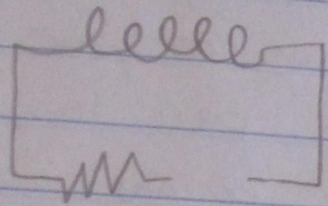
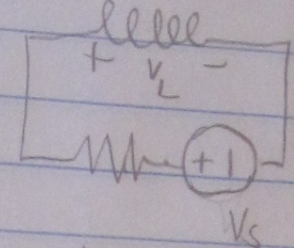


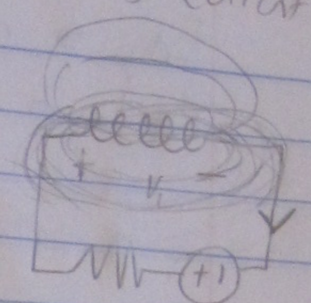
→

• Zero current



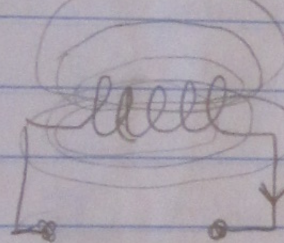
→

• Lots of current



• $V_L = V_s$ (KVL)
(Rapidly Increasing magnetic field)
• Acts like open

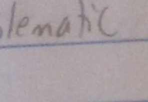
• $V_L = 0$
(Constant magnetic field)
• Acts like short

→ 

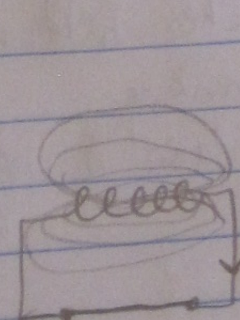
What happens here?
We've suddenly stopped the current through inductor
This is like shorting the terminals of charged capacitor
' Infinite voltage for infinitesimal time

- Mathematically ill-behaved, don't do this!

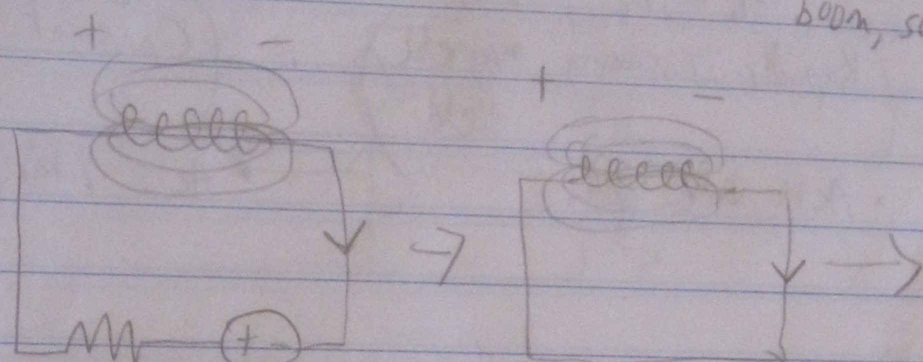
In the real world you do see this! Huge voltage spikes due to "back emf" are problematic

What is the correct equivalent of  ?

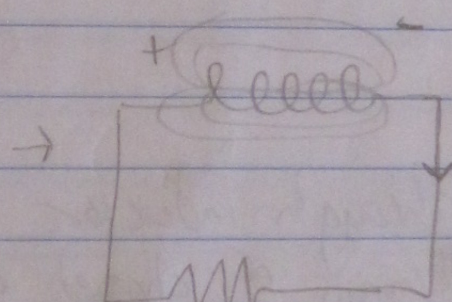
Question:
Why, if capacitors and inductors are duals, do we only hear about energy storage with capacitors and not inductors?

Simply! 

Easy to make effectively infinite resistance.
Hard to make zero resistance. (but possible, just convince your electron pairs to act like bosons by forming Cooper pairs — boom, superconductivity).



$i_L = I$



- $i_L = I$
- Sudden large negative voltage (back emf) across inductor to changing field as current drops.

- Only natural that voltage is opposite of the voltage that created magnetic field

$$V \propto \frac{dB}{dt} \propto \frac{dI}{dt}$$

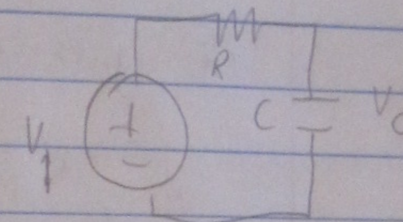
Voltage was positive when field was growing
Now negative when shrinking
(also, inconsistent w/ Ohm's law if induced voltage is positive).

$V \propto \frac{dB}{dt}$ means V is proportional to change in B.

Solving 1st order ODEs

$$\frac{V_c - V_1}{R} + C V_c' = 0$$

Above is our node voltage equation for



Steps for solving:

0. Put ODE in standard form
I. Find homogeneous solution $y_H(t)$
II. Find particular solution $y_p(t)$
III. Form complete solution $y(t) = y_H(t) + y_p(t)$
IV. Use initial conditions to find unknown coefficients

0. Putting ODE in standard form

Desired form is: $y'(t) = T(t)\, y(t) + \underbrace{f(t)}_{\text{Forcing Function}}$

Forcing Function
- represents effect of sources

For our example, this is just:

$$V_c' = -\frac{V_c}{RC} + \frac{V_1}{RC}$$

Here our state variable $y(t)$ is $V_c(t)$.

I. Find Homogeneous Solution $y_{y,H}(t)$

A. Set $f(t)=0$

B. Replace state variable $y(t)$ with 1

Replace state variable $y'(t)$ with s.

This gives us the characteristic polynomial $p(s)$.

- For a first order ODE, solving this polynomial requires no effort
- For a second order, you'll just have to use $s = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$V_c' = -\frac{V_c}{RC} + \underbrace{\frac{V_1}{RC}}_{f(t)}$$

A. $V_c' = -\frac{V_c}{RC}$

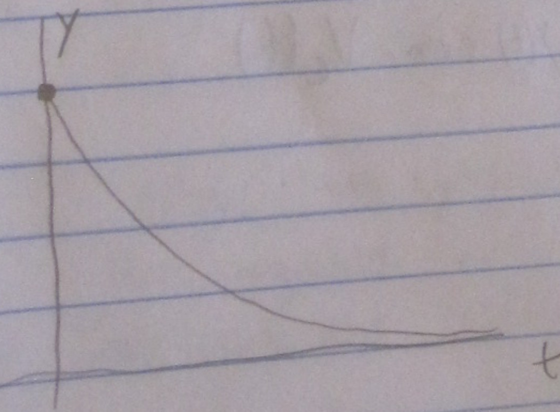
B. $s = -\frac{1}{RC}$

C. The homogeneous solution is just $Y_H = Ae^{st}$.

C. $V_{c,H} = Ae^{-t/RC}$

The homogeneous solution is also called the natural response. It tells us how the circuit acts in the absence of sources.

1st order circuit:



II. Find Particular Solution

A. Guess a form for $y_p(t)$

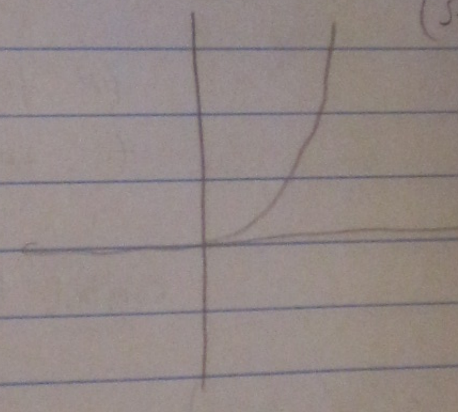
Usually just $y_p(t) = A\cdot f(t) + B\cdot f'(t) + C\cdot f''(t) + \ldots$

B. Plug in to standard form equation and find $A, B, C \ldots$

C. Verify your answer in B by re-plugging in to SFE.

A. Choosing form for $y_p(t)$.

Let's suppose $V_1(t) = \begin{cases} 0 & t<0 \\ 3t^2 & t\geq 0 \end{cases}$



Our SFE was:

$$V_c' = -\frac{V_c}{RC} + \underbrace{\frac{V_1}{RC}}_{f(t)}$$

Since we

D because C already appears in our ODE

Assume $V_{c,p}(t) = At^2 + Bt + D$

Then $V_{c,p}'(t) = 2At + B$

B. Plugging in to standard form:

$$2At + B = \frac{-At^2 + Bt + D}{RC} + \frac{3t^2}{RC}$$

For t^2 terms to cancel A must be 3!

$$6t + B = \frac{Bt + D}{RC}$$

B must be 6RC

For t terms to balance,

$$6t + 6RC = \frac{6RCt + D}{RC}$$

For constant terms to balance, must be $6R^2C^2$

$$V_{C,p}(t) = 3t^2 + 6RCt + 6R^2C^2$$

$$V'_{C,p}(t) = 6t + 6RC$$

$$V_C' = -\frac{V_C}{RC} + \frac{V_i}{RC}$$

$$6t + 6RC = -\frac{3t^2 + 6RCt + 6R^2C^2}{RC} + \frac{3t^2}{RC}$$

t^2 term cancels $\quad -3t^2 = 3t^2$

t term cancels $\quad 6RC = \frac{6RCt}{RC}$

constant term cancels! $\quad \frac{6R^2C^2}{RC} = 6RC$

So we're good.

Particular solution also called the <u>forced response</u>, since it tells us how circuit reacts to the sources which are trying to force the circuit into some non-zero state.

III. Forming the complete solution $y(t)$

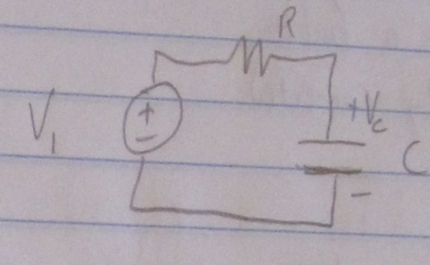
Just add $y_p(t)$ and $y_H(t)$, so $y(t) = y_p(t) + y_H(t)$

Example:

$$V_C(t) = \underbrace{Ae^{-t/RC}}_{\text{natural response}} + \underbrace{6t + 6RC}_{\text{forced response}}$$

IV. Use initial conditions to find unknown coefficients.

We need an initial condition, so let's suppose that
$V_c(0) = 0$



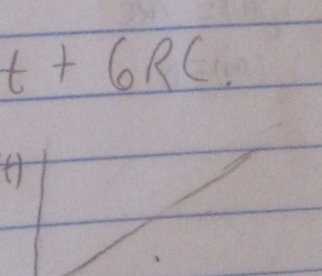
Plug in $V_c = 0$ for $t=0$, and we get:

$$V_c(0) = Ae^0 + 6 \cdot 0 + 6RC = A + 6RC = 0$$

so $A = -6RC$

This gives our final solution

$$V_c(t) = -6RCe^{-t/RC} + 6t + 6RC.$$

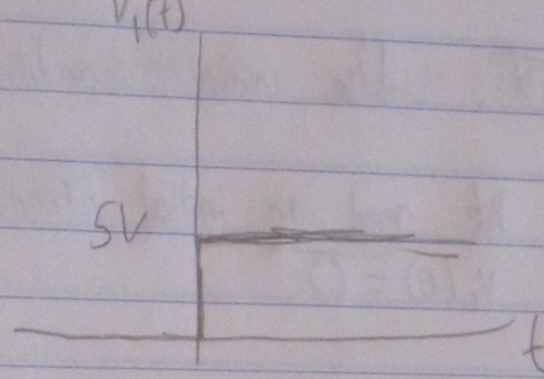


Why is $V_c(t)$ running off to infinity?

Because our source is!

A more typical source is a step input.

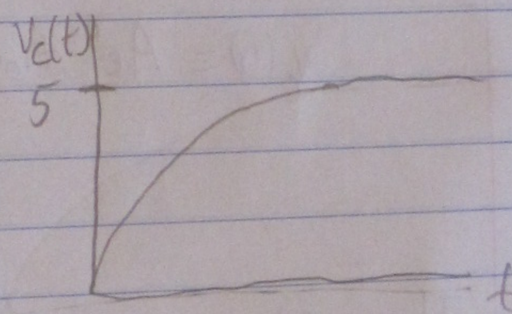
Think of as a source being switched on or a battery suddenly connected



Try the exercise we just did, but now with the source above assuming $V_c(0)=0$

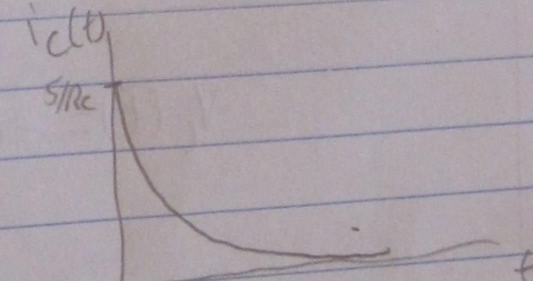
You should get : $V_c(t) = 5 - 5e^{-t/RC}$

At $t=0$ $V_c(0)=0$
$t=\infty$ $V_c(\infty)=0$



$i_c(t) = C \cdot V_c'(t) = \frac{5}{RC} e^{-t/RC}$

$i_c(0) = 5/RC$
$i_c(\infty) = 0$



Acts like a short at $t=0$, open at $t=\infty$