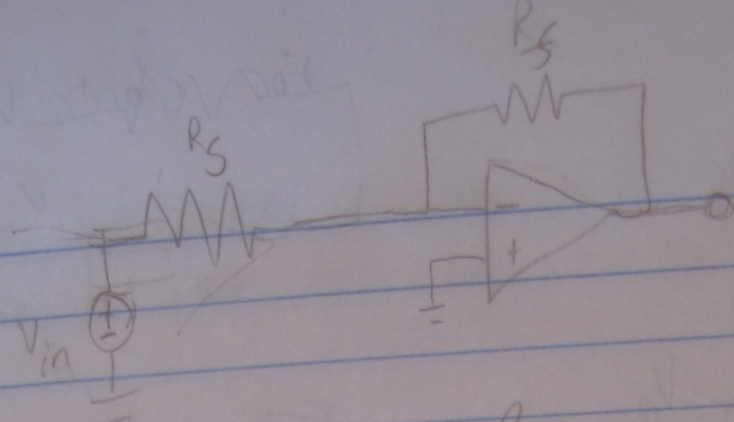
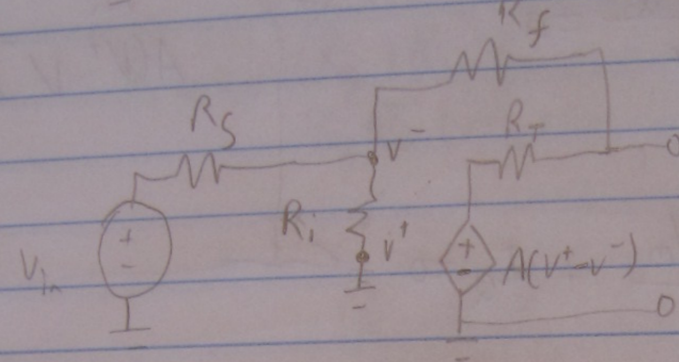
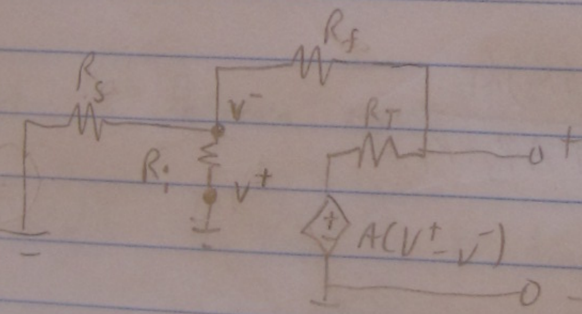


This is very dry stuff. Sorry!

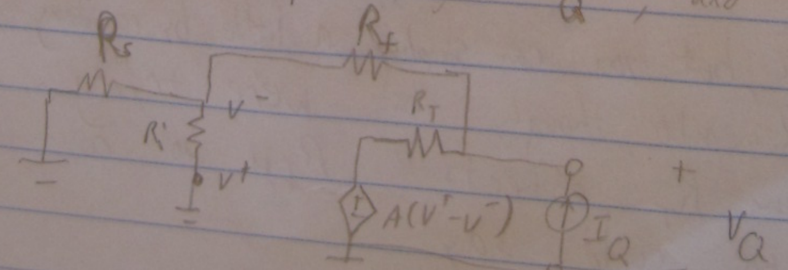


To find resistance looking into output terminals, we eliminate the INDEPENDENT voltage source.

Why? Well, output resistance is supposed to represent the resistance that a load is competing with, i.e. the Thevenin resistance of the Thevenin equivalent circuit



Apply a made up source I_Q, and find V_Q



For clarity, $V^- = V_m$

$V^+ = V_p$

Node voltage:

$$\frac{V_m}{R_s} + \frac{V_m}{R_i} + \frac{V_m - V_Q}{R_f} = 0$$

$$A(\overset{0}{\cancel{V^+}} - V^-) = -AV_m$$

$$\frac{V_Q + AV_m}{R_T} + \frac{V_Q - V_m}{R_f} + I_Q = 0$$

Goal: Find $\frac{V_Q}{I_Q} = R_{TH}$

Can we use summing point constraint?

No! Node voltage equations get stupid since $V_m = 0$

- Intuitively: We care about the minor deviation when computing thev resistance

In practical circuits, R_i is tiny compared to R_f and R_s. $R_i \sim 10^{12}\ \Omega$, so unless you're using resistors made of glass, you're good. Thus we can set $V_m/R_i = 0$.

Lots of Algebra:

$$V_Q = \frac{I_Q(R_f + R_s)R_T}{R_f + R_s + A \cdot R_s + R_T}$$

$$R_{TH} = \frac{(R_f + R_s)R_T}{R_f + R_s + A \cdot R_s + R_T}$$

This answer is fine, but you can simplify a little by considering what happens as A gets large. Remember, though, that we're trying to understand how R_T affects R_{TH}. So as A gets large, we could say

$$\frac{(R_f + R_s)R_T}{R_f + R_s + AR_s + R_T} \to 0$$

, but that's boring and just means that strong enough amplification can overcome R_T

So, let's instead consider what happens as A is big, but not tooo big.

$$R_{TH} = \frac{(R_f + R_s) R_T}{R_f + R_s + AR_s + R_T} \qquad R_f + R_s + AR_s + R_T \approx AR_s$$

$$R_{TH} \approx \frac{(R_f + R_s) R_T}{AR_s} = \frac{R_T}{A \frac{R_s}{R_f + R_s}}$$

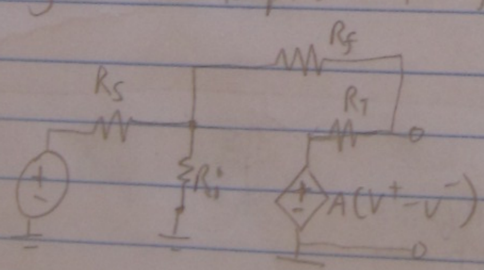
Typical R_T is $1{,}000\ \Omega$. For $A = 10^6$, $R_f = 10^5$, $R_s = 10^4$

$$R_{TH} = \frac{1{,}000}{10^6 \cdot \frac{10^4}{10^5 + 10^4}} \approx 10^{-2} = 0.01\ \Omega$$

Not too shabby

What is Thevenin equivalent circuit?

You might be tempted to find V_{oc} of:



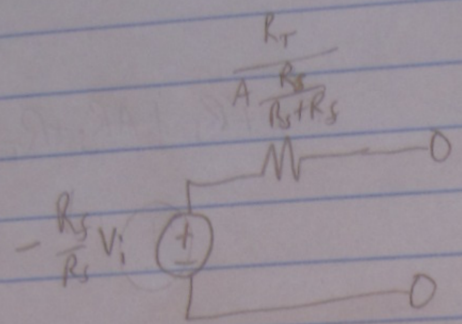
And you could, giving the most accurate answer:

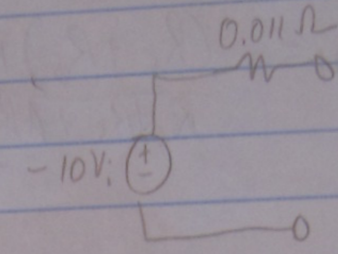
$$V_{oc} = \frac{-(AR_f - R_T) V_i}{R_f + R_s + A \cdot R_s + R_T}$$, which in the large A limit, gives our summing point constraint answer:

$$V_{oc} = -\frac{R_f}{R_s} V_i$$

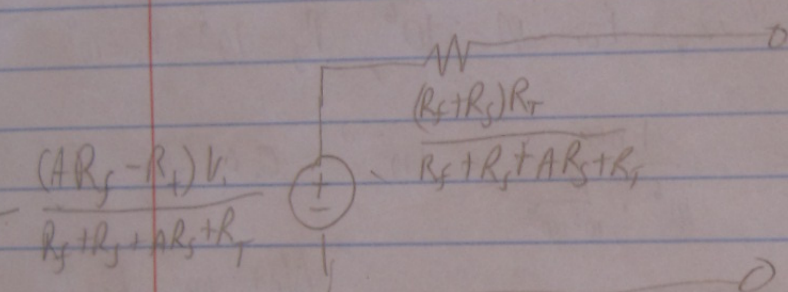
Thus our Thevenin circuit, assuming very large R_i and A is:

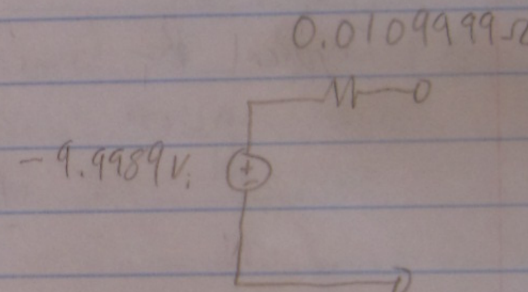
$A = 10^6$ $R_f = 10^5$ $R_s = 10^4$ $R_T = 10^3$





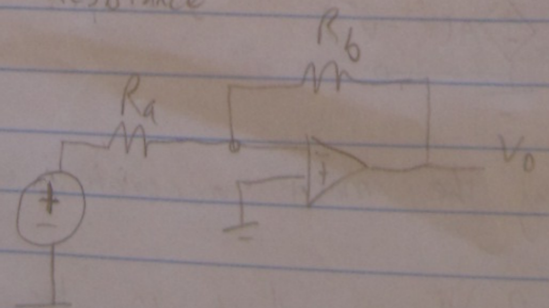
Or if we want to be really accurate and consider what happens if A isn't "big enough":





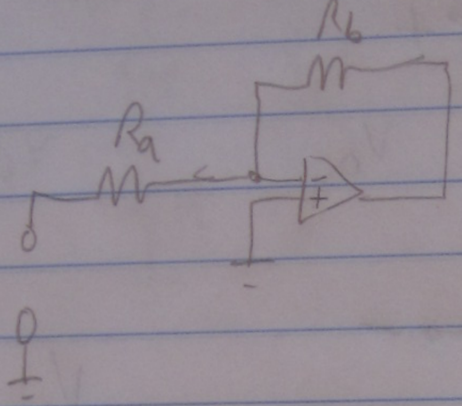
Unless a problem specifically says otherwise, assume your op-amp is ideal.

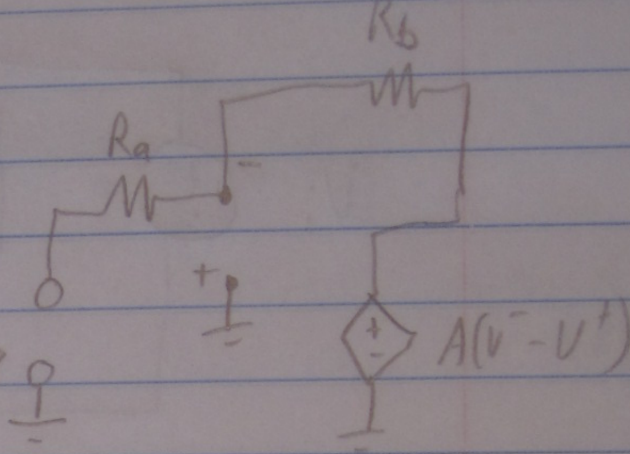
Input Resistance



Find input resistance of ideal op-amp.

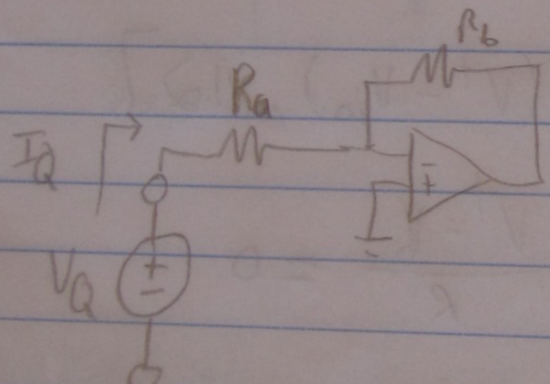
We're interested in the resistance that the input voltage source sees:





Want R_{eq}

Keep in mind that finding R_{eq} is the same as applying a made up voltage V_Q and finding I_Q, ie.

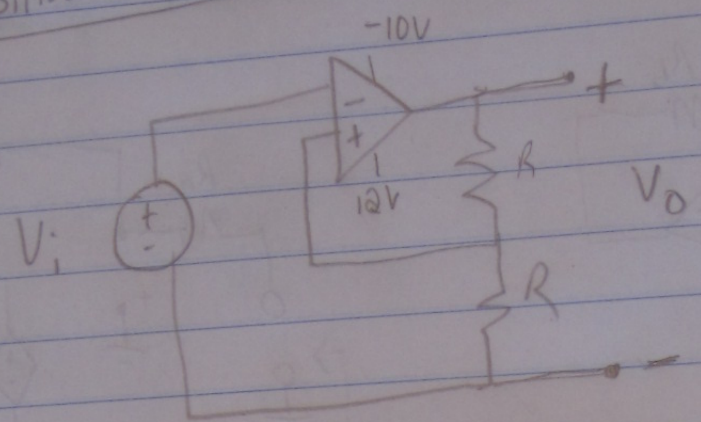


Which for ideal op-amp is just $I_Q = \frac{V_Q}{R_a}$

So $R_{eq} = \frac{V_Q}{I_Q} = \frac{V_Q}{V_Q/R_a} = \boxed{R_a}$ Easy!

Can repeat for non-ideal op-amp, but I won't,
See Sec. 15.42 for the non-ideal case

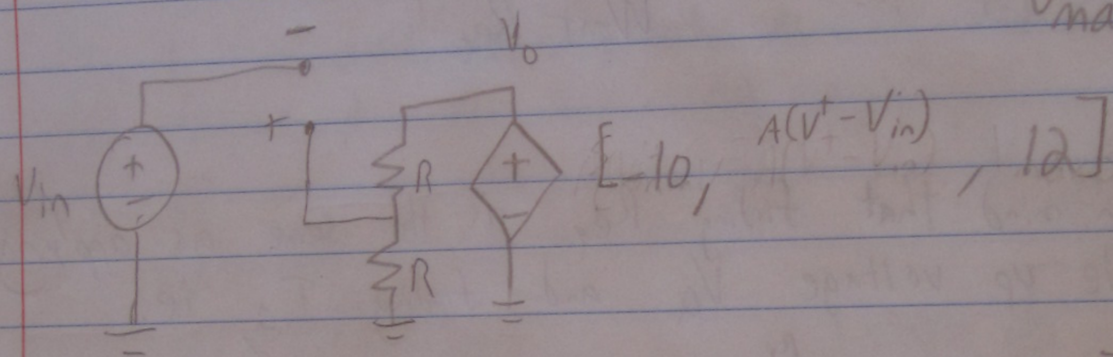
Positive Feedback:



$F_{-10,12}$

$V_{min} = -10\text{ V}$

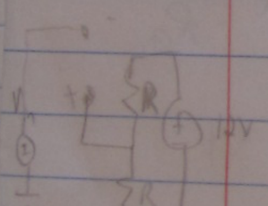
$V_{max} = 12\text{V}$



$$V_o = [-10, \; A(V^+ - V_{in}), \; 12]$$

V^+ node: $\dfrac{V^+}{R} + \dfrac{V^+ - V_o}{R} = 0$

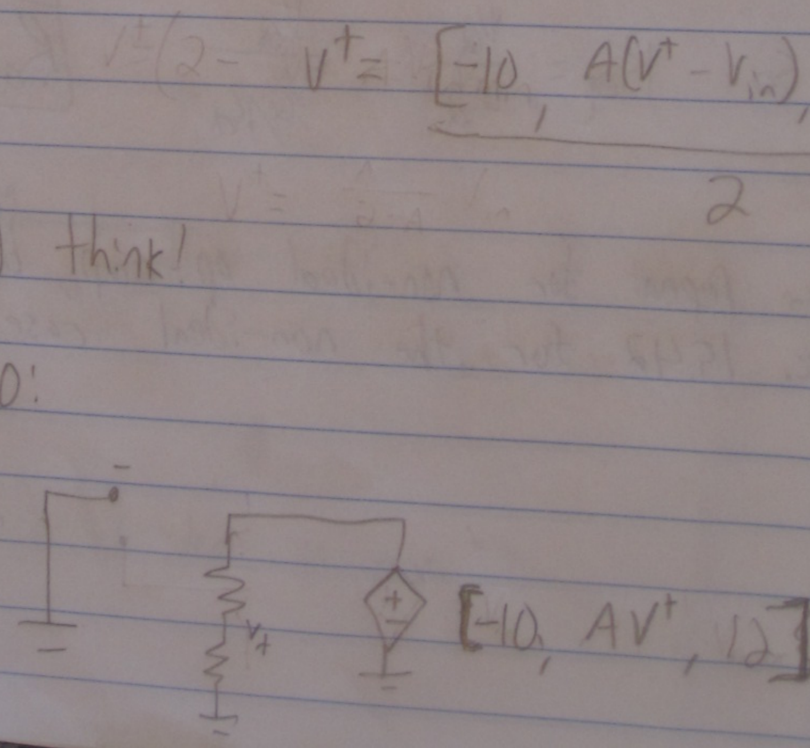
$$\frac{V^+}{R} + \frac{V^+ - [-10, A(V^+ - V_{in}), 12]}{R} = 0$$



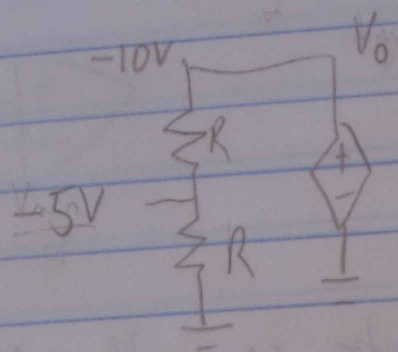
$$V^+ = \frac{[-10, \; A(V^+ - V_{in}), \; 12]}{2}$$

Step back and think!

Consider $V_{in} = 0$:



If $V^+ = -5V$



Voltage divider says $V_0 = -10V$

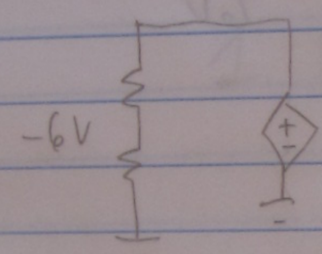
Op-amp says:

$V_0 = [-10V, -5\cdot A, 12V] =$

gain, not amps!

Everything is consistent!

If $V^+ = -6V$



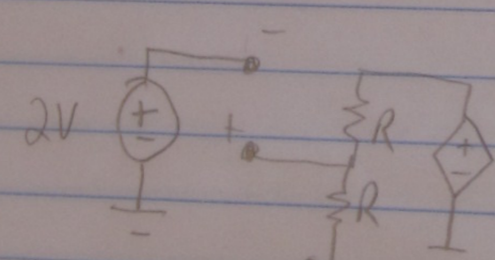
Voltage divider says $V_0 = -12V$

Op-amp says:

$V_0 = [-10V, -6\cdot A, 12V] = -10V$

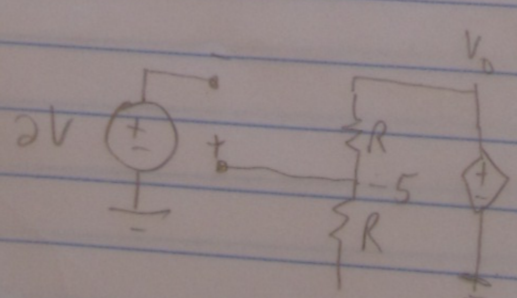
disagreement.

Interestingly, if we have:



$[-10, A(V^+ - 2), 12V]$, then:

If $V^+ = -5V$



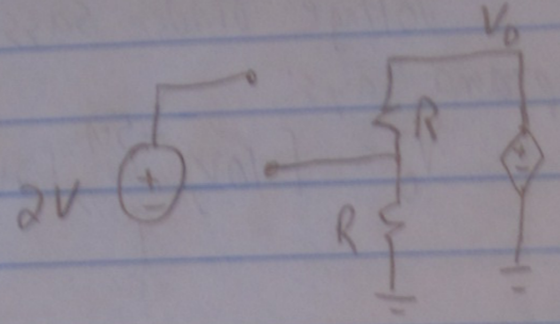
Current divider says $V_0 = -10V$

Op-amp says:

$[-10, A\cdot(-7), 12V] = -10V$

OK!

And if we assume $V^+ = 6V$



Current divider says $V_o = 12V$

op amp says:

$$[-10, \; A\cdot(3V), \; 12] = 12V$$

Consistent! V^+ can be $-5V$ or $6V$