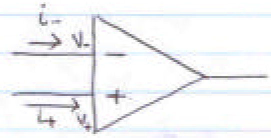
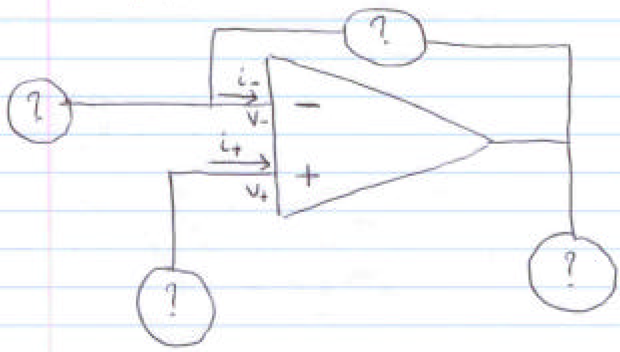
Ideal op-amp



The ideal op-amp operates on the following principles:

1) $i_+ = 0$
2) $i_- = 0$
3) $V_+ = V_-$

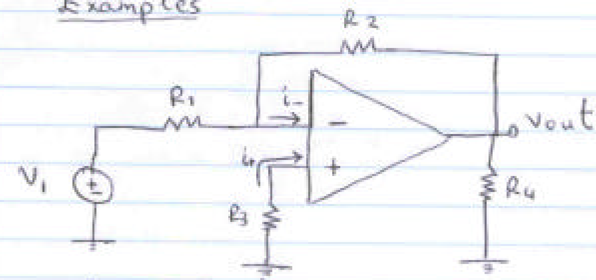
So even if we connect other elements on the amplifier's terminals, these rules still apply:



$i_+ = 0$
$i_- = 0$
$V_+ = V_-$

Examples

(1)



Find $\frac{V_{out}}{V_1}$

Step 1

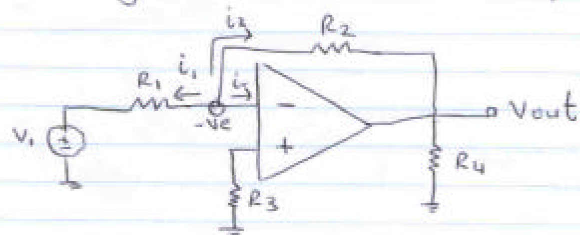
$i_+ = 0 \Rightarrow$ The voltage across $R_3 = 0 \Rightarrow V_+ = 0$

Step 2

$V_+ = V_- \Rightarrow V_- = 0$

step 3

Use the fact that $i_- = 0$ and do nodal analysis on the -ve node.

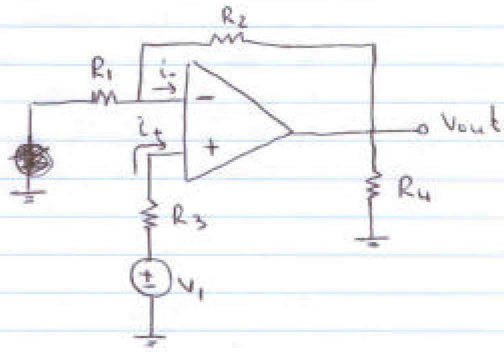


The voltage at the -ve terminal is $V_- = 0$ (We will need this)

So: $i_1 + i_2 + i_- = 0 \Rightarrow \frac{0 - V_1}{R_1} + \frac{0 - V_{out}}{R_2} + 0 = 0$

$\Rightarrow -\frac{V_1}{R_1} - \frac{V_{out}}{R_2} = 0 \Rightarrow \boxed{\frac{V_{out}}{V_1} = -\frac{R_2}{R_1}}$

(2)

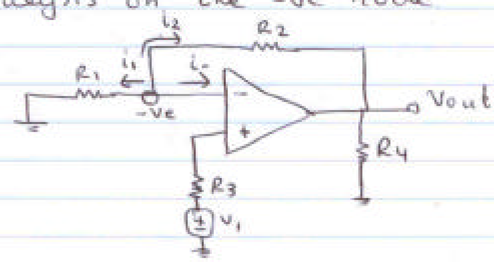


Find $\frac{V_{out}}{V_1}$

Step 1
$i_+ = 0 \Rightarrow$ The voltage across $R_3 = 0 \Rightarrow V_+ = V_1$

Step 2
$V_+ = V_- \Rightarrow V_- = V_1$

Step 3
Use the fact that $i_- = 0$ and do nodal analysis on the -ve node

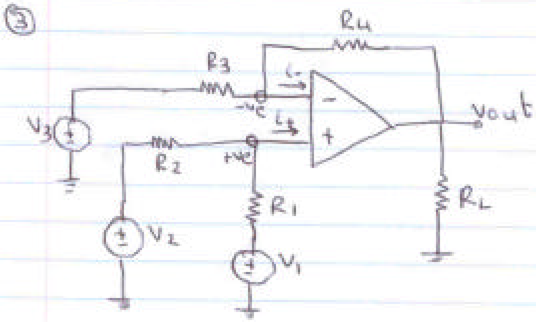


The voltage at the -ve node is $V_- = V_1$

$i_1 + i_2 + i_- = 0 \Rightarrow \frac{V_1 - 0}{R_1} + \frac{V_1 - V_{out}}{R_2} + 0 \Rightarrow \boxed{\frac{V_{out}}{V_1} = 1 + \frac{R_2}{R_1}}$

Notes
1) Example (1) has a negative gain, so it is called the inverting configuration. Example (2) has a positive gain so it is called the non-inverting configuration

3



Find Vout in terms of $V_1, V_2, V_3, R_1, R_2, R_3, R_4$

Step 1

$$i_+ = 0 \Rightarrow \frac{V_{+ve} - V_1}{R_1} + \frac{V_{+ve} - V_2}{R_2} = 0 \Rightarrow V_{+ve} = \left(\frac{R_2 \cdot V_1 + R_1 \cdot V_2}{R_1 + R_2}\right)$$

All I did was nodal analysis at the +ve terminal

$$V_+ = V_{+ve} = \left(\frac{R_2 \cdot V_1 + R_1 \cdot V_2}{R_1 + R_2}\right)$$

Step 2

Use $V_+ = V_- \Rightarrow V_- = \dfrac{R_2 \cdot V_1 + R_1 \cdot V_2}{R_1 + R_2}$

Step 3

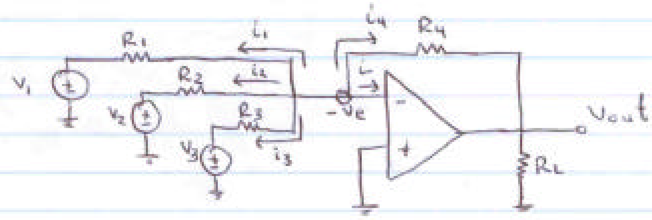
Nodal analysis at the -ve node (Remember: $i_- = 0$)

$$\frac{V_- - V_3}{R_3} + \frac{V_- - V_{out}}{R_4} = 0 \Rightarrow V_{out} = \underbrace{\left(-\frac{R_4}{R_3}\right)}_{\text{inverting}} \cdot V_3 + \underbrace{\left(1 + \frac{R_4}{R_3}\right)}_{\text{non-inverting}} \cdot V_-$$

Now finally, if we subtitute $V_- = \dfrac{R_2 V_1 + R_1 \cdot V_2}{R_1 + R_2}$ we get:

$$V_{out} = \left(-\frac{R_4}{R_3}\right) V_3 + \left(1 + \frac{R_4}{R_3}\right)\left(\frac{R_2 \cdot V_1 + R_1 \cdot V_2}{R_1 + R_2}\right)$$

(4)



Find Vout in terms of $V_1, V_2, V_3, R_1, R_2, R_3$

Again: $V_+ = 0 \Rightarrow V_- = 0$

So $V_{-ve} = 0$

And by doing nodal analysis:

$i_1 + i_2 + i_3 + i_4 + i_- = 0 \rightarrow i_1 + i_2 + i_3 + i_4 = 0$ since $i_- = 0$

$$\Rightarrow \frac{0 - V_1}{R_1} + \frac{0 - V_2}{R_2} + \frac{0 - V_3}{R_3} + \frac{0 - V_{out}}{R_4} = 0$$

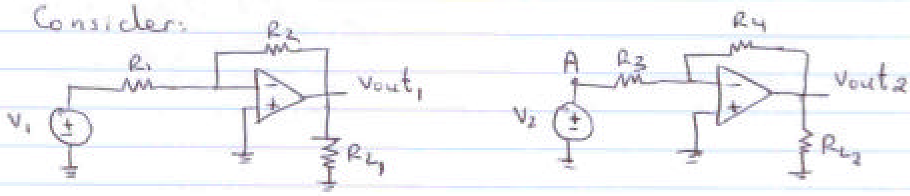
$$\Rightarrow V_{out} = \underbrace{\left(-\frac{R_4}{R_1} \cdot V_1\right)}_{\text{inverting}} \underbrace{\left(-\frac{R_4}{R_2} \cdot V_2\right)}_{\text{inverting}} \underbrace{\left(-\frac{R_4}{R_3} \cdot V_3\right)}_{\text{inverting}}$$

Note

In all of the above examples we were able to find Vout without ever considering ~~about~~ the value of R_L. That is the good thing about ideal op-amps: The load resistor (R_L) does not affect the output voltage

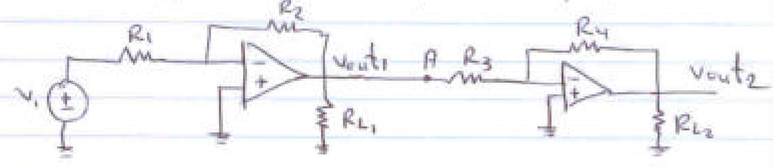
Cascading op-amps

Consider:



We have shown in example ① that:
$V_{out_1} = V_1 \cdot \left(-\frac{R_2}{R_1}\right)$ and $V_{out2} = V_2 \cdot \left(-\frac{R_4}{R_3}\right)$

What is going to happen if instead of using the voltage source V_2, I connect point A to the output of the first amplifier?



Well, for the first amplifier: $V_{out1} = V_1\left(-\frac{R_2}{R_1}\right)$
Remember that whatever we decide to connect on the output is not going to affect the value of V_{out1}.
Now for the second amplifier:

$V_+ = 0 \Rightarrow V_- = 0$
And by using $i_- = 0$ and doing nodal analysis on the -ve node: $\frac{0 - V_A}{R_3} + \frac{0 - V_{out2}}{R_4} + 0 = 0 \Rightarrow V_{out2} = \left(-\frac{R_4}{R_3}\right) V_A$

What is V_A though? $V_A = V_{out1} = V_1\left(-\frac{R_2}{R_1}\right)$

$$\Rightarrow V_{out2} = \underbrace{\left(-\frac{R_4}{R_3}\right)}_{\text{gain of 2nd amplifier}} V_1 \underbrace{\left(-\frac{R_2}{R_1}\right)}_{\text{gain of 1st amplifier}} \Rightarrow \boxed{V_{out2} = \left(\frac{R_2 R_4}{R_1 R_3}\right) V_1}$$