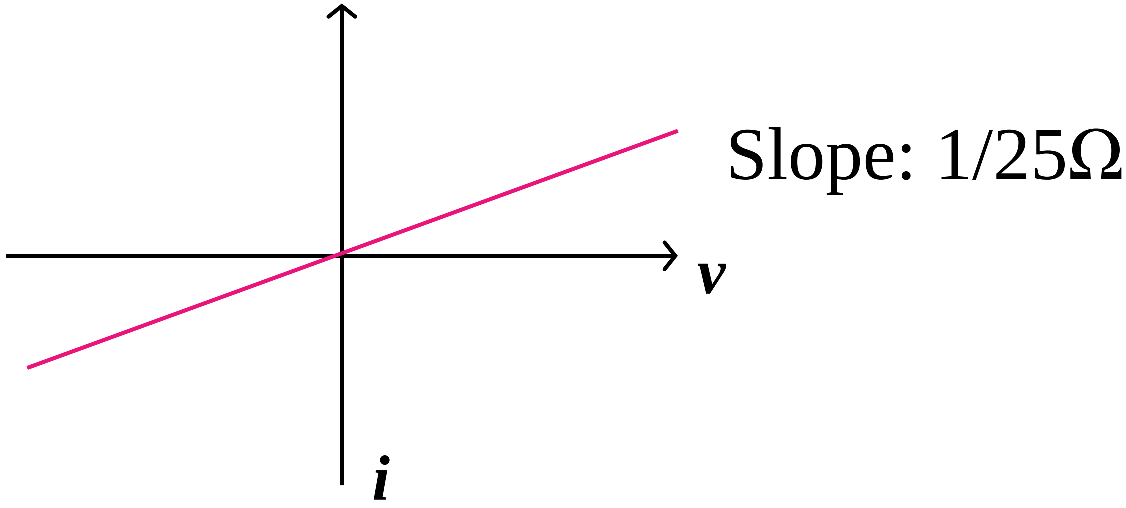
- If you add a source to any two terminals in a purely resistive circuit
  - The added source will "see" the resistive circuit as a single resistor



Alternate Viewpoint

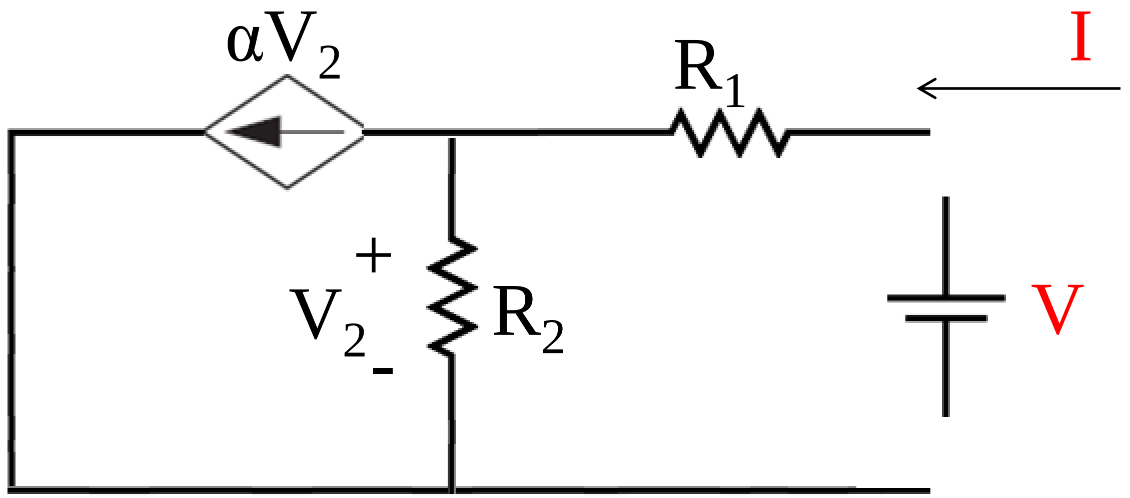


We can think of the circuit above as a two terminal circuit element with an I-V characteristic



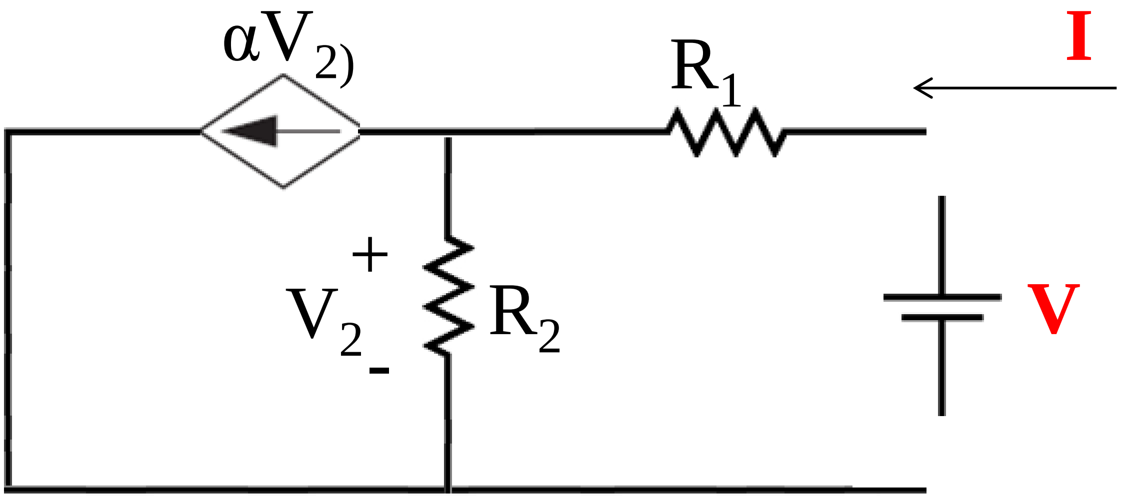
Equivalent Resistance

- Let's consider the IV characteristic of the following circuit:



$$\frac{V_2 - V}{R_1} + \frac{V_2}{R_2} + \alpha V_2 = 0$$

Equivalent Resistance



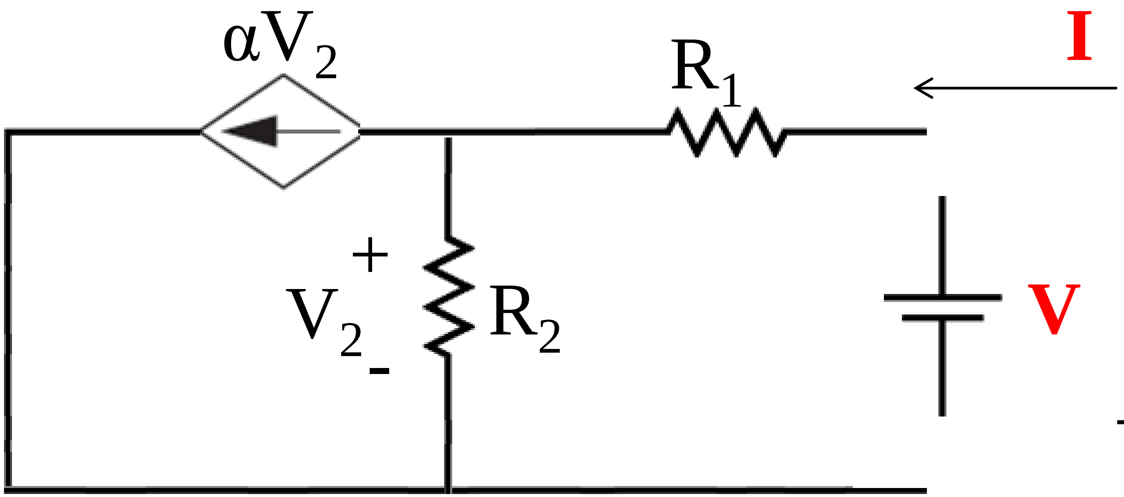
$$\frac{V_2 - V}{R_1} + \frac{V_2}{R_2} + \alpha V_2 = 0$$

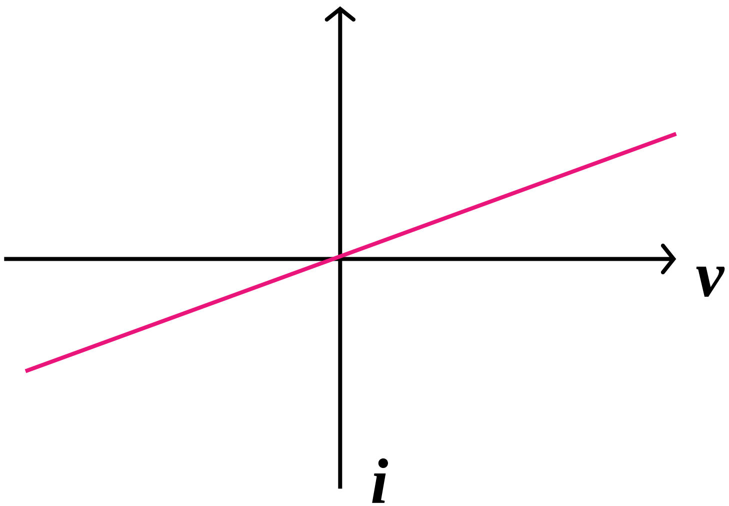
$$V_2 = \frac{R_2 V}{R_1 + R_2 + \alpha R_1 R_2}$$

$$I = \frac{V - \frac{R_2 V}{R_1 + R_2 + \alpha R_1 R_2}}{R_1}$$

$$\frac{I}{V} = \frac{1 + \alpha R_2}{R_1 + R_2 + \alpha R_1 R_2}$$

Equivalent Resistance





$$\frac{I}{V} = \frac{1 + \alpha R_2}{R_1 + R_2 + \alpha R_1 R_2}$$

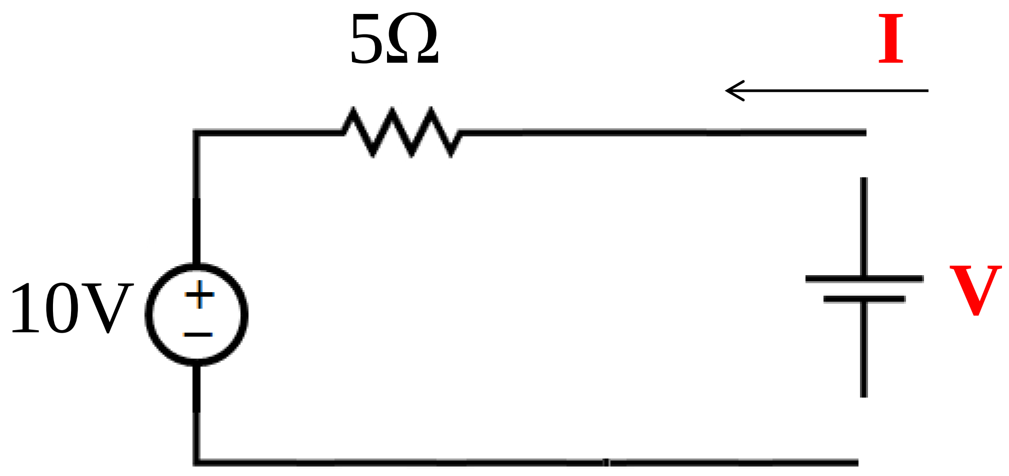
$$= 1/R_{eq}$$

Slope: $\frac{1+\alpha R_2}{R_1+R_2+\alpha R_1 R_2}$

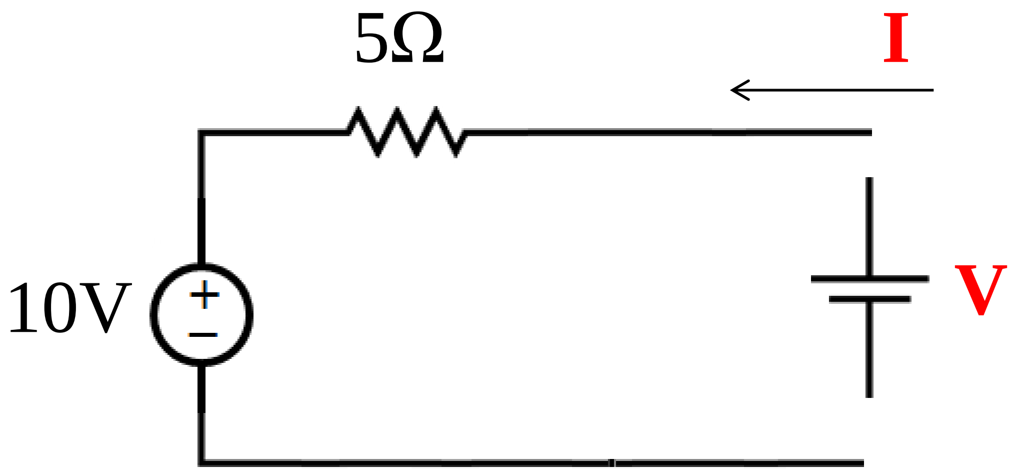
This circuit just acts like a resistor!

Equivalent Resistance Summary So Far

- Purely resistive networks have an I-V characteristic that looks just like their equivalent resistance
- **Purely resistive networks which also include dependent sources also act like resistors**
- Let's see what happens with a circuit with an independent source

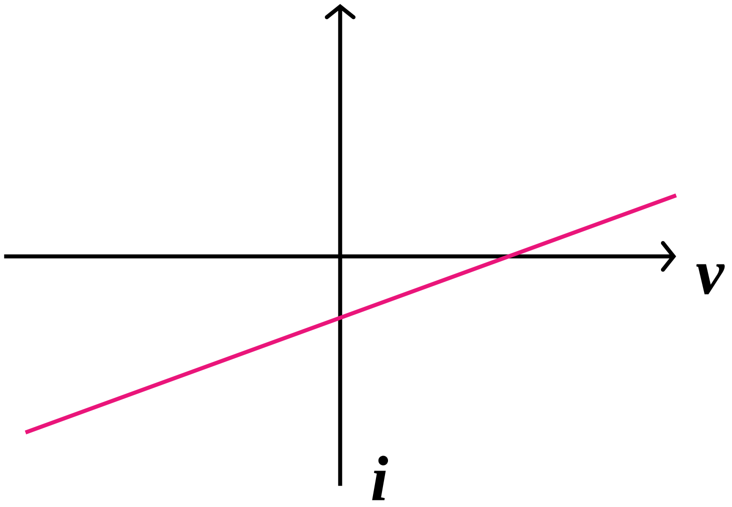


Equivalent Resistance Summary So Far

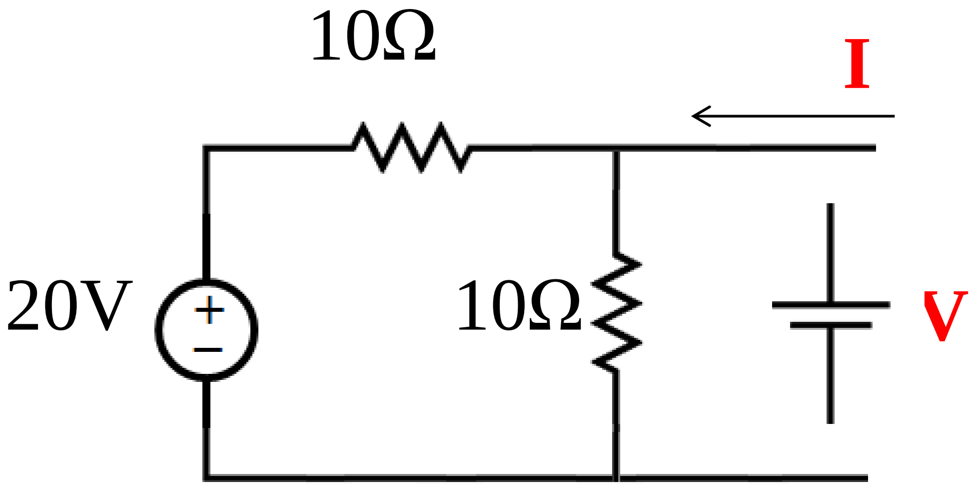


$$I = \frac{V - 10}{5} = \frac{V}{5} - 2$$

Doesn't match our basic I-V characteristics… good!

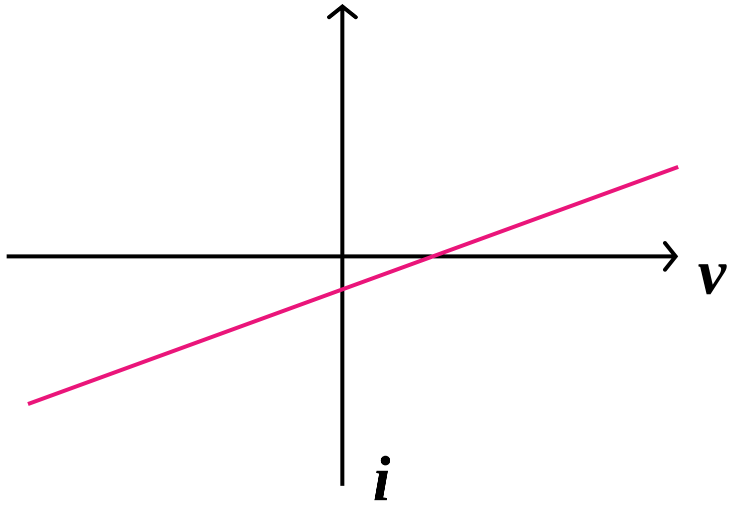


Interestingly…

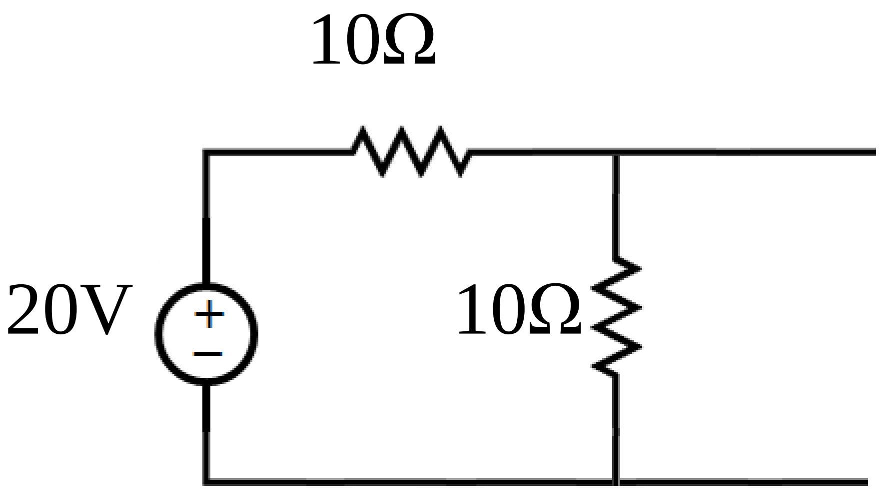


$$I = \frac{V - 20}{10} + \frac{V}{10}$$

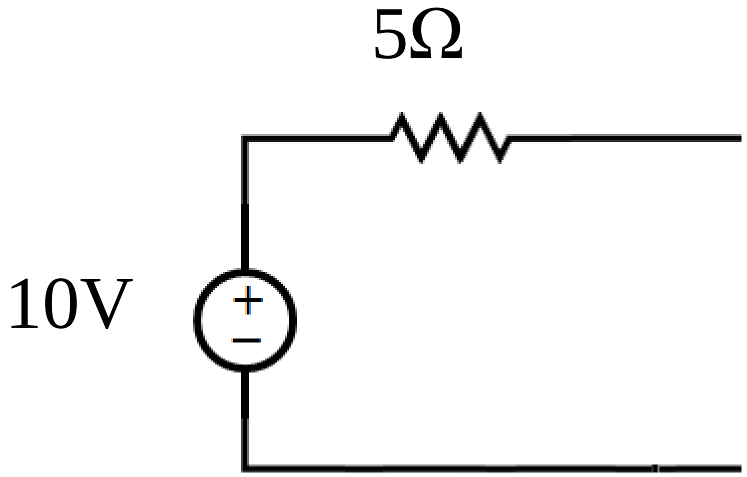
$$= \frac{V}{5} - 2$$



Equivalent Circuits



Has the exact same I-V characteristic as:

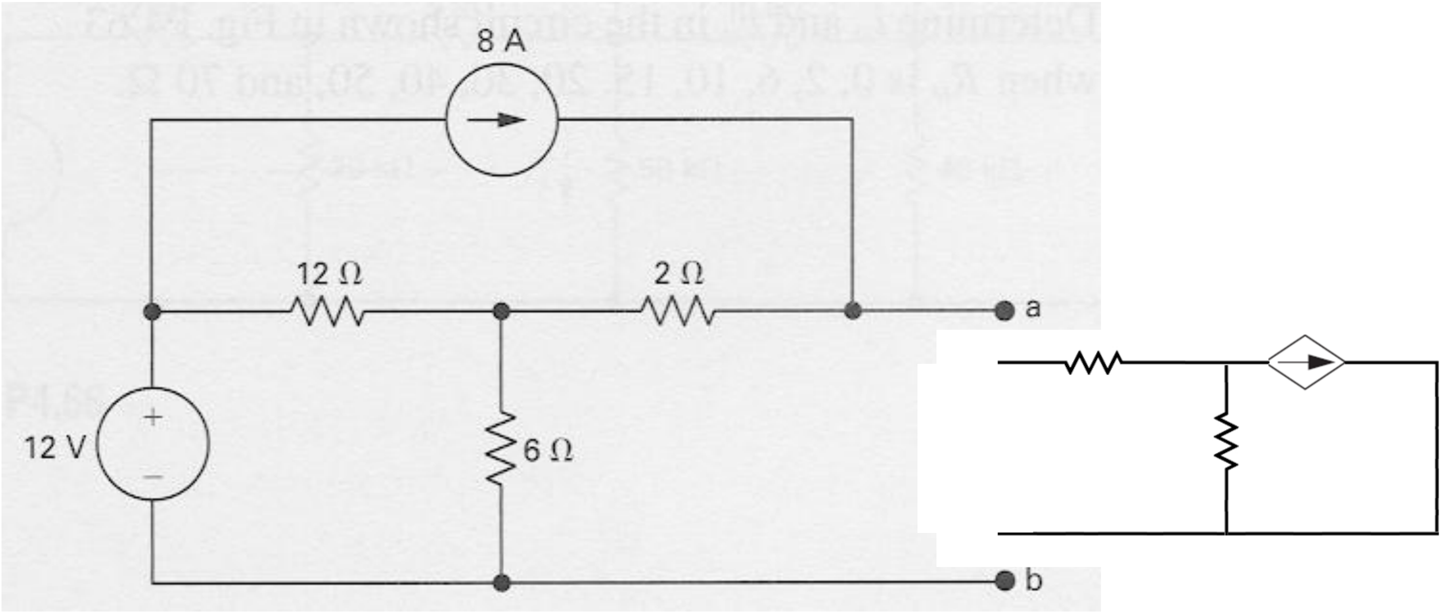


Thevenin Equivalents

- We saw before that we can replace a network of resistors (and dependent sources) with a single equivalent resistance
- Now, we have that **we can replace any circuit we can build so far with a single voltage source and resistor**
  - Not proven, but it's true, trust me
- This two element network is known as a **Thevenin equivalent**
- Generalization of the idea of equivalent resistance

Again: Discovered twice, named after the second guy!

Why is this useful?



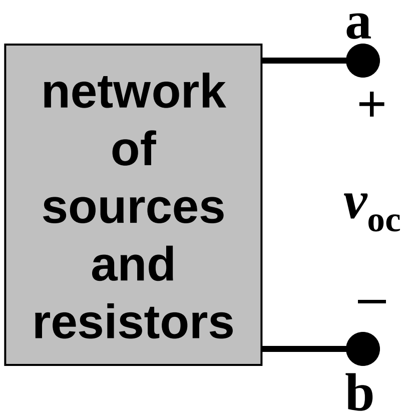
- Can swap out elements and not have to resolve a big circuit again
- Captures the fundamental operation of the circuit as a whole (at chosen two terminals only!)

Thevenin Algorithm for Independently Sourced Circuits

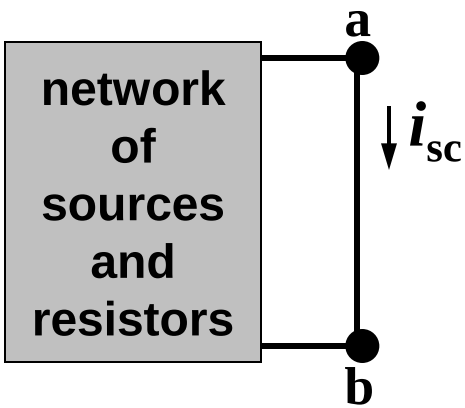
- What you're ultimately doing is finding the I-V characteristic of the circuit
- You can do this by attaching a made up V, and calculating I as on slides 49 and 50
  - Often called a "Test Voltage"
- This is equivalent to:
  - Finding the "open circuit" voltage
  - Finding the "short circuit" current

Calculating a Thévenin Equivalent

1. Calculate the ***open-circuit voltage***, v_{oc}



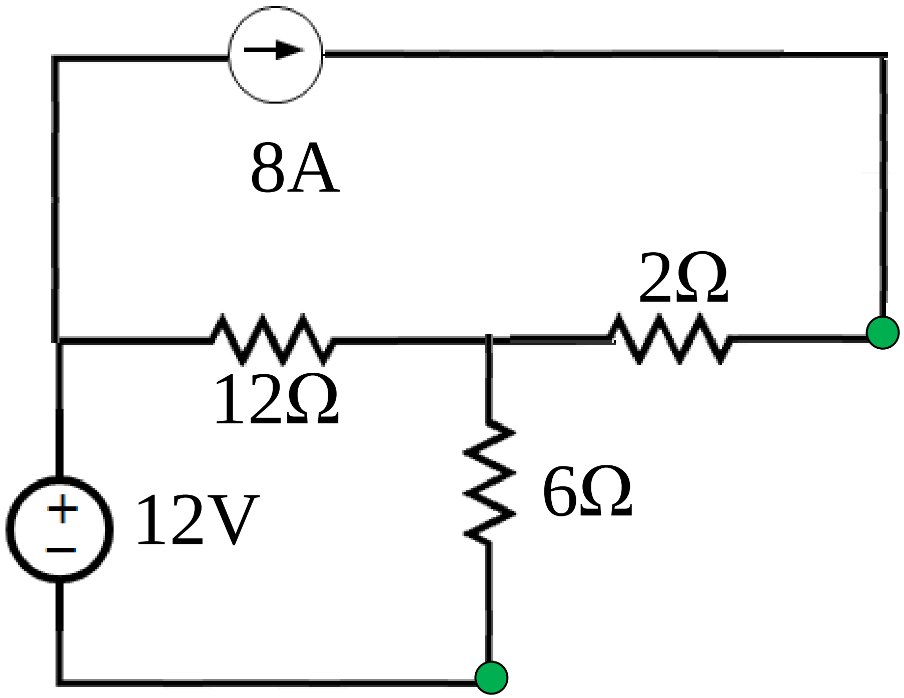
2. Calculate the ***short-circuit current***, i_{sc}
   - Note that i_{sc} is in the direction of the open-circuit voltage drop across the terminals a,b !



$$V_{Th} = v_{oc}$$

$$R_{Th} = \frac{v_{oc}}{i_{sc}}$$

Example – On the Board



Find the Thevenin Equivalent circuit, by finding

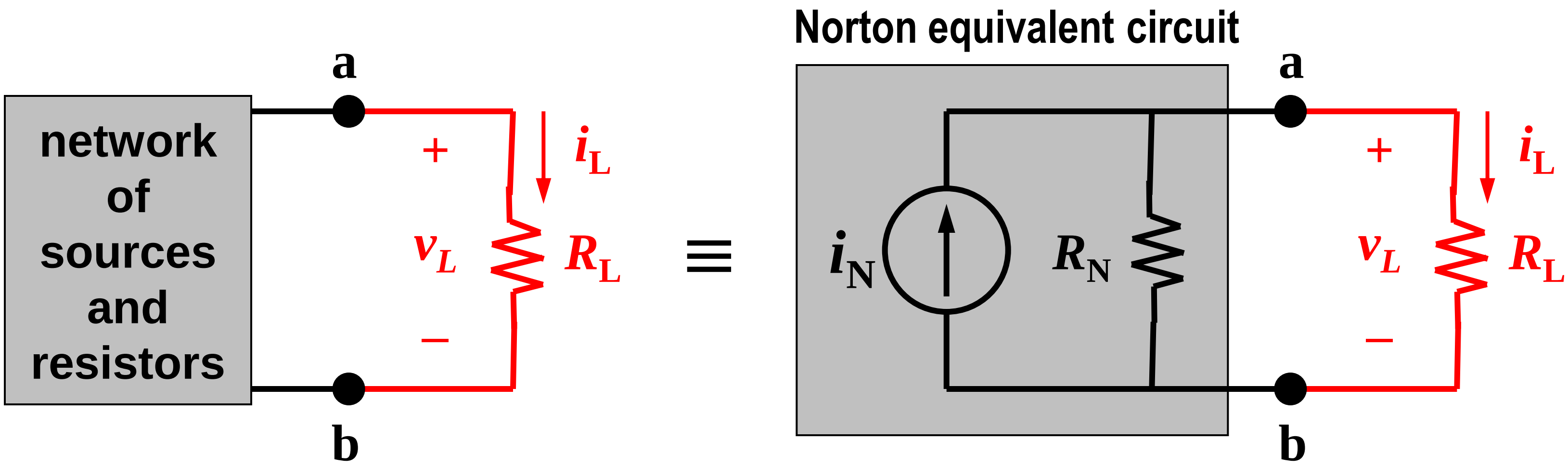
1. V_{OC}
2. I_{SC}

Finding Thevenin Resistance Directly

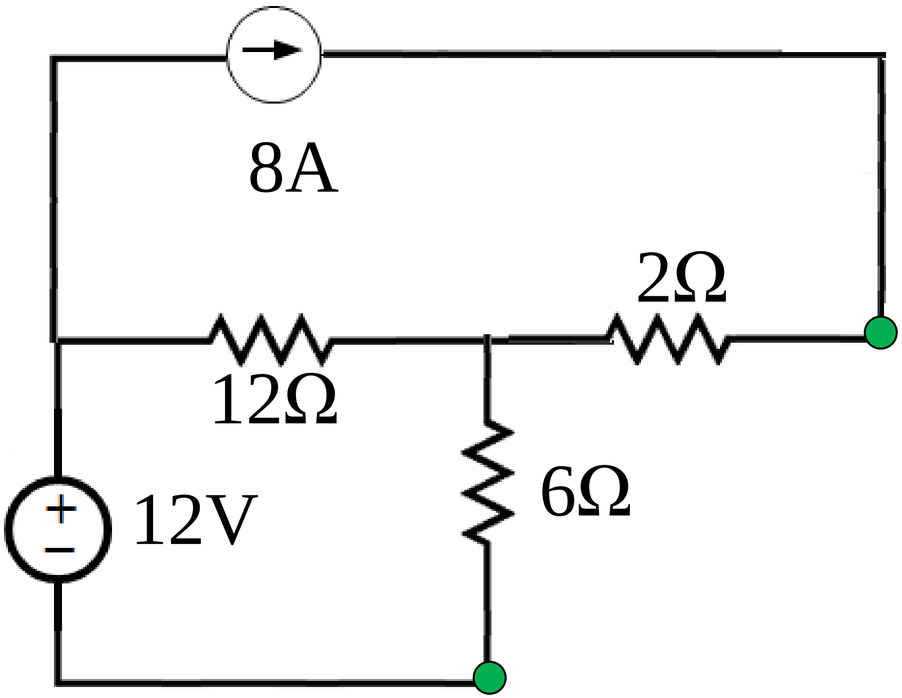
- If there are no dependent sources in the circuit, we can find the Thevenin Resistance directly
- Algorithm is easy:
  – Set all **independent** sources to zero
    - Voltage source becomes short circuit
    - Current source becomes open circuit
  – Leave dependent sources intact
  – Find equivalent resistance between terminals of interest

Norton Equivalent Circuit

- Any network of voltage sources, current sources, and resistors can also be replaced by an equivalent circuit consisting of an independent current source in parallel with a resistor without affecting the operation of the rest of the circuit.



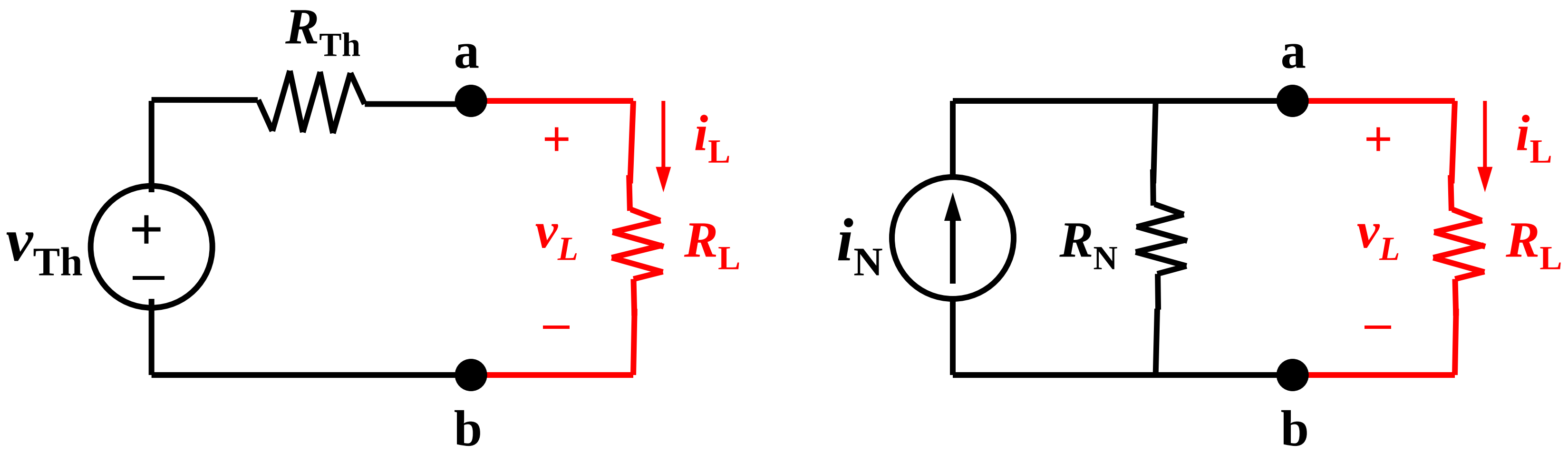
Example – On the Board



Find the Thevenin Equivalent resistance directly

Finding I_N and R_N

- We can derive the Norton equivalent circuit from a Thévenin equivalent circuit simply by making a source transformation:



$$i_L = \frac{v_{Th}}{R_{Th} + R_L}$$

$$i_L = \frac{R_N}{R_N + R_L} i_N$$

$$R_N = R_{Th} = \frac{v_{oc}}{i_{sc}}; \quad i_N = \frac{v_{Th}}{R_{Th}} = i_{sc}$$

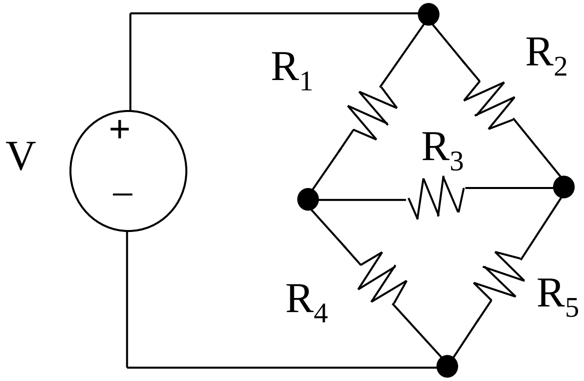
Circuit Simulation

- Automated equation solvers use our algorithm:
  - Choose a ground node
  - Assign node voltage labels to all nodes
  - Write out a system of N-1 linear equations
  - Solve (using standard linear algebra techniques)
- One pretty handy tool is falstad.com's circuit simulator
- Let's try a live demo

Extra Slides

Delta-Wye Conversion

Equivalent Resistances



Are there any circuit elements in series? parallel?

No

Are there any circuit elements in parallel?

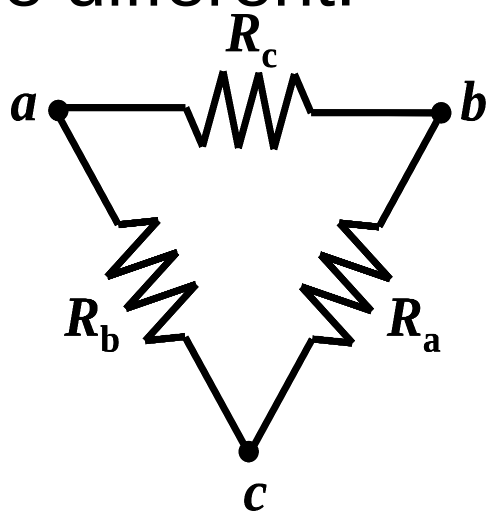
No

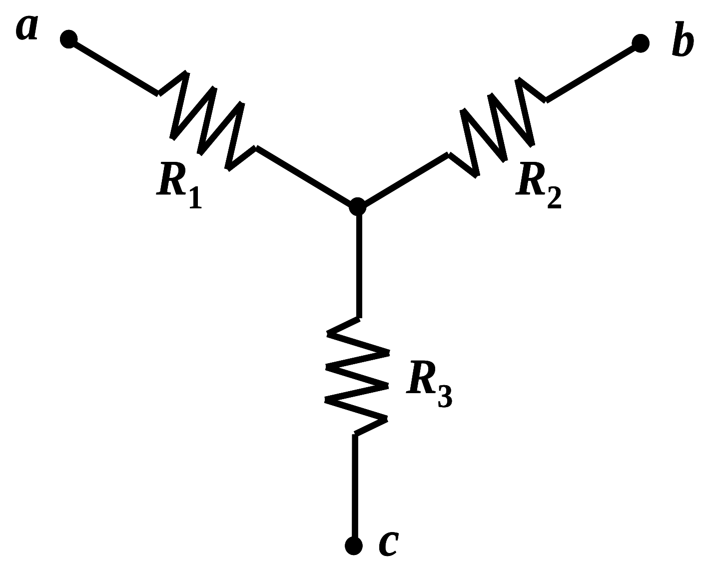
What do we do?

1. Be clever and find I-V characteristic directly
2. Apply weirder transformation rules than series or parallel

Y-Delta Conversion

- These two resistive circuits are equivalent for voltages and currents external to the Y and Δ circuits. Internally, the voltages and currents are different.





$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \qquad R_2 = \frac{R_a R_c}{R_a + R_b + R_c} \qquad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

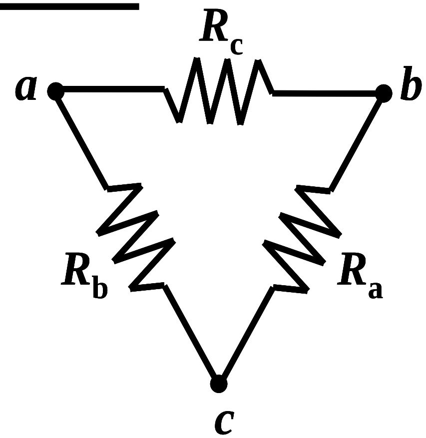
Δ-Y and Y-Δ Conversion Formulas

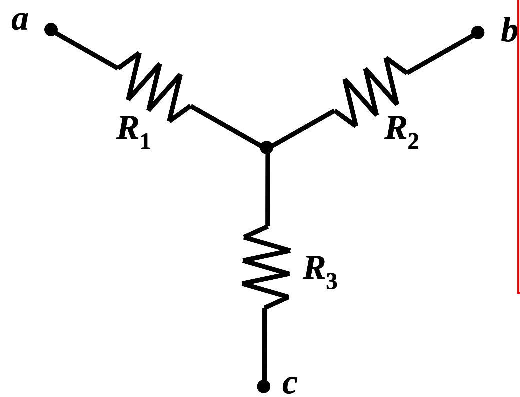
Delta-to-Wye conversion

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$





Wye-to-Delta conversion

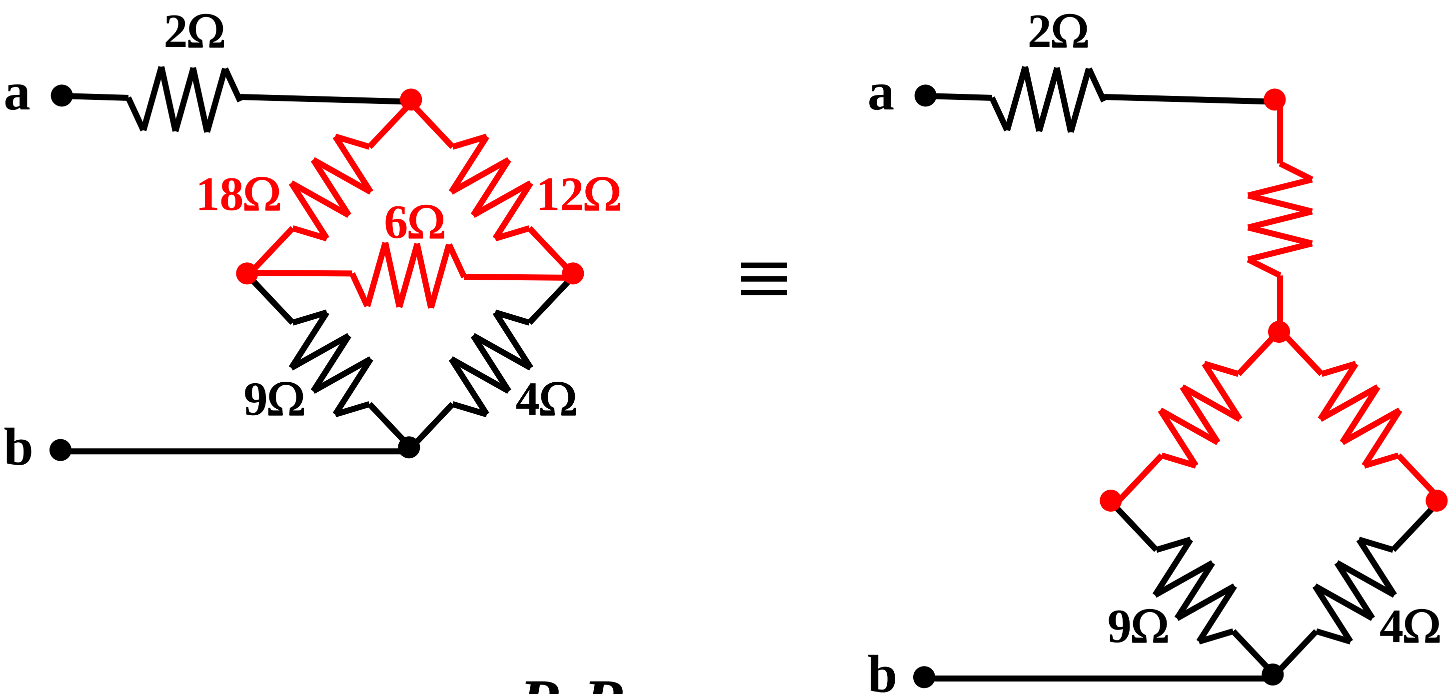
$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

Circuit Simplification Example

Find the equivalent resistance R_{ab}:



$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = 6$$

R_b=18

R_a=12

R_c=6

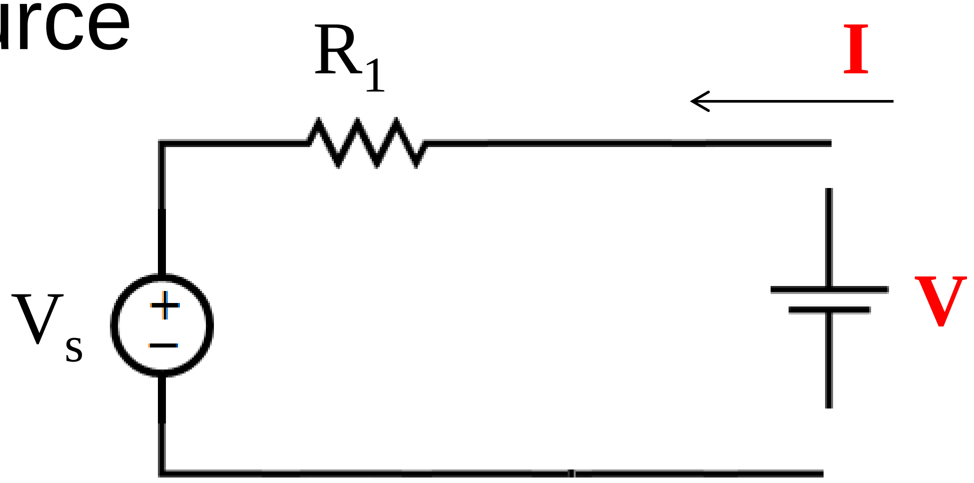
3

R2=2

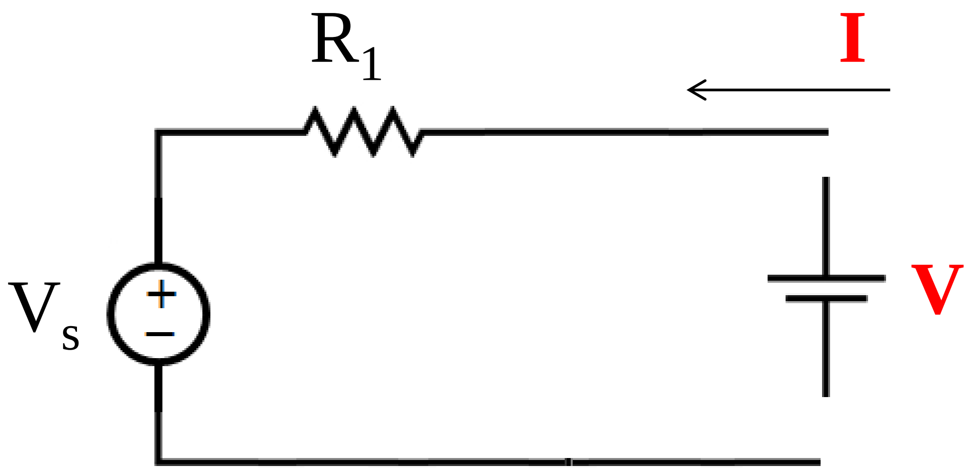
General Versions of Thevenin Slides

Equivalent Resistance Summary So Far

- Purely resistive networks have an I-V characteristic that look just like their equivalent resistance
- Networks which include dependent sources also act like resistors
- Let’s see what happens with a circuit with a source

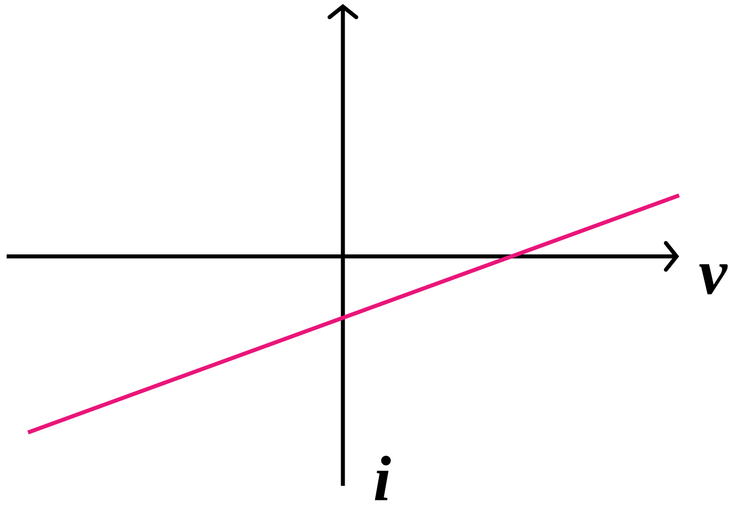


Equivalent Resistance Summary So Far

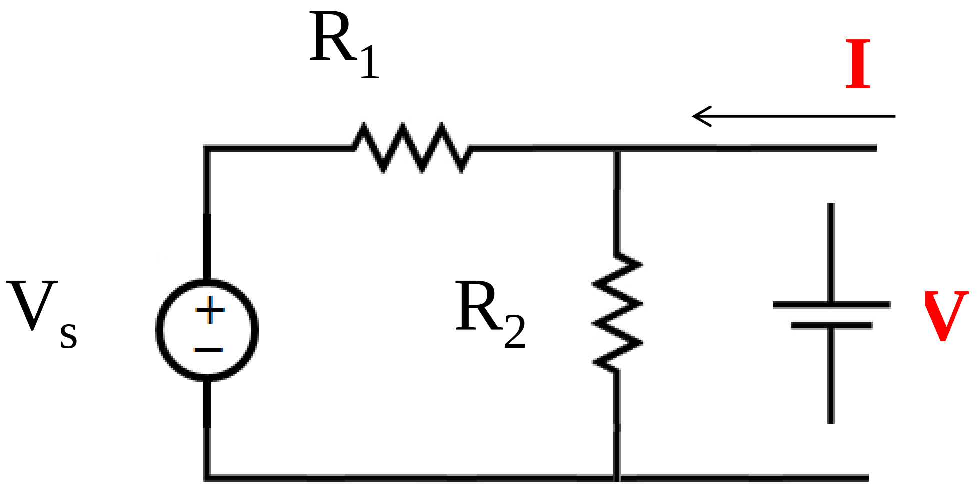


$$I = \frac{V - V_s}{R_1}$$

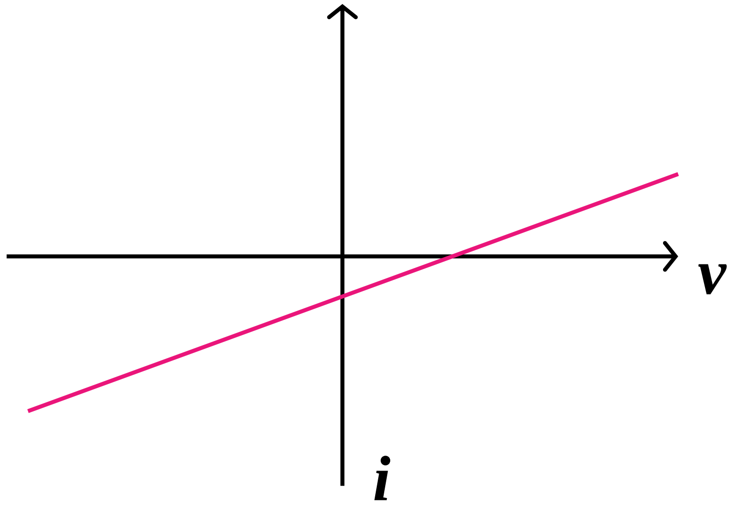
Doesn't match our basic I-V characteristics… good!

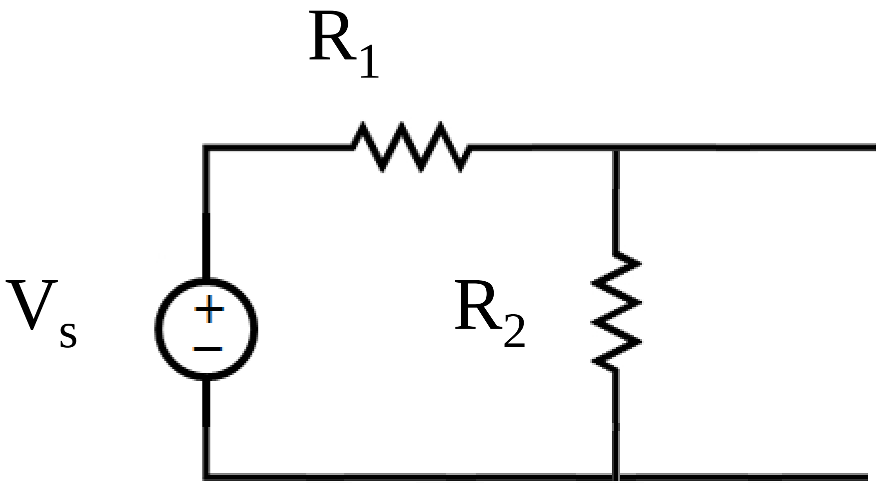


Interestingly…



$$I = \frac{V - V_s}{R_1} + \frac{V}{R_2}$$





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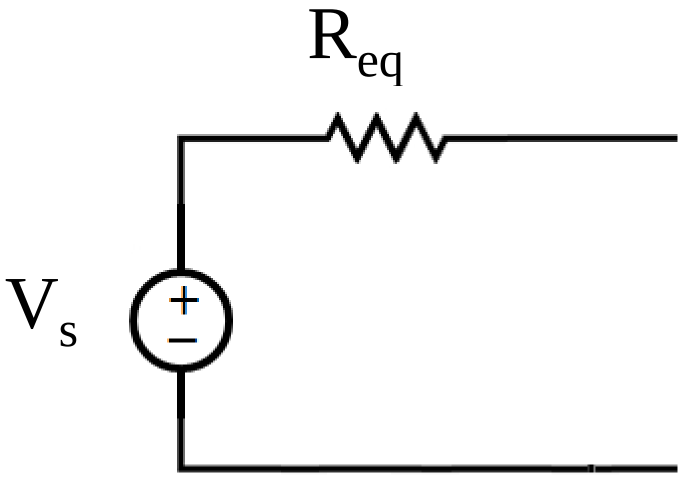

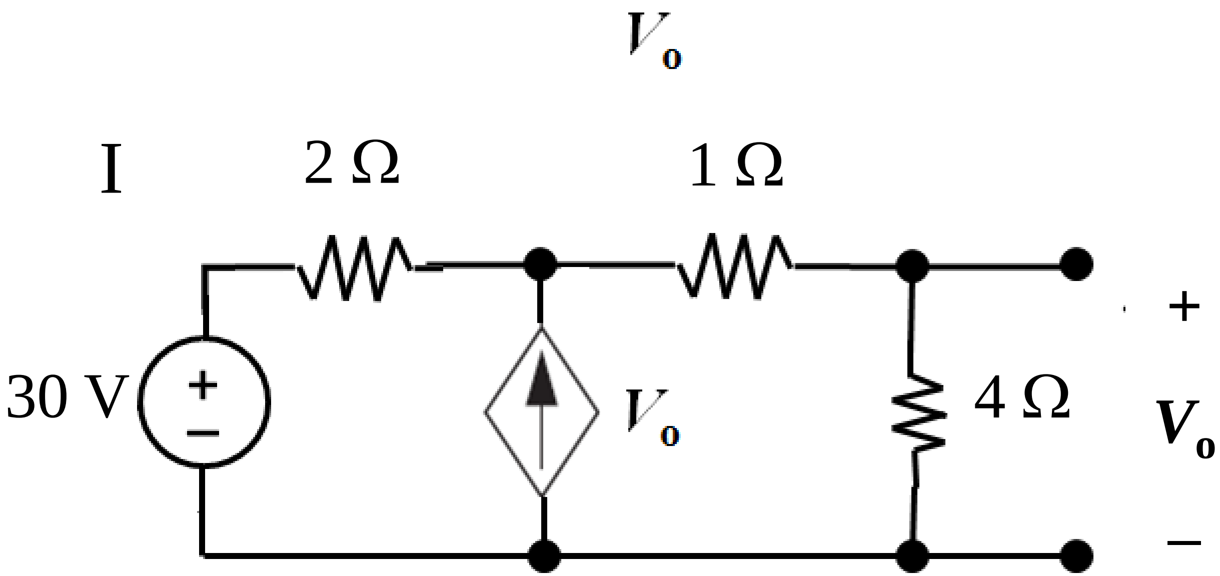
V_o
I
2 Ω
1 Ω
30 V
+
−
V_o
4 Ω
+
V_o
−