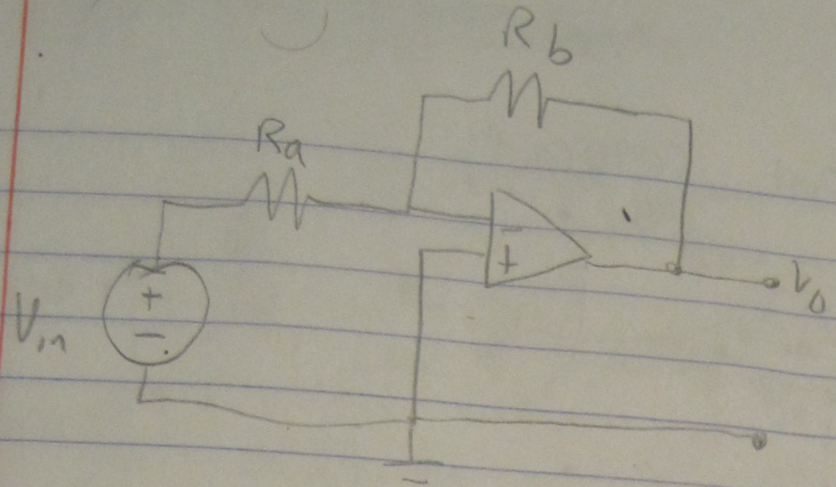
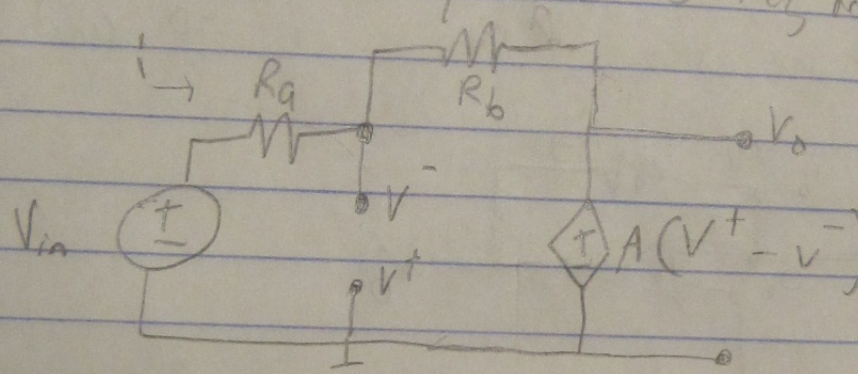


Goal, find $\frac{V_o}{V_{in}}$.

Two approaches!

(Slower approach, but familiar)

#1: Replace w/ dependent source, and use node voltage!



where $A > 1{,}000{,}000$

Get V^+ for free, is ground, $V^+ = 0$.

Get V_o for free, source connected to ground, $V_o = A(V^+ - V^-) = -AV^-$

or $V^- = -V_o/A$

Node V^-: $\frac{V^- - V_{in}}{R_a} + \frac{V^- - V_o}{R_b} = 0$

Substitute

$$\frac{-V_o/A - V_{in}}{R_a} + \frac{-V_o/A - V_o}{R_b} = 0$$

$$\frac{-V_o \cdot R_b}{A} - R_b \cdot V_{in} - \frac{V_o \cdot R_a}{a} - V_o \cdot R_a = 0$$

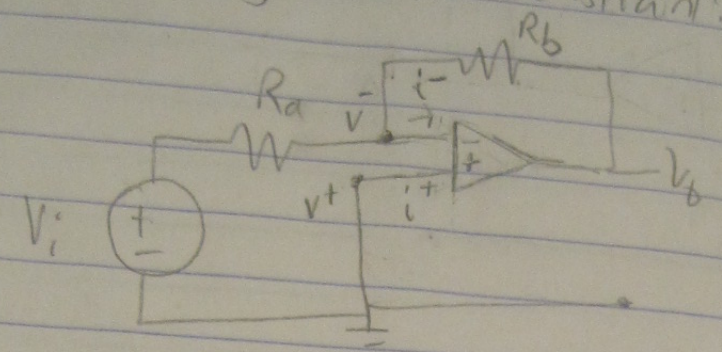
If A very large, then:

$$V_o\left(\frac{R_b}{A} + \frac{R_a}{A} + R_a\right) = -R_b \cdot V_{in} \qquad V_o = \frac{R_b V_{in}}{\frac{R_b}{A} + \frac{R_a}{A} + R_a}$$

$$\boxed{V_o = -\frac{R_b}{R_a} V_{in}}$$

Approach

#2: Or with Summing point constraint:

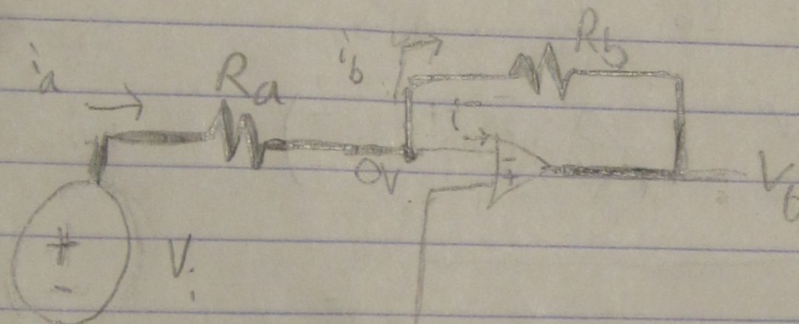


Summing point constraint gives us:

$$v^- = v^+$$

and v^+ is ground, so $v^- = v^+ = 0$.

Further, we also know $i^- = 0$ and $i^+ = 0$



Drew op-amp lightly here to emphasize that all it does is force v^- to zero and i^- and i^+ to zero.

$$i_a = i_b = \frac{V_i}{R_a}$$

$$V_o = 0 - i_b \cdot R_b = -\frac{R_b}{R_a} V_i$$

Done in 2 lines
No algebra.

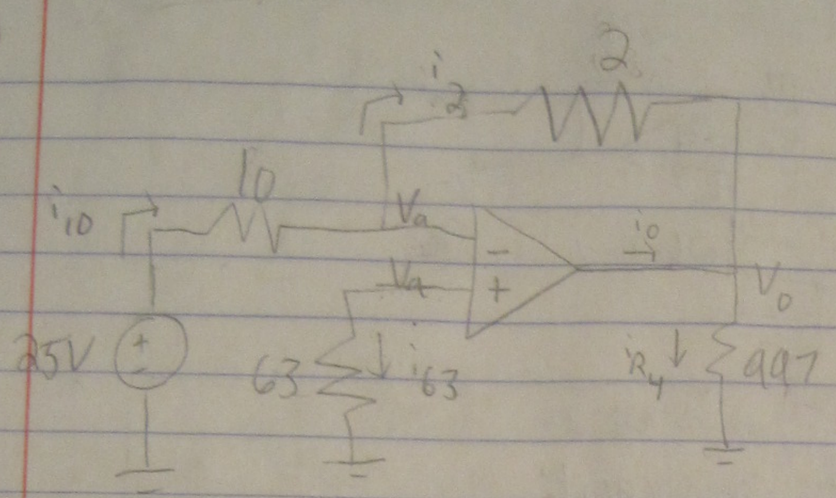
Also, $i_b = i_a$ because i^- is zero in case you were confused.

That was a lot easier, right?

God bless you, Harold Stephen Black.

Another Summing Point Constraint Example:

Find V_o



First, set

$V^- = V^+ = V_a$

Second, note that

$i_{10} = i_2$ and $i_{63} = 0$

Now can use node voltage if you don't want to think anymore, which will give you:

V^-: $\frac{V_a - 25}{10} + \frac{V_a - V_o}{2} = 0$

V^+: $\frac{V_a}{63} = 0$

Enough to solve, giving

$V_a = 0$

$V_o = -5\,V$

V_o: $\frac{V_o - V_a}{2} + \frac{V_o}{997} + i_o = 0$ — Just gives i_o... which we don't need.

Note, i_o is usually not zero! Function of the load!

Faster way:

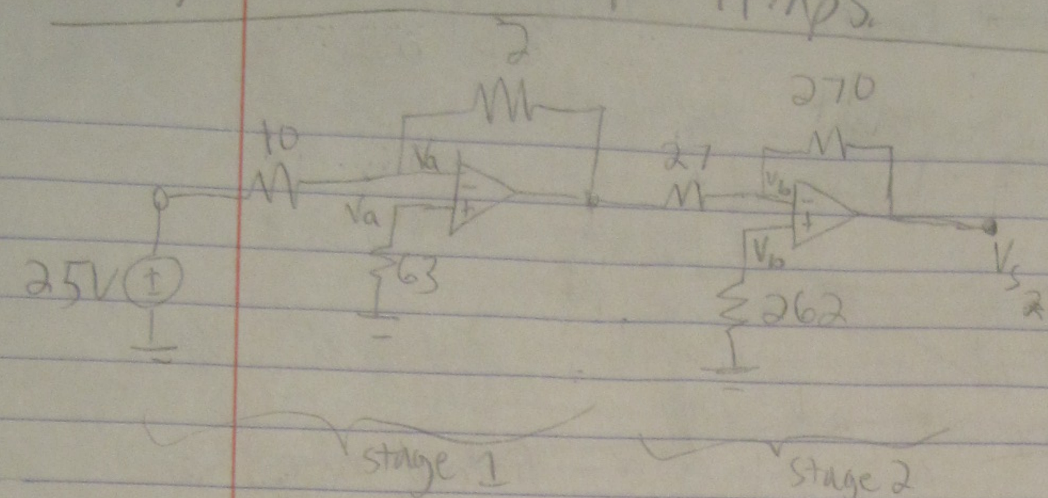
Just realize immediately that i_{63} is zero and so V_a must be zero.

Then realize that $i_{10} = i_2 = \frac{25V}{10\Omega} = 2.5\,A$, regardless of the 997Ω resistor or 2Ω resistor.

Then, $V_o = 0 - 2 \cdot 2.5 = -5V$.

This is pretty much what we did on the previous page.

Composition of Op-Amps



Summing point constraint:

because no current through 63Ω

$$V^-_{stage_1} = V^+_{stage_1} = V_a = 0$$

$V^-_{stage_1}$ is just the − terminal of the op-amp in Stage 1

$$i_{10} = i_2 = \frac{25V}{10\Omega} = 2.5A$$

$$V_{s_1} = -2.5A \cdot 2\Omega = -5V.$$

Seem familiar? Op-amps provide isolation! Doesn't matter what you connect them to!

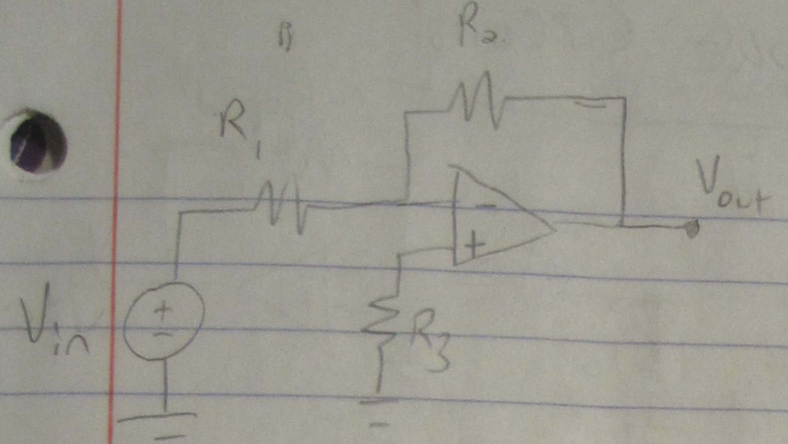
Continuing w/ Stage 2:

no current through 262Ω

$$V^-_{stage_2} = V^+_{stage_2} = V_b = 0$$

$$i_{27} = \frac{V_{s_1} - 0}{27} = \frac{-5V}{27\Omega}$$

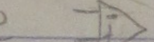
$$V_{s_2} = 0 - \frac{-5V}{27\Omega} \cdot 270\Omega = 50V$$

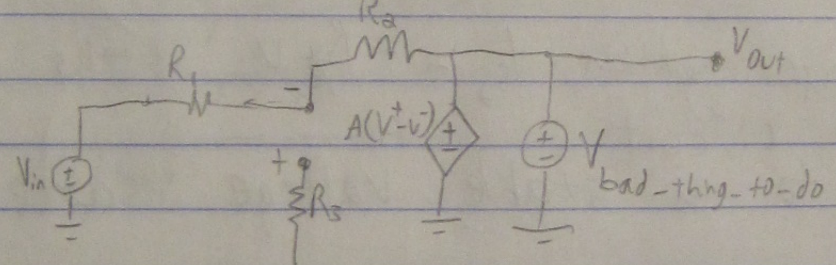


Earlier, we showed

$$V_{out} = -\frac{R_2}{R_1} \cdot V_{in}$$

It doesn't matter what kind of load we put at V_{out}, except that we shouldn't "cross the streams", i.e. connect a voltage source from V_{out} to ground.

Why? – because  – is a voltage source, and that would be 2 voltage sources in parallel:

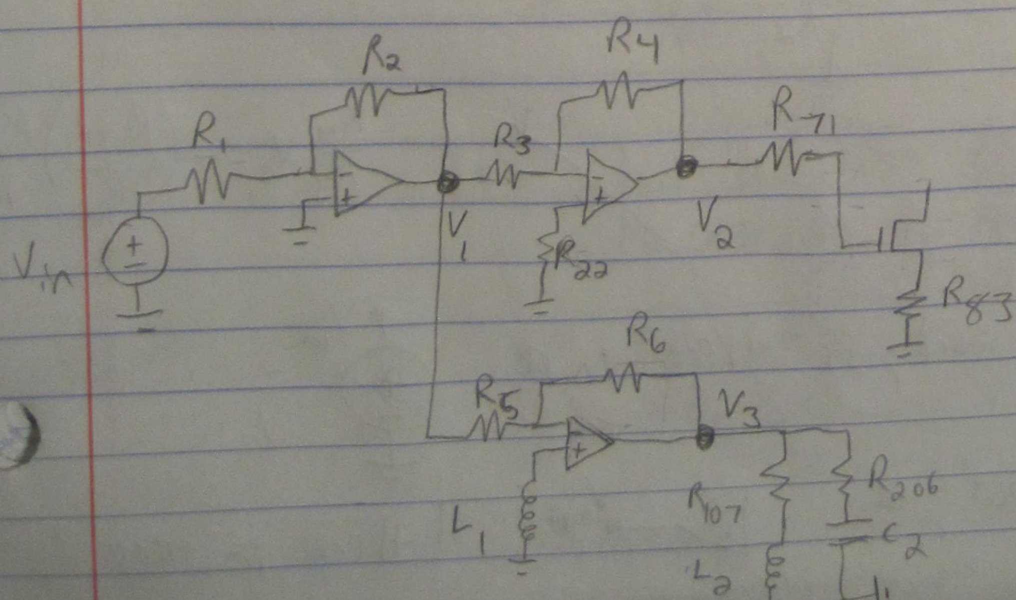


So what this means is we can analyze each stage individually, and that loads don't matter, i.e.:

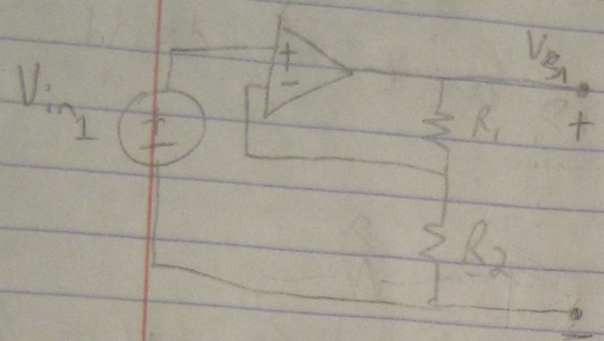
$$V_1 = -\frac{R_2}{R_1} \cdot V_{in}$$

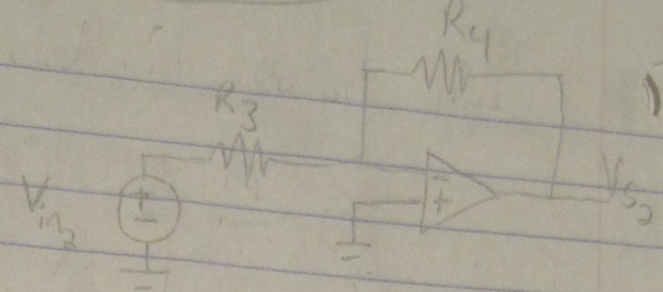
$$V_2 = \frac{R_4 \cdot R_2}{R_1 \cdot R_3} V_{in}$$

$$V_3 = \frac{R_6 \cdot R_2}{R_5 \cdot R_1} V_{in}$$



When is it safe to compose circuits?





$$V_{s_1} = V_{in_1}\left(\frac{R_2+R_1}{R_2}\right)$$

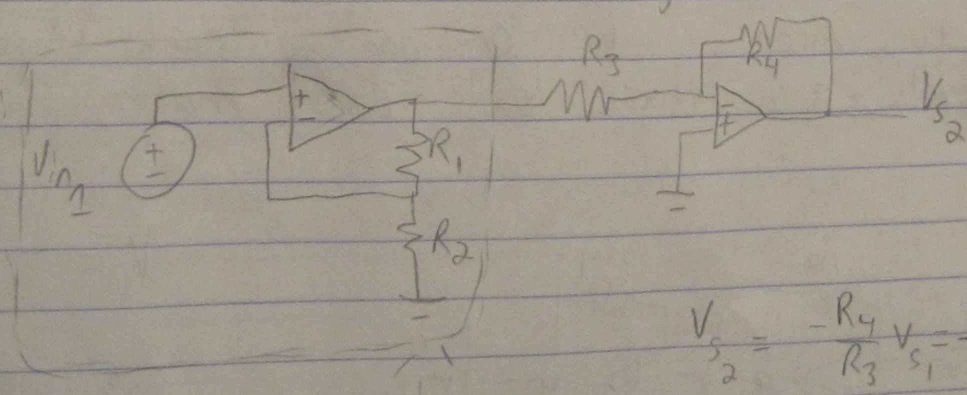
$$V_{s_2} = -\frac{R_4}{R_3}V_{in_2}$$

How do we know it's safe to compose these circuits?

Well our equation $V_{s_2} = -\frac{R_4}{R_3}V_{in_2}$ tells us how the circuit reacts to a stable voltage sarce.

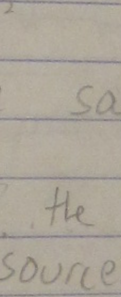
And what does our circuit on the left do? Provide a stable voltage source!

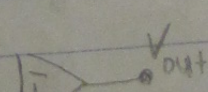
So:



Acts like V_{in_2}

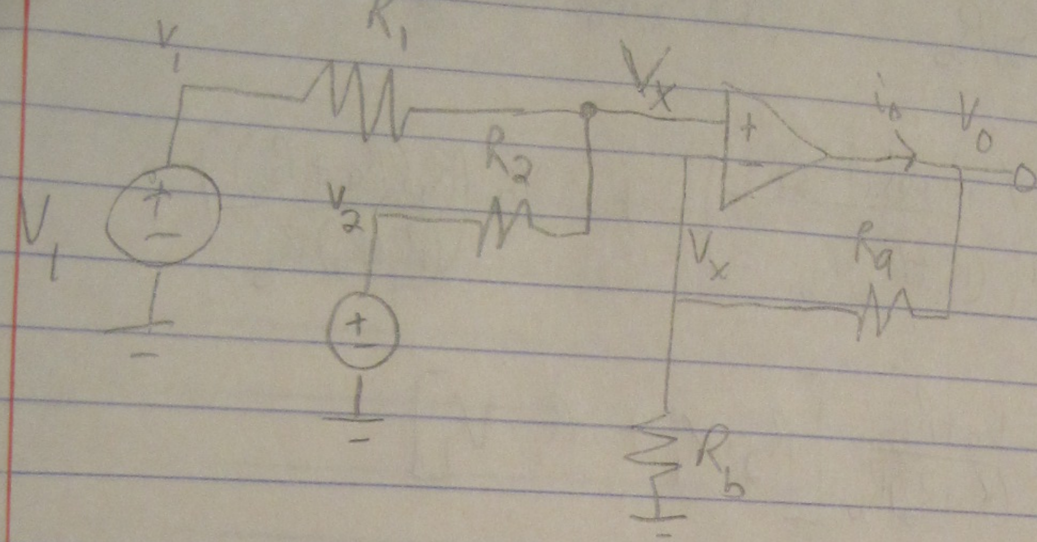
$$V_{s_2} = \frac{-R_4}{R_3}V_{s_1} = -\left(\frac{R_2+R_1}{R_2}\right)\frac{R_4}{R_3}V_{in}$$

In other words, if there's a ,

you can put a  there instead

if V_{out} is a known voltage

Multi-Source op-amp example



Goal: Find V_0 as function of V_1, V_2

Complicated enough that I'd suggest just using node voltage

First, using summing point and get that $V^+ = V^- = V_x$ and $i^+ = i^- = 0$.

Node Voltage:

V^+: $\frac{V_x - V_1}{R_1} + \frac{V_x - V_2}{R_2} = 0$ $\quad$ V^-: $\frac{V_x}{R_b} + \frac{V_x - V_0}{R_a} = 0$

Again, you might be tempted to write KCL for V_0, giving:

$$V_0: \quad \frac{V_0 - V_x}{R_a} - i_0 = 0$$

but all this gives you is i_0.
No need to do this!

From KCL @ V^+: $V_x R_2 - V_1 R_2 + V_x R_1 - V_2 R_1 = 0$

$$V_x = \frac{V_1 R_2 + V_2 R_1}{R_1 + R_2}$$

From KCL @ V^-: $$\frac{V_1R_2 + V_2R_1}{(R_1+R_2)R_b} + \frac{V_1R_2+V_2R_1}{(R_1+R_2)R_a} = \frac{V_0}{R_a}$$

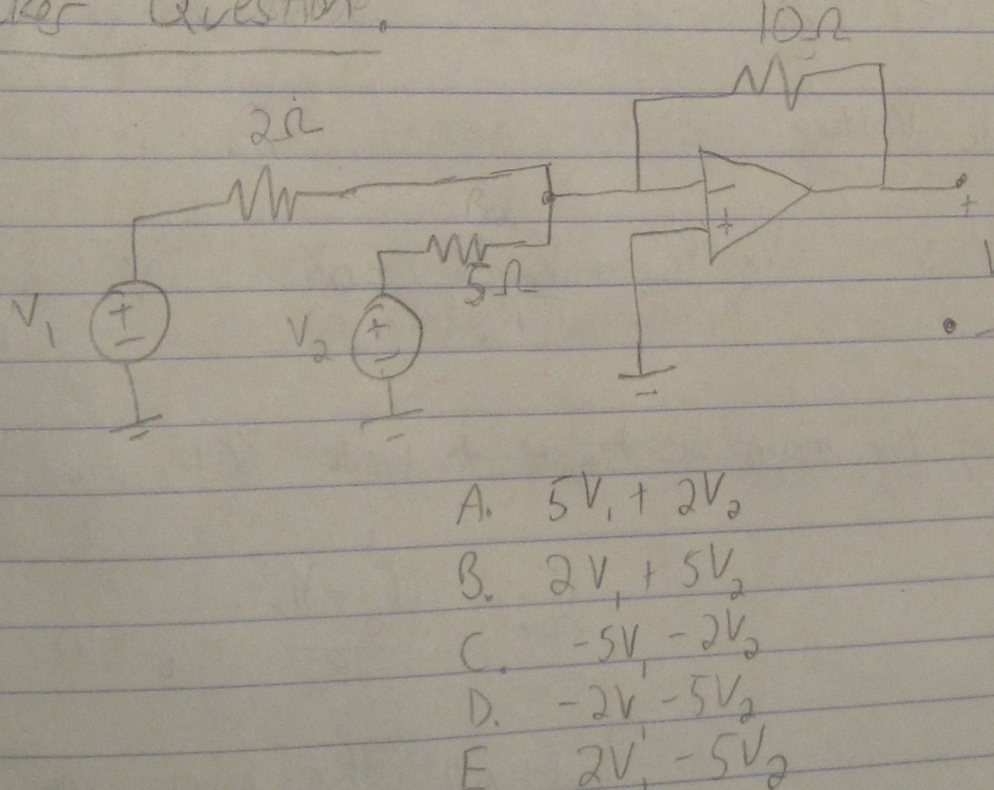
$$V_0 = V_1\left(\frac{R_2(R_a+R_b)}{(R_1+R_2)R_b}\right) + V_2\left(\frac{R_1(R_a+R_b)}{(R_1+R_2)R_b}\right)$$

$$= \frac{R_a+R_b}{(R_1+R_2)R_b}\left[R_2 \cdot V_1 + R_1 \cdot V_1\right]$$

Performs weighted sums of voltages.

Provides a reliable way to add 2 voltages.
Can also make a subtraction

<u>iClicker Question:</u>



A. $5V_1 + 2V_2$

B. $2V_1 + 5V_2$

C. $-5V_1 - 2V_2$

D. $-2V_1 - 5V_2$

E. $2V_1 - 5V_2$

$i_2 = \frac{V_1}{2}$ $i_5 = \frac{V_2}{5}$ $i_{10} = i_2 + i_5$ $V_{10} = -i_{10} \cdot 10 = -5V_1 - 2V_2$