

CIRCUIT ANALYSIS CONTINUED

Lecture 6 review:

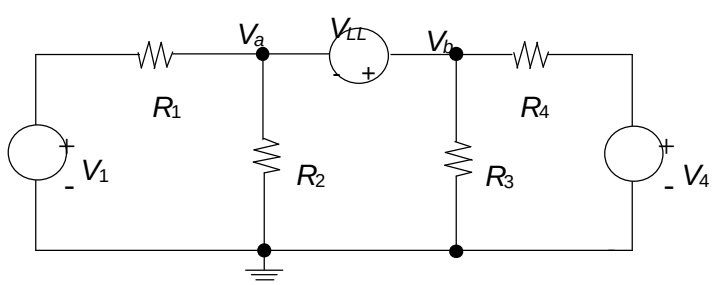
- Formal nodal analysis (including floating Voltage sources)
- Voltage divider example (generalized)

Today:

- Examples of circuits and nodal analysis
- Parallel resistors
- Current dividers
- Simplifying circuits
- Real voltmeters
- Real ammeters
- Series and parallel capacitors

NODAL ANALYSIS EXAMPLE

Find V_a, V_b if $R_1 = R_2 = R_3 = R_4 = 1M\Omega$, and $V_1 = V_4 = 1.5V$ with $V_{LL} = 1V$



Solution: At supernode enclosing nodes a and b :

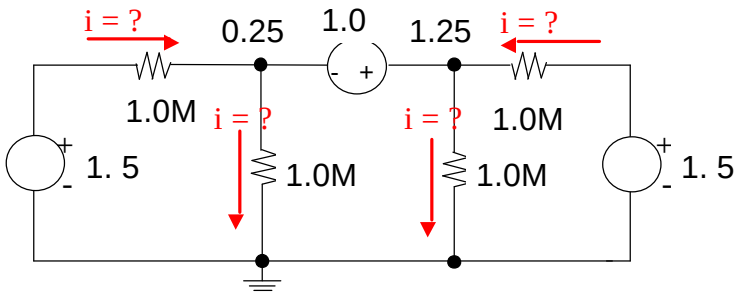
$(V_1 - V_a)/R_1 - V_a/R_2 =$ _____ *and*

$V_b =$ _____ *Thus:* $V_a =$ _____ Be sure to check

$V_b =$ _____ answer with KCL!

CHECK ANSWER WITH KCL

Is V_a= 0.25 and V_b = 1.25 if R_1= R_2 = R_3 = R_4 = 1MΩ, and V_1 = V_4 =1.5V with V_{LL} = 1V ????



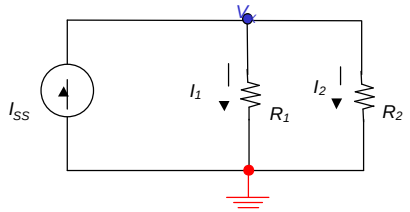
KCL at the Supernode: 0.25 -1.25 + 1.25 - 0.25 =0

Clearly the current into the supernode is zero and we have verified that the solution is correct. :

RESISTORS IN PARALLEL

1 Select Reference Node

2 Define unknown node voltages



Note: I_{ss} = I_1 + I_2, i.e.,

$$I_{SS} = \frac{V_X}{R_1} + \frac{V_X}{R_2} \quad \Rightarrow V_X = I_{SS} \cdot \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \quad = I_{SS} \cdot \frac{R_1 R_2}{R_1 + R_2}$$

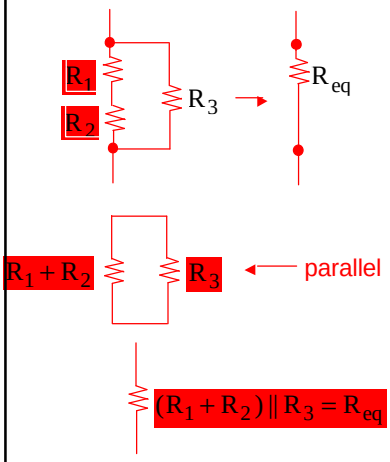
RESULT 1 EQUIVALENT RESISTANCE: $R_{\|} \equiv R_1 \| R_2 =$ _____?

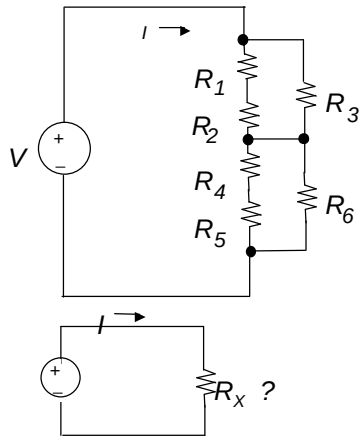
RESULT 2 CURRENT DIVIDER: $I_1 = \frac{V_X}{R_1} = I_{SS} \times$ _____

$I_2 = \frac{V_X}{R_2} = I_{SS} \times$ _____

IDENTIFYING SERIES AND PARALLEL COMBINATIONS

Use series/parallel equivalents to simplify a circuit before starting KVL/KCL





$R_1 = R_2 = 10\,K$

$R_3 = 20\,K$

$R_4 = 5 = 5\,K$

$R_6 = 10\,K$

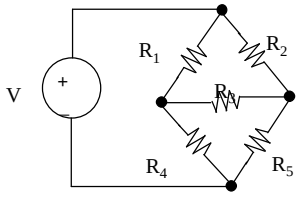
$R_X =$ _____

$= 15\,K$

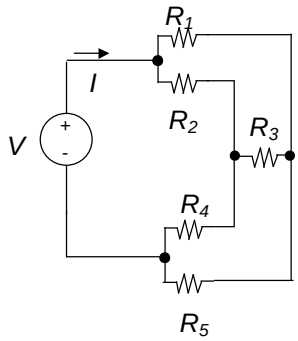
Please note the order of math operators here!

IDENTIFYING SERIES AND PARALLEL COMBINATIONS (cont.)

Some circuits *must* be analyzed (not amenable to simple inspection)



R_1 and R_2 are **not** in ||



R_1 and R_5 are **not** in series

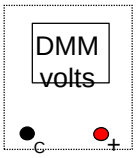
Special cases:

$R_3 = 0$ **OR** $R_3 = \infty$

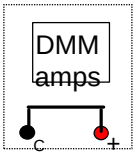
Example: $R_3 = 0 \Rightarrow R_1 \| R_2$; $R_4 \| R_5$ in series; $R_{eq} =$ _____

OR IF $R_3 = \infty \Rightarrow$ _____

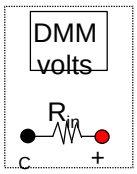
IDEAL AND NON-IDEAL METERS



IDEAL



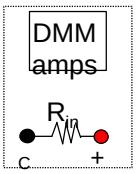
IDEAL



MODEL OF REAL DIGITAL VOLTMETER

Note: R_{in} may depend on range

R_{in} typically $\tilde{>}$ 10 MΩ



MODEL OF REAL DIGITAL AMMETER

Note: R_{in} usually depends on current range

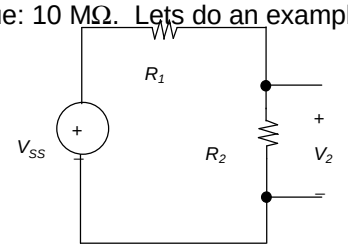
R_{in} typically $\tilde{<}$ 1 Ω

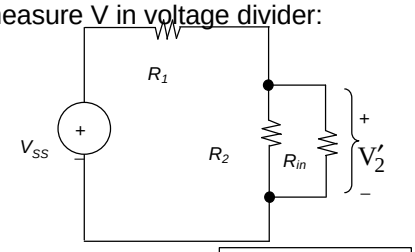
REAL VOLTMETERS

Concept of "Loading" as Application of Parallel Resistors

How is voltage measured? Modern answer: Digital multimeter (DMM)

Problem: Connecting leads from voltmeter across two nodes changes the circuit. The voltmeter is characterized by how much current it draws at a given voltage → "voltmeter input resistance," R_{in}. Typical value: 10 MΩ. Lets do an example; measure V in voltage divider:





$$V_2 = V_{SS}\left[\frac{R_2}{R_1 + R_2}\right]$$

$$V_2' = \boxed{}$$

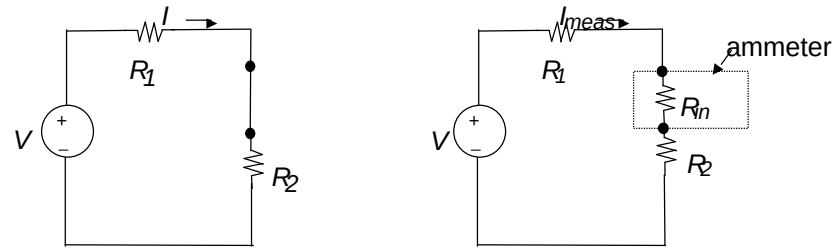
Example: $V_{SS} = 10\text{V}, R_2 = 100\text{K}, R_1 = 900\text{K} \Rightarrow V_2 = 1\text{V}$

But if $R_{in} = 10\text{M}, V_2' = 0.991\text{V}$, a 1% error

MEASURING CURRENT

Insert DMM (in current measurement mode) into circuit. But ammeters disturb the circuit. (Note: Ammeters are characterized by their "ammeter input resistance," R_{in}. Ideally this should be very low. Typical value (in mA range) 1Ω.)

Potential measurement error due to non-zero input resistance:



undisturbed circuit

$$I = \frac{V_1}{R_1 + R_2}$$

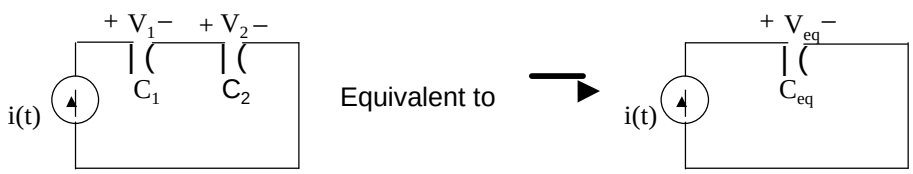
with ammeter

$$I_{meas} = \boxed{}$$

Example V = 1 V: R1 + R2 = 1 KΩ , Rin = 1Ω

$$I = 1\text{mA} \quad , \quad I_{meas} = \frac{1}{1\text{K} + 1\Omega} \cong 0.999\ \text{mA} \qquad (0.1\%\ \text{error})$$

CAPACITORS IN SERIES



Equivalent capacitance defined by

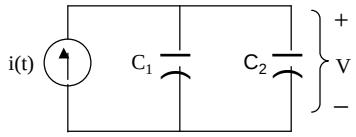
$$V_{eq} = V_1 + V_2 \ \text{ and } \ i = C_{eq}\frac{dV_{eq}}{dt} = C_{eq}\frac{d(V_1 + V_2)}{dt}$$

$$i = C_1\frac{dV_1}{dt} = C_2\frac{dV_2}{dt}$$

So $\frac{dV_1}{dt} = \frac{i}{C_1}$, $\frac{dV_2}{dt} = \frac{i}{C_2}$, so $\frac{dV_{eq}}{dt} = \boxed{} = \frac{i}{C_{eq}}$

Clearly, $C_{eq} =$ CAPACITORS IN SERIES

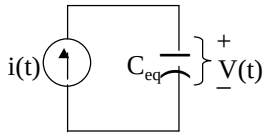
CAPACITORS IN PARALLEL



$$i(t) = C_1 \frac{dV}{dt} + C_2 \frac{dV}{dt}$$

Equivalent capacitance defined by

$$i = C_{eq} \frac{dV}{dt}$$



Clearly, $C_{eq} =$ _____ CAPACITORS IN PARALLEL