7/12/2010
Lecture 8

Capacitors and Inductors

Congratulations to Paul the Octopus on getting 8/8 world cup predictions correct

So far…

- All circuits we’ve dealt with have reacted instantaneously
  - Change a resistance, voltage, or current, and everything else reacts instantly

- Obviously this isn’t a complete model for electronic device. Why?

For the next 2 weeks

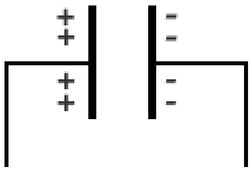
- We’ll be talking about elements with memory
  - Capacitors
  - Inductors
- Our first 6 lectures taught us how we can take a circuit schematic containing memoryless elements and convert them into algebraic equations
- In the next 2 lectures we’ll talk about how to convert circuits with memory into differential equations
- The next 3 after that will be about how we can use algebraic equations for circuits with memory if we have AC sources

Announcements

- Midterm #2 will be on the 28th
  - Elements with memory
- Make sure you do pre-lab before lab tomorrow
  - Who has finished it?

The Capacitor

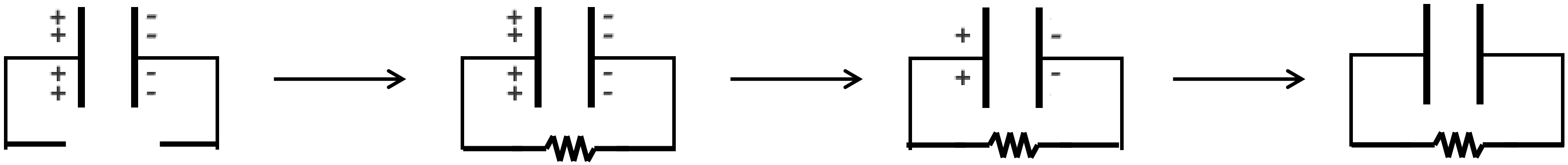
- The basic idea is pretty simple
  - Imagine you have two parallel metal plates, both of which have equal and opposite excess charges
  - Plates are separated by an insulating layer (air, glass, wood, etc)



- The charges would love to balance out
- Insulator blocks them (just as the ground blocks you from falling into the center of the earth)

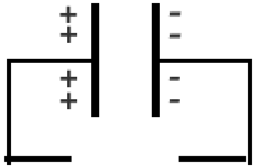
The Capacitor

- If you were to connect a resistive wire to the plates
  - Charges would flow through the wire
    - Charge flow is current
    - $P = I^2R$
    - Energy is released as heat



The Capacitor

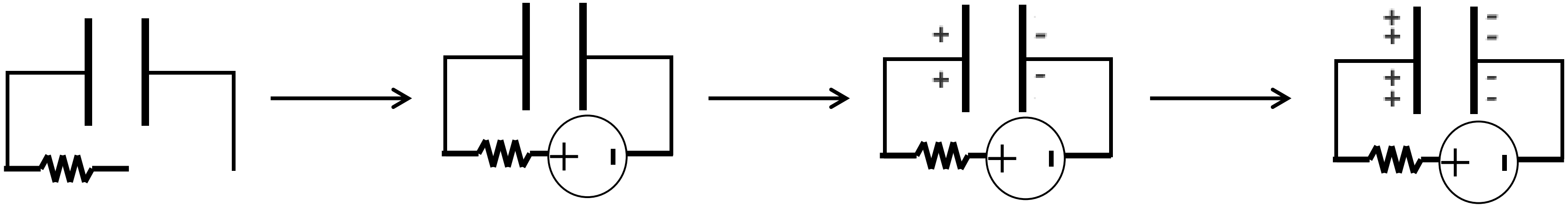
- Remember that a voltage is the electrical potential between two points in space



- Here, we have an imbalance of charge, and thus an electric field, and thus a voltage $V = EL$
  - Field strength is dependent on number and distribution of charges as well as material properties
  - Field length is dependent on size of capacitor
  - Capacitor size and material properties lumped into single “capacitance” C
    - $V = Q/C$

The Capacitor

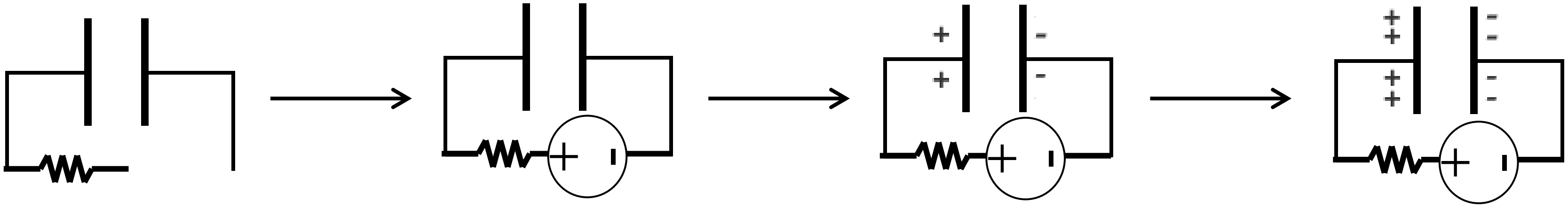
- Thus, if you connect a voltage source to the plates
  - Like charges will move to get away from the source
    - Charge flow is current
    - Current will stop once charges reach equilibrium with voltage source, i.e. $V_c = V_s$
    - Energy has been stored



The Capacitor

Lots of current

Zero current



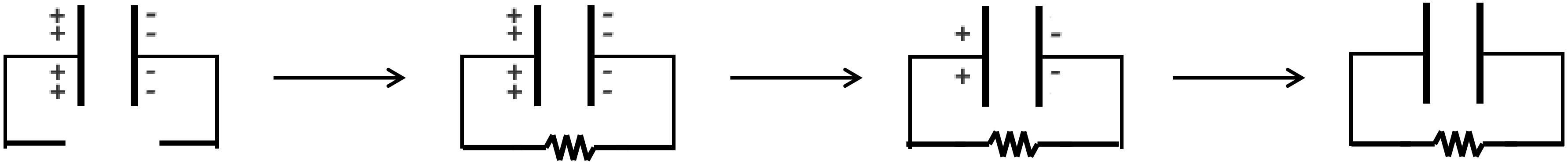
Zero V_C

$V_C=V_S$

$$I = C\frac{dV}{dt}$$

Lots of current
[the other way]

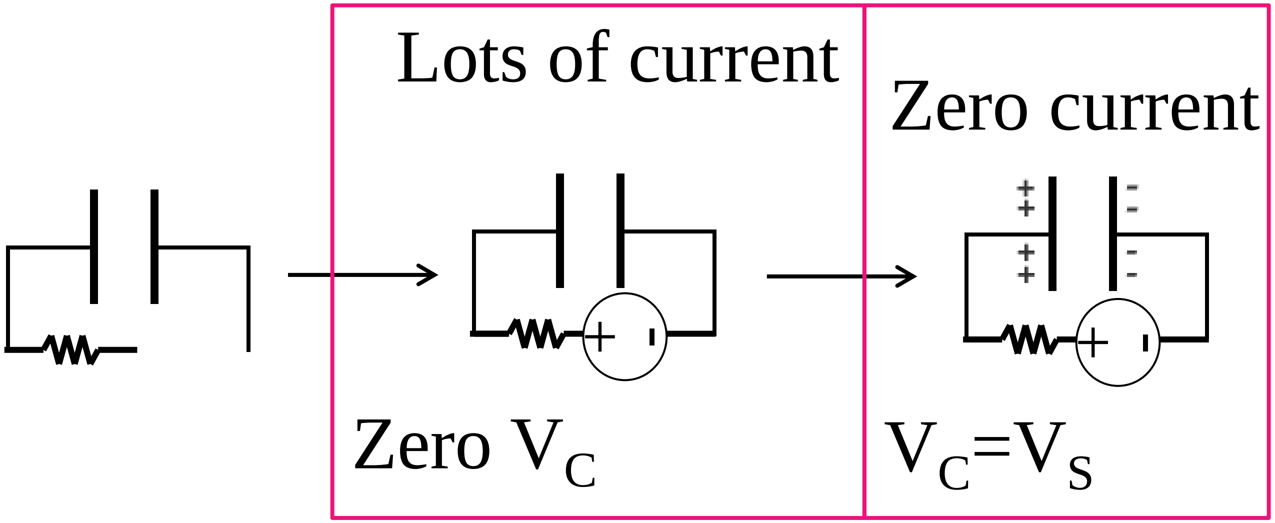
Zero current



$V_C=V_S$

Zero V_C

iClicker



Lots of current

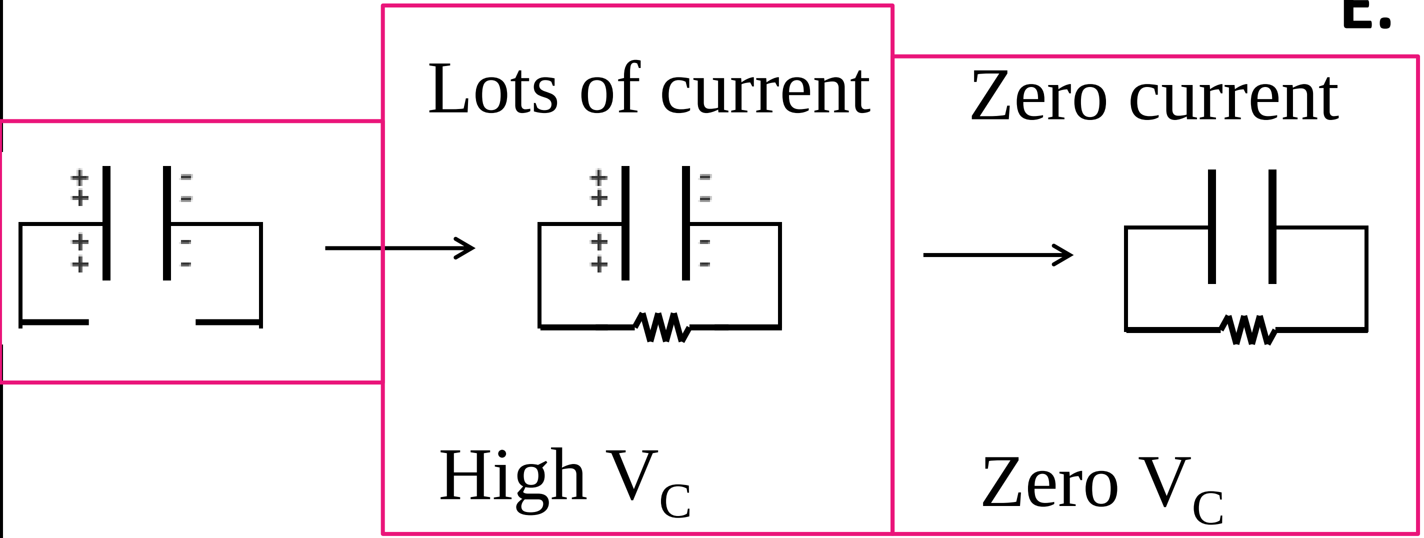
Zero V_C

Zero current

$V_C=V_S$

Acts like a:

- **A. Short circuit**
- **B. Open circuit**
- **C. Resistor**
- **D. Voltage source**
- **E. Current source**



Lots of current

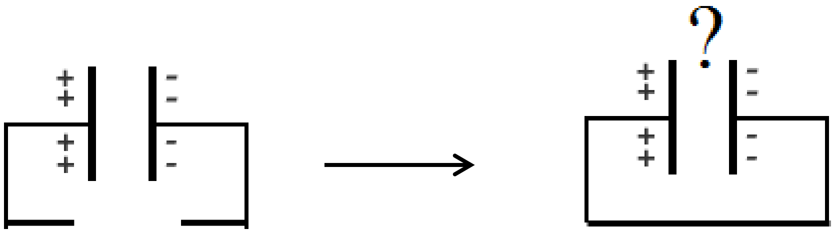
High V_C

Zero current

Zero V_C

Extreme Corner Case

- What happens if we short a charged capacitor?



- Think of it is as the limit as resistance goes to zero:
  - – Infinite current
  - – Lasts only a very short time ($P = V(t)I(t)$) until energy is released

- Mathematically poorly behaved
  - – Don't do this

How much energy is stored?

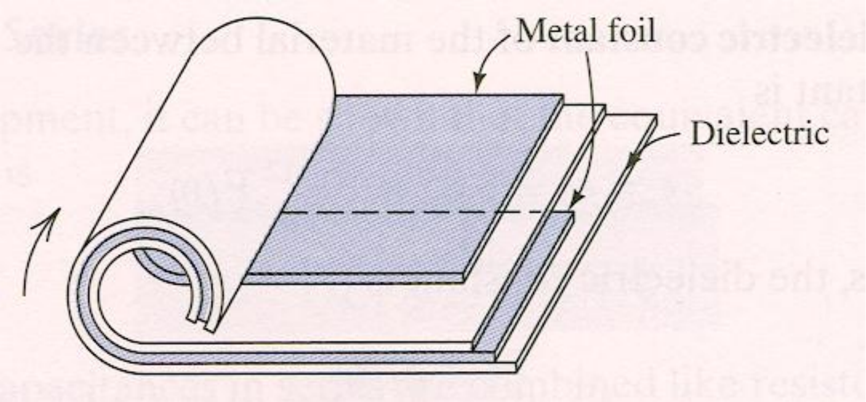
- $P(t) = V(t)I(t)$
- $P(t) = V_C(t)C\frac{dV_c(t)}{dt}$
- $E = \int_0^t P(t)dt$

$$= \frac{1}{2}CV_C(t)^2$$

Strictly speaking we shouldn't use t as our integration variable and also the limit that we're integrating to, but you know what I mean…

Practical Capacitors

- A capacitor can be constructed by interleaving the plates with two dielectric layers and rolling them up, to achieve a compact size.



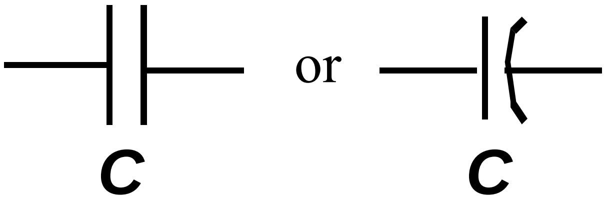
- To achieve a small volume, a very thin dielectric with a high dielectric constant is desirable. However, dielectric materials break down and become conductors when the electric field (units: V/cm) is too high.
  - Real capacitors have maximum voltage ratings
  - An engineering trade-off exists between compact size and high voltage rating

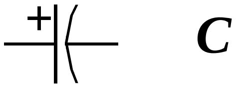
Capacitors

- Useful for
  - Storing Energy
  - Filtering
  - Modeling unwanted capacitive effects, particularly delay

Capacitor

Symbol:





Electrolytic (polarized) capacitor

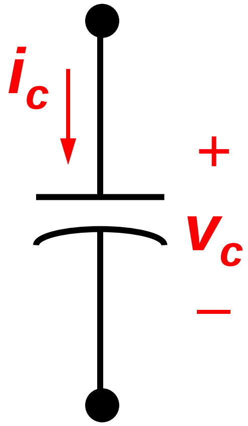
These have high capacitance and cannot support voltage drops of the wrong polarity

Units: **Farads (Coulombs/Volt)**

(typical range of values: 1 pF to 1 μF; for "supercapacitors" up to a few F!)

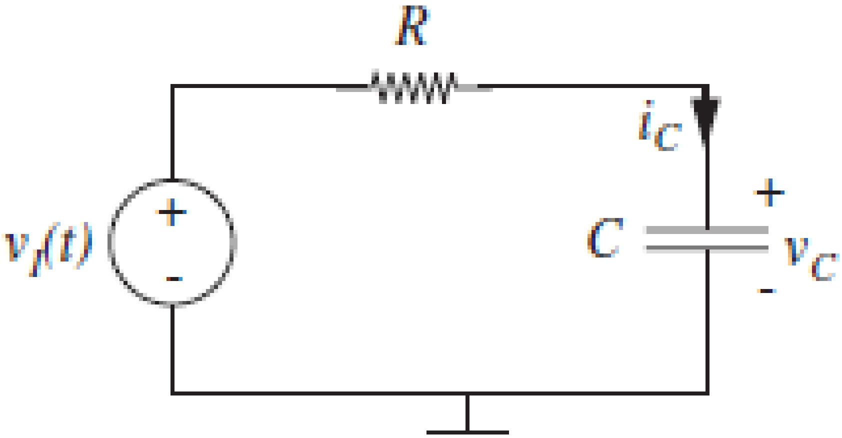
Current-Voltage relationship:

$$i_c = \frac{dQ}{dt} = C\frac{dv_c}{dt}$$

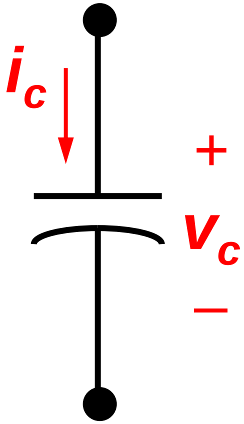


Note: **v_c must be a continuous function of time since the charge stored on each plate cannot change suddenly**

Node Voltage with Capacitors



$$i_c = \frac{dQ}{dt} = C\frac{dv_c}{dt}$$



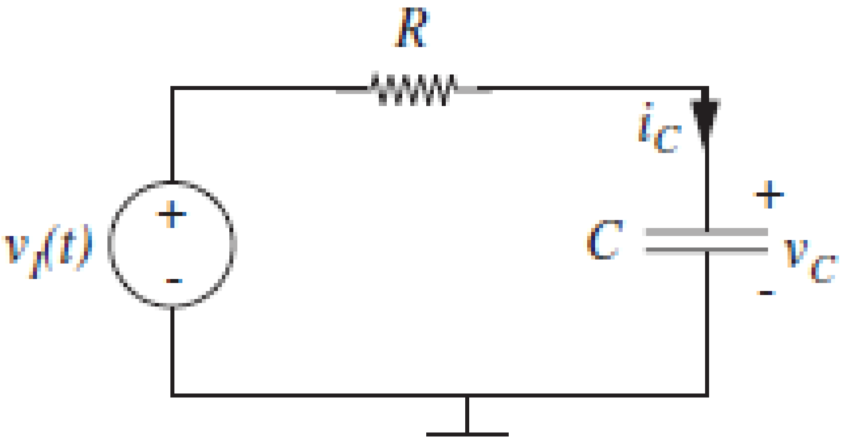
- At the top right node, we write KCL
  - Current to the left through resistor:

$$\frac{V_C(t) - V_I(t)}{R}$$

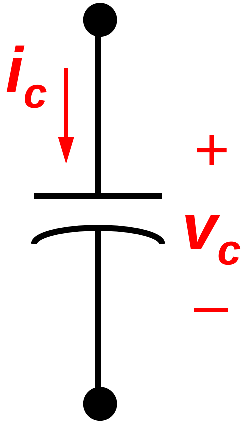
  - Current down through the capacitor

$$C\frac{dV_C(t)}{dt}$$

Node Voltage with Capacitors



$$i_c = \frac{dQ}{dt} = C\frac{dv_c}{dt}$$



- Or in other words

$$\frac{V_C(t) - V_I(t)}{R} + C\frac{dV_C(t)}{dt} = 0$$

- Or more compactly:

$$\frac{V_C - V_I}{R} + CV_C' = 0$$

ODEs

$$\frac{V_C - V_I}{R} + C V_C' = 0$$

- Later today, we'll talk about how to solve ODEs…
- For now, let's talk about inductors

Inductors

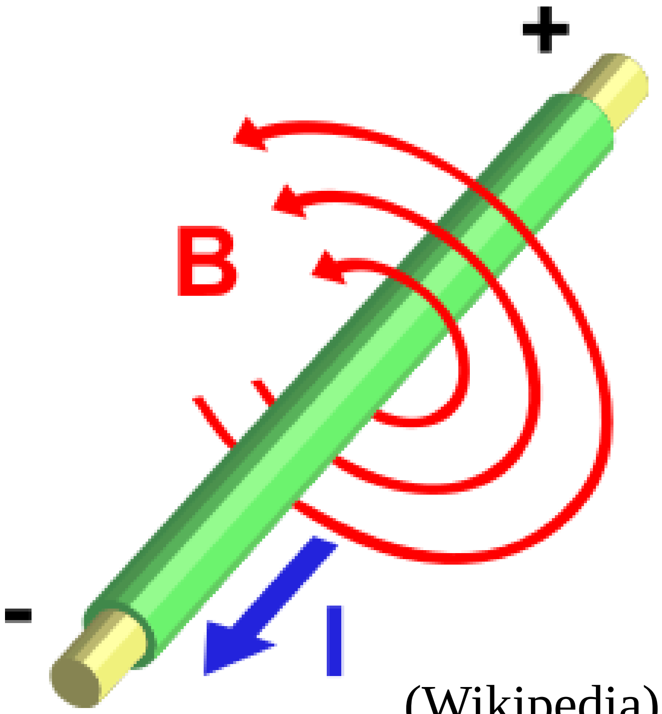
- Capacitors are a piece of cake to understand, just rely on Coulomb's Law

$$F = k_e \frac{q_1 q_2}{d^2}$$

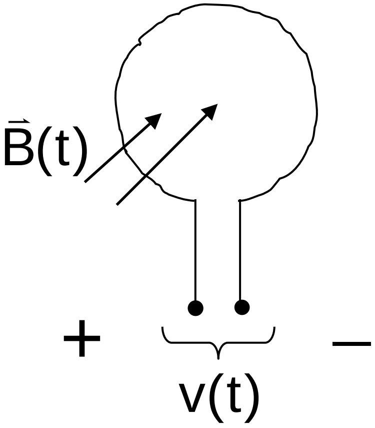
- Inductors, by contrast, involve magnetic fields, and rely instead on Faraday's Law
  - Comprehension comes with greater difficulty
- Thus, we'll treat inductors as mathematical objects and leave the derivation to Physics 7B (or page 467 of the book)

Two Fundamental Principles

- The flow of current induces a magnetic field (Ampere’s Law)
- A change in magnetic field through a loop of wire induces a voltage (Faraday’s Law)

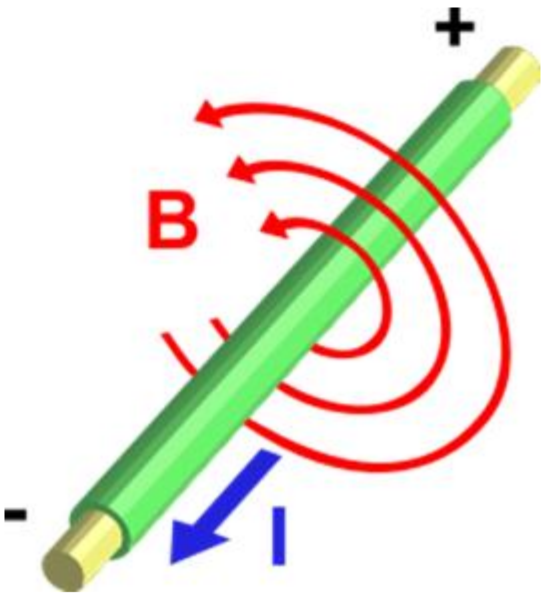


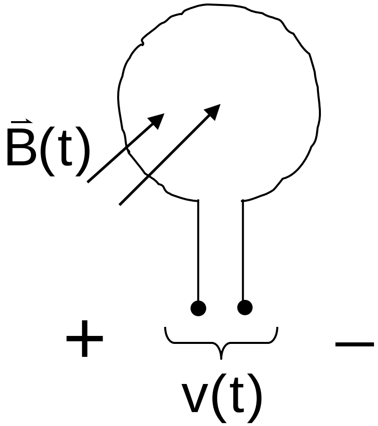
(Wikipedia)



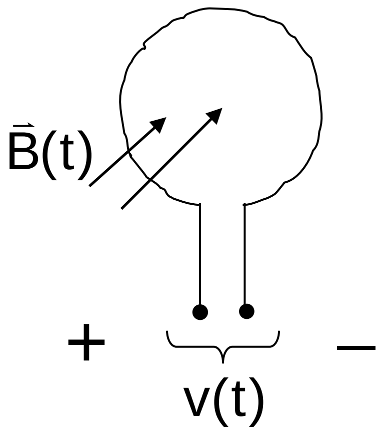
Inductor Basics (1)

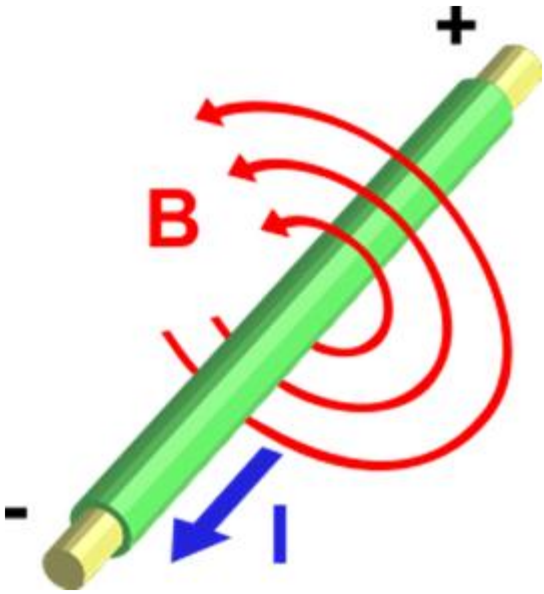
- When we connect a voltage source to a wire, current clearly takes a little time to get moving
- Thus, the magnetic field builds to some maximum strength over time

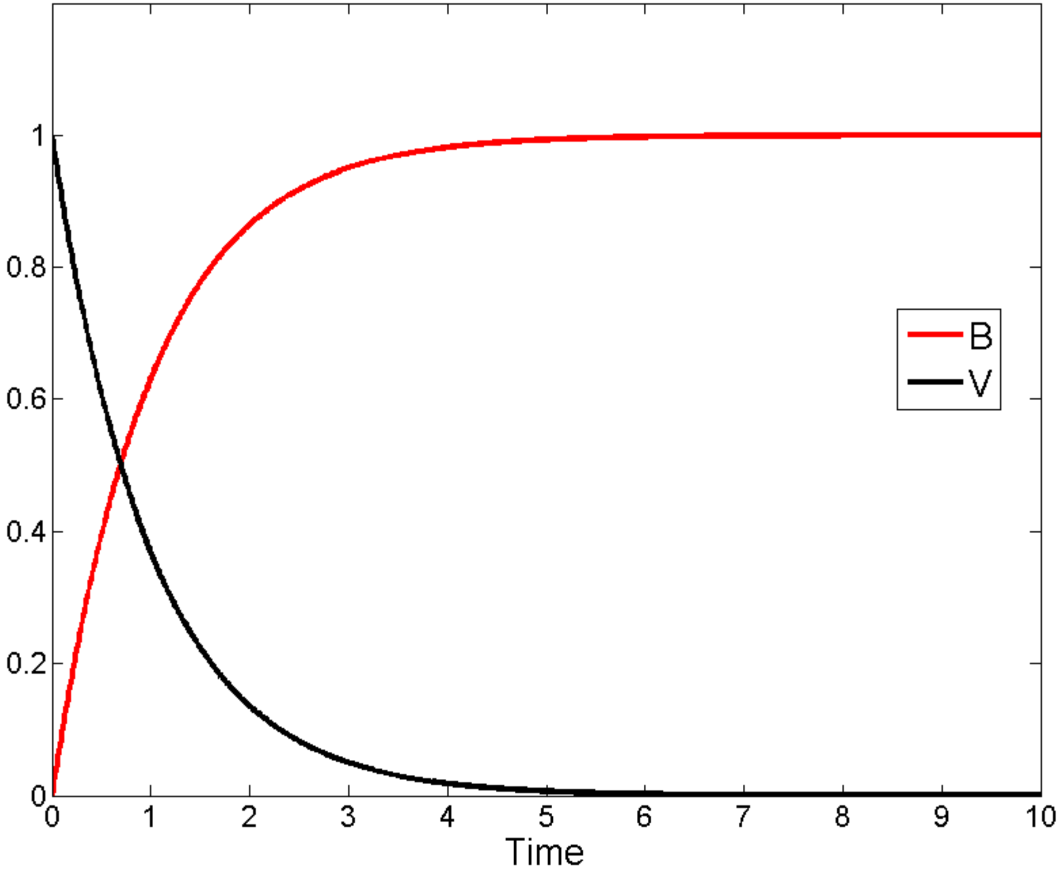




Inductor Basics (2)



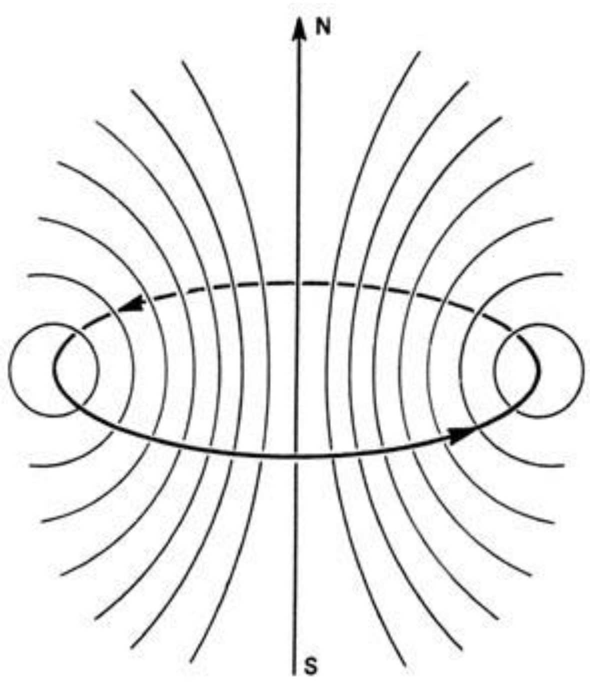




Current in a wire ~~causes~~ induces a voltage in any nearby circuit

Inductors Basics (3)

- If we make a loop, the entire loop of wire will all contribute to the magnetic field through the loop



- What's more, this field will go through the loop producing the current!
  - Self induced voltage
  - **Self inductance**

From: Dr. Richard F.W. Bader Professor of Chemistry / McMaster University

Inductors Basics (4)

- More loops
  - More magnetic field generated
  - More circuit to receive magnetic field

University of Surrey

http://personal.ee.surrey.ac.uk/Personal/H.M/UGLabs/components/inductors.htm

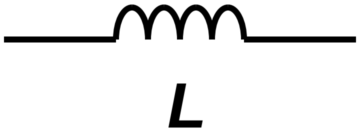
- Inductors are literally just loops of wire
- Just like capacitors are just two conductors separated by an insulator (or a gap)
- Just like resistors are just stuff with wires stuck to the ends

Inductors

- Capacitors hold a voltage in the form of stored charge
- Inductors hold a current in the form of stored magnetic field [dude…]

- For webcast viewers, see drawings on board/notes for more comparison to capacitor

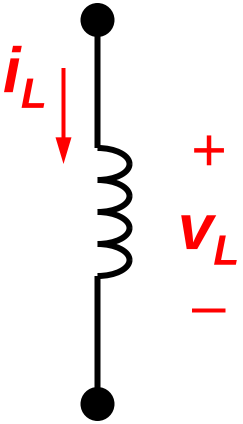
Symbol:



Units: Henrys (Volts • second / Ampere)

(typical range of values: μH to 10 H)

Current in terms of voltage:



$$di_L = \frac{1}{L} v_L(t)dt$$

$$i_L(t) = \frac{1}{L}\int_{t_0}^{t} v_L(\tau)d\tau + i(t_0)$$

Note: i_L must be a continuous function of time

Summary

Capacitor

$$i = C\frac{dv}{dt}; w = \frac{1}{2}Cv^2$$

v cannot change **instantaneously**

i can change instantaneously

Do not short-circuit a charged capacitor (-> infinite current!)

In steady state (not time-varying), a capacitor behaves like an open circuit.

Inductor

$$v = L\frac{di}{dt}; w = \frac{1}{2}Li^2$$

i cannot change **instantaneously**

v can change instantaneously

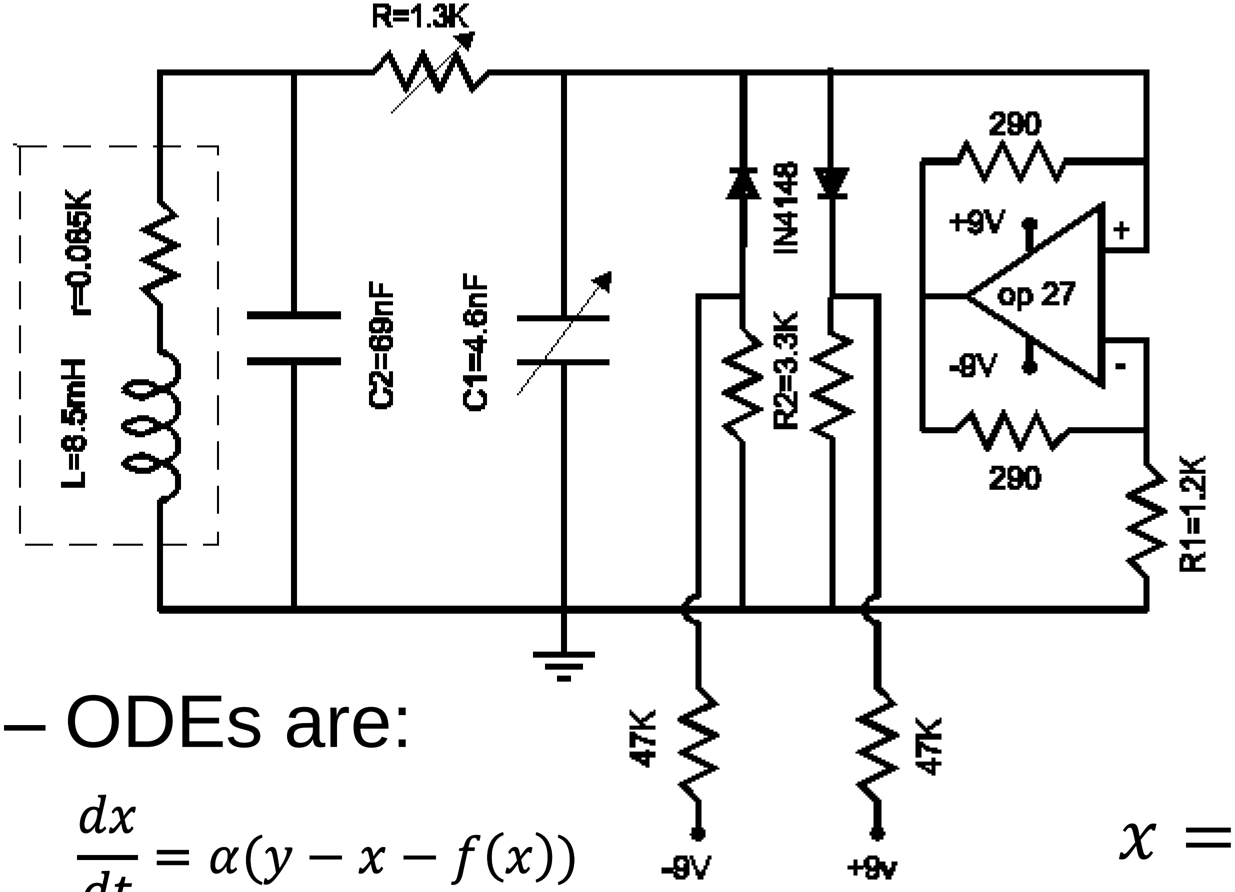
Do not open-circuit an inductor with current (-> infinite voltage!)

In steady state, an inductor behaves like a short circuit.

Ordinary Differential Equations

- Inductors, too, give us a simple 1st order relationship between voltage and current
- Node Voltage with memoryless circuits gave us algebraic equations
- Node voltage with elements with memory will give us Ordinary Differential Equations (ODEs)
- Next week will be a bunch of setting up and solving 1st and 2nd order linear ODEs
- Higher order and especially nonlinear ODEs are tough to solve. For example…

Chua’s Circuit



$f(x)$ – response of diodes and resistor block

– ODEs are:

$$\frac{dx}{dt} = \alpha(y - x - f(x))$$

$$\frac{dy}{dt} = x - y + z$$

$$\frac{dz}{dt} = -\beta y$$

$$x = v_{C1}$$

$$y = v_{C2}$$

$$z = i_I$$

Chua's Circuit

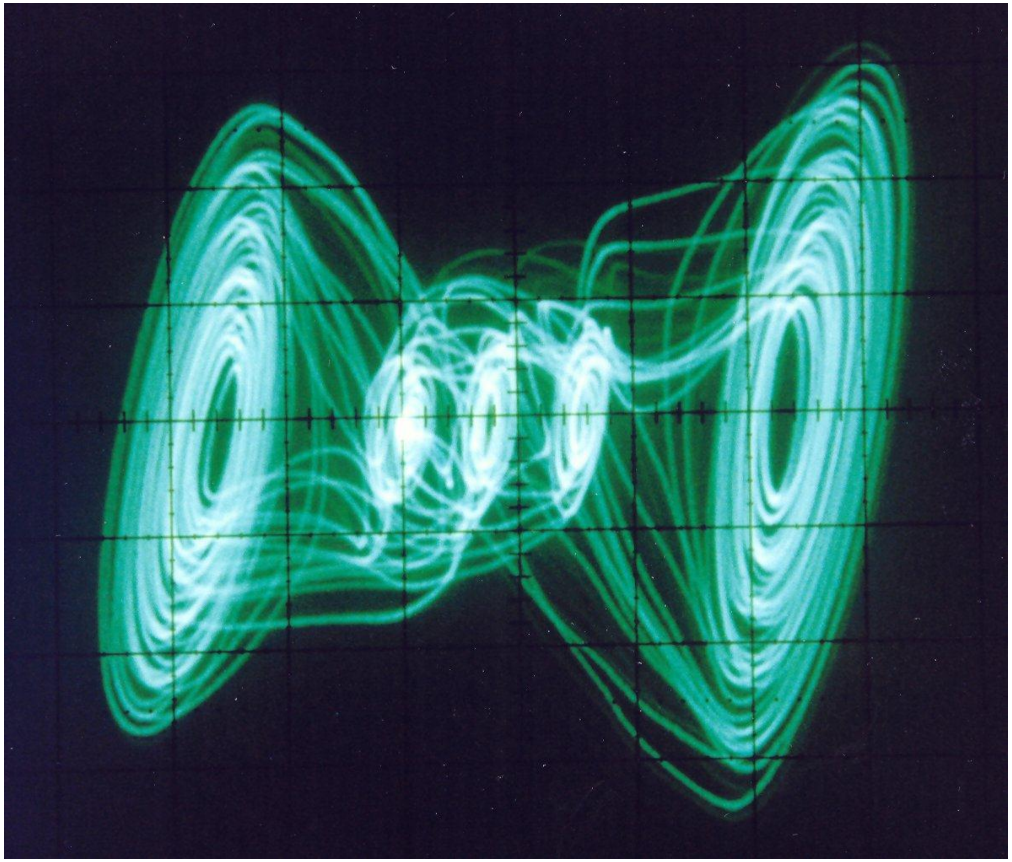
- Despite simplicity of ODEs

$$\frac{dx}{dt} = \alpha(y - x - f(x))$$

$$\frac{dy}{dt} = x - y + z$$

$$\frac{dz}{dt} = -\beta y$$

- Exhibits chaos!



Invented by current UC Berkeley EECS professor Leon Chua in 1983